Origin of flavor structures for elementary particles

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Plan of my talk

1. Introduction
   - Standard model for particle physics
   - Neutrino oscillation and lepton mixing
2. Non-Abelian discrete symmetry
3. Flavor model in lepton sector
   - A_4 neutrino model
   - Modified A_4 model
   - Toward the minimal model
4. Summary

1. Introduction

- Standard model for particle physics

Particle	First	Second	Third	Mixing matrix
Quark	$\begin{pmatrix} u \\ d \end{pmatrix}_L$ u^c_R d^c_R	$\begin{pmatrix} c \\ s \end{pmatrix}_L$ c^c_R s^c_R	$\begin{pmatrix} t \\ b \end{pmatrix}_L$ t^c_R b^c_R	CKM matrix (Cabibbo-Kobayashi-Maskawa)
Lepton	$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$ e^c_R	$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$ μ^c_R	$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$ τ^c_R	PMNS matrix (Pontecorvo-Maki-Nakagawa-Sakata)

- Generation Mysteries

- Masses of elementary particles are different each generation.
- Lepton flavor mixing is quite different from quark one.

- Spontaneous symmetry breaking and Higgs mechanism

$$\mathcal{L}_{\rm SM} \supset i\bar{\psi}_i \partial\!\!\!/ \psi_i + y_{ij}\bar{\psi}_i\psi_j h$$

Kinetic terms   Yukawa interactions (y_{ij} : Yukawa couplings)

↓ Spontaneous symmetry breaking

$$h \to v + \delta h$$

$$\mathcal{L}_{\rm SM} \supset i\bar{\psi}_i \partial\!\!\!/ \psi_i + y_{ij}v\bar{\psi}_i\psi_j$$

Kinetic terms   Mass terms

- Masses of fermions

$$m_f = y_{ij}v$$ ··· however not mass eigenstates

moving from gauge eigenstates to mass eigenstates

- Charged current interaction for left-handed quarks

$$u_i = (U_u)_{ik} u_k^m, \quad d_i = (U_d)_{ik} d_k^m$$

u_i, d_i : gauge eigenstate $\qquad u_i^m, d_i^m$: mass eigenstate

W_μ : Weak boson $\qquad g$: Gauge coupling

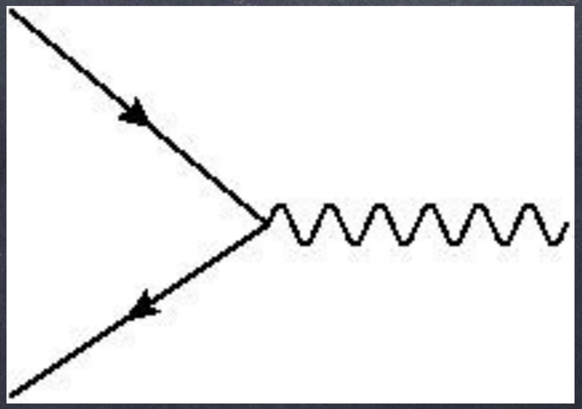
$$\mathcal{L}_{CC} = \frac{g}{\sqrt{2}} W_\mu^+ (\bar{u}_L, \bar{c}_L, \bar{t}_L) \gamma^\mu \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} + h.c.$$

$$= \frac{g}{\sqrt{2}} W_\mu^+ (\bar{u}_L^m, \bar{c}_L^m, \bar{t}_L^m) \gamma^\mu U_u^\dagger U_d \begin{pmatrix} d_L^m \\ s_L^m \\ b_L^m \end{pmatrix} + h.c.$$

- CKM matrix

$$V_{\mathrm{CKM}} \equiv U_u^\dagger U_d$$

u_L



W

d_L

- CP symmetry
  - CP transformation:

$$\psi_L \xrightarrow{C} (i\gamma^2\psi^*)_R \xrightarrow{P} (\gamma^0 i\gamma^2\psi^*)_L$$

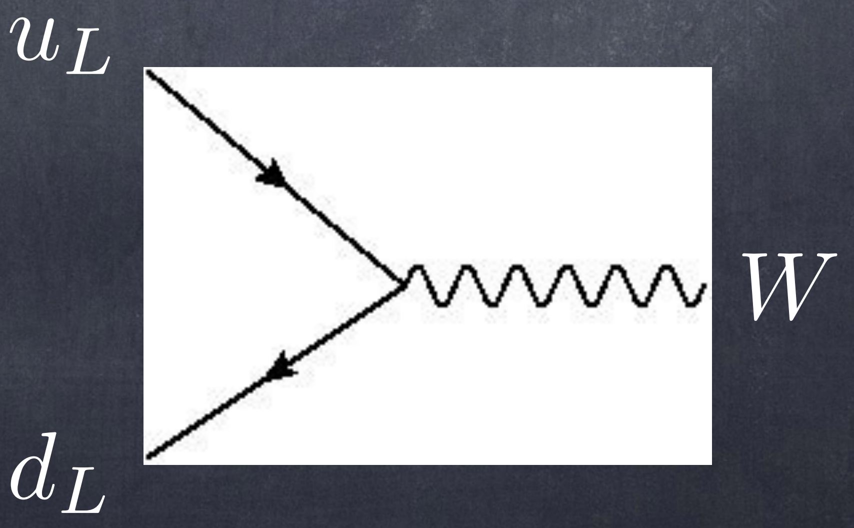
$$\mathcal{L}_{CC} = \frac{g}{\sqrt{2}} V_{ij} W_\mu^+ \bar{u}_{Li}^m \gamma^\mu d_{Lj}^m + \frac{g}{\sqrt{2}} V_{ij}^* W_\mu^- \bar{d}_{Lj}^m \gamma^\mu u_{Li}^m$$
$$\xrightarrow{CP} \frac{g}{\sqrt{2}} V_{ij} W_\mu^- \bar{d}_{Lj}^m \gamma^\mu u_{Li}^m + \frac{g}{\sqrt{2}} V_{ij}^* W_\mu^+ \bar{u}_{Li}^m \gamma^\mu d_{Lj}^m$$

- Condition of the CP violation

CKM matrix V_{ij} is complex

⬇

$\not{CP}$



- Parameters of CKM matrix (N generations case)

$$V_{ij} = \begin{pmatrix} V_{ud} & \cdots & V_{uj} \\ \vdots & & \vdots \\ V_{id} & \cdots & V_{ij} \end{pmatrix}$$

- # of real parameters for $N \times N$ complex matrix: $N^2 \times 2$
- Unitarity condition: $\sum_k V_{ik}^* V_{jk} = \delta_{ij} \ \rightarrow \ N^2$
  - for diagonal elements $(i = j)$: N
  - for off-diagonal elements $(i \neq j)$: $N(N-1)$
- # of removing phases for quark fields: $2N - 1$
- # of physical parameters:

$$2N^2 - N^2 - (2N-1) = (N-1)^2$$

- We consider the mixing angles and phase parameters:
  - # of mixing angles (θ_{ij}) : ${}_N C_2 = \frac{1}{2}N(N-1)$
  - # of phase parameters:

$$(N-1)^2 - {}_N C_2 = \frac{1}{2}(N-1)(N-2)$$

- CP violation occurs at least 3 generations
  - Kobayashi-Maskawa Theory

$$V_{\rm CKM} \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{\rm CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{\rm CP}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\rm CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{\rm CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{\rm CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{\rm CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{\rm CP}} & c_{23}c_{13} \end{pmatrix}$$

- Neutrino oscillation and lepton mixing

- Neutrino flavor, mass eigenstates and time evolution

$$|\nu_\alpha\rangle = \sum_i U_{\alpha j}|\nu_j\rangle, \quad |\nu_\alpha(t)\rangle = \sum_i U_{\alpha j}|\nu_j\rangle e^{-iE_j t}$$

- We consider 2 generations

$$|\nu_e(t)\rangle = \cos\theta|\nu_1\rangle e^{-iE_1 t} + \sin\theta|\nu_2\rangle e^{-iE_2 t}$$
$$|\nu_\mu(t)\rangle = -\sin\theta|\nu_1\rangle e^{-iE_1 t} + \cos\theta|\nu_2\rangle e^{-iE_2 t}$$

- Transition probability of $\nu_e \to \nu_\mu$

$$P(\nu_e \to \nu_\mu; t) = |\langle\nu_\mu|\nu_e(t)\rangle|^2 = \sin^2 2\theta \sin^2 \frac{E_2 - E_1}{2} t$$
$$\simeq \sin^2 2\theta \sin^2 \frac{\Delta m^2 L}{4E}, \quad \Delta m^2 = m_2^2 - m_1^2$$
$$E_j = \sqrt{p^2 + m_j^2} \simeq p + \frac{m_j^2}{2E}$$

- Neutrino mass squared differences

$$\Delta m^2_{\rm sol} \equiv m_2^2 - m_1^2, \quad \left|\Delta m^2_{\rm atm}\right| \equiv \left|m_3^2 - m_1^2\right|.$$

- Leptons get masses through Higgs mechanism

$$\mathcal{L}_Y = y\bar{\psi}_L H \psi_R \to y\langle H\rangle \bar{\psi}_L \psi_R = m_f \bar{\psi}_L \psi_R.$$

  - In the SM, because there are not right-handed neutrinos, neutrino is massless.

- Seesaw mechanism

Minkowski '77; Gell-Mann, Ramond, Slansky; Yanagida; Glashow; Mohapatra, Senjanovic '79

  - Adding three right-handed Majorana neutrinos

$$M = \begin{pmatrix} 0 & M_D \\ M_D^T & M_N \end{pmatrix} \xrightarrow{\text{diagonalized}} M_\nu \simeq -M_D^T M_N^{-1} M_D$$

  - If $M_N >> M_D$, left-handed Majorana neutrinos get non-zero and small masses

- Parameters of PMNS matrix (N generations case)
- # of real parameters for $N \times N$ complex matrix: $N^2 \times 2$
- Unitarity condition: $\sum_k V_{ik}^* V_{jk} = \delta_{ij} \ \rightarrow \ N^2$
  - for diagonal elements $(i = j)$: N
  - for off-diagonal elements $(i \neq j)$: $N(N-1)$
- # of removing phases for lepton fields: N
- # of physical parameters:

$$2N^2 - N^2 - N = N(N-1)$$

- We consider the mixing angles and phase parameters
  - # of mixing angles: ${}_N C_2 = \frac{1}{2} N(N-1)$
  - # of phase parameters: $N(N-1) - {}_N C_2 = \frac{1}{2} N(N-1)$

- Phase parameters (3 generations case)

$$\frac{1}{2}N(N-1)$$ → 3 (1 Dirac phase and 2 Majorana phases)

- Lepton flavor mixing matrix (PMNS matrix)

$$U \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{\rm CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{\rm CP}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\alpha} & 0 & 0 \\ 0 & e^{i\beta} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\rm CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{\rm CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{\rm CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{\rm CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{\rm CP}} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\alpha} & 0 & 0 \\ 0 & e^{i\beta} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- $\delta_{\rm CP}$ is Dirac phase and α , β are Majorana phases

- Neutrino mass hierarchy

  - Normal hierarchy (NH) → $m_1 < m_2 < m_3$
  - Inverted hierarchy (IH) → $m_3 < m_1 < m_2$
  - Quasi-degenerated (QD) → $m_1 \sim m_2 \sim m_3$

- Experimental situations

- Reactor neutrino experiments indicate non-zero θ_{13}

  - Experimental result by Daya Bay

$$\sin^2 2\theta_{13} = 0.084 \pm 0.005$$

  - Consistent with RENO, Double Chooz, and T2K experiments

  - Global fit of the neutrino oscillation

M. C. Gonzalez-Garcia, M. Maltoni, T. Schwetz, JHEP 1411 (2014) 052

parameter	best fit	1σ	3σ
$\sin^2\theta_{12}$	0.304	0.292-0.317	0.270-0.344
$\sin^2\theta_{23}$	0.452 0.579	0.424-0.504 0.542-0.604	0.382-0.643 0.389-0.644
$\sin^2\theta_{13}$	0.0218 0.0219	0.0208-0.0228 0.0209-0.0230	0.0186-0.0250 0.0188-0.0251
$\Delta m^2_{\rm sol}$ $[10^{-5}{\rm eV}^2]$	7.50	7.33-7.69	7.02-8.09
$\lvert\Delta m^2_{\rm atm}\rvert$ $[10^{-3}{\rm eV}^2]$	2.457 2.449	2.410-2.504 2.401-2.496	2.317-2.607 2.307-2.590
$\delta_{\rm CP}$ $[^\circ]$	306 254	236-345 192-317	0-360

- # Global fit of the neutrino oscillation

NuFIT 4.1 (2019)

without SK atmospheric data

	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 6.2$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.310^{+0.013}_{-0.012}$	$0.275 \to 0.350$	$0.310^{+0.013}_{-0.012}$	$0.275 \to 0.350$
$\theta_{12}/^\circ$	$33.82^{+0.78}_{-0.76}$	$31.61 \to 36.27$	$33.82^{+0.78}_{-0.76}$	$31.61 \to 36.27$
$\sin^2\theta_{23}$	$0.558^{+0.020}_{-0.033}$	$0.427 \to 0.609$	$0.563^{+0.019}_{-0.026}$	$0.430 \to 0.612$
$\theta_{23}/^\circ$	$48.3^{+1.1}_{-1.9}$	$40.8 \to 51.3$	$48.6^{+1.1}_{-1.5}$	$41.0 \to 51.5$
$\sin^2\theta_{13}$	$0.02241^{+0.00066}_{-0.00065}$	$0.02046 \to 0.02440$	$0.02261^{+0.00067}_{-0.00064}$	$0.02066 \to 0.02461$
$\theta_{13}/^\circ$	$8.61^{+0.13}_{-0.13}$	$8.22 \to 8.99$	$8.65^{+0.13}_{-0.12}$	$8.26 \to 9.02$
$\delta_{CP}/^\circ$	222^{+38}_{-28}	$141 \to 370$	285^{+24}_{-26}	$205 \to 354$
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.39^{+0.21}_{-0.20}$	$6.79 \to 8.01$	$7.39^{+0.21}_{-0.20}$	$6.79 \to 8.01$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.523^{+0.032}_{-0.030}$	$+2.432 \to +2.618$	$-2.509^{+0.032}_{-0.030}$	$-2.603 \to -2.416$

with SK atmospheric data

	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 10.4$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.310^{+0.013}_{-0.012}$	$0.275 \to 0.350$	$0.310^{+0.013}_{-0.012}$	$0.275 \to 0.350$
$\theta_{12}/^\circ$	$33.82^{+0.78}_{-0.76}$	$31.61 \to 36.27$	$33.82^{+0.78}_{-0.75}$	$31.61 \to 36.27$
$\sin^2\theta_{23}$	$0.563^{+0.018}_{-0.024}$	$0.433 \to 0.609$	$0.565^{+0.017}_{-0.022}$	$0.436 \to 0.610$
$\theta_{23}/^\circ$	$48.6^{+1.0}_{-1.4}$	$41.1 \to 51.3$	$48.8^{+1.0}_{-1.2}$	$41.4 \to 51.3$
$\sin^2\theta_{13}$	$0.02237^{+0.00066}_{-0.00065}$	$0.02044 \to 0.02435$	$0.02259^{+0.00065}_{-0.00065}$	$0.02064 \to 0.02457$
$\theta_{13}/^\circ$	$8.60^{+0.13}_{-0.13}$	$8.22 \to 8.98$	$8.64^{+0.12}_{-0.13}$	$8.26 \to 9.02$
$\delta_{CP}/^\circ$	221^{+39}_{-28}	$144 \to 357$	282^{+23}_{-25}	$205 \to 348$
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.39^{+0.21}_{-0.20}$	$6.79 \to 8.01$	$7.39^{+0.21}_{-0.20}$	$6.79 \to 8.01$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.528^{+0.029}_{-0.031}$	$+2.436 \to +2.618$	$-2.510^{+0.030}_{-0.031}$	$-2.601 \to -2.419$

NuFIT 5.0 (2020)

without SK atmospheric data

	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 2.7$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$
$\theta_{12}/^\circ$	$33.44^{+0.78}_{-0.75}$	$31.27 \to 35.86$	$33.45^{+0.78}_{-0.75}$	$31.27 \to 35.87$
$\sin^2\theta_{23}$	$0.570^{+0.018}_{-0.024}$	$0.407 \to 0.618$	$0.575^{+0.017}_{-0.021}$	$0.411 \to 0.621$
$\theta_{23}/^\circ$	$49.0^{+1.1}_{-1.4}$	$39.6 \to 51.8$	$49.3^{+1.0}_{-1.2}$	$39.9 \to 52.0$
$\sin^2\theta_{13}$	$0.02221^{+0.00068}_{-0.00062}$	$0.02034 \to 0.02430$	$0.02240^{+0.00062}_{-0.00062}$	$0.02053 \to 0.02436$
$\theta_{13}/^\circ$	$8.57^{+0.13}_{-0.12}$	$8.20 \to 8.97$	$8.61^{+0.12}_{-0.12}$	$8.24 \to 8.98$
$\delta_{CP}/^\circ$	195^{+51}_{-25}	$107 \to 403$	286^{+27}_{-32}	$192 \to 360$
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.514^{+0.028}_{-0.027}$	$+2.431 \to +2.598$	$-2.497^{+0.028}_{-0.028}$	$-2.583 \to -2.412$

with SK atmospheric data

	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 7.1$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.304^{+0.012}_{-0.012}$	$0.269 \to 0.343$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$
$\theta_{12}/^\circ$	$33.44^{+0.77}_{-0.74}$	$31.27 \to 35.86$	$33.45^{+0.78}_{-0.75}$	$31.27 \to 35.87$
$\sin^2\theta_{23}$	$0.573^{+0.016}_{-0.020}$	$0.415 \to 0.616$	$0.575^{+0.016}_{-0.019}$	$0.419 \to 0.617$
$\theta_{23}/^\circ$	$49.2^{+0.9}_{-1.2}$	$40.1 \to 51.7$	$49.3^{+0.9}_{-1.1}$	$40.3 \to 51.8$
$\sin^2\theta_{13}$	$0.02219^{+0.00062}_{-0.00063}$	$0.02032 \to 0.02410$	$0.02238^{+0.00063}_{-0.00062}$	$0.02052 \to 0.02428$
$\theta_{13}/^\circ$	$8.57^{+0.12}_{-0.12}$	$8.20 \to 8.93$	$8.60^{+0.12}_{-0.12}$	$8.24 \to 8.96$
$\delta_{CP}/^\circ$	197^{+27}_{-24}	$120 \to 369$	282^{+26}_{-30}	$193 \to 352$
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.517^{+0.026}_{-0.028}$	$+2.435 \to +2.598$	$-2.498^{+0.028}_{-0.028}$	$-2.581 \to -2.414$

- Deference of mixing matrices

- Lepton mixing: PMNS mixing matrix

$$|U_{\rm PMNS}| \simeq \begin{pmatrix} 0.825 & 0.545 & 0.148 \\ 0.462 & 0.587 & 0.665 \\ 0.326 & 0.598 & 0.732 \end{pmatrix}$$

The lepton mixing is large except for reactor angle θ_{13}

$$\sin\theta_{13} \simeq 0.148$$

- Quark mixing: CKM mixing matrix (PDG)

$$|V_{\rm CKM}| \simeq \begin{pmatrix} 0.974 & 0.225 & 0.00355 \\ 0.225 & 0.973 & 0.0414 \\ 0.00886 & 0.0405 & 0.999 \end{pmatrix}$$

The quark mixing is small except for Cabibbo angle λ_C

$$\lambda_C \simeq 0.225$$

- I want to solve difference of lepton and quark flavor mixing

2. Non-Abelian discrete symmetry

- Before reactor experiments were reported θ_{13} , tri-bimaximal mixing (TBM) was good scheme

P. F. Harrison, D. H. Perkins and W. G. Scott, Phys. Lett. B 530 (2002) 167

$$U_{\text{PMNS}} = V_{\text{TBM}} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix},$$

$$|U_{e2}| = \frac{1}{\sqrt{3}}, \quad |U_{\mu 3}| = \frac{1}{\sqrt{2}}, \quad |U_{e3}| = 0$$

- The left-handed Majorana neutrino mass matrix

$$M_\nu^{\text{TBM}} = \frac{m_1+m_3}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \frac{m_2-m_1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{m_1-m_3}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

- TBM is realized by non-Abelian discrete group

H. Ishimori, T. Kobayashi, H. Ohki, H. Okada, Y. S., and M. Tanimoto,
Prog. Theor. Phys. Suppl. 183 (2010) 1; Lect. Notes Phys. 858 (2012) 1, Springer

- Flavor Symmetry

- Abelian or non-Abelian

  Abelian: discriminate between generations
  non-Abelian: connect different generations

- continuous or discrete

  continuous: free rotation between generations
  discrete: definite meaning of generations

- We introduce non-Abelian discrete symmetry as flavor symmetry c.f. gauge symmetry: $SU(3)_Q \times SU(2)_L \times U(1)_Y$

- Typical non-Abelian discrete group

- Symmetry group S_n (# of group elements is n!)

	Number of elements	Geometry	Irreducible representations
S_3	6	Regular triangle	$1_S, 1_A, 2$
S_4	24	Octahedron	$1, 1', 2, 3, 3'$

- Even permutation group A_n (# of group elements is n!/2)

	Number of elements	Geometry	Irreducible representations
A_4	12	Tetrahedron	$1, 1', 1'', 3$

- T' group

	Number of elements	Geometry	Irreducible representations
T'	24	Double tetrahedron	$1, 1', 1'', 2, 2', 2'', 3$

- A_4 group

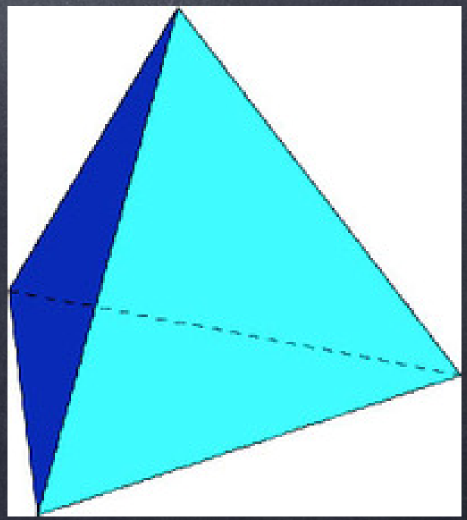
the symmetry group of tetrahedron or even permutation of four elements. Number of elements is 12.

- Irreducible representations of A_4 are $\mathbf{1}$, $\mathbf{1'}$, $\mathbf{1''}$, $\mathbf{3_S}$, $\mathbf{3_A}$

- Multiplication rules are

$$(a_1, a_2, a_3)_{\mathbf{3}} \otimes (b_1, b_2, b_3)_{\mathbf{3}} = (a_1 b_1 + a_2 b_3 + a_3 b_2)_{\mathbf{1}} \oplus (a_3 b_3 + a_1 b_2 + a_2 b_1)_{\mathbf{1'}}$$
$$\oplus (a_2 b_2 + a_1 b_3 + a_3 b_1)_{\mathbf{1''}} \oplus \frac{1}{3} \begin{pmatrix} 2a_1 b_1 - a_2 b_3 - a_3 b_3 \\ 2a_3 b_3 - a_1 b_2 - a_2 b_1 \\ 2a_2 b_2 - a_1 b_3 - a_3 b_1 \end{pmatrix}_{\mathbf{3_S}} \oplus \frac{1}{2} \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_1 b_2 - a_2 b_1 \\ a_1 b_3 - a_3 b_1 \end{pmatrix}_{\mathbf{3_A}}$$

- A_4 invariant representation is $\mathbf{1}$



3. Flavor model in lepton sector

- A_4 neutrino model

- Tri-bimaximal mixing can be derived by A_4 symmetry

  G. Altarelli and F. Feruglio, Nucl. Phys. B 720 (2005) 64

- Charge assignments of $SU(2)$, A_4, Z_3: $(\omega = e^{\frac{2\pi i}{3}})$

	(l_e, l_μ, l_τ)	e^c	μ^c	τ^c	$h_{u,d}$	ϕ_l	ϕ_ν	ξ
$SU(2)$	2	1	1	1	2	1	1	1
A_4	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}''$	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$
Z_3	ω	ω^2	ω^2	ω^2	1	1	ω	ω

- $A_4 \times Z_3$ invariant Lagrangian for the Yukawa interaction

$$\mathcal{L}_\ell = y^e e^c l \phi_l h_d/\Lambda + y^\mu \mu^c l \phi_l h_d/\Lambda + y^\tau \tau^c l \phi_l h_d/\Lambda$$
$$+ (y^\nu_{\phi_\nu} \phi_\nu + y^\nu_\xi \xi) l l h_u h_u/\Lambda^2$$

- Taking VEVs: $\langle h_{u,d} \rangle = v_{u,d}, \quad \langle \xi \rangle = \alpha_\xi \Lambda$
- VEV alignments: $\langle \phi_l \rangle = \alpha_l \Lambda (1,0,0), \quad \langle \phi_\nu \rangle = \alpha_\nu \Lambda (1,1,1)$
- Charged lepton mass matrix: (diagonal)

$$M_l = \alpha_l v_d \begin{pmatrix} y^e & 0 & 0 \\ 0 & y^\mu & 0 \\ 0 & 0 & y^\tau \end{pmatrix}$$

- Mass matrix of the left-handed Majorana neutrino ($a = -3b$)

$$M_\nu = a \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + b \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + c \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

$$a = \frac{y^\nu_{\phi_\nu} \alpha_\nu v_u^2}{\Lambda}, \qquad b = -\frac{y^\nu_{\phi_\nu} \alpha_\nu v_u^2}{3\Lambda}, \qquad c = \frac{y^\nu_\xi \alpha_\xi v_u^2}{\Lambda}$$

- Altarelli et al introduce only $\mathbf{1}$ in A_4 model

$$M_\nu = a\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} + b\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix} + c\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}$$

- The reactor experiments reported $\theta_{13} \neq 0$
- We introduce not only the trivial singlet $\mathbf{1}$ but also $\mathbf{1}'$ or $\mathbf{1}''$, then we discuss the deviation from the tri-bimaximal mixing

$$\mathbf{1}:\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix},\quad \mathbf{1}':\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix},\quad \mathbf{1}'':\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix}$$

$$\mathbf{1}'':\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix} = \begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix} - \begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix} - \begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}$$

- The left-handed Majorana neutrino mass matrix ($\mathbf{1}'$)

$$M_\nu = a\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} + b\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix} + c\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix} + d\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix}$$

- By rotating tri-bimaximal mixing

$$M_\nu = V_{\text{tri-bi}}\begin{pmatrix}a+c-\frac{d}{2} & 0 & \frac{\sqrt{3}}{2}d\\ 0 & a+3b+c+d & 0\\ \frac{\sqrt{3}}{2}d & 0 & a-c+\frac{d}{2}\end{pmatrix} V_{\text{tri-bi}}^T$$

- Neutrino mass squared differences

$$\Delta m_{31}^2 = -4a\sqrt{c^2+d^2-cd}\ ,$$

$$\Delta m_{21}^2 = (a+3b+c+d)^2 - (a+\sqrt{c^2+d^2-cd})^2$$

- The lepton mixing matrix

$$U_{\text{PMNS}} = V_{\text{tri-bi}} \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix},$$

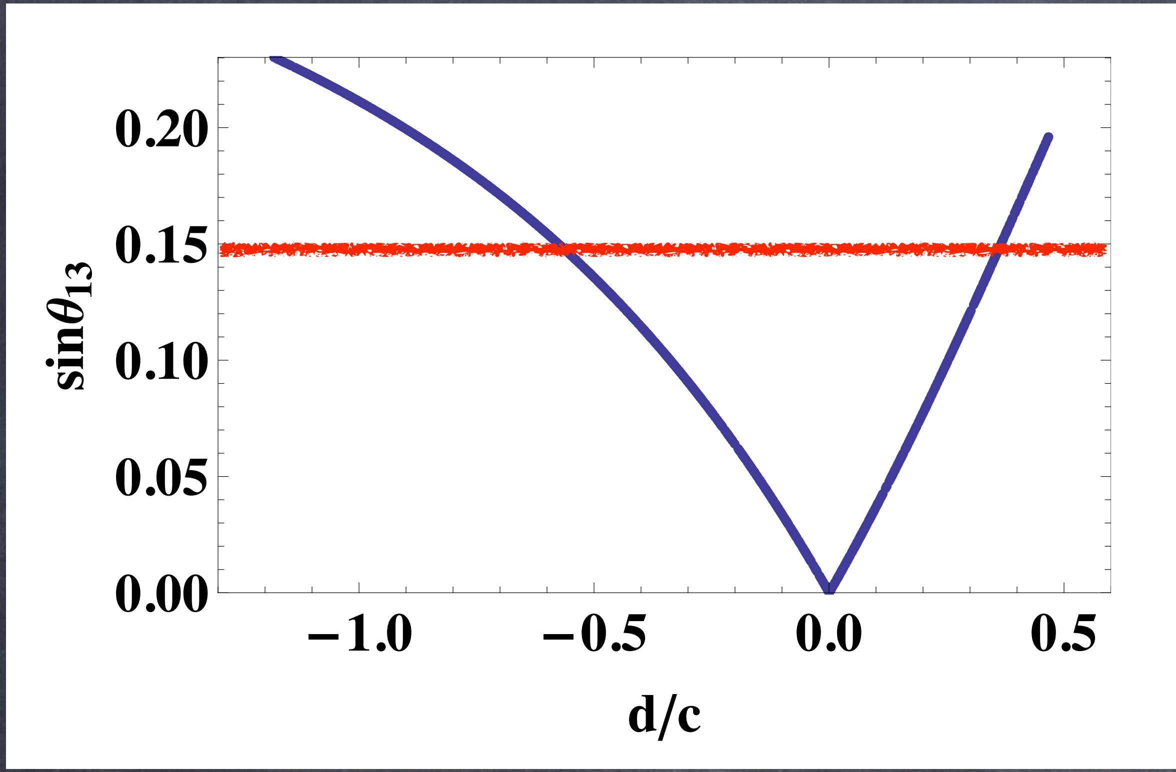
$$\tan 2\theta = \frac{\sqrt{3}d}{-2c + d}$$

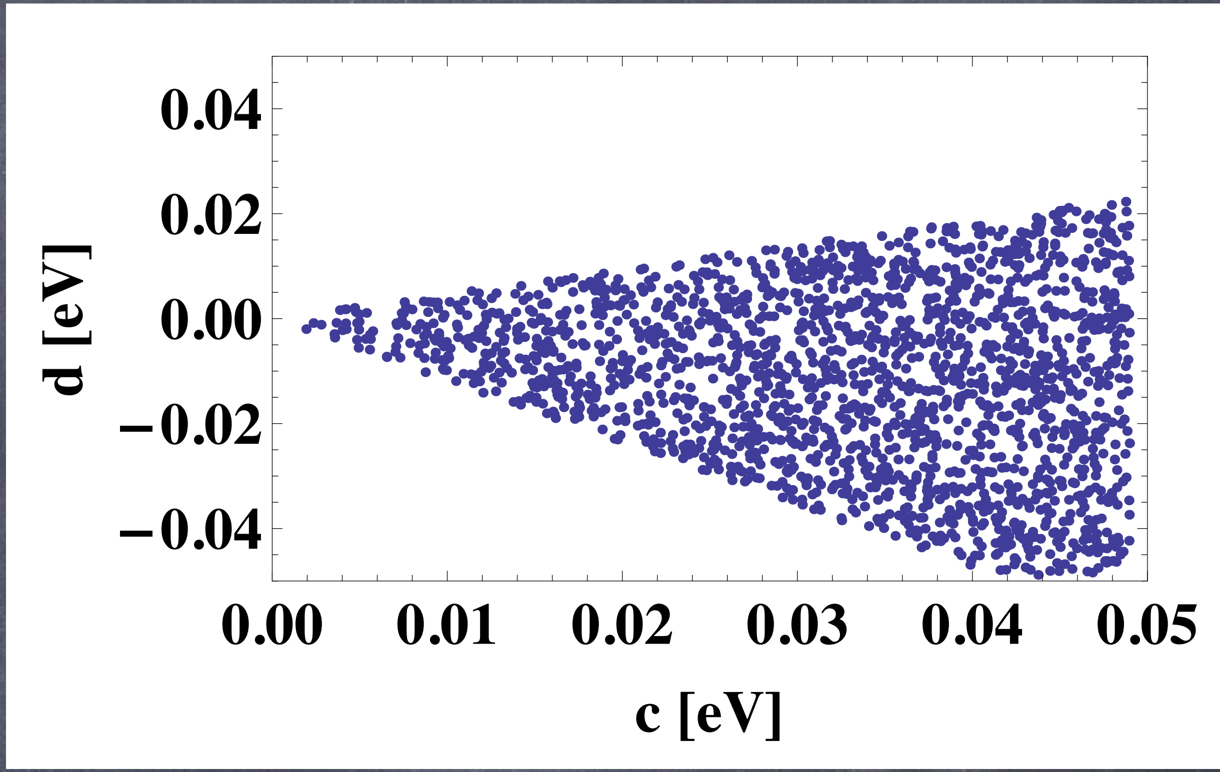
- The relevant lepton mixing matrix elements

$$|U_{e2}| = \frac{1}{\sqrt{3}},\ |U_{e3}| = \frac{2}{\sqrt{6}}|\sin\theta|,\ |U_{\mu 3}| = \left|-\frac{1}{\sqrt{6}}\sin\theta - \frac{1}{\sqrt{2}}\cos\theta\right|$$

- We obtain the non-zero θ_{13}

- Numerical results





- We have predicted the magnitude of the θ_{13} before neutrino reactor experiments were reported

Y.S., M. Tanimoto and A. Watanabe, Prog. Theor. Phys. 126 (2011) 81

↓

20th Outstanding Paper Award of the Physical Society of Japan (2015)

- Modified Altarelli model

Y.S., M. Tanimoto and A. Watanabe, Prog. Theor. Phys. 126 (2011) 81

	l	e_R^c	μ_R^c	τ_R^c	ν_R^c	$h_{u,d}$	ϕ_T	ϕ_S	ξ	ξ'
$SU(2)$	2	1	1	1	1	2	1	1	1	1
A_4	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}''$	$\mathbf{1}'$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}'$
Z_3	ω	ω^2	ω^2	ω^2	ω^2	1	1	ω^2	ω^2	ω^2

- $A_4 \times Z_3$ invariant Lagrangian for the Yukawa interaction

$$\mathcal{L}_Y \equiv \mathcal{L}_\ell + \mathcal{L}_D + \mathcal{L}_N,$$
$$\mathcal{L}_\ell = y_e \phi_T l e_R^c h_d/\Lambda + y_\mu \phi_T l \mu_R^c h_d/\Lambda + y_\tau \phi_T l \tau_R^c h_d/\Lambda,$$
$$\mathcal{L}_D = y_D l \nu_R^c h_u,$$
$$\mathcal{L}_N = y_{\phi_S} \phi_S \nu_R^c \nu_R^c + y_\xi \xi \nu_R^c \nu_R^c + y_{\xi'} \xi' \nu_R^c \nu_R^c$$

- Charged lepton and Dirac neutrino mass matrices

$$M_\ell = \frac{v_d v_T}{\Lambda} \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}, \quad M_D = y_D v_u \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

- Modified Altarelli model

Y.S., M. Tanimoto and A. Watanabe, Prog. Theor. Phys. 126 (2011) 81

- Right-handed Majorana neutrino mass matrix

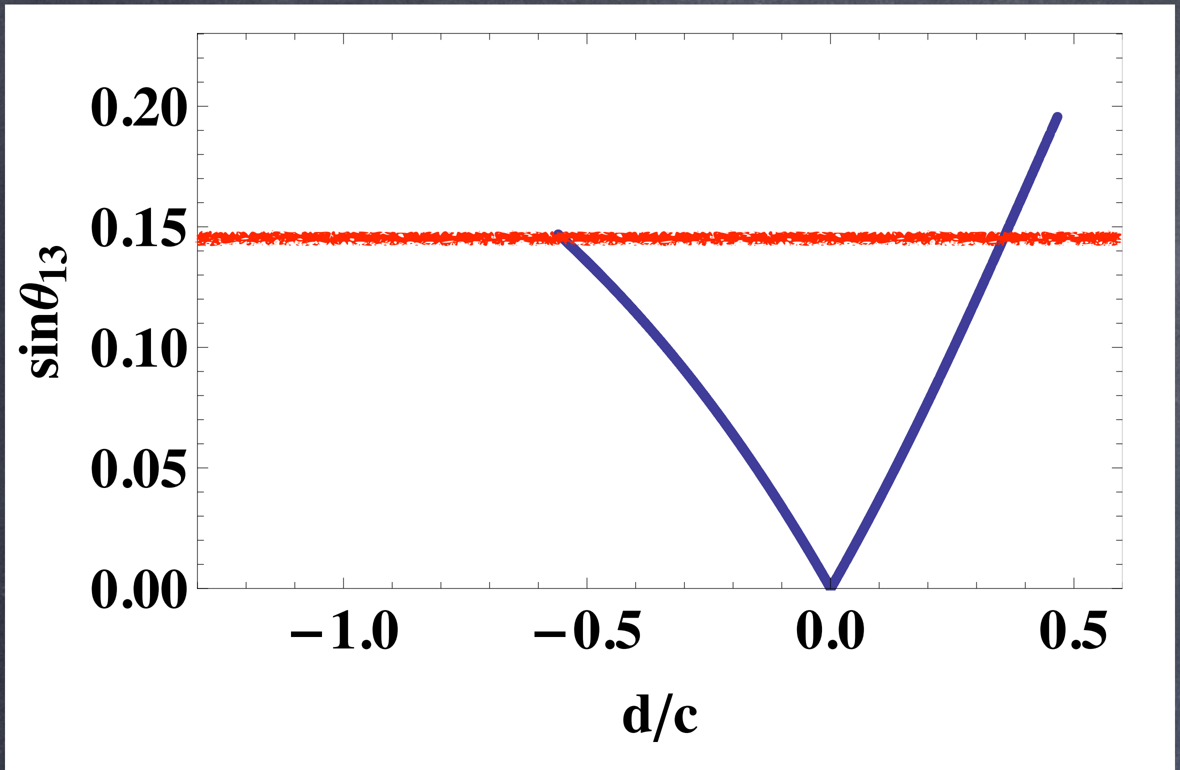
$$M_N = y_{\phi_S} v_S \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} + y_\xi u \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + y_{\xi'} u' \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

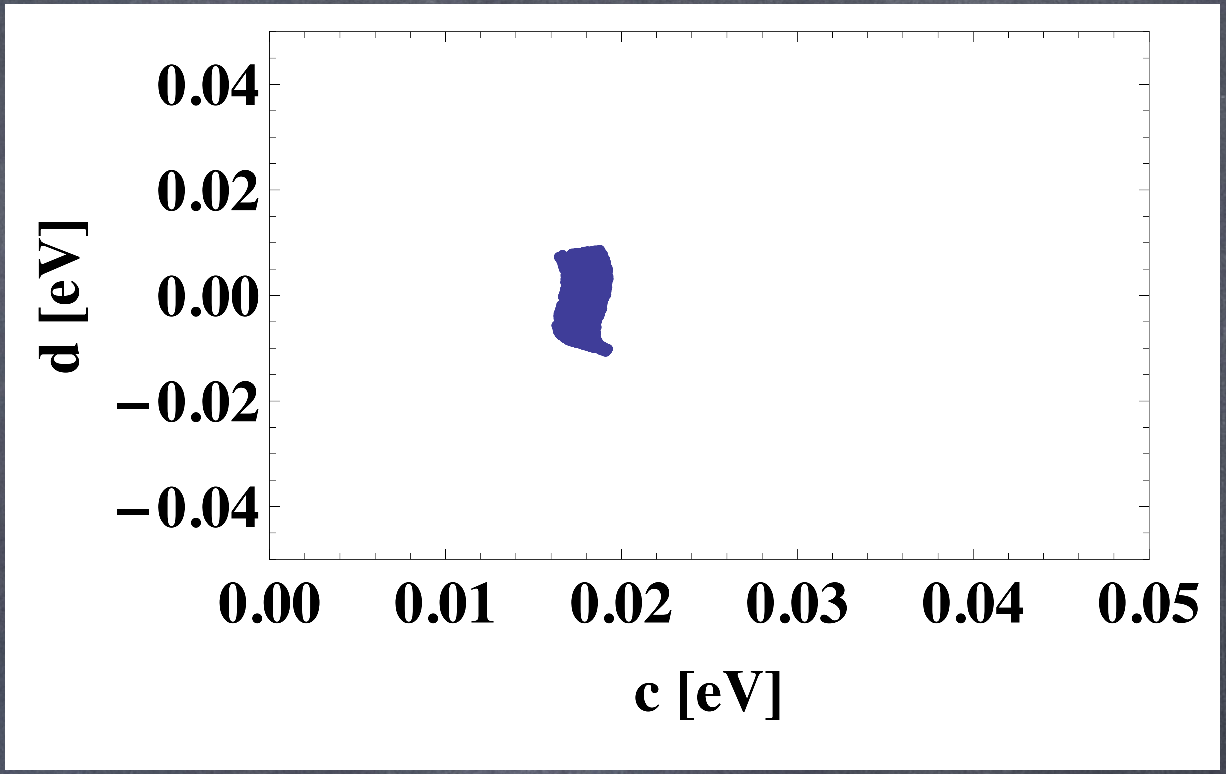
- By using seesaw mechanism, the left-handed Majorana Neutrino mass matrix is obtained

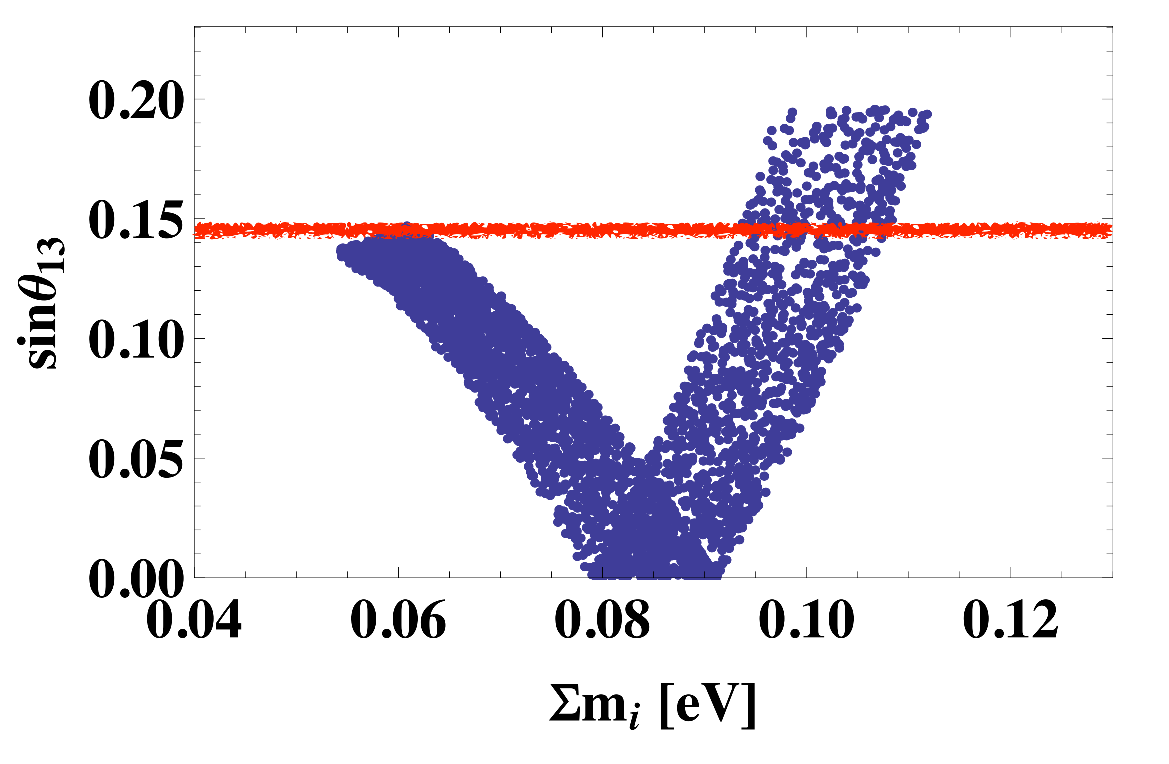
$$M_\nu = a \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + b \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + c \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + d \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

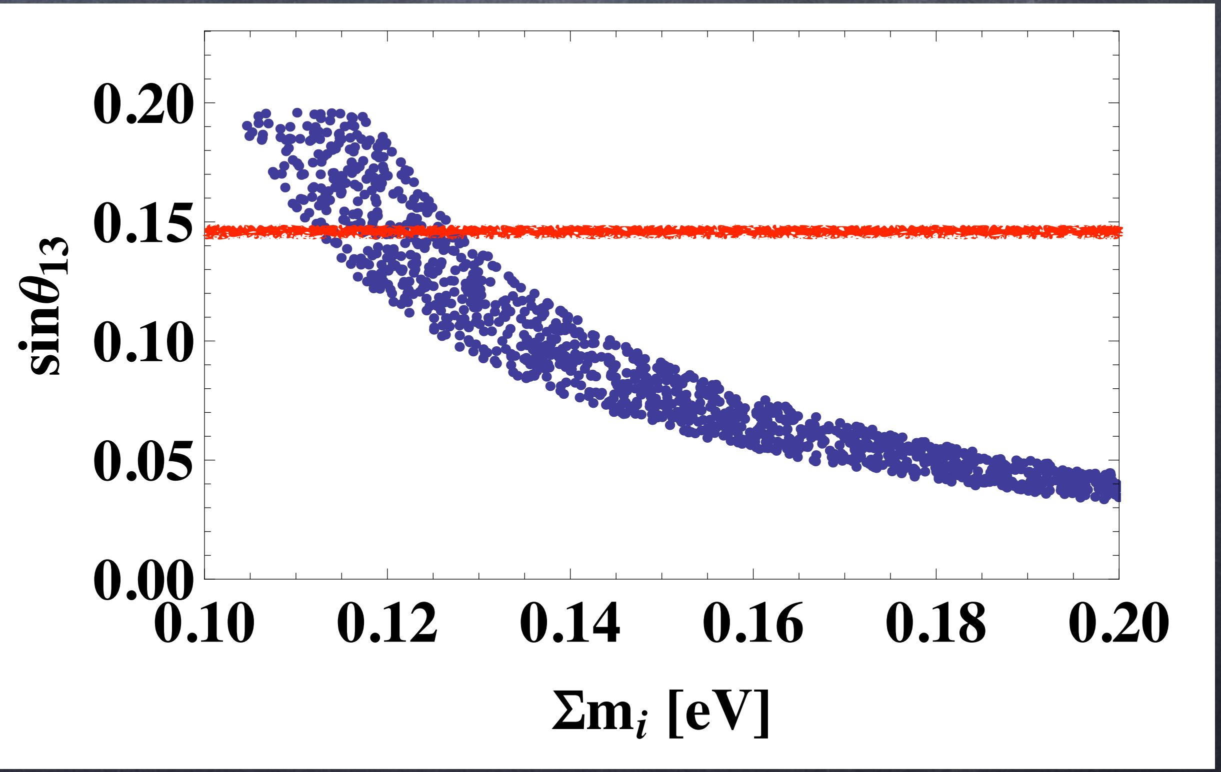
- We obtain the non-zero θ_{13}

- Numerical results









- Toward the minimal model

- As we shown, we use non-Abelian discrete symmetry as flavor symmetry to naturally explain mass hierarchy and flavor mixing for elementary particles.
- We introduce non-Abelian discrete symmetry and the scalar fields (so-called "flavons").
- We derive large mixing for lepton sector by using VEV of flavon and its alignment.
- Altarelli and Feruglio introduced two A4 triplet flavons (ϕT, ϕS) in non-SUSY framework. However they derived misalignment for their potential analysis.

$$\langle \phi_T \rangle \sim (1,0,0), \quad \langle \phi_S \rangle \sim (1,0,0)$$

or

$$\langle \phi_T \rangle \sim (1,1,1), \quad \langle \phi_S \rangle \sim (1,1,1)$$

- Toward the minimal model

- They applied the A4 model to the SUSY. They obtained correct alignments for flavons by introducing so-called "driving fields" such as （ϕ0T, ϕ0S）. G. Altarelli, F. Feruglio, 2006

$$\langle \phi_T \rangle \sim (1,0,0), \quad \langle \phi_S \rangle \sim (1,1,1)$$
$$\langle \phi_T^0 \rangle \sim (0,0,0), \quad \langle \phi_S^0 \rangle \sim (0,0,0)$$

- We consider minimal model beyond the SM i.e. non-SUSY model, because there are no signals for new particles e.g. SUSY particles in the LHC experiment.

ϕ(A4, Z3)→ϕ(3, ω)

	$\overline{l_L} = \begin{pmatrix} \overline{e_L} \\ \overline{\mu_L} \\ \overline{\tau_L} \end{pmatrix}$	e_R	μ_R	τ_R	$\nu_R = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$
A_4	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1''}$	$\mathbf{1'}$	$\mathbf{3}$
Z_3	ω^2	1	1	1	1

§Model

Y. Kawamura, Y. Matsuo, Y.S., and S. Takahashi, arXiv:20XX.XXXX

- Potential of flavon ϕ

$$V_\phi = -M_\phi^2 \phi^* \phi + \mu_\phi(\phi^3 + \mathbf{h.c.}) + \lambda_\phi(\phi^*\phi)^2$$

- Alignment of flavon ϕ

$$\langle \phi \rangle \sim (v_a, v_b, v_b)$$

ϕ(A4, Z3)→ϕ(3, ω)

	$\overline{l_L} = \begin{pmatrix} \overline{e_L} \\ \overline{\mu_L} \\ \overline{\tau_L} \end{pmatrix}$	e_R	μ_R	τ_R	$\nu_R = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$
A_4	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1''}$	$\mathbf{1'}$	$\mathbf{3}$
Z_3	ω^2	1	1	1	1

§Model

Y. Kawamura, Y. Matsuo, Y.S., and S. Takahashi, arXiv:20XX.XXXX

- Lagrangian for lepton sector：H (1,1)

$$\mathscr{L}_e^Y = -\frac{y_e}{\Lambda}(\bar{l}_L\phi)_1 H e_R - \frac{y_\mu}{\Lambda}(\bar{l}_L\phi)_1 H \mu_R - \frac{y_\tau}{\Lambda}(\bar{l}_L\phi)_1 H \tau_R$$

$$\mathscr{L}_\nu^D = -\frac{y_{\nu S}}{\Lambda}(\bar{l}_L\nu_R)_{3S}\tilde{H}\phi - \frac{y_{\nu A}}{\Lambda}(\bar{l}_L\nu_R)_{3A}\tilde{H}\phi$$

$$\mathscr{L}_\nu^M = -M_R(\overline{(\nu_R)^c}\nu_R)_1$$

ϕ(A4, Z3)→ϕ(3, ω)

	$\overline{l_L} = \begin{pmatrix} \overline{e_L} \\ \overline{\mu_L} \\ \overline{\tau_L} \end{pmatrix}$	e_R	μ_R	τ_R	$\nu_R = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$
A_4	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}''$	$\mathbf{1}'$	$\mathbf{3}$
Z_3	ω^2	1	1	1	1

§Model

Y. Kawamura, Y. Matsuo, Y.S., and S. Takahashi

$$\mathcal{L}_e^Y = -\frac{y_e}{\Lambda}(\bar{l}_L\phi)_1 H e_R - \frac{y_\mu}{\Lambda}(\bar{l}_L\phi)_1 H \mu_R - \frac{y_\tau}{\Lambda}(\bar{l}_L\phi)_1 H \tau_R$$

$$\mathcal{L}_\nu^D = -\frac{y_{\nu S}}{\Lambda}(\bar{l}_L \nu_R)_{3S}\tilde{H}\phi - \frac{y_{\nu A}}{\Lambda}(\bar{l}_L \nu_R)_{3A}\tilde{H}\phi$$

$$\mathcal{L}_\nu^M = -M_R(\overline{(\nu_R)^c}\nu_R)_1$$

- Mass matrix for lepton sector: $\langle\phi\rangle \sim (v_a, v_b, v_b)$

$$M_l = \frac{v_H}{\Lambda}\begin{pmatrix} y_e v_a & y_\mu v_b & y_\tau v_b \\ y_e v_b & y_\mu v_a & y_\tau v_b \\ y_e v_b & y_\mu v_b & y_\tau v_a \end{pmatrix},$$

$$M_D = \frac{v_H}{\Lambda}\begin{pmatrix} \frac{2}{3}y_{\nu S}v_a & -\frac{1}{3}y_{\nu S}v_b + \frac{1}{2}y_{\nu A}v_b & -\frac{1}{3}y_{\nu S}v_b - \frac{1}{2}y_{\nu A}v_b \\ -\frac{1}{3}y_{\nu S}v_b - \frac{1}{2}y_{\nu A}v_b & \frac{2}{3}y_{\nu S}v_b & -\frac{1}{3}y_{\nu S}v_a + \frac{1}{2}y_{\nu A}v_a \\ -\frac{1}{3}y_{\nu S}v_b + \frac{1}{2}y_{\nu A}v_b & -\frac{1}{3}y_{\nu S}v_a - \frac{1}{2}y_{\nu A}v_a & \frac{2}{3}y_{\nu S}v_b \end{pmatrix},$$

$$M_N = M_R\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

ϕ(A4, Z3)→ϕ(3, ω)

	$\overline{l_L} = \begin{pmatrix}\overline{e_L}\\ \overline{\mu_L}\\ \overline{\tau_L}\end{pmatrix}$	e_R	μ_R	τ_R	$\nu_R = \begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}$
A_4	**3**	**1**	$\mathbf{1''}$	$\mathbf{1'}$	**3**
Z_3	ω^2	1	1	1	1

§Model

Y. Kawamura, Y. Matsuo, Y.S., and S. Takahashi

$\langle\phi\rangle \sim (v_a, v_b, v_b)$

$$M_l = \frac{v_H}{\Lambda}\begin{pmatrix} y_e v_a & y_\mu v_b & y_\tau v_b \\ y_e v_b & y_\mu v_a & y_\tau v_b \\ y_e v_b & y_\mu v_b & y_\tau v_a \end{pmatrix},$$

$$M_D = \frac{v_H}{\Lambda}\begin{pmatrix} \frac{2}{3}y_{\nu S}v_a & -\frac{1}{3}y_{\nu S}v_b + \frac{1}{2}y_{\nu A}v_b & -\frac{1}{3}y_{\nu S}v_b - \frac{1}{2}y_{\nu A}v_b \\ -\frac{1}{3}y_{\nu S}v_b - \frac{1}{2}y_{\nu A}v_b & \frac{2}{3}y_{\nu S}v_b & -\frac{1}{3}y_{\nu S}v_a + \frac{1}{2}y_{\nu A}v_a \\ -\frac{1}{3}y_{\nu S}v_b + \frac{1}{2}y_{\nu A}v_b & -\frac{1}{3}y_{\nu S}v_a - \frac{1}{2}y_{\nu A}v_a & \frac{2}{3}y_{\nu S}v_b \end{pmatrix},$$

$$M_N = M_R\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

- Seesaw mechanism: $m^\nu_{eff} = -M_D M_N^{-1} M_D^T$

P. Minkowski (1977); T. Yanagida (1979); M. Gell-Mann, P. Ramond and R. Slansky (1979); R. N. Mohapatra and G. Sen- janovic,(1980)

$$m^\nu_{\rm TBM} = V^T_{\rm TBM} m^\nu_{eff} V_{\rm TBM}$$

$$= -\frac{v_H^2}{M_R\Lambda^2}\begin{pmatrix} \mathcal{V}_{11} & \frac{(v_a-v_b)(v_a+2v_b)(4y_{\nu S}^2+3y_{\nu A}^2)}{18\sqrt{2}} & \frac{y_{\nu S}y_{\nu A}v_b(2v_a+v_b)}{\sqrt{3}} \\ \frac{(v_a-v_b)(v_a+2v_b)(4y_{\nu S}^2+3y_{\nu A}^2)}{18\sqrt{2}} & \mathcal{V}_{22} & 0 \\ \frac{y_{\nu S}y_{\nu A}v_b(2v_a+v_b)}{\sqrt{3}} & 0 & \mathcal{V}_{33} \end{pmatrix}$$

$$V_{\rm TBM} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

- Free parameters: $\frac{v_H^2}{M_R\Lambda^2}$ v_a v_b $y_{\nu S}$ $y_{\nu A}$

$$U_\nu^\dagger \left(m^\nu_{\rm TBM} m^{\nu\dagger}_{\rm TBM}\right) U_\nu = \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix}$$

ϕ(A4, Z3)→ϕ(3, ω)

	$\overline{l_L} = \begin{pmatrix} \overline{e_L} \\ \overline{\mu_L} \\ \overline{\tau_L} \end{pmatrix}$	e_R	μ_R	τ_R	$\nu_R = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$
A_4	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1''}$	$\mathbf{1'}$	$\mathbf{3}$
Z_3	ω^2	1	1	1	1

§Model

Y. Kawamura, Y. Matsuo, Y.S., and S. Takahashi

$\langle\phi\rangle \sim (v_a, v_b, v_b)$

$$M_l = \frac{v_H}{\Lambda}\begin{pmatrix} y_e v_a & y_\mu v_b & y_\tau v_b \\ y_e v_b & y_\mu v_a & y_\tau v_b \\ y_e v_b & y_\mu v_b & y_\tau v_a \end{pmatrix} ,$$

$$M_D = \frac{v_H}{\Lambda}\begin{pmatrix} \frac{2}{3}y_\nu S v_a & -\frac{1}{3}y_\nu S v_b + \frac{1}{2}y_\nu A v_b & -\frac{1}{3}y_\nu S v_b - \frac{1}{2}y_\nu A v_b \\ -\frac{1}{3}y_\nu S v_b - \frac{1}{2}y_\nu A v_b & \frac{2}{3}y_\nu S v_b & -\frac{1}{3}y_\nu S v_a + \frac{1}{2}y_\nu A v_a \\ -\frac{1}{3}y_\nu S v_b + \frac{1}{2}y_\nu A v_b & -\frac{1}{3}y_\nu S v_a - \frac{1}{2}y_\nu A v_a & \frac{2}{3}y_\nu S v_b \end{pmatrix} ,$$

$$M_N = M_R\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} ,$$

- Charged lepton: $H_l = M_l M_l^\dagger$ $h_i \equiv y_i^2$ $i = e, \mu, \tau$

$$H_l = \frac{v_H^2}{\Lambda^2}\begin{pmatrix} X & b & c \\ b & Y & d \\ c & d & Z \end{pmatrix}$$

$$X = v_a^2 h_e + v_b^2(h_\mu + h_\tau) , \quad Y = v_a^2 h_\mu + v_b^2(h_t \quad + h_e) ,$$
$$Z = v_a^2 h_\tau + v_b^2(h_e + h_\mu) , \quad b = v_a v_b(h_e + h_\mu) + v_b^2 h_\tau ,$$
$$c = v_a v_b(h_\tau + h_e) + v_b^2 h_\mu , \quad d = v_a v_b(h_\mu + h_\tau) + v_b^2 h_e .$$

$$U_l^\dagger H_l U_l = \begin{pmatrix} m_e^2 & 0 & 0 \\ 0 & m_\mu^2 & 0 \\ 0 & 0 & m_\tau^2 \end{pmatrix}$$

- Free parameters: h_i v_a v_b

ϕ(A4, Z3)→ϕ(3, ω)

	$\overline{l_L} = \begin{pmatrix} \overline{e_L} \\ \overline{\mu_L} \\ \overline{\tau_L} \end{pmatrix}$	e_R	μ_R	τ_R	$\nu_R = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$
A_4	**3**	**1**	**1″**	**1′**	**3**
Z_3	ω^2	1	1	1	1

§Model

Y. Kawamura, Y. Matsuo, Y.S., and S. Takahashi

- Lepton mixing matrix: $V_{\rm TBM} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$

$$U_\nu^\dagger \left(m^\nu_{\rm TBM} m^{\nu\dagger}_{\rm TBM}\right) U_\nu = \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix}$$

$$U_l^\dagger H_l U_l = \begin{pmatrix} m_e^2 & 0 & 0 \\ 0 & m_\mu^2 & 0 \\ 0 & 0 & m_\tau^2 \end{pmatrix}$$

$$U_{\rm PMNS} \equiv U_l^\dagger V_{\rm TBM} U_\nu$$

$y_{\nu S}$ → Real : y_S^D

$y_{\nu A}$ → Complex: y_A^D, ϕ_A^D

- Free parameters: $\frac{v_H^2}{M_R \Lambda^2}$ v_a v_b $y_{\nu S}$ $y_{\nu A}$ h_i

ϕ(A4, Z3)→ϕ(3, ω)

	$\overline{l_L} = \begin{pmatrix} \overline{e_L} \\ \overline{\mu_L} \\ \overline{\tau_L} \end{pmatrix}$	e_R	μ_R	τ_R	$\nu_R = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$
A_4	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1''}$	$\mathbf{1'}$	$\mathbf{3}$
Z_3	ω^2	1	1	1	1

§Global fit of the neutrino oscillation

NuFIT 4.1 (2019)

without SK atmospheric data

	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 6.2$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.310^{+0.013}_{-0.012}$	$0.275 \to 0.350$	$0.310^{+0.013}_{-0.012}$	$0.275 \to 0.350$
$\theta_{12}/^\circ$	$33.82^{+0.78}_{-0.76}$	$31.61 \to 36.27$	$33.82^{+0.78}_{-0.76}$	$31.61 \to 36.27$
$\sin^2\theta_{23}$	$0.558^{+0.020}_{-0.033}$	$0.427 \to 0.609$	$0.563^{+0.019}_{-0.026}$	$0.430 \to 0.612$
$\theta_{23}/^\circ$	$48.3^{+1.1}_{-1.9}$	$40.8 \to 51.3$	$48.6^{+1.1}_{-1.5}$	$41.0 \to 51.5$
$\sin^2\theta_{13}$	$0.02241^{+0.00066}_{-0.00065}$	$0.02046 \to 0.02440$	$0.02261^{+0.00067}_{-0.00064}$	$0.02066 \to 0.02461$
$\theta_{13}/^\circ$	$8.61^{+0.13}_{-0.13}$	$8.22 \to 8.99$	$8.65^{+0.13}_{-0.12}$	$8.26 \to 9.02$
$\delta_{CP}/^\circ$	222^{+38}_{-28}	$141 \to 370$	285^{+24}_{-26}	$205 \to 354$
$\frac{\Delta m^2_{21}}{10^{-5}\ \text{eV}^2}$	$7.39^{+0.21}_{-0.20}$	$6.79 \to 8.01$	$7.39^{+0.21}_{-0.20}$	$6.79 \to 8.01$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \text{eV}^2}$	$+2.523^{+0.032}_{-0.030}$	$+2.432 \to +2.618$	$-2.509^{+0.032}_{-0.030}$	$-2.603 \to -2.416$

with SK atmospheric data

	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 10.4$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.310^{+0.013}_{-0.012}$	$0.275 \to 0.350$	$0.310^{+0.013}_{-0.012}$	$0.275 \to 0.350$
$\theta_{12}/^\circ$	$33.82^{+0.78}_{-0.76}$	$31.61 \to 36.27$	$33.82^{+0.78}_{-0.75}$	$31.61 \to 36.27$
$\sin^2\theta_{23}$	$0.563^{+0.018}_{-0.024}$	$0.433 \to 0.609$	$0.565^{+0.017}_{-0.022}$	$0.436 \to 0.610$
$\theta_{23}/^\circ$	$48.6^{+1.0}_{-1.4}$	$41.1 \to 51.3$	$48.8^{+1.0}_{-1.2}$	$41.4 \to 51.3$
$\sin^2\theta_{13}$	$0.02237^{+0.00066}_{-0.00065}$	$0.02044 \to 0.02435$	$0.02259^{+0.00065}_{-0.00065}$	$0.02064 \to 0.02457$
$\theta_{13}/^\circ$	$8.60^{+0.13}_{-0.13}$	$8.22 \to 8.98$	$8.64^{+0.12}_{-0.13}$	$8.26 \to 9.02$
$\delta_{CP}/^\circ$	221^{+39}_{-28}	$144 \to 357$	282^{+23}_{-25}	$205 \to 348$
$\frac{\Delta m^2_{21}}{10^{-5}\ \text{eV}^2}$	$7.39^{+0.21}_{-0.20}$	$6.79 \to 8.01$	$7.39^{+0.21}_{-0.20}$	$6.79 \to 8.01$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \text{eV}^2}$	$+2.528^{+0.029}_{-0.031}$	$+2.436 \to +2.618$	$-2.510^{+0.030}_{-0.031}$	$-2.601 \to -2.419$

NuFIT 5.0 (2020)

without SK atmospheric data

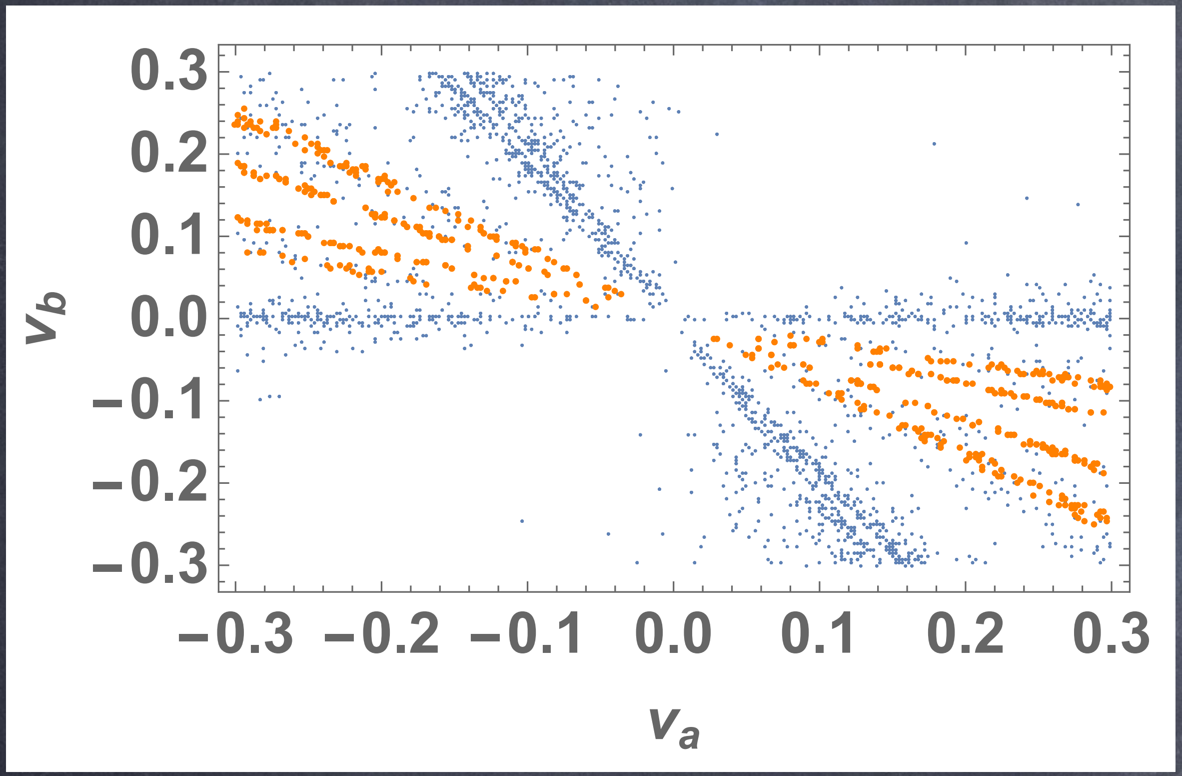
	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 2.7$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$
$\theta_{12}/^\circ$	$33.44^{+0.78}_{-0.75}$	$31.27 \to 35.86$	$33.45^{+0.78}_{-0.75}$	$31.27 \to 35.87$
$\sin^2\theta_{23}$	$0.570^{+0.018}_{-0.024}$	$0.407 \to 0.618$	$0.575^{+0.017}_{-0.021}$	$0.411 \to 0.621$
$\theta_{23}/^\circ$	$49.0^{+1.1}_{-1.4}$	$39.6 \to 51.8$	$49.3^{+1.0}_{-1.2}$	$39.9 \to 52.0$
$\sin^2\theta_{13}$	$0.02221^{+0.00068}_{-0.00062}$	$0.02034 \to 0.02430$	$0.02240^{+0.00062}_{-0.00062}$	$0.02053 \to 0.02436$
$\theta_{13}/^\circ$	$8.57^{+0.13}_{-0.12}$	$8.20 \to 8.97$	$8.61^{+0.12}_{-0.12}$	$8.24 \to 8.98$
$\delta_{CP}/^\circ$	195^{+51}_{-25}	$107 \to 403$	286^{+27}_{-32}	$192 \to 360$
$\frac{\Delta m^2_{21}}{10^{-5}\ \text{eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \text{eV}^2}$	$+2.514^{+0.028}_{-0.027}$	$+2.431 \to +2.598$	$-2.497^{+0.028}_{-0.028}$	$-2.583 \to -2.412$

with SK atmospheric data

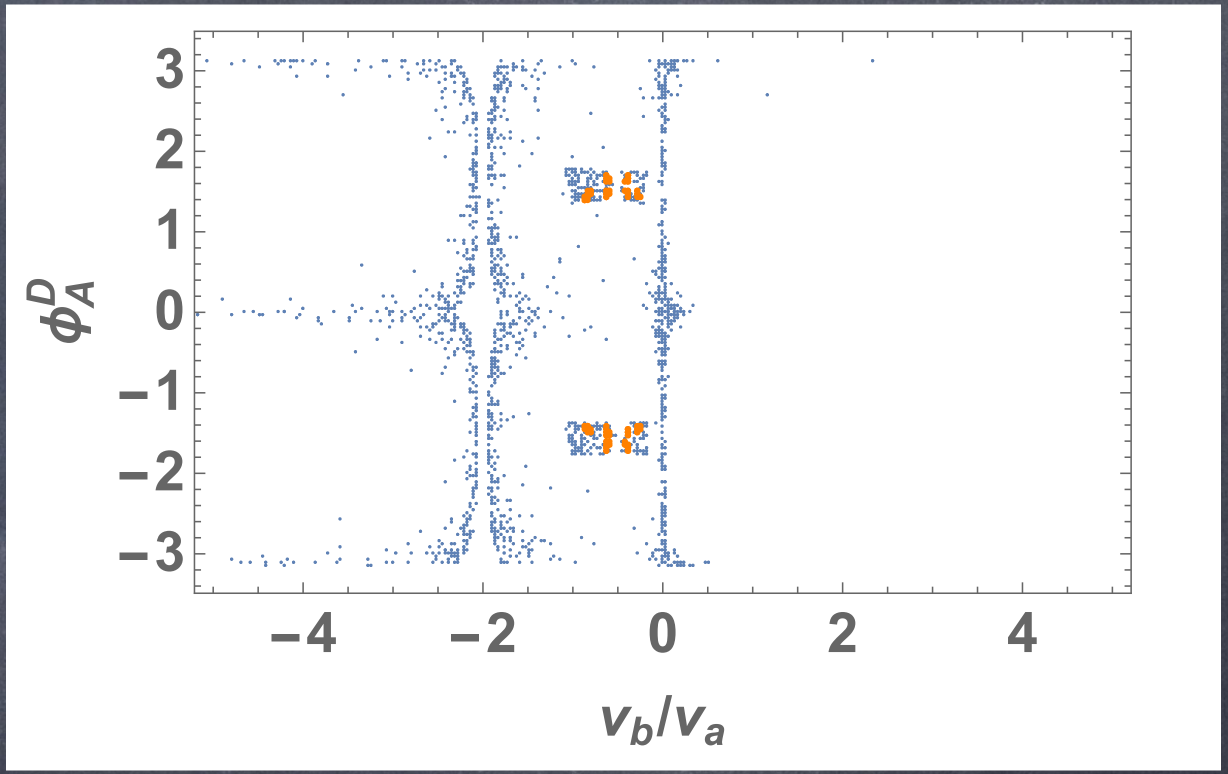
	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 7.1$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.304^{+0.012}_{-0.012}$	$0.269 \to 0.343$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$
$\theta_{12}/^\circ$	$33.44^{+0.77}_{-0.74}$	$31.27 \to 35.86$	$33.45^{+0.78}_{-0.75}$	$31.27 \to 35.87$
$\sin^2\theta_{23}$	$0.573^{+0.016}_{-0.020}$	$0.415 \to 0.616$	$0.575^{+0.016}_{-0.019}$	$0.419 \to 0.617$
$\theta_{23}/^\circ$	$49.2^{+0.9}_{-1.2}$	$40.1 \to 51.7$	$49.3^{+0.9}_{-1.1}$	$40.3 \to 51.8$
$\sin^2\theta_{13}$	$0.02219^{+0.00062}_{-0.00063}$	$0.02032 \to 0.02410$	$0.02238^{+0.00063}_{-0.00062}$	$0.02052 \to 0.02428$
$\theta_{13}/^\circ$	$8.57^{+0.12}_{-0.12}$	$8.20 \to 8.93$	$8.60^{+0.12}_{-0.12}$	$8.24 \to 8.96$
$\delta_{CP}/^\circ$	197^{+27}_{-24}	$120 \to 369$	282^{+26}_{-30}	$193 \to 352$
$\frac{\Delta m^2_{21}}{10^{-5}\ \text{eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \text{eV}^2}$	$+2.517^{+0.026}_{-0.028}$	$+2.435 \to +2.598$	$-2.498^{+0.028}_{-0.028}$	$-2.581 \to -2.414$

§Numerical analyses (preliminary)

- The relation among VEVs



- The relation between ratio of the VEVs and phase

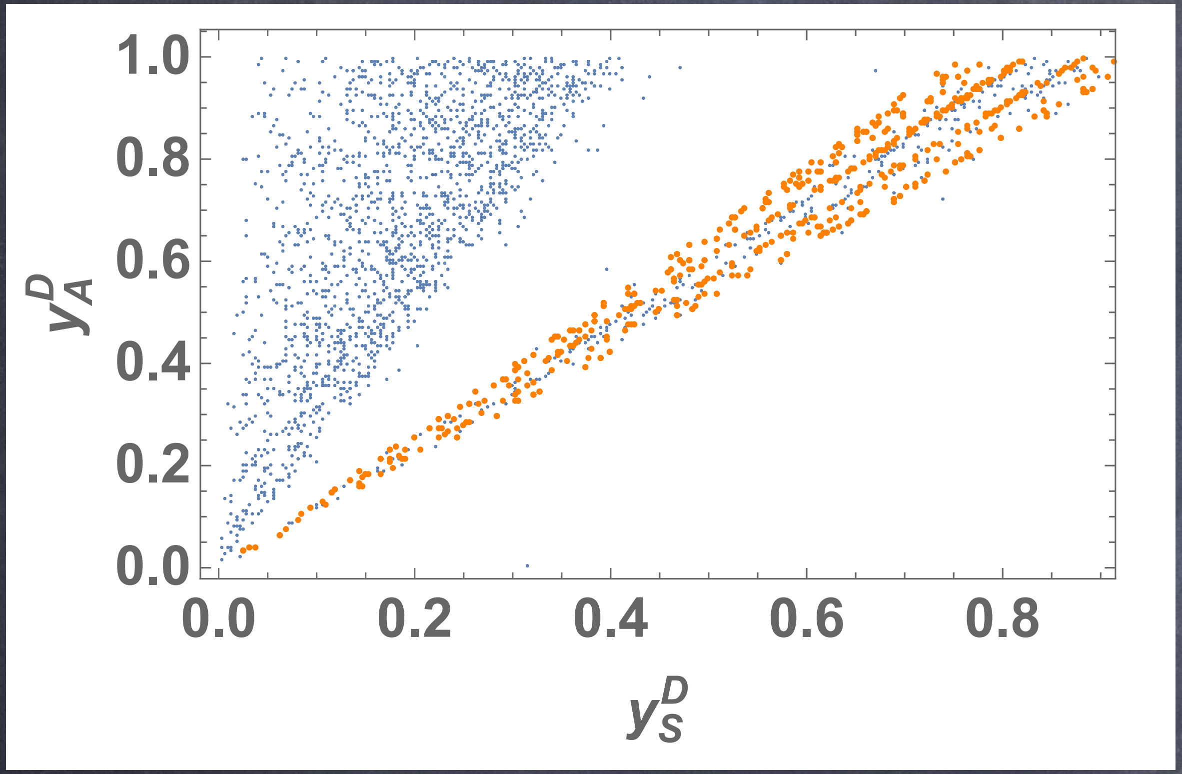


Orange:within 3σ range, Blue: only neutrino mass squared differences within 3σ range except for lepton mixing angles

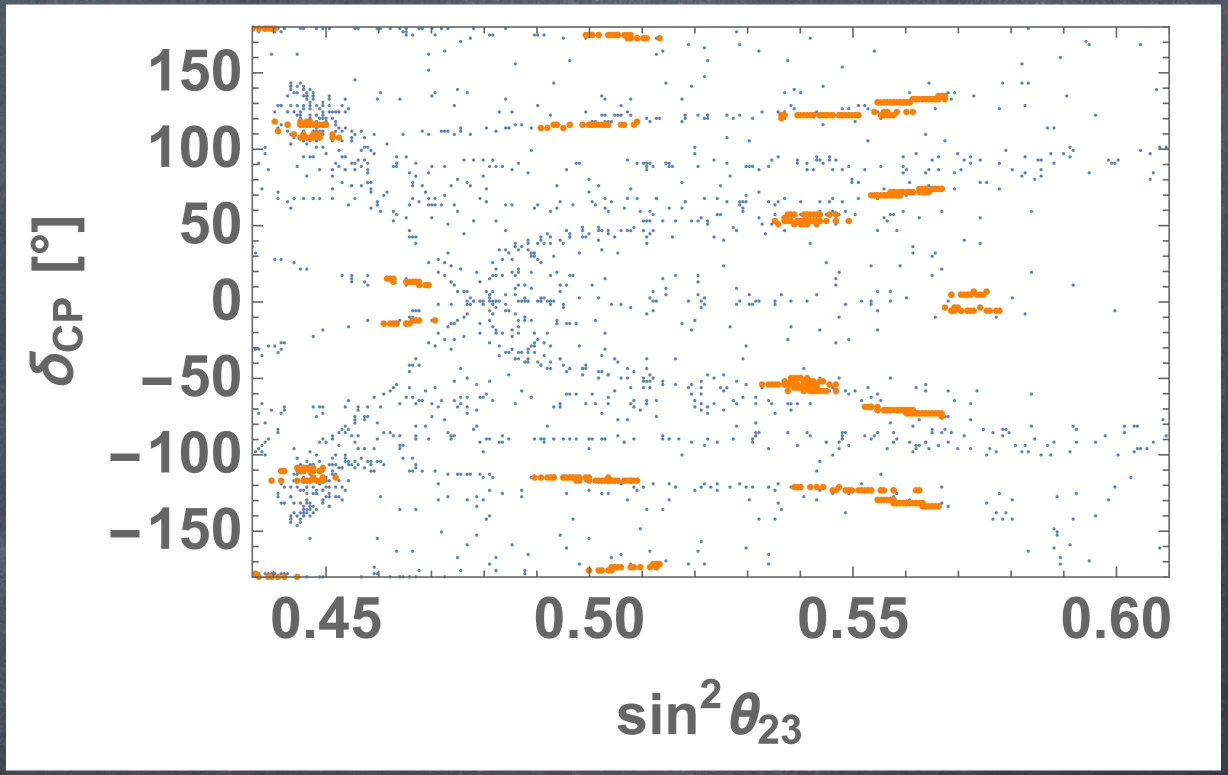
VEVs are opposite sign for each other and the phase is limited.

§Numerical analyses (preliminary)

- The elation among Yukawa couplings



- The relation between θ 23 and δ CP

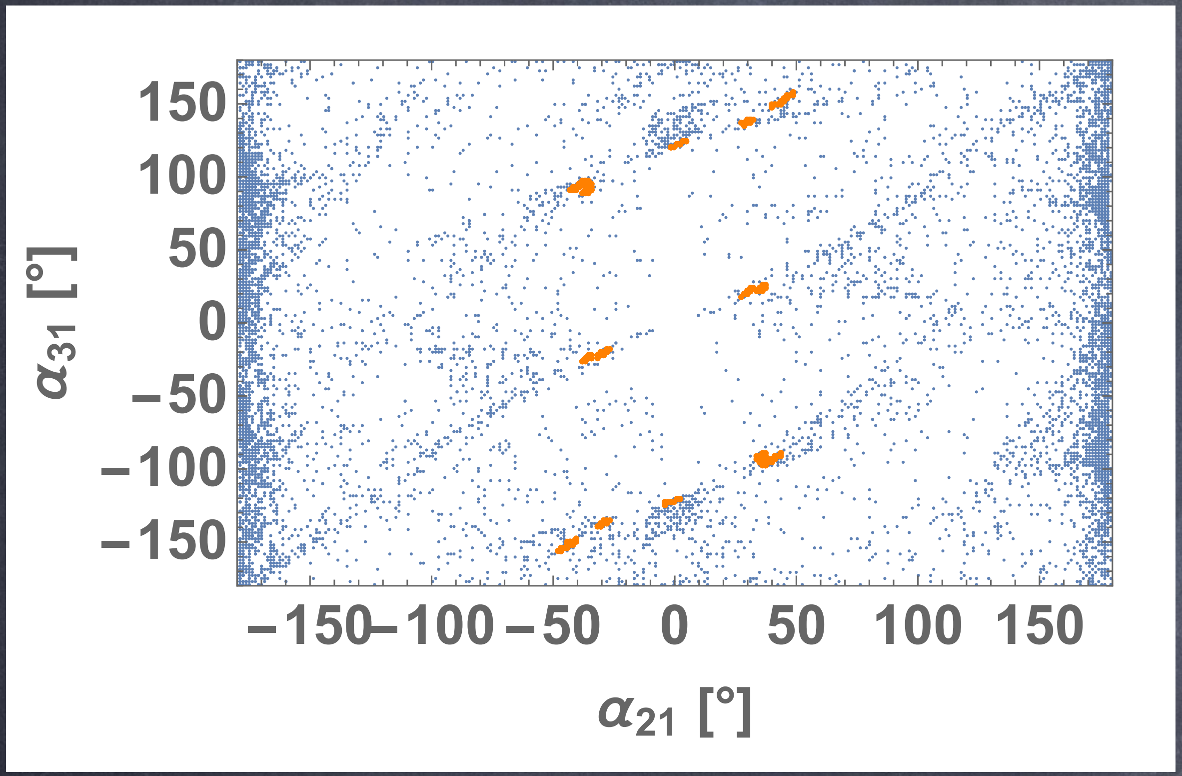


Orange:within 3σ range, Blue: only neutrino mass squared differences within 3σ range except for lepton mixing angles

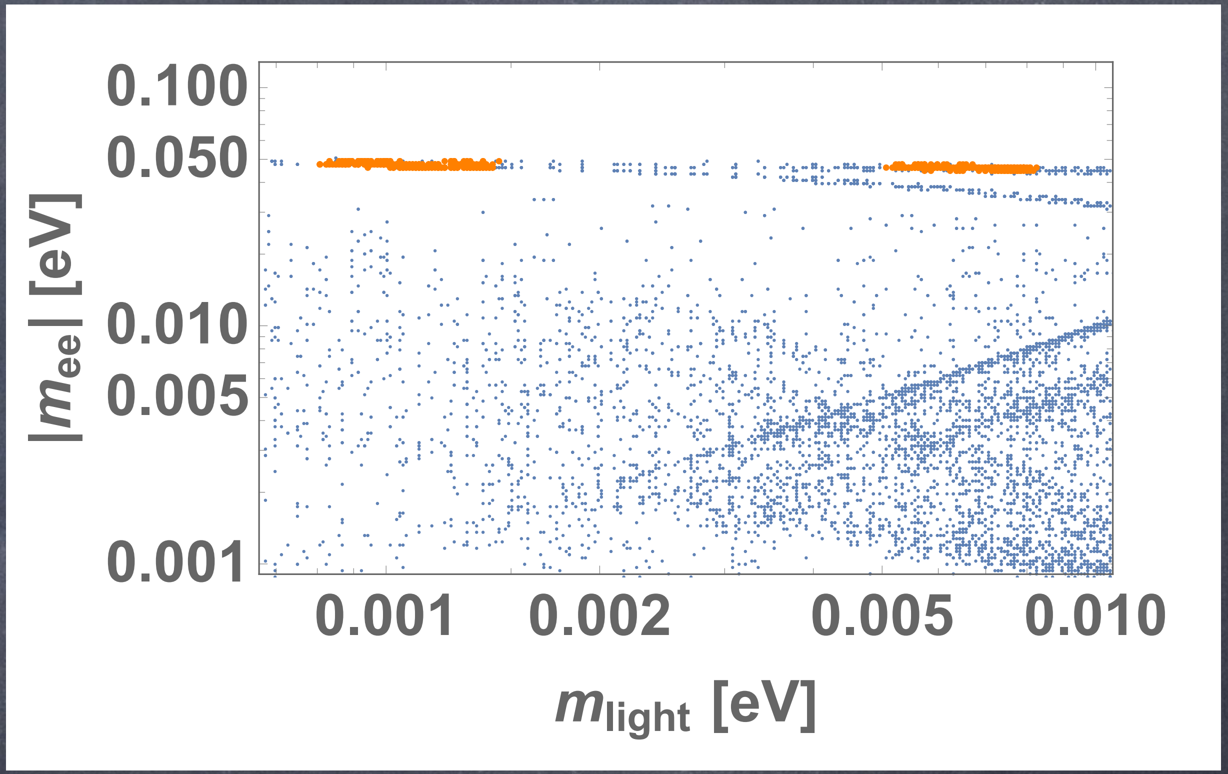
Symmetric Yukawa coupling is proportional to anti-symmetric one.
There is relation between θ 23 and δ CP.

§Numerical analyses (preliminary)

- The relation between $\alpha 21$ and $\alpha 31$



- The relation between mlight and |mee|



Orange:within 3σ range, Blue: only neutrino mass squared differences within 3σ range except for lepton mixing angles

Majorana phases are limited
|mee| is 0.050 eV

4. Summary

- We discussed CP violation and quark mixing matrix (CKM matrix)
- We discussed neutrino oscillation and lepton mixing matrix (PMNS matrix)
- We presented successful model by using non-Abelian discrete symmetry A_4 .
- We will apply non-Abelian discrete symmetry to the quark sector and consider origin of the flavor symmetry.