Modular A_4 flavor model

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Plan of my talk

1. Introduction

2. Modular symmetry

3. Modular A_4 flavor model

4. Summary

1. Introduction

- Standard model for particle physics

Particle	First	Second	Third	Mixing matrix
Quark	$\begin{pmatrix} u \\ d \end{pmatrix}_L$ u^c_R d^c_R	$\begin{pmatrix} c \\ s \end{pmatrix}_L$ c^c_R s^c_R	$\begin{pmatrix} t \\ b \end{pmatrix}_L$ t^c_R b^c_R	CKM matrix (Cabibbo-Kobayashi-Maskawa)
Lepton	$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$ e^c_R	$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$ μ^c_R	$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$ τ^c_R	PMNS matrix (Pontecorvo-Maki-Nakagawa-Sakata)

- Generation Mysteries

- Masses of elementary particles are different each generation.
- Lepton flavor mixing is quite different from quark one.

For previous talk

- As we shown, we use non-Abelian discrete symmetry as flavor symmetry to naturally explain mass hierarchy and flavor mixing for elementary particles.

- We introduce non-Abelian discrete symmetry and the scalar fields (so-called "flavons").

- We derive large mixing for lepton sector by using VEV of flavon and its alignment.

- Those vacuum expectation values determine the flavor structure of quarks and leptons. However, the breaking sector of flavor symmetry typically produces many unknown parameters.

- Superstring theory with certain compactifications can lead to non-Abelian discrete flavor symmetries. For example, heterotic orbifold models lead to D_4, $\Delta(54)$, etc.

  T. Kobayashi, H. P. Nilles, F. Ploger, S. Raby and M. Ratz, Nucl. Phys. B 768, 135 (2007)

- Similar flavor symmetries are also derived in type II magnetized and intersecting D-brane models.

  H. Abe, K. S. Choi, T. Kobayashi and H. Ohki, Nucl. Phys. B 820, 317 (2009);
  M. Berasaluce-Gonzalez, P. G. Camara, F. Marchesano, D. Regalado and A. M. Uranga, JHEP 1209, 059 (2012)

- It is interesting that the modular group includes S_3, A_4, S_4, and A_5 as its finite subgroups, $\Gamma(N)$. However, there is a difference between the modular symmetry and the usual flavor symmetry. Yukawa couplings are written as modular forms, functions of the modulus τ, and transform non-trivially under the modular symmetry as well as fields. On the other hand, Yukawa couplings are invariants in the usual flavor symmetries. In this aspect, an attractive ansatz was proposed by taking $\Gamma(3) \simeq A_4$.

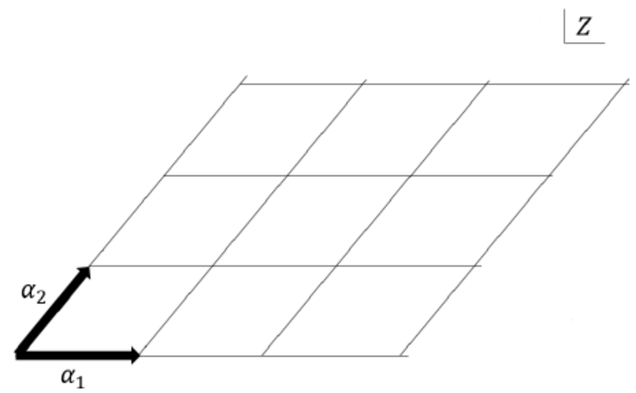
  F. Feruglio, arXiv:1706.08749

2. Modular symmetry

The torus compactification is the simplest compactification. For example, the two-dimensional torus T^2 can be constructed as division of $\mathbb{R}^2$ by a two-dimensional lattice Λ, i.e. $T^2 = \mathbb{R}^2/\Lambda$. Here, we use the complex coordinate on $\mathbb{R}^2$ with the lattice spanned by two lattice vectors, $\alpha_1 = 2\pi R$ and $\alpha_2 = 2\pi R\tau$; where R is real and τ is a complex modulus parameter. However, there is some ambiguity in choice of the basis vectors. The same lattice can be spanned by the following basis vectors,

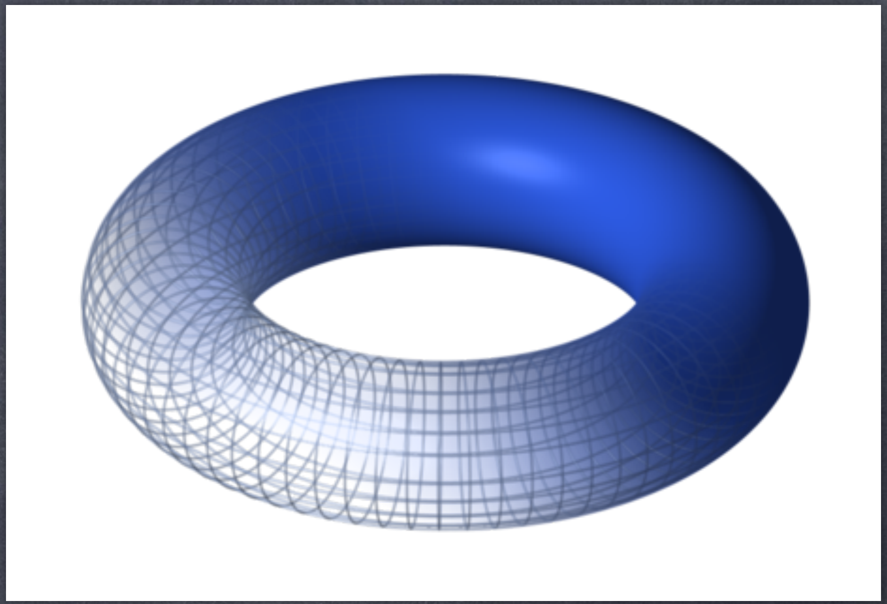
$$\begin{pmatrix} \alpha_2' \\ \alpha_1' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \alpha_2 \\ \alpha_1 \end{pmatrix}$$

where a, b, c, d are integer with satisfying $ad - bc = 1$.



A lattice spaned by basis vectors $\alpha_1 = 2\pi R$ and $\alpha_2 = 2\pi R\tau$ in a 2D complex plane. These are parametrized by $R \in \mathbb{R}$ and $\tau \in \mathbb{C}$.

K. Takagi's Ph.D. thesis (2020)



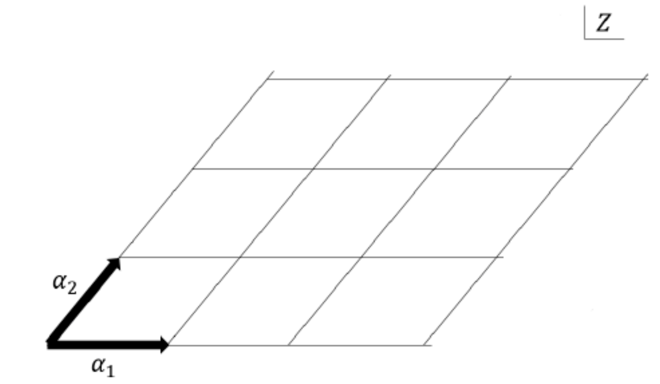
Wikipedia

That is the $SL(2, \mathbb{Z})$ transformation.

2. Modular symmetry

$$\begin{pmatrix} \alpha'_2 \\ \alpha'_1 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \alpha_2 \\ \alpha_1 \end{pmatrix}$$

where a, b, c, d are integer with satisfying $ad - bc = 1$.



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K. Takagi's Ph.D. thesis (2020)

Under the above transformation, the modulus parameter transforms as

$$\tau \longrightarrow \tau' = \frac{a\tau + b}{c\tau + d} ,$$

and this modular transformation is generated by S and T,

$$S : \tau \longrightarrow -\frac{1}{\tau} ,$$
$$T : \tau \longrightarrow \tau + 1 .$$

They satisfy the following algebraic relations,

$$S^2 = \mathbb{I} , \qquad (ST)^3 = \mathbb{I} .$$

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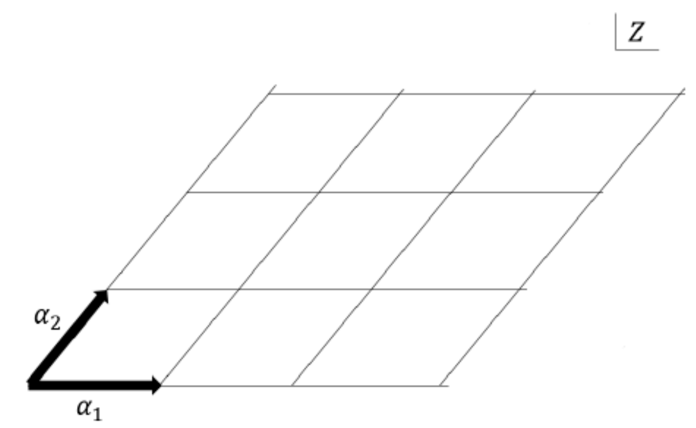

A lattice spaned by basis vectors $\alpha_1 = 2\pi R$ and $\alpha_2 = 2\pi R\tau$ in a 2D complex plane. These are parametrized by $R \in \mathbb{R}$ and $\tau \in \mathbb{C}$.

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If we impose $T^N = \mathbb{I}$ furthermore, we obtain finite subgroups $\Gamma(N)$. $\Gamma(N)$ with $N = 2, 3, 4, 5$ are isomorphic to S_3, A_4, S_4 and A_5, respectively . Indeed, $\Gamma(N)$ is a quotient of the modular group by the so-called congruence subgroup $\bar{\Gamma}(N)$. Holomorphic functions which transform as

$$f(\tau) \to (c\tau + d)^k f(\tau) ,$$

under the modular transformation are called modular forms of weight k.

Superstring theory on the torus T^2 or orbifold T^2/Z_N has the modular symmetry. Its l energy effective field theory is described in terms of supergravity theory, and string-deri supergravity theory has also the modular symmetry. Under the modular transformation Eq.

2. Modular symmetry

Superstring theory on the torus T^2 or orbifold T^2/Z_N has the modular symmetry. Its low-energy effective field theory is described in terms of supergravity theory, and string-derived supergravity theory has also the modular symmetry.

$$\tau \longrightarrow \tau' = \frac{a\tau + b}{c\tau + d}$$

Under the modular transformation chiral superfields $\phi^{(I)}$ transform as

$$\phi^{(I)} \to (c\tau + d)^{-k_I} \rho^{(I)}(\gamma) \phi^{(I)},$$

where $-k_I$ is the so-called modular weight and $\rho^{(I)}(\gamma)$ denotes a unitary representation matrix of $\gamma \in \Gamma(N)$. The kinetic terms of their scalar components are written by

$$\sum_I \frac{|\partial_\mu \phi^{(I)}|^2}{\langle -i\tau + i\bar{\tau} \rangle^{k_I}} ,$$

which is invariant under the modular transformation.

2. Modular symmetry

The Dedekind eta-function $\eta(\tau)$ is one of famous modular forms, which is written by

$$\eta(\tau) = q^{1/24} \prod_{n=1}^{\infty} (1 - q^n) \ ,$$

where $q = e^{2\pi i\tau}$ and $\eta(\tau)^{24}$ is a modular form of weight 12. By use of $\eta(\tau)$ and its derivative, A_4 triplet modular forms (Y_1, Y_2, Y_3) of modular weight 2 are written by

$$\begin{aligned}
Y_1(\tau) &= \frac{i}{2\pi}\left(\frac{\eta'(\tau/3)}{\eta(\tau/3)} + \frac{\eta'((\tau+1)/3)}{\eta((\tau+1)/3)} + \frac{\eta'((\tau+2)/3)}{\eta((\tau+2)/3)} - \frac{27\eta'(3\tau)}{\eta(3\tau)}\right), \\
Y_2(\tau) &= \frac{-i}{\pi}\left(\frac{\eta'(\tau/3)}{\eta(\tau/3)} + \omega^2\frac{\eta'((\tau+1)/3)}{\eta((\tau+1)/3)} + \omega\frac{\eta'((\tau+2)/3)}{\eta((\tau+2)/3)}\right), \\
Y_3(\tau) &= \frac{-i}{\pi}\left(\frac{\eta'(\tau/3)}{\eta(\tau/3)} + \omega\frac{\eta'((\tau+1)/3)}{\eta((\tau+1)/3)} + \omega^2\frac{\eta'((\tau+2)/3)}{\eta((\tau+2)/3)}\right),
\end{aligned}$$

where $\omega = e^{2\pi i/3}$. The overall coefficient is one choice and cannot be determined essentially.

F. Feruglio, arXiv:1706.08749

$$Y = \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \\ Y_3(\tau) \end{pmatrix} = \begin{pmatrix} 1 + 12q + 36q^2 + 12q^3 + \dots \\ -6q^{1/3}(1 + 7q + 8q^2 + \dots) \\ -18q^{2/3}(1 + 2q + 5q^2 + \dots) \end{pmatrix}, \qquad q = e^{2\pi i\tau}.$$

3. Modular A4 flavor model

	L	e_R, μ_R, τ_R	ν_R	H_u	H_d	Y
$SU(2)$	2	1	1	2	2	1
A_4	3	$1,\ 1'',\ 1'$	3	1	1	3
$-k_I$	$-1\ (1)$	$-1\ (-3)$	-1	0	0	$k=2$

The modular invariant mass terms of the leptons are given as the following superpotentials:

$$
\begin{aligned}
w_e &= \alpha e_R H_d(LY) + \beta \mu_R H_d(LY) + \gamma \tau_R H_d(LY)\ , \\
w_D &= g(\nu_R H_u L Y)_1\ , \\
w_N &= \Lambda(\nu_R \nu_R Y)_1\ ,
\end{aligned}
$$

- Charged lepton mass matrix is written as

$$
M_E = \mathrm{diag}[\alpha, \beta, \gamma] \begin{pmatrix} Y_1 & Y_3 & Y_2 \\ Y_2 & Y_1 & Y_3 \\ Y_3 & Y_2 & Y_1 \end{pmatrix}_{RL} .
$$

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$$w_D = g(\nu_R H_u LY)_1 \ ,$$
$$w_N = \Lambda(\nu_R \nu_R Y)_1 \ ,$$

- Superpotential for the Dirac neutrino is decomposed as

$$\begin{aligned} w_D =& v_u \begin{pmatrix} \nu_{R1} \\ \nu_{R2} \\ \nu_{R3} \end{pmatrix} \otimes \left[g_1 \begin{pmatrix} 2\nu_e Y_1 - \nu_\mu Y_3 - \nu_\tau Y_2 \\ 2\nu_\tau Y_3 - \nu_e Y_2 - \mu Y_1 \\ 2\nu_\mu Y_2 - \nu_\tau Y_1 - \nu_e Y_3 \end{pmatrix} \oplus g_2 \begin{pmatrix} \nu_\mu Y_3 - \nu_\tau Y_2 \\ \nu_e Y_2 - \nu_\mu Y_1 \\ \nu_\tau Y_1 - \nu_e Y_3 \end{pmatrix} \right] \\ =& v_u g_1 \left[\nu_{R1}(2\nu_e Y_1 - \nu_\mu Y_3 - \nu_\tau Y_2) + \nu_{R2}(2\nu_\mu Y_2 - \nu_\tau Y_1 - \nu_e Y_3) + \nu_{R3}(2\nu_\tau Y_3 - \nu_e Y_2 - \nu_\mu Y_1) \right] \\ & + v_u g_2 \left[\nu_{R1}(\nu_\mu Y_3 - \nu_\tau Y_2) + \nu_{R2}(\nu_\tau Y_1 - \nu_e Y_3) + \nu_{R3}(\nu_e Y_2 - \nu_\mu Y_1) \right] . \end{aligned}$$

3. Modular A4 flavor model

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w_D &= g(\nu_R H_u L Y)_1 \ , \\
w_N &= \Lambda(\nu_R \nu_R Y)_1 \ ,
\end{aligned}
$$

- Dirac neutrino mass matrix is written as

$$
M_D = v_u \begin{pmatrix} 2g_1 Y_1 & (-g_1+g_2)Y_3 & (-g_1-g_2)Y_2 \\ (-g_1-g_2)Y_3 & 2g_1 Y_2 & (-g_1+g_2)Y_1 \\ (-g_1+g_2)Y_2 & (-g_1-g_2)Y_1 & 2g_1 Y_3 \end{pmatrix}_{RL} .
$$

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$$w_D = g(\nu_R H_u L Y)_1 ,$$
$$w_N = \Lambda(\nu_R \nu_R Y)_1 ,$$

- Superpotential for the right-handed Majorana neutrino is decomposed as

$$\begin{aligned} w_N =& \Lambda \begin{pmatrix} 2\nu_{R1}\nu_{R1} - \nu_{R2}\nu_{R3} - \nu_{R3}\nu_{R2} \\ 2\nu_{R3}\nu_{R3} - \nu_{R1}\nu_{R2} - \nu_{R2}\nu_{R1} \\ 2\nu_{R2}\nu_{R2} - \nu_{R3}\nu_{R1} - \nu_{R1}\nu_{R3} \end{pmatrix} \otimes \begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \end{pmatrix} \\ =& \Lambda \left[(2\nu_{R1}\nu_{R1} - \nu_{R2}\nu_{R3} - \nu_{R3}\nu_{R2})Y_1 + (2\nu_{R3}\nu_{R3} - \nu_{R1}\nu_{R2} - \nu_{R2}\nu_{R1})Y_3 \right. \\ & \left. + (2\nu_{R2}\nu_{R2} - \nu_{R3}\nu_{R1} - \nu_{R1}\nu_{R3})Y_2 \right] . \end{aligned}$$

3. Modular A_4 flavor model

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$SU(2)$	2	1	1	2	2	1
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$$w_D = g(\nu_R H_u LY)_1 \ ,$$
$$w_N = \Lambda(\nu_R \nu_R Y)_1 \ ,$$

- The right-handed Majorana neutrino mass matrix is written as

$$M_N = \Lambda \begin{pmatrix} 2Y_1 & -Y_3 & -Y_2 \\ -Y_3 & 2Y_2 & -Y_1 \\ -Y_2 & -Y_1 & 2Y_3 \end{pmatrix}_{RR} .$$

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$$w_D = g(\nu_R H_u L Y)_{\mathbf{1}} \ ,$$
$$w_N = \Lambda(\nu_R \nu_R Y)_{\mathbf{1}} \ ,$$

- We obtain the left-handed Majorana neutrino mass matrix by using type I seesaw mechanism, $M_\nu = -M_D^{\mathrm{T}} M_N^{-1} M_D \ .$

§Numerical analyses

The coefficients α/γ and β/γ in the charged lepton mass matrix are given only in terms of τ after inputting the observed values m_e/m_τ and m_μ/m_τ as shown in Appendix B. Then, we have two free parameters, g_1/g_2 and the modulus τ apart from the overall factors in the neutrino sector. Since these are complex, we set

$$\tau = \mathrm{Re}[\tau] + i\ \mathrm{Im}[\tau]\ , \qquad \frac{g_2}{g_1} = g\ e^{i\phi_g}\ .$$

In practice, we restrict our parametric search in $\mathrm{Re}[\tau] \in [-1.5, 1.5]$ and $\mathrm{Im}[\tau] > 0.6$. We also take $\phi_g \in [-\pi, \pi]$. These four parameters are fixed by the observed $\Delta m^2_{\rm sol}/\Delta m^2_{\rm atm}$ and three mixing angles θ_{23}, θ_{12} and θ_{13}.

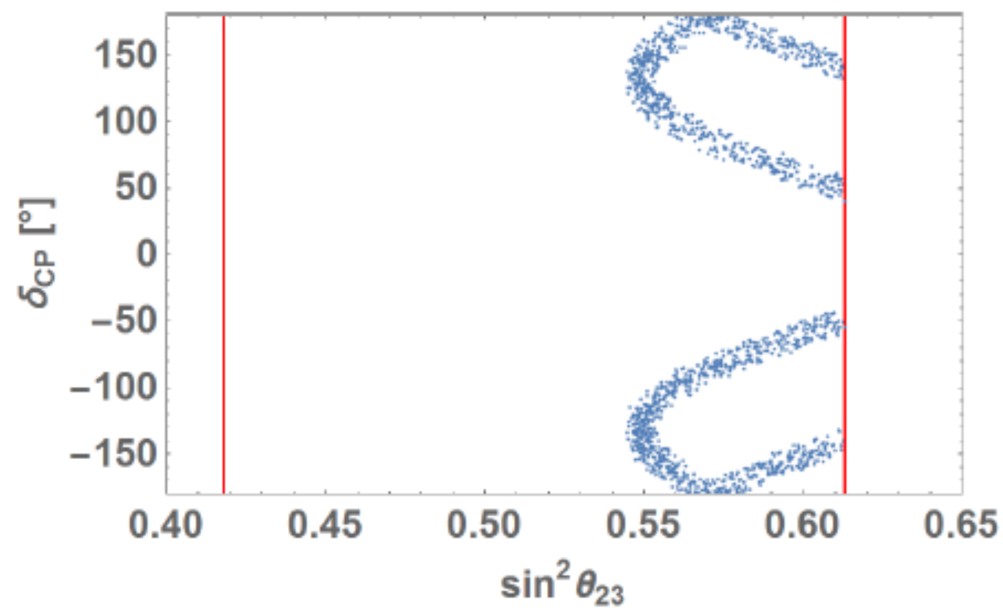


Figure 1: The prediction of δ_{CP} versus $\sin^2\theta_{23}$ for NH in model I(a). The vertical red lines represent the upper and lower bounds of the experimental data with 3 σ.

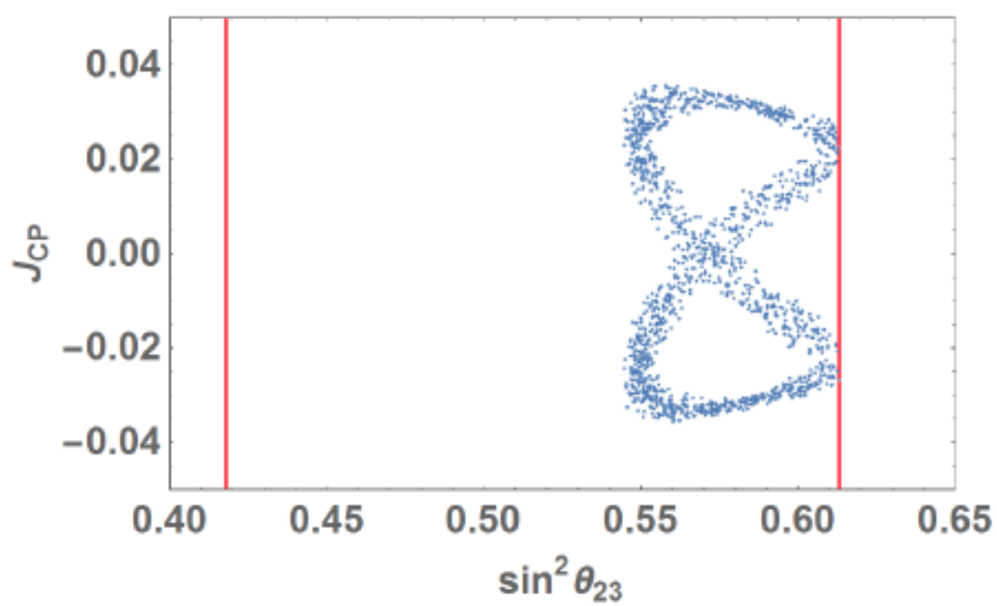


Figure 2: The prediction of J_{CP} versus $\sin^2\theta_{23}$ for NH in model I(a). The vertical red lines represent the upper and lower bounds of the experimental data with 3 σ.

§Global fit of the neutrino oscillation

NuFIT 4.1 (2019)

		Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 6.2$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
without SK atmospheric data	$\sin^2\theta_{12}$	$0.310^{+0.013}_{-0.012}$	$0.275 \to 0.350$	$0.310^{+0.013}_{-0.012}$	$0.275 \to 0.350$
	$\theta_{12}/^\circ$	$33.82^{+0.78}_{-0.76}$	$31.61 \to 36.27$	$33.82^{+0.78}_{-0.76}$	$31.61 \to 36.27$
	$\sin^2\theta_{23}$	$0.558^{+0.020}_{-0.033}$	$0.427 \to 0.609$	$0.563^{+0.019}_{-0.026}$	$0.430 \to 0.612$
	$\theta_{23}/^\circ$	$48.3^{+1.1}_{-1.9}$	$40.8 \to 51.3$	$48.6^{+1.1}_{-1.5}$	$41.0 \to 51.5$
	$\sin^2\theta_{13}$	$0.02241^{+0.00066}_{-0.00065}$	$0.02046 \to 0.02440$	$0.02261^{+0.00067}_{-0.00064}$	$0.02066 \to 0.02461$
	$\theta_{13}/^\circ$	$8.61^{+0.13}_{-0.13}$	$8.22 \to 8.99$	$8.65^{+0.13}_{-0.12}$	$8.26 \to 9.02$
	$\delta_{CP}/^\circ$	222^{+38}_{-28}	$141 \to 370$	285^{+24}_{-26}	$205 \to 354$
	$\frac{\Delta m^2_{21}}{10^{-5}\ \text{eV}^2}$	$7.39^{+0.21}_{-0.20}$	$6.79 \to 8.01$	$7.39^{+0.21}_{-0.20}$	$6.79 \to 8.01$
	$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \text{eV}^2}$	$+2.523^{+0.032}_{-0.030}$	$+2.432 \to +2.618$	$-2.509^{+0.032}_{-0.030}$	$-2.603 \to -2.416$
		Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 10.4$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
with SK atmospheric data	$\sin^2\theta_{12}$	$0.310^{+0.013}_{-0.012}$	$0.275 \to 0.350$	$0.310^{+0.013}_{-0.012}$	$0.275 \to 0.350$
	$\theta_{12}/^\circ$	$33.82^{+0.78}_{-0.76}$	$31.61 \to 36.27$	$33.82^{+0.78}_{-0.75}$	$31.61 \to 36.27$
	$\sin^2\theta_{23}$	$0.563^{+0.018}_{-0.024}$	$0.433 \to 0.609$	$0.565^{+0.017}_{-0.022}$	$0.436 \to 0.610$
	$\theta_{23}/^\circ$	$48.6^{+1.0}_{-1.4}$	$41.1 \to 51.3$	$48.8^{+1.0}_{-1.2}$	$41.4 \to 51.3$
	$\sin^2\theta_{13}$	$0.02237^{+0.00066}_{-0.00065}$	$0.02044 \to 0.02435$	$0.02259^{+0.00065}_{-0.00065}$	$0.02064 \to 0.02457$
	$\theta_{13}/^\circ$	$8.60^{+0.13}_{-0.13}$	$8.22 \to 8.98$	$8.64^{+0.12}_{-0.13}$	$8.26 \to 9.02$
	$\delta_{CP}/^\circ$	221^{+39}_{-28}	$144 \to 357$	282^{+23}_{-25}	$205 \to 348$
	$\frac{\Delta m^2_{21}}{10^{-5}\ \text{eV}^2}$	$7.39^{+0.21}_{-0.20}$	$6.79 \to 8.01$	$7.39^{+0.21}_{-0.20}$	$6.79 \to 8.01$
	$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \text{eV}^2}$	$+2.528^{+0.029}_{-0.031}$	$+2.436 \to +2.618$	$-2.510^{+0.030}_{-0.031}$	$-2.601 \to -2.419$

NuFIT 5.0 (2020)

		Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 2.7$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
without SK atmospheric data	$\sin^2\theta_{12}$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$
	$\theta_{12}/^\circ$	$33.44^{+0.78}_{-0.75}$	$31.27 \to 35.86$	$33.45^{+0.78}_{-0.75}$	$31.27 \to 35.87$
	$\sin^2\theta_{23}$	$0.570^{+0.018}_{-0.024}$	$0.407 \to 0.618$	$0.575^{+0.017}_{-0.021}$	$0.411 \to 0.621$
	$\theta_{23}/^\circ$	$49.0^{+1.1}_{-1.4}$	$39.6 \to 51.8$	$49.3^{+1.0}_{-1.2}$	$39.9 \to 52.0$
	$\sin^2\theta_{13}$	$0.02221^{+0.00068}_{-0.00062}$	$0.02034 \to 0.02430$	$0.02240^{+0.00062}_{-0.00062}$	$0.02053 \to 0.02436$
	$\theta_{13}/^\circ$	$8.57^{+0.13}_{-0.12}$	$8.20 \to 8.97$	$8.61^{+0.12}_{-0.12}$	$8.24 \to 8.98$
	$\delta_{CP}/^\circ$	195^{+51}_{-25}	$107 \to 403$	286^{+27}_{-32}	$192 \to 360$
	$\frac{\Delta m^2_{21}}{10^{-5}\ \text{eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$
	$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \text{eV}^2}$	$+2.514^{+0.028}_{-0.027}$	$+2.431 \to +2.598$	$-2.497^{+0.028}_{-0.028}$	$-2.583 \to -2.412$
		Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 7.1$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
with SK atmospheric data	$\sin^2\theta_{12}$	$0.304^{+0.012}_{-0.012}$	$0.269 \to 0.343$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$
	$\theta_{12}/^\circ$	$33.44^{+0.77}_{-0.74}$	$31.27 \to 35.86$	$33.45^{+0.78}_{-0.75}$	$31.27 \to 35.87$
	$\sin^2\theta_{23}$	$0.573^{+0.016}_{-0.020}$	$0.415 \to 0.616$	$0.575^{+0.016}_{-0.019}$	$0.419 \to 0.617$
	$\theta_{23}/^\circ$	$49.2^{+0.9}_{-1.2}$	$40.1 \to 51.7$	$49.3^{+0.9}_{-1.1}$	$40.3 \to 51.8$
	$\sin^2\theta_{13}$	$0.02219^{+0.00062}_{-0.00063}$	$0.02032 \to 0.02410$	$0.02238^{+0.00063}_{-0.00062}$	$0.02052 \to 0.02428$
	$\theta_{13}/^\circ$	$8.57^{+0.12}_{-0.12}$	$8.20 \to 8.93$	$8.60^{+0.12}_{-0.12}$	$8.24 \to 8.96$
	$\delta_{CP}/^\circ$	197^{+27}_{-24}	$120 \to 369$	282^{+26}_{-30}	$193 \to 352$
	$\frac{\Delta m^2_{21}}{10^{-5}\ \text{eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$
	$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \text{eV}^2}$	$+2.517^{+0.026}_{-0.028}$	$+2.435 \to +2.598$	$-2.498^{+0.028}_{-0.028}$	$-2.581 \to -2.414$

§Numerical analyses

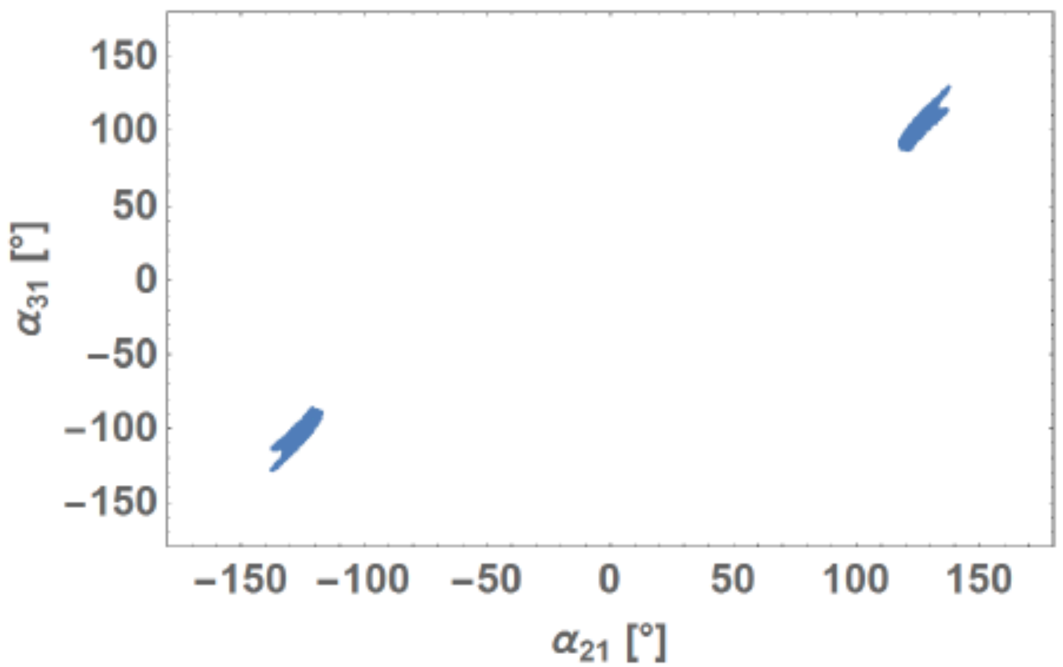


Figure 3: The prediction of Majorana phases α_{21} and α_{31} for NH in model I(a).

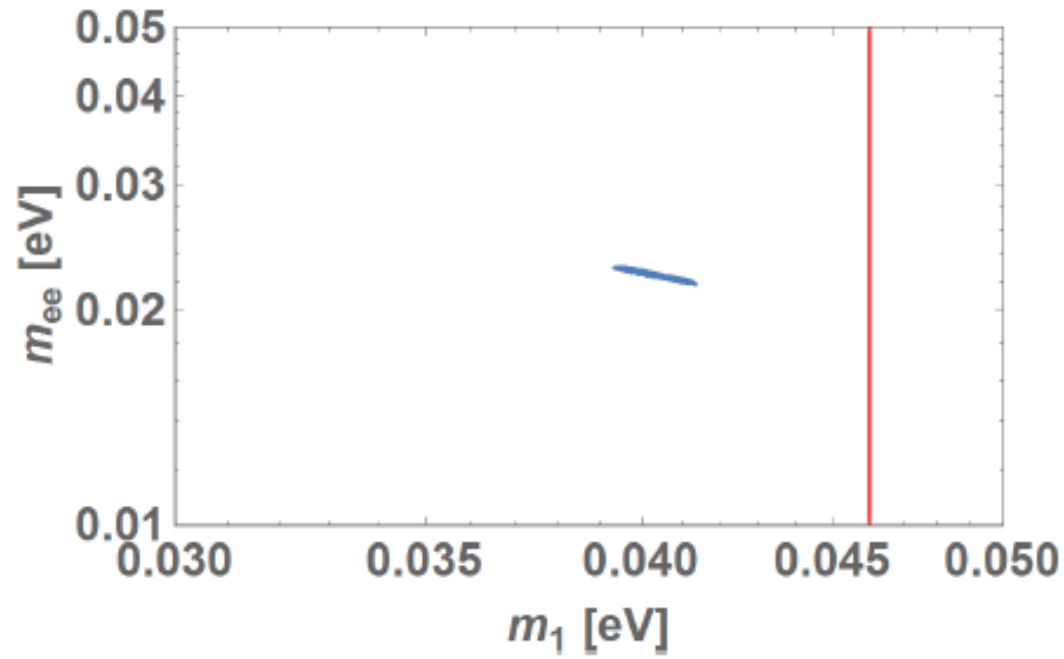


Figure 4: The prediction of m_{ee} versus m_1 for NH in model I(a). The red vertical line denotes the upper-bound of m_1.

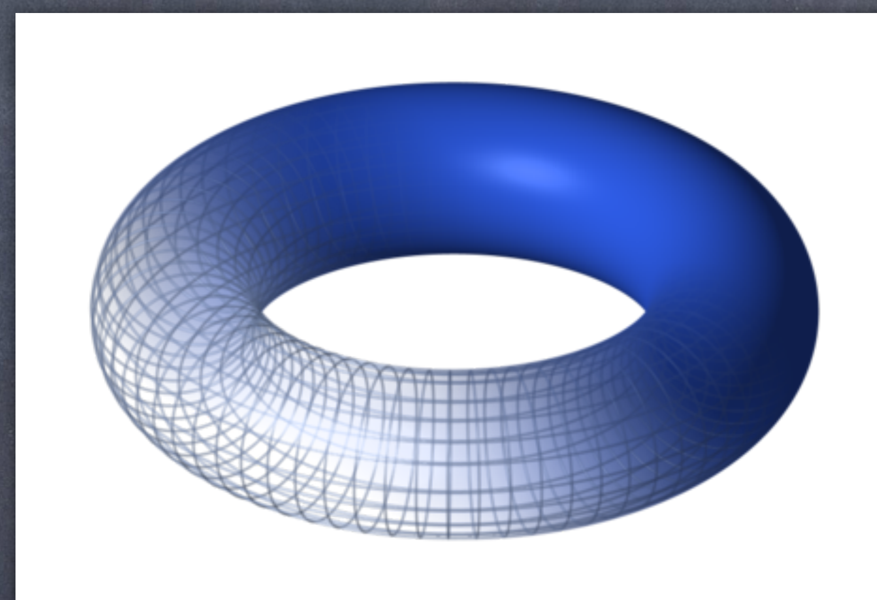
Im[τ]	Re[τ]	g	ϕ_g	α/γ	β/γ
$0.66-0.73$ $1.17-1.32$	$\pm(0.25-0.31)$, $\pm(0.46-0.54)$, $\pm(0.66-0.75)$, $\pm(1.25-1.31)$, $\pm(1.46-1.50)$	$1.20-1.22$	$\pm(87-88)°$ $\pm(92-93)°$	$202-203$	$3286-3306$

Table 4: The parameter regions consistent with the experimental data of Table 3 for model I(a). Results do not change under the exchange of α/γ and β/γ.

4. Summary

We study the phenomenological implications of the modular symmetry $\Gamma(3) \simeq A_4$ facing recent experimental data of neutrino oscillations. The mass matrices of neutrinos and charged leptons are essentially given by fixing the expectation value of the modulus τ, which is the only source of modular invariance breaking. We introduce no flavons in contrast with conventional flavor models with the A_4 symmetry.

From summary in previous talk


Wikipedia

answer?

- We will apply non-Abelian discrete symmetry to the quark sector and consider origin of the flavor symmetry.