Phase structure in Gross–Neveu model at finite density on torus

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Outline

- Introduction

  quantum chromodynamics, phase diagram, effective model, effective potential

- Analysis

  background spacetime, assumption&conditions

- Result

  phase diagram on μ-T plane,

  thermodynamic quantities (dynamically generated fermion mass, particle number density)

- Summary

Pre-introduction: What we did

We analyzed **a QCD-like model**

- Non-trivial chiral condensation emerged in strongly coupled fermions system

  $\rightarrow$ to analyze non-trivial vacuum structures
- Phase diagrams with effects of finite size (topology) and finite particle density

  $\rightarrow$ to consider effects from non-trivial environments
- Thermodynamic quantities: Dynamically generated mass, particle number density

  $\rightarrow$ to see effects to physical phenomena

Introduction: Quantum chromodynamics(QCD)

Quarks are considered as fundamental components of nature

$\rightarrow$ described by Quantum Chromodynamics(QCD)

$$\mathcal{L}_{\mathrm{QCD}} = \bar{\psi} i \gamma^{\mu} \left(\partial_{\mu} - i g G_{\mu}^{a} T^{a} \right) \psi - \bar{\psi} m \psi - \frac{1}{4} F^{\mu\nu a} F_{\mu\nu}^{a}$$

- Strong interactions $\rightarrow$ chiral condensation
- Asymptotic freedom (at high energies)
- Color confinement

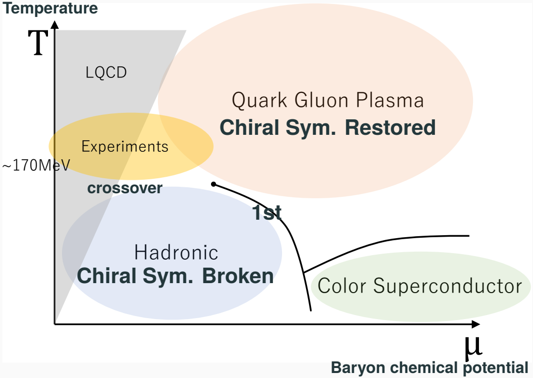
Introduction: QCD phase diagram

One of main issues in QCD: investigating the QCD phase structure

Why focus on chiral symmetry breaking??

$\Rightarrow$ One of clues

to investigate the QCD phase structure



It is important to reveal non-trivial vacuum structures at low energies

e.g. Hadrons obtain their masses due to spontaneous breaking of chiral symmetry

Y. Nambu and G. Jona-Lasinio (1961)

$\longrightarrow$ Perturbative QCD does NOT work

Introduction: Effective model

Four-fermion interaction model : one of effective models

Nambu–Jona Lasinio(NJL) model, Gross–Neveu(GN) model J. Gross and A. Neveu,(1974), ...

Used in the context of nuclear physics, particle physics, condensed matter physics, ..

we used **Gross–Neveu model** in this work

$$S_{\mathrm{GN}} = \int \mathrm{d}^D x \left[\bar{\psi}(x) i \gamma^\mu \partial_\mu \psi(x) + \frac{\lambda}{2N} \left(\bar{\psi}(x)\psi(x) \right)^2 \right]$$

λ: coupling of the four-fermion interaction, N: the number of copies of fermions

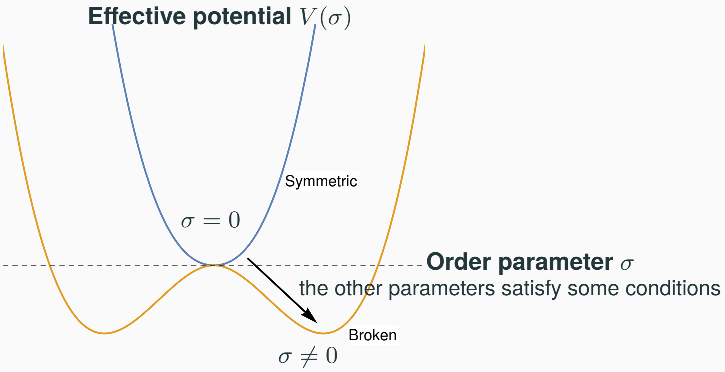
- Invariant under a discrete chiral transformation: $\psi \longrightarrow \gamma^5 \psi$
- Very simple model
- Can describe spontaneous chiral symmetry breaking

Introduction: Effective potential analysis

Derive an effective potential $V(\sigma)$ (an order parameter σ) from the effective action

$\rightarrow$ Find the value of the order parameter at the minimum of the effective potential

(equivalent to solving the gap equation)



Gross–Neveu model

$$S_{\mathrm{GN}} = \int \mathrm{d}^2x \left[\bar{\psi}(x) i\gamma^\mu \partial_\mu \psi(x) + \frac{\lambda}{2N} \left(\bar{\psi}(x)\psi(x)\right)^2\right]$$
$$\to \int \mathrm{d}^2x \left[\bar{\psi}(x) \left(i\gamma^\mu \partial_\mu - \sigma(x)\right) \psi(x) - \frac{N}{2\lambda}\sigma(x)^2\right]$$

Assuming the homogeneous chiral condensation,

we can solve this model at the leading order of the $1/N$ expansion

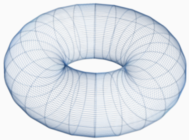
$$\frac{V_{\mathrm{eff}}\left(\sigma; \lambda_r, \Lambda\right)}{\Lambda^2} = \frac{1}{2\lambda_r}\left(\frac{\sigma}{\Lambda}\right)^2 - \frac{1}{4\pi}\left(3 - \ln\left(\frac{\sigma}{\Lambda}\right)^2\right)\left(\frac{\sigma}{\Lambda}\right)^2$$

Order parameter is introduced as $\sigma \simeq -\frac{\lambda}{N}\left\langle\bar{\psi}\psi\right\rangle$ (Chiral condensation)

Analysis: Background spacetime

Gross–Neveu model on 2-dimensional spacetime $T^2 \simeq S^1 \times S^1$ (torus)

Not a trivial spacetime $\mathbb{R}^2$



- A compactified space S^1
- Finite temperature (a compactified imaginary time) S^1
- Solvable analytically

  $\rightarrow$ similar to 4-dimensional case due to an even dimension

One of extreme situations

Analysis: Assumption & Conditions

Assumption: **Homogeneous chiral condensation** (order parameter σ)

- Leading order of the $1/N$ expansion (semi-classical approximation)
- Particle number density $\psi^\dagger\psi$ (chemical potential μ)
- Boundary conditions of fermion
  - Spacial direction: $\mathrm{U}(1)$-valued $\psi(x_1 + L, x_2) = e^{-i\pi\delta}\psi(x_1, x_2)$

    L: the length of the compactified space, δ: the phase
  - Imaginary-time direction: Anti-periodic $\psi(x_1, x_2 + 1/T) = -\psi(x_1, x_2)$

    T: temperature

Effective potential analysis with the model parameters: L, δ, T, μ

Previous studies and our work

Previous studies ($\mathrm{U}(1)$-valued boundary conditions)

- Relativistic anyon-like systems on $\mathbb{R}^{1\text{or }2} \times S^1$ D. Y. Song (1993), S. Huang and B. Schreiber (1994)
- Revised the effective potential analysis on $\mathbb{R}^{D-1} \times S^1$ from the above studies
  T. Inagaki, Y. Matsuo, and H.S. (2019)

$\longrightarrow$ At zero temperature and particle number density

Our work

Extended to finite temperature and particle number density

Phase structures of the chiral condensation in a QCD-like theory
(a strongly coupled fermions system)

Brief summary

We evaluate

- the Gross–Neveu model
- at the leading order of the $1/N$ expansion
- at finite particle number density
- under the $\mathrm{U}(1)$-valued and anti-periodic boundary conditions
- on the torus spacetime(finite size and temperature)

and then investigate the phase structure.

Through thermodynamic quantities

we see characteristics of phases and phase transitions

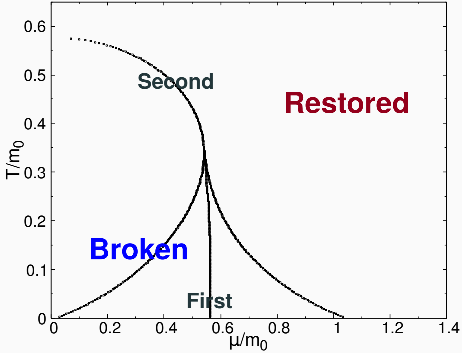
Result: Expression of the effective potential

$$
\begin{aligned}
\frac{V_{\text{eff}}(\sigma; L, \delta, T, \mu)}{m_0^2} = & -\frac{1}{4\pi}\left[1 - \ln\left(\frac{\sigma}{m_0}\right)^2\right]\left(\frac{\sigma}{m_0}\right)^2 \\
& -\frac{1}{\pi}\frac{1}{Lm_0}\int_0^\infty \frac{\mathrm{d}q}{m_0}\ln\left(2\frac{\cosh L\sqrt{q^2+\sigma^2} - \cos\pi\delta}{\exp L\sqrt{q^2+\sigma^2}}\right) \\
& -\frac{T}{Lm_0^2}\sum_{n=-\infty}^{\infty}\ln\left(2\frac{\cosh\sqrt{k_{\delta,n}^2+\sigma^2}/T + \cosh\mu/T}{\exp\sqrt{k_{\delta,n}^2+\sigma^2}/T}\right)
\end{aligned}
$$

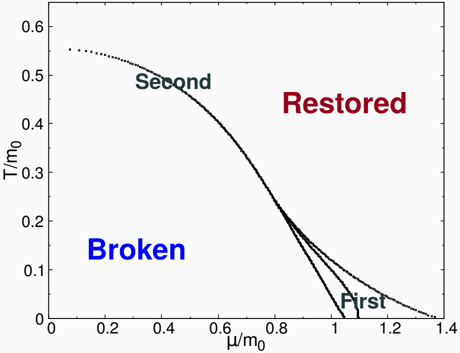
a dynamical mass at the trivial spacetime $\mathbb{R}^2$: $\frac{m_0}{\Lambda} = \exp\left(1 - \frac{\pi}{\lambda_r}\right)$, a discrete momentum: $k_{\delta,n} = \frac{2n+\delta}{L}\pi$

Result: Phase diagrams: μ-T plane (smaller size)

Size $Lm_0 = 3.0$ (the critical chemical potential at $\mathbb{R}^2$: $\frac{\mu_c}{m_0} = \frac{1}{\sqrt{2}} \simeq 0.71$)



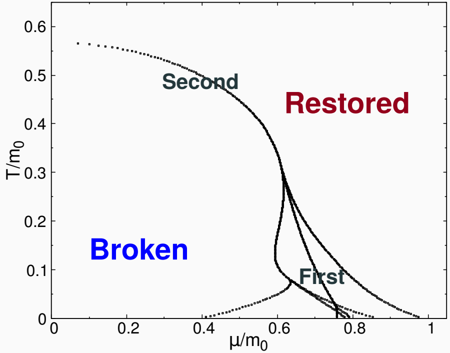
Periodic boundary condition



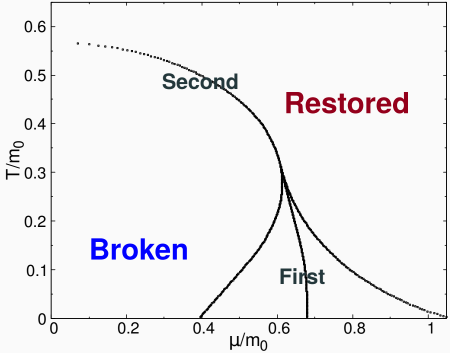
Anti-periodic boundary condition

Result: Phase diagrams: μ-T plane (larger size)

Size $Lm_0 = 8.0$



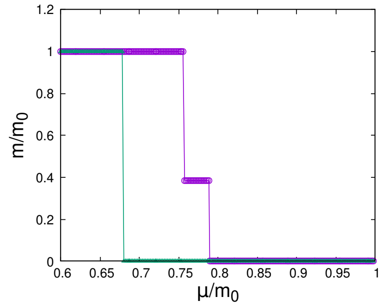
Periodic boundary condition



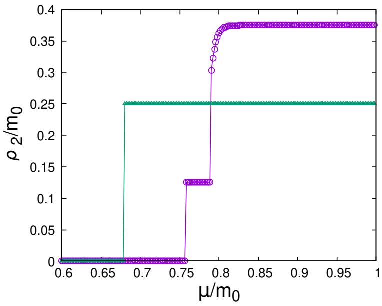
Anti-periodic boundary condition

Result: Phase diagrams: Thermodynamic quantities

Size $Lm_0 = 8.0$, Temperature $T/m_0 = 0.005$



Dynamically generated fermion mass



Particle number density

Purple: periodic boundary condition, Green: anti-periodic boundary condition

Summary

- The finite temperature and particle number density Gross–Neveu model with a compactified space under the $\mathrm{U}(1)$-valued boundary condition has been considered.
- The effective potential derived from the Gross–Neveu action at the leading order of the $1/N$ expansion has been evaluated.

We have showed the chiral phase diagrams with the effects of size and boundary conditions on a μ-T plane and showed the thermodynamic quantities, the dynamically generated fermion mass and particle number density including contributions of the chiral phase transition.

Future works: the magnetic field, flavors(u,d,s), current masses, ...

I expect to associated with the dimensional reduction under a strong magnetic field: $3+1 \to 1+1$

V. P. Gusynin, V. A. Miransky and I.A. Shovkovy (1995)

Backups: Chiral symmetry

Dirac fermions, ψ, decomposed into left- and right-handed states, ψ_L and ψ_R

$$\psi = \psi_L + \psi_R$$

Eigenstates of chirality operator γ^5 ($\gamma^\mu\gamma^5 = -\gamma^5\gamma^\mu, \quad \left(\gamma^5\right)^2 = 1$):

$$\gamma^5\psi_L = -\psi_L, \quad \gamma^5\psi_R = +\psi_R$$

Discrete chiral transformation: $\psi \longrightarrow \gamma^5\psi$

Simple product of Dirac fermions appearing as a mass term

$$\bar{\psi}\psi = \bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L \quad \xrightarrow{\text{chiral trans.}} \quad -\bar{\psi}\psi$$ Not invariant

Backups: energy scales

$197\mathrm{MeVfm} \simeq 1 \longrightarrow 100\mathrm{MeV} \cdot 0.5\mathrm{fm} \sim 1$ or $1\mathrm{MeV} \cdot 5\mathrm{pm} \sim 1$ or $1\mathrm{keV} \cdot 5\mathrm{nm} \sim 1$

$0.7 \sim 170\mathrm{MeV}/240\mathrm{MeV}$ ($m_0 \sim 240\mathrm{MeV}$)

$1\mathrm{T}(= 10^4\mathrm{G}) \simeq 195\mathrm{eV}^2$ (Neutron star: $\sim 10^8\mathrm{T}$, Magnetar: $\sim 10^{11}\mathrm{T}$)

$100\mathrm{MeV} \sim 1.2 \times 10^{12}\mathrm{K}$