Degeneracies in Long-baseline Neutrino Oscillations

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Neutrino Oscillations with Natural Sources

- Solar neutrino deficit is explained in terms of neutrino oscillations with $\Delta m^2_{\rm sol} : (10^{-5}, 10^{-4})$ eV^2 and $\theta_{\rm sol} \approx 35^\circ$.
- Atmospheric neutrino deficit is also explained in terms of neutrino oscillations with $\Delta m^2_{\rm atm} : (10^{-3}, 10^{-2})$ eV^2 and $\theta_{\rm atm} \approx 45^\circ$.
- LEP showed that there are three flavours of light, active neutrinos $\Longrightarrow$ three neutrino mass eigenstates.
- Two independent mass-squared differences: $\Delta m^2_{21} = m^2_2 - m^2_1$ and $\Delta m^2_3 = m^2_3 - m^2_1$.
- Without loss of generality, can identify $\Delta m^2_{\rm sol} = \Delta m^2_{21}$ and $\Delta m^2_{31} = \Delta m^2_{\rm atm} \approx \Delta m^2_{32} = m^2_3 - m^2_2$.
- Three flavour oscillations can explain both Solar and Atmospheric neutrino deficits.

Neutrino Oscillations (20th Century)

- The PMNS matrix, connecting the flavour eigenstates to mass eigenstates, is parametrized in terms of three mixing angles θ_{12}, θ_{13} and θ_{23} and one CP-violating phase $\delta_{\rm CP}$.
- The reactor neutrino experiments, CHOOZ and Palo Verde, did not see any evidence for oscillations.
- Interpretation of this result, in terms of three flavour oscillations, leads to the strong constraint

$$\sin^2 2\theta_{13} \leq 0.1.$$

- Combining it with solar neutrino data leads to $\theta_{13} \ll 1 \implies \theta_{\rm sol} \approx \theta_{12}$ and $\theta_{\rm atm} \approx \theta_{23}$.

Neutrino Oscillations with Man-made Sources

- Efforts started to verify neutrino oscillations with man-made neutrino sources.
- In particular, the aim was to measure the neutrino spectrum and identify the spectral distortion due to oscillations.
- Nuclear reactors produce $\bar{\nu}_e$ and we can measure only $\bar{\nu}_e$ survival probability.
- Accelerators produce $\nu_\mu/\bar{\nu}_\mu$ and we can measure both their survival probabilities and their oscillation probabilities to $\nu_e/\bar{\nu}_e$.
- A difference in the oscillation probabilities of neutrinos and anti-neutrinos is the signature for CP-violation in neutrino oscillations.

Effective Two-flavour Oscillations

- Reactor neutrino energies are in the range $(3, 10)$ MeV.
- Matter effects are utterly negligible for such small energies.
- Matter effects are also negligible in the survival probabilites of accelerator neutrinos, if the baseline is of order 1000 km or less.
- Hence, the data of survival probabilities can be analyzed using vacuum probabilities.
- For long-baseline reactor neutrino experiment KamLAND, we can use the approximation $\theta_{13} = 0$.
- Survival probability has effective two flavour form with Δm^2_{21} and θ_{12}.
- Identifying the spectral distortion leads to a precise measurement

$$\Delta m^2_{21} = (7.58 \pm 0.22) \times 10^{-5} \ \mathrm{eV}^2 \ \text{and} \ \tan^2\theta_{12} = 0.56 \pm 0.14.$$

Effective Two-flavour Oscillations

- For ~ 1 km baseline reactor neutrino experiments, Double-CHOOZ, Daya Bay and RENO, we can use the approximation $\Delta m^2_{21} = 0$.
- Again the survival probability reduces to effective two flavour form with Δm^2_{31} and θ_{13}.
- Very high statistics data from Daya Bay and RENO lead to

$$\Delta m^2_{31} = (2.54 \pm 0.07) \times 10^{-3}\ \mathrm{eV}^2 \text{ and } \sin^2 2\theta_{13} = 0.086 \pm 0.003.$$

- For the accelerator experiment MINOS, with $L = 730$ km and $E_\nu \sim 3.5$ GeV, we set $\Delta m^2_{21} = 0 = \theta_{13}$.
- The effective two flavour form of the survival probability gives

$$\Delta m^2_{31} = (2.32 \pm 0.12) \times 10^{-3}\ \mathrm{eV}^2 \text{ and } \sin^2 2\theta_{23} \geq 0.94.$$

The Unknowns in Neutrino Oscillations

- The two mass-squared differences are measured to a precision of about 3%.
- The mixing angles $\sin^2\theta_{12}$ and $\sin^2 2\theta_{13}$ are also measured to a similar precision.
- Survival probabilities give **no information** on $\delta_{\rm CP}$.
- Matter effects are crucial to explain the solar neutrino problem. They also fix that $\Delta m^2_{\rm sol} = \Delta m^2_{21}$ is positive.
- At present, there is no information whether Δm^2_{31} is positive or negative.
- Both atmospheric and accelerator data prefer $\sin^2 2\theta_{23} \simeq 1$ but there is no information if $\theta_{23} < \pi/4$ or $\theta_{23} > \pi/4$.

Goals of Current Neutrino Oscillation Experiments

- Look for evidence of matter effects in atmospheric/accelerator experiments and determine the sign of Δm^2_{31}.
- The case of Δm^2_{31} positive is called Normal Hierarchy (NH) and that of Δm^2_{31} negative is called Inverted Hierarchy (IH).
- Determine whether $\theta_{23} < \pi/4$ or $\theta_{23} > \pi/4$.
- The first case is called Lower Octant (LO) and the second case is called Higher Octant (HO).
- Look for evidence for CP-violation in neutrino oscillations and measure $\delta_{\rm CP}$.

How to Determine the Unknowns

- Till 2014, all the experiments measured only survival probabilities.
- They are not sensitive to the unknowns.
- Good News: The $\nu_\mu \to \nu_e$ oscillation probability is sensitive to all the three unknowns.
- Can measure all three in one experiment.
- Bad News: The $\nu_\mu \to \nu_e$ oscillation probability is sensitive to all the three unknowns.
- The change in the probability, induced by the change in one unknown can be cancelled by the change in another unknown.
- Have to do a set of very careful measurements to disentangle the effects due to different unknowns.

Matter Effects and Sign of Δm^2

- Matter effects can be included in two flavour oscillations in a straight forward manner, by means of the Wolfenstein matter term

$$A(\text{in eV}^2) = 2\sqrt{2} G_F N_e E = 0.76 \times 10^{-4} \rho \, (\text{in gm/cc}) E \, (\text{in GeV}).$$

- With this inclusion, the $\nu_\mu \to \nu_e$ oscillation probability becomes

$$\begin{aligned} P^m(\nu_\mu \to \nu_e) &= \sin^2 2\theta_m \sin^2 \left(\frac{\Delta m_m^2 L}{4E} \right), \text{ where} \\ \Delta m_m^2 &= \sqrt{(\Delta m^2 \cos 2\theta - A)^2 + (\Delta m^2 \sin 2\theta)^2} \\ \sin 2\theta_m &= \sin 2\theta \frac{\Delta m^2}{\Delta m_m^2} \end{aligned}$$

- This probability is sensitive to the sign of the product $(\Delta m^2 \cos 2\theta)$.

Matter Effects in Three Flavour Oscillations

- In two flavour oscillations, we can not determine sign of Δm^2 and octant of θ separately.
- A physically meaningful question to ask is: Is the overlap of the lighter mass eigenstate with electron neutrino larger/smaller than 0.5?
- This is equivalent to asking if $(\Delta m^2 \cos 2\theta)$ positive or negative?
- That is not so in three flavour oscillations for two of the three mixing angles.
- Inclusion of matter effects in three flavour oscillations is more complicated.
- Usually, the calculations are done numerically.
- Since $\Delta m^2_{21} \ll \Delta m^2_{31}$, we can treat the terms proportional to Δm^2_{21} perturbatively, in the neutrino Hamiltonian.

Perturbative Algebraic Expression for $P^m_{\mu e} \equiv P^{\rm mat}(\nu_\mu \to \nu_e)$

- In this manner, we obtain

$$
\begin{aligned}
P^m_{\mu e} &= \sin^2\theta_{23}\sin^2 2\theta_{13}\frac{\sin^2[(1-\hat{A})]\hat{\Delta}}{(1-\hat{A})^2} \\
&+ \alpha\tilde{J}\cos(\hat{\Delta}+\delta_{\rm CP})\frac{\sin(1-\hat{A})\hat{\Delta}}{[(1-\hat{A})]}\frac{\sin(\hat{A}\hat{\Delta})}{\hat{A}},
\end{aligned}
$$

  where $\alpha = \Delta m^2_{21}/\Delta m^2_{31}$ and $\tilde{J} = \cos\theta_{13}\sin 2\theta_{12}\sin 2\theta_{13}\sin 2\theta_{23}$.
- In the above equation, we have used the notation $\hat{A} = A/\Delta m^2_{31}$ and $\hat{\Delta} = (\Delta m^2_{31} L)/(4E)$.
- In the case of anti-neutrinos, we can obtain $P^{\rm mat}(\bar{\nu}_\mu \to \bar{\nu}_e) \equiv P^m_{\bar{\mu}\bar{e}}$ by the replacements $\hat{A} \to -\hat{A}$ and $\delta_{\rm CP} \to -\delta_{\rm CP}$.

Sensitivity of $P^m_{\mu e}$ and $P^m_{\bar{\mu}\bar{e}}$ to Unknowns

- We need $\theta_{13} \neq 0$ for $P^m_{\mu e}$ (and $P^m_{\bar{\mu}\bar{e}}$) to be non-zero.
- The $\sin^2\theta_{23}$ factor in the leading term makes it sensitive to the octant of θ_{23}.
- If the sign of Δm^2_{31} is changed, $\hat{A} \to -\hat{A}$ and $\alpha \to -\alpha$, making it sensitive to hierarchy.
- The δ_{CP} dependence appears only in the sub-leading term.
- This is a consequence of the fact that CP-violation is unoservable if any one of the mass-squared differences vanishes.

Eight-fold Degeneracy of $P_{\mu e}$

- Before the measurement of θ_{13}, it was argued that there is an eight-fold degeneracy in $P_{\mu e}^{m}$.
- If the value of θ_{13} is not precisely known, then a measured value of $P_{\mu e}^{m}$ can be reproduced by eight combinations of the unknowns.
  1. θ_{13}^{0} (true value), true hierarchy, true octant, true δ_{CP}
  2. θ_{13}^{1} (wrong value), wrong hierarchy, true octant, true δ_{CP}
  3. θ_{13}^{2} (wrong value), true hierarchy, wrong octant, true δ_{CP}
  4. θ_{13}^{3} (wrong value), true hierarchy, true octant, wrong δ_{CP}
- There will four more cases of wrong value of θ_{13}: three where two of the unknowns are wrong and the third is correct and the fourth one where all the three unknowns take the wrong values.
- With the precision measurement of θ_{13}, it seems as if all the wrong solutions can be ruled out.

Degeneracies in $P_{\mu e}$ with fixed θ_{13}

- However, depending on the values of the unknowns, there are leftover degeneracies.
- To understand these, let us define a standard value of $P_{\mu e}$ where we set all the unknowns equal to ZERO.
- That is we consider $P_{\mu e}$ for vacuum oscillations with $\theta_{23} = \pi/4$ and $\delta_{\mathrm{CP}} = 0$.
- For the sake of simplicity we consider all the unknowns to be binary variables.
- Hierarchy, of course, is a binary variable. Octant also becomes a binary variable if the value of $\sin 2\theta_{23}$ is measured accurately through survival probability. We assume δ_{CP} takes only the maximal CP-violating values $\pm\pi/2$.

Degeneracies in $P_{\mu e}$ with fixed θ_{13}

- Compared to the standard value, $P^m_{\mu e}$ increases if
  1. Hierarchy is normal or
  2. Octant is higher or
  3. $\delta_{\rm CP} = -\pi/2$.
- $P^m_{\mu e}$ decreases if the unknowns take the opposite binary value.
- If all the unknowns take values that increase $P^m_{\mu e}$, we have the largest possible increase with respect to the standard value.
- If the measured value is equal to this the highest predicted value, we have a unique solution for the three unknowns.
- Similarly if measured $P_{\mu e}$ is much below the the standard value, it means that all the three unknowns take values which reduce $P_{\mu e}$.

Degeneracies in $P_{\mu e}$ with fixed θ_{13}

- But suppose the measured $P_{\mu e}$ is a little higher than the standard value.
- This happens if two of the unknowns take values which increase $P_{\mu e}$ and the third one takes the value which decreases it.
- There are three possible ways this can happen and hence there is a three fold degenerate set of solutions.
- Such a three fold degeneracy also occurs for the case where the measured $P_{\mu e}$ is a little smaller than the standard value, where two of the unknowns take values to decrease $P_{\mu e}$ and the third one takes value to increase it.
- In such a situation, the anti-neutrino data is helpful in partially lifting this three fold degeneracy.

Degeneracies in $P_{\mu e}$ and $P_{\bar{\mu}\bar{e}}$

- For Normal Hierarchy, $P_{\mu e}$ increases and $P_{\bar{\mu}\bar{e}}$ decreases relative to the standard value.
- Situation is reversed for Inverted Hierarchy.
- For Higher Octant (HO) both $P_{\mu e}$ and $P_{\bar{\mu}\bar{e}}$ increase and they both decrease for Lower Octant (LO).
- For $\delta_{\rm CP} = -\pi/2$, $P_{\mu e}$ increases and $P_{\bar{\mu}\bar{e}}$ decreases relative to the standard value.
- Situation is reversed for $\delta_{\rm CP} = +\pi/2$.
- Thus hierarchy and $\delta_{\rm CP}$ have opposite effects on $P_{\mu e}$ and on $P_{\bar{\mu}\bar{e}}$ but octant has the same effect on both of them.

Resolving the Octant Degeneracy with Anti-Neutrino Data

- Suppose the measured $P_{\mu e}$ is a little higher than the standard value.
- That means two of the unknowns take value to increase $P_{\mu e}$ and the third takes value to decrease it.
- If the unknown causing the decrease is the octant, then the predicted value of $P_{\bar{\mu}\bar{e}}$ also should be smaller than the standard value.
- Hence a measurement of $P_{\bar{\mu}\bar{e}}$ will uniquely determine if the unknown causing the decrease is the octant or not.
- If the unknown causing the decrease is either the hierarchy or δ_{CP}, then the anti-neutrino data is not of any help in determining which is responsible.
- Very similar arguments also apply when the measured $P_{\mu e}$ is a little smaller than the standard value.

Resolution of Hierarchy-$\delta_{\rm CP}$ Degeneracy

- We saw that if the hierarchy increases $P_{\mu e}$ and $\delta_{\rm CP}$ decreases it, then hierarchy decreases $P_{\bar{\mu}\bar{e}}$ and $\delta_{\rm CP}$ increases it.
- A similar situation occurs when the hierarchy decreases $P_{\mu e}$ and $\delta_{\rm CP}$ increases it.
- This hierarchy-$\delta_{\rm CP}$ degeneracy is one of the most difficult one to resolve.
- The hierarchy sensitivity of atmospheric neutrinos is independent of $\delta_{\rm CP}$.
- Determining hierarchy with an atmospheric neutrino data of INO or Hyper-Kamiokande and $\delta_{\rm CP}$ with an accelerator neutrino experiment is one possibility.

Resolution of Hierarchy-δ_{CP} Degeneracy

- Accelerator neutrino experiments typically tune their energy such that $(\Delta m^2_{31} L)/(4E) \simeq \pi/2$ (called first maximum), so that the $\nu_\mu \to \nu_e$ appearance probability is maximized.
- DUNE plans to employ a wide band energy beam of neutrinos so that there is appreciable flux at energy $(\Delta m^2_{31} L)/(4E) = 3\pi/2$ (called second maximum).
- DUNE plans to use a two pronged strategy:
  1. Have a longer baseline so that the change in $P_{\mu e}$ due to hierarchy at the first maximum is more than that due to δ_{CP}.
  2. Arrange the energies such that change in $P_{\mu e}$ due to hierarchy at the smaller energy of the second maximum is less than that due to δ_{CP}.
- Thus the combined data from the first and second maxima can resolve the hierarchy-δ_{CP} degeneracy.

Current Long-Baseline Experiments: T2K and NOvA

- T2K has a narrow band beam of neutrinos, with peak flux at $E = 0.6$ GeV, from JPARC to Super-Kamiokande detector 295 km away.
- They have taken data in neutrino mode with 14.9×10^{20} POT and in anti-neutrino mode with 16.4×10^{20} POT. (Protons on Target)
- NOvA far detector receives narrow band neutrino beam, with peak flux at $E = 2$ GeV, from Fermilab 810 km away.
- So far, NOvA collected data with 13.6×10^{20} POT in neutrino mode and 12.5×10^{20} POT in anti-neutrino mode.
- Both experiments have near detectors to monitor the neutrino fluxes and to measure the interaction cross sections.
- Their analysis of $\nu_\mu/\bar{\nu}_\mu$ disappearance (based on the survival probabilities) agree with each other.
- T2K: $\Delta m^2_{31} = (2.54 \pm 0.07) \times 10^{-3}$ eV^2 and $\sin^2\theta_{23} = 0.53 \pm 0.04$.
- NOvA: $\Delta m^2_{31} = (2.48 \pm 0.07) \times 10^{-3}$ eV^2 and $\sin^2\theta_{23} = 0.57 \pm 0.03$.

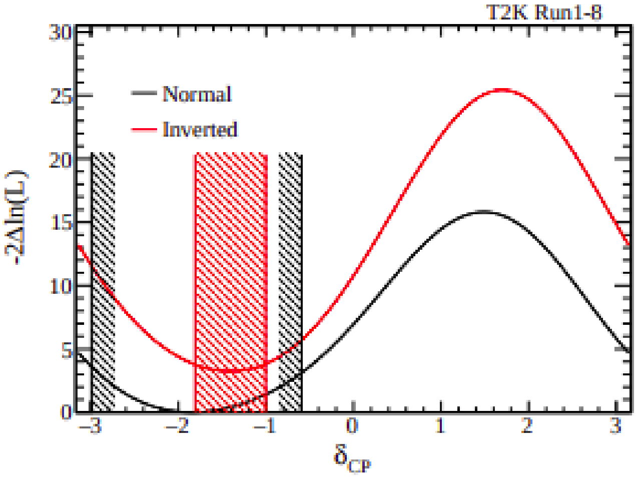
Appearance Data of T2K

- T2K observes a total of 90 ν_e appearance events and 15 $\bar{\nu}_e$ appearance events.
  Nature 580 (2020) 7803, 339-344, Nature 583 (2020) 7814, E16 (erratum); e-Print: 1910.03887
- The ν_e events are much larger than the standard number of 60, whereas the $\bar{\nu}_e$ events are a little lower than the standard number of 17.2.
- According to the arguments made before, this data indicates that the hierarchy is normal, the octant is higher and $\delta_{\rm CP} \simeq -\pi/2$.
- Based on their data, T2K claim to rule out $\delta_{\rm CP} = 0$ at 99.79% (3 σ) confidence level.

Appearance Data of NOvA

- NOvA observes 82 ν_e appearance events and 33 $\bar{\nu}_e$ appearance events.
- The ν_e event number is somewhat larger than the standard number of 62 and the $\bar{\nu}_e$ event number is also somewhat larger than the standard number of 23.
- The best fit point is (NH, HO with $\sin^2\theta_{23} = 0.57$, $\delta_{\rm CP} = 5\pi/6$).
- But, multiple solutions with (NH, LO), (NH, HO), (IH, LO) and (IH, HO) are all acceptable at 68% ($1\ \sigma$) confidence level.
- So there is a strong tension between the appearance data of T2K and NOvA.
- Eventhough, the best-fit points of both T2K and NOvA choose normal hierarchy, a fit to the combined data of the two experiments prefers inverted hierarchy
  K. J. Kelly, P. A. Machado, S. J. Parke, Y. F. Perez Gonzalez and R. Zukanovich-Funchal, [arXiv:2007.08526 [hep-ph]].

T2K Results from 2018



NOvA Results from Neutrino-2020

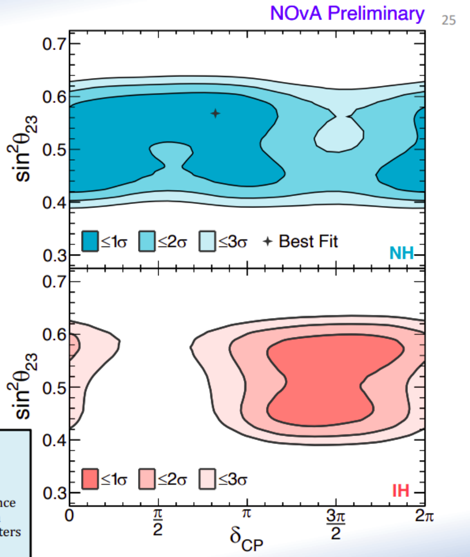
Best Fit

Normal hierarchy

$\Delta m^2_{32} = (2.41 \pm 0.07) \times 10^{-3}\ \text{eV}^2$

$\sin^2\theta_{23} = 0.57^{+0.04}_{-0.03}$

$\delta = 0.82\pi$



Posters

83. Long-baseline neutrino oscillation results from NOvA
   – Liudmila Kolupaeva & Karl Warburton

262. Accelerating Calculation of Confidence Intervals for NOvA's Neutrino Oscillation Parameter Estimation with Supercomputers
   – Steven Calvez, Tarak Thakore

NOvA Results from Neutrino-2020

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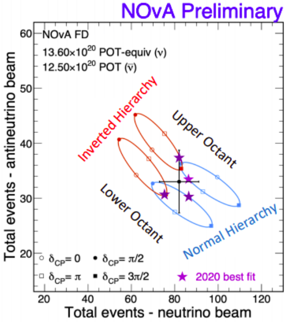

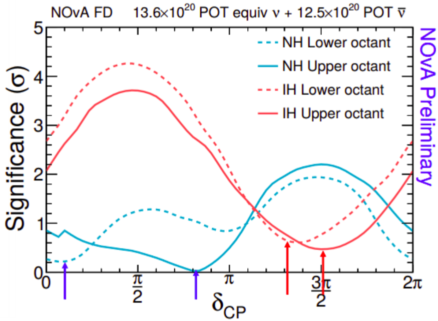

- We see no strong asymmetry in the rates of appearance of ν_e and $\bar{\nu}_e$
- Disfavor hierarchy-δ combinations which would produce that asymmetry
- Consistent with hierarchy-octant-δ combinations which include some "cancellation."
  - Since such options exist for both octants and hierarchies, results show no strong preferences.

Exclude IH $\delta = \pi/2$ at $>3\sigma$

Disfavor NH $\delta = 3\pi/2$ at $\sim 2\sigma$

Prefer...

Normal Hierarchy at 1.0σ

Upper Octant at 1.2σ

Tension Between Appearance Data of T2K and NOvA

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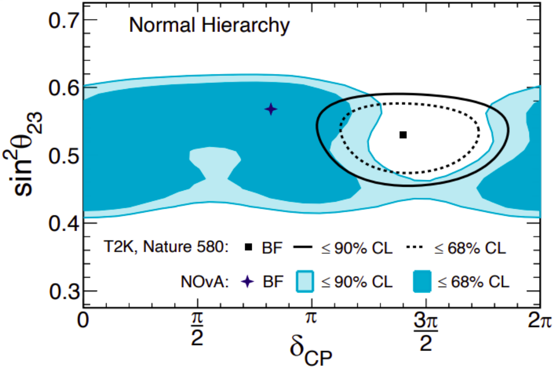
Comparison to T2K

NOvA Preliminary



- Clear tension with T2K's preferred region.
- Quantifying consistency requires a joint fit of the data from the two experiments, which is already in the works.
  - Semi-annual workshops, regular joint group meetings, and a signed joint agreement.

Conclusions

- Precision measurement of $\sin^2 2\theta_{13}$ removes a large number of degeneracies in $P_{\mu e}$.
- Combined data on $P_{\mu e}$ and $P_{\bar{\mu}\bar{e}}$ can lift the degeneracies related to the θ_{23} octant.
- Overcoming the hierarchy-δ_{CP} degeneracy is quite difficult and various different proposals are being pursued.
- The ν_e and $\bar{\nu}_e$ appearance data of the current long-baseline accelerator neutrino experiments seem to be in severe tension.
- A joint analysis team, with represenatives from the two experiments, is looking into the causes for this tension.