Dirac Neutrino Lepton Number in a Unified Framework

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Based on: arXiv:2020.07664(retracted)
& arXiv:2020.XXXXX

Outline

- Introduction
  - Cosmic Backgrounds
- Lepton Number
  - Continuity and Expectation Value
- Numerical Results
  - Momentum dependence
- Conclusion

Introduction

- Observation of Cosmic Microwave Background (PLANCK 2015)
  - Further supports Big Bang Inflation Cosmology
  - Photons decoupled from early universe thermal bath
- Similarly, Inflation Cosmology predicts a Neutrino Background
  - Cosmic Neutrino background (CnuB)
- After expansion both backgrounds would cool
  - Leading to a blackbody background of Photons
  - Non-relativistic background of Neutrinos
    - Thermal energy of $\mathcal{O}(10^{-4})\mathrm{eV}$

Planck 2015 results - XIII. Cosmological parameters
Planck Collaboration, P. A. R. Ade, etc.
DOI: https://doi.org/10.1051/0004-6361/201525830
A&A, 594 (2016) A13

Introduction

- Neutrino oscillations have been observed
  - Indicates Neutrinos are massive
- These oscillations should appear in the CnuB
  - Cosmology predicts the CnuB to be non-relativistic
- Current theory of oscillations is for relativistic Neutrinos

Goal: Design a model to explore non-relativistic Neutrinos

Y. Fukuda et al. (Super-Kamiokande Collaboration)
Phys. Rev. Lett. 81, 1562
DOI: 10.1103/PhysRevLett.81.1562

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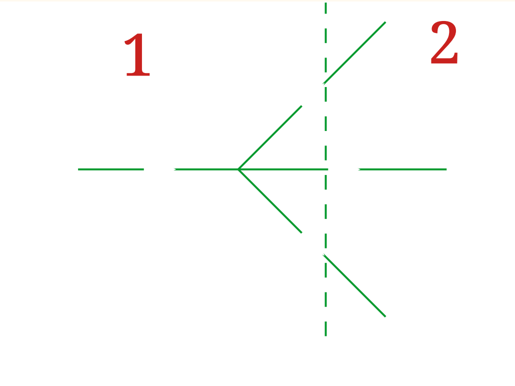
Methods

- Set-up for the calculation
  - Start with a definite flavor state (1)
  - After sometime the mass term is turned-on (2)
- We demand continuity at this time
  - Step-function is used to enforce this demand



Source: "Lepton number violation in a unified framework"; Y. Kawamura, Y. Matsuo, T. Morozumi, A. S. Adam, Y. Shimizu et al. PTEP 2020 (2020) 9, 093B07 DOI: 10.1093/ptep/ptaa107

Methods: Setup

- We consider two different chiral neutrino fields
  - Possible to construct a Dirac type mass term
- Continuity is enforced by a step-function

$$\mathcal{L} = \sum_{\alpha=e}^{\tau} \overline{\nu}_{L\alpha} i\partial\!\!\!/ \nu_{L\alpha} + \sum_{\alpha=e}^{\tau} \overline{\nu}_{R\alpha} i\partial\!\!\!/ \nu_{R\alpha} - \theta(t) \sum_{\alpha\beta=e\mu} \overline{\nu}_{R\alpha} m_{\alpha\beta} \nu_{L\alpha} - h.c.$$

Left-Handed Chiral Field — Right-Handed Chiral Field — Dirac Mass Term

Unitary transformations into the mass basis

$$\begin{aligned} \nu_{L\beta} &= V_{\beta j} \nu_{Lj} \\ \nu_{R\alpha} &= U_{\alpha i} \nu_{Ri} \end{aligned}$$

Preliminary

Methods

- Fourier expansion of the field in-terms of operators

$$\psi_i(x) = \int \frac{d^3p}{(2\pi)^3 2E_i(p)} \sum_{\lambda} \Big(u_i(p,\lambda) a_i(p,\lambda) e^{-ip\cdot x} + v_i(p,\lambda) b_i^\dagger(p,\lambda) e^{ip\cdot x} \Big)$$

helicity

- Construction of relationships between flavor and mass
  - Use the step- function continuity condition from before

projection operators

$$\begin{aligned} L\psi_i(t=0_+) &= V^*_{\beta i}\nu_{L\beta}(t=0_-) \\ R\psi_i(t=0_+) &= U^*_{\beta i}\nu_{R\beta}(t=0_-) \end{aligned}$$

Preliminary

Methods

- **Results in the following relationships**

flavor operators | mass operators

$$
\begin{aligned}
\frac{1}{\sqrt{2|p|}} a_{L\beta}(p) &= \sum_i^3 V_{\beta i} \frac{N_{ip}}{2E_i(p)} \left(a_i(p,-) + i c_i b_i^\dagger(-p,-) \right) \\
\frac{1}{\sqrt{2|p|}} b^\dagger_{L\beta}(-p) &= \sum_i^3 V_{\beta i} \frac{N_{ip}}{2E_i(p)} \left(b_i^\dagger(-p,+) + i c_i a_i(p,+) \right) \\
&\dots
\end{aligned}
$$

$$
\begin{aligned}
\frac{1}{\sqrt{2|p|}} a_{R\beta}(p) &= \sum_i^3 U_{\beta i} \frac{N_{ip}}{2E_i(p)} \left(a_i(p,+) + i c_i b_i^\dagger(-p,+) \right) \\
\frac{1}{\sqrt{2|p|}} b^\dagger_{R\beta}(-p) &= \sum_i^3 U_{\beta i} \frac{N_{ip}}{2E_i(p)} \left(b_i^\dagger(-p,-) + i c_i a_i(p,-) \right) \\
&\dots
\end{aligned}
$$

Preliminary

Methods

- Neutrino family Lepton Number is prepared as such

left-handed state

$$L^L_\alpha(t) = \int \frac{d^3p}{(2)^3 2\,|p|} \left(a^\dagger_{L\alpha}(p,t)a_{L\alpha}(p,t) - b^\dagger_{L\alpha}(p,t)b_{L\alpha}(p,t)\right) + \int \frac{d^3p}{(2)^3 2\,|p|} \left(a^\dagger_{L\alpha}(-p,t)a_{L\alpha}(-p,t) - b^\dagger_{L\alpha}(-p,t)b_{L\alpha}(-p,t)\right)$$

- This results in two complex equations
  - Left-handed and Right-handed
- The combination of these equations is conserved

$$\sum_{\alpha=e}^{\tau} \left(L^L_\alpha(t) + L^R_\alpha(t)\right) = \sum_{\alpha=e}^{\tau} \left(L^L_\alpha(0) + L^R_\alpha(0)\right)$$

Methods

- At this point we are ready to study the expectation value
  - We consider the time evolution of a single flavor Eigenstate

$$
\begin{aligned}
\langle\sigma_L(q)|L_\alpha^L(t)|\sigma_L(q)\rangle \quad = \quad & \textstyle\sum_{i,j}^3 V_{\alpha i}^* V_{\sigma i} V_{\alpha j} V_{\sigma j}^* \Big(\cos(E_i t)\cos(E_j t) + \frac{(1-c_i^2)}{(1+c_i^2)}\frac{(1-c_j^2)}{(1+c_j^2)}\sin(E_i t)\sin(E_j t) \\
& + i\left(\frac{(1-c_i^2)}{(1+c_i^2)}\sin(E_i t)\cos(E_j t) - \frac{(1-c_j^2)}{(1+c_j^2)}\cos(E_i t)\sin(E_j t)\right)\Big)
\end{aligned}
$$

  - Where,

$$c_i = \frac{m_i}{E_i(q)+|q|}$$

$$\langle\sigma_L(q)|a_{L\beta}^\dagger(p)a_{L\gamma}(p)|\sigma_L(q)\rangle = \delta_{\sigma\beta}\delta_{\gamma\sigma}(2)^3 2|q|\delta^{(3)}(p-q)$$

$$|\sigma_L(q)\rangle = \frac{a_{L\sigma}^\dagger(q)|0\rangle}{\sqrt{\langle 0|a_{L\sigma}(q)a_{L\sigma}^\dagger(q)|0\rangle}}$$

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Numerical Results

- Numerically we investigate the time evolution
  - Considered a dimensionless time τ
  - Values of PMNS matrix and mass from Nufit v5.0 2020
  - Mass of lightest neutrino is set as $m_{lightest} = 0.01\mathrm{eV}$
- We consider the following three cases,

$$\langle e_L(q)|L_e^L(\tau)|e_L(q)\rangle \qquad \langle e_L(q)|L_\mu^L(\tau)|e_L(q)\rangle$$

$$\langle e_L(q)|L_\tau^L(\tau)|e_L(q)\rangle$$

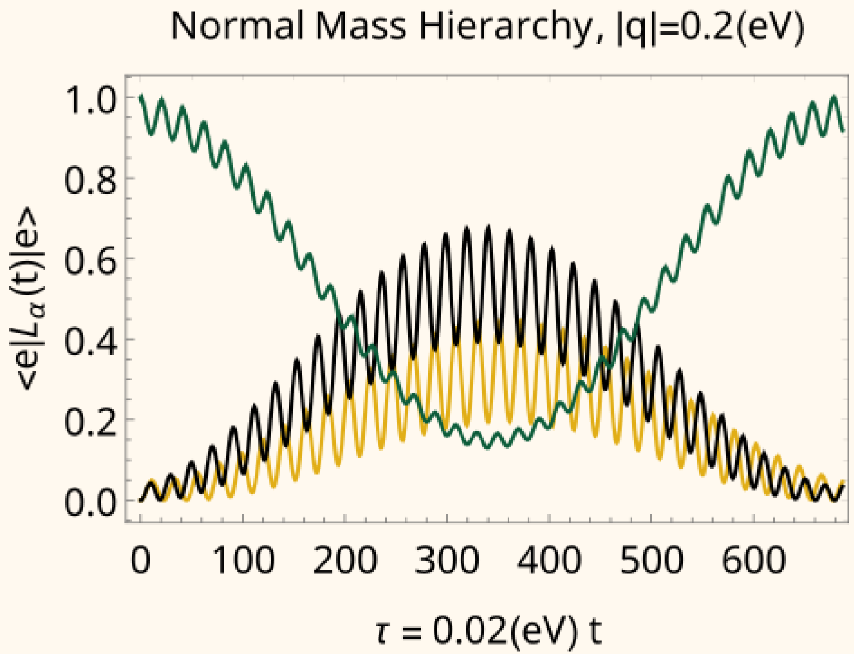
Numerical Results

- **Lastly, we modify the equation for numerical analysis**

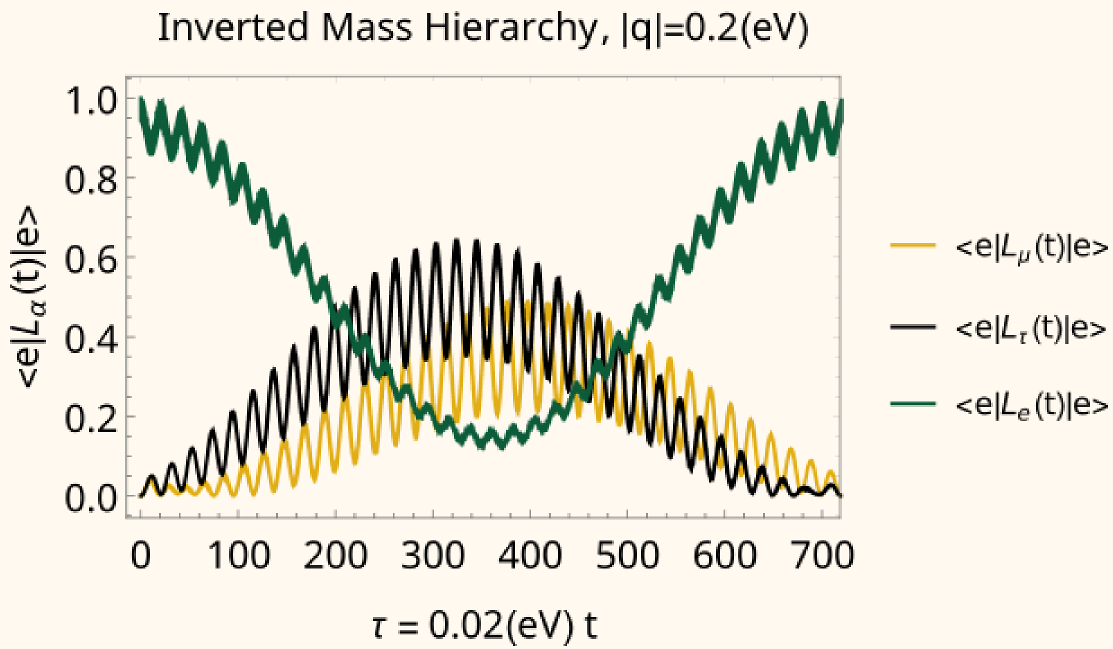
$$
\begin{aligned}
\langle e_L(q)|L_\alpha^L(\tau)|e_L(q)\rangle \quad = \quad & \frac{1}{2}\sum_{i,j}^{3} \Re\left(V_{\alpha i}^* V_{1i} V_{\alpha j} V_{1j}^*\right)\left(\left(1+\frac{1}{E_i(q)}\frac{1}{E_j(q)}\right)\cos([E_i-E_j]\tau)\right. \\
& \left.+\left(1-\frac{1}{E_i(q)}\frac{1}{E_j(q)}\right)\cos([E_i+E_j]\tau)\right) \\
- \quad & \frac{1}{2}\sum_{i,j}^{3} \Im\left(V_{\alpha i}^* V_{1i} V_{\alpha j} V_{1j}^*\right)\left(\left(\frac{1}{E_i(q)}-\frac{1}{E_j(q)}\right)\sin([E_i+E_j]\tau)\right. \\
& \left.+\left(\frac{1}{E_i(q)}+\frac{1}{E_j(q)}\right)\sin([E_i-E_j]\tau)\right)
\end{aligned}
$$

Numerical Results (q=0.2 eV)

- **Normal Mass**

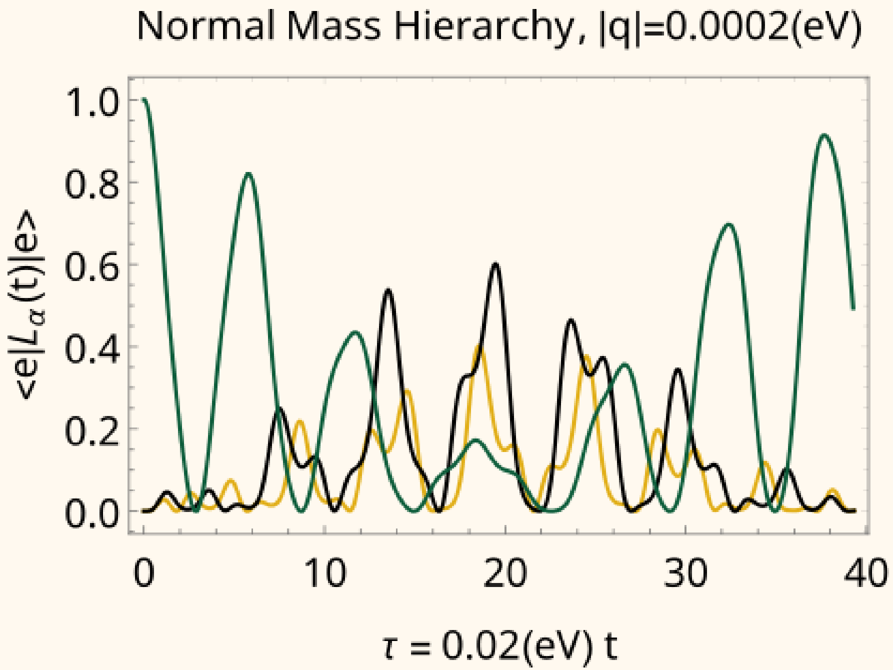


- **Inverted Mass**

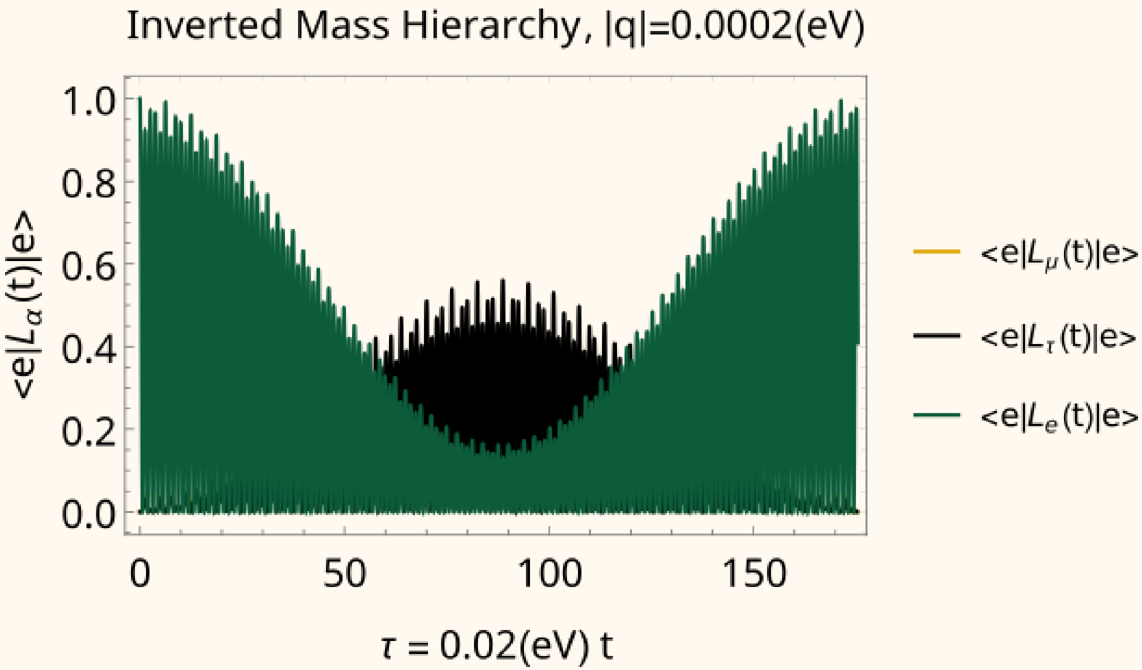


Numerical Results (q=0.0002 eV)

- **Normal Mass**



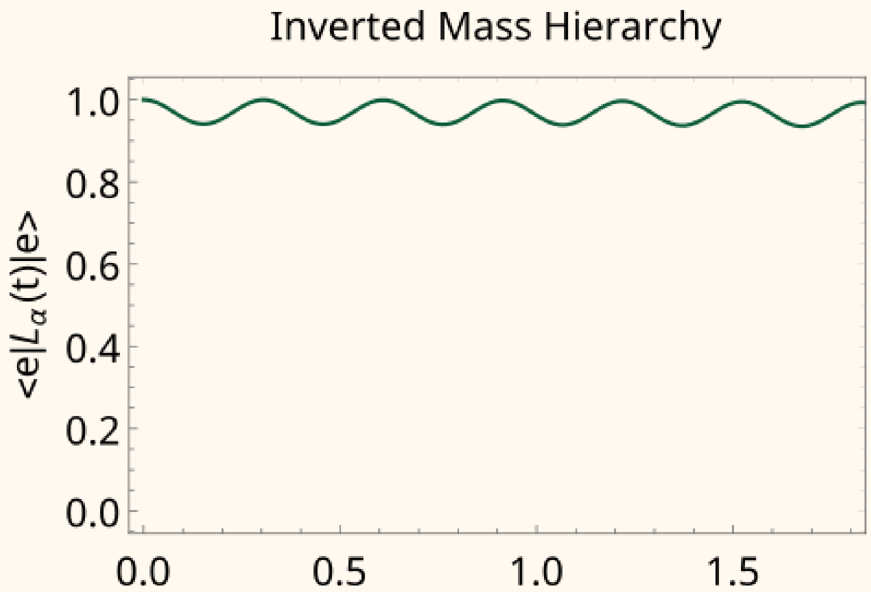
- **Inverted Mass**



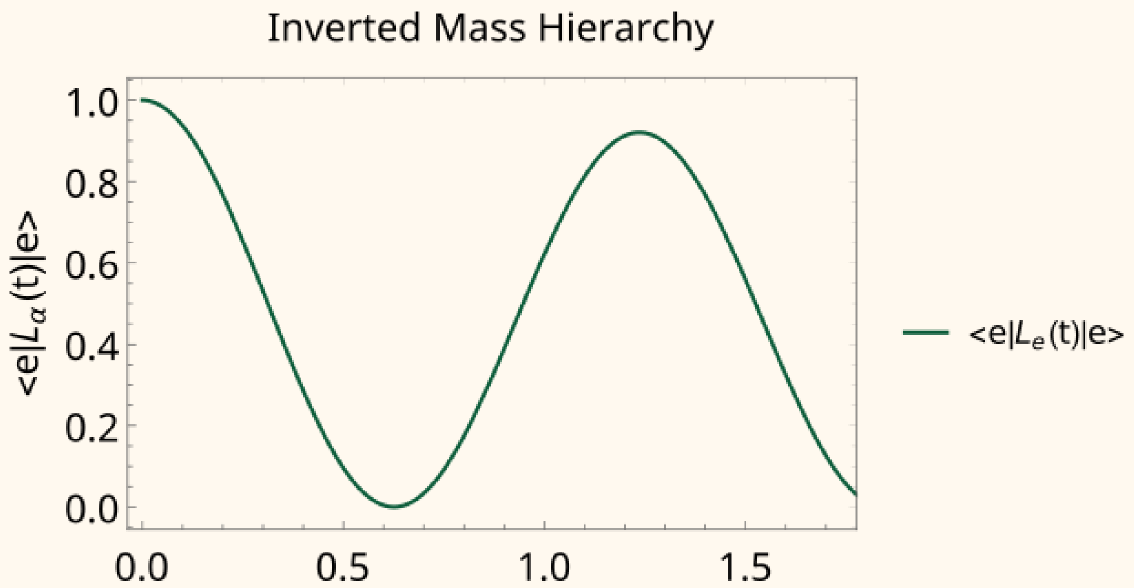
large differences between Normal vs Inverted Hierarchies

Numerical Results (zoom)

- q=0.2 eV



- q=0.0002 eV



where does this change come from?

Numerical Results

- Categorize the periods to investigate this phenomena
  - Source is energy values inside Sine and Cosine functions

Source of periods

$$\cos([E_i - E_j]t)$$
$$\cos([E_i + E_j]t)$$
$$\sin([E_i - E_j]t)$$
$$\sin([E_i + E_j]t)$$

Normal Mass Hierarchy $(m_3 \gg m_2 > m_1)$	$(m_1 \sim m_2)$	Invertered Mass Hierarchy $(m_2 > m_1 \gg m_3)$
$T_{-,11} = T_{-,22} = T_{-,33} = 0$ $T_{-,12} = \frac{2\pi}{[E_1 - E_2]}$	Largest	$T_{-,11} = T_{-,22} = T_{-,33} = 0$ $T_{-,12} = \frac{2\pi}{[E_1 - E_2]}$
$T_{-,23} = \frac{2\pi}{[E_2 - E_3]}$ $T_{-,31} = \frac{2\pi}{[E_3 - E_1]}$	Med	$T_{-,23} = \frac{2\pi}{[E_2 - E_3]}$ $T_{-,31} = \frac{2\pi}{[E_3 - E_1]}$
$T_{+,11} = \frac{2\pi}{[E_1 + E_1]}$	Smaller	$T_{+,33} = \frac{2\pi}{[E_3 + E_3]}$
$T_{+,12} = \frac{2\pi}{[E_1 + E_2]}$		$T_{+,31} = \frac{2\pi}{[E_3 + E_1]}$
$T_{+,22} = \frac{2\pi}{[E_2 + E_2]}$		$T_{+,23} = \frac{2\pi}{[E_2 + E_3]}$
$T_{+,31} = \frac{2\pi}{[E_3 + E_1]}$		$T_{+,11} = \frac{2\pi}{[E_1 + E_1]}$
$T_{+,23} = \frac{2\pi}{[E_2 + E_3]}$		$T_{+,12} = \frac{2\pi}{[E_1 + E_2]}$
$T_{+,33} = \frac{2\pi}{[E_3 + E_3]}$	Smallest	$T_{+,22} = \frac{2\pi}{[E_2 + E_2]}$

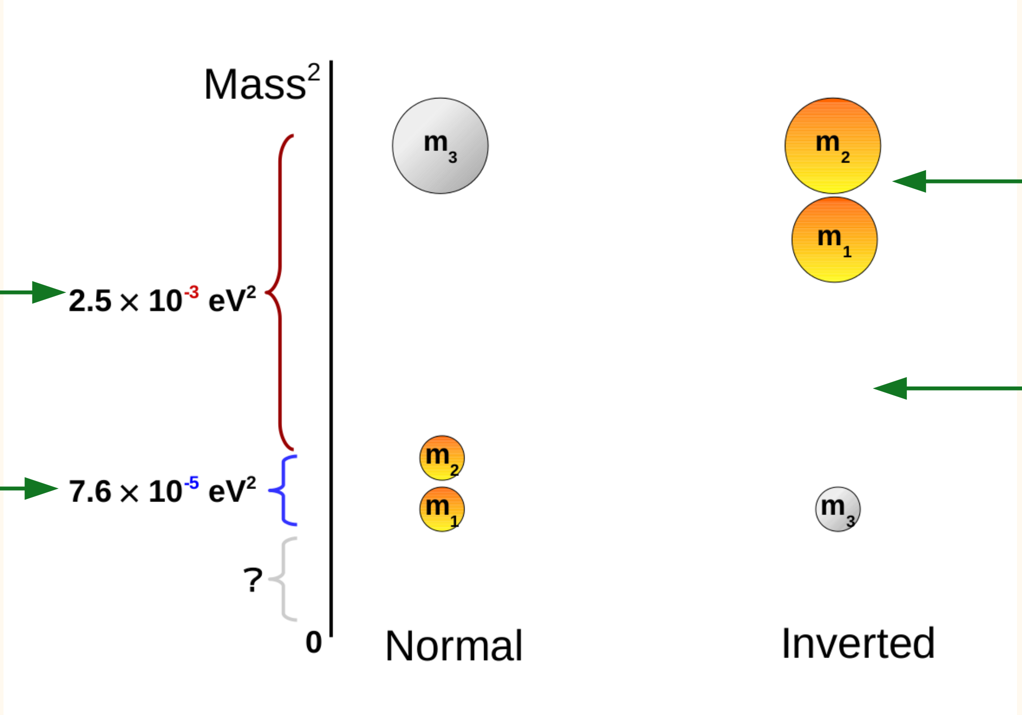
Preliminary

Numerical Results

- **Neutrino Mass Hierarchies**

Large mass difference

Small mass difference



Small mass difference

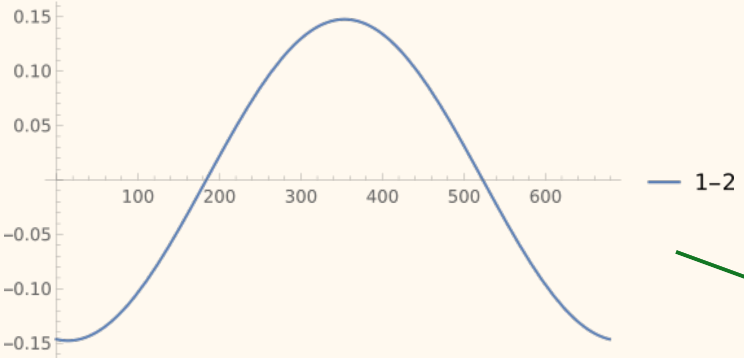
Large mass difference

Source: http://www.hyper-k.org/en/physics/phys-hierarchy.html

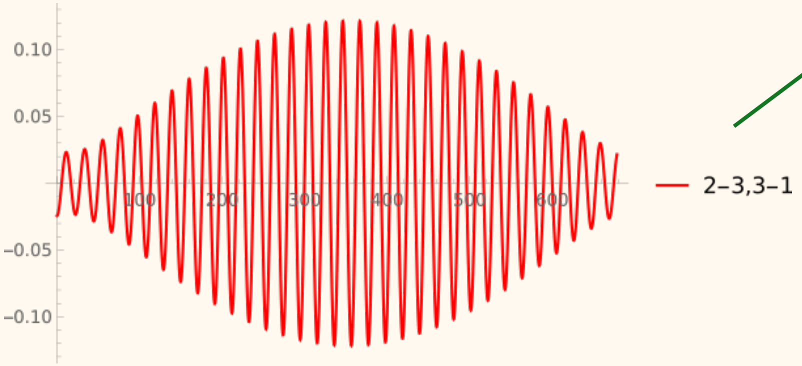
Numerical Results

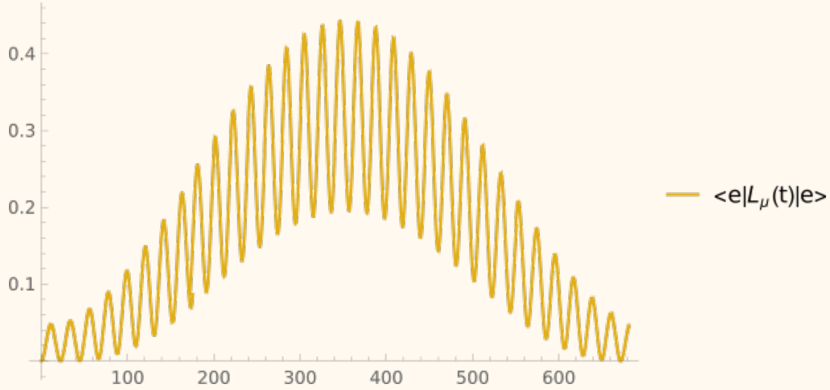
- Focus on the larger periods for a single flavor, NH

$$T_{-,12} = \frac{2\pi}{[E_1 - E_2]}$$



$$T_{-,23} = \frac{2\pi}{[E_2 - E_3]}$$
$$T_{-,31} = \frac{2\pi}{[E_3 - E_1]}$$





q=0.2 eV

Numerical Results

- **What happens when we move to smaller momentum?**

depressed closer to 1

$$\langle \sigma_L(q) | L^L_\alpha(t) | \sigma_L(q) \rangle = \tfrac{1}{2}\sum_{i,j}^{3} \Re\left(V^*_{\alpha i} V_{\sigma i} V_{\alpha j} V^*_{\sigma j}\right) \left(\left(1 + \frac{q}{E_i(q)}\frac{q}{E_j(q)}\right) \cos([E_i - E_j]t) \right.$$

enhanced closer to 1

remains close to 0

$$\left. + \left(1 - \frac{q}{E_i(q)}\frac{q}{E_j(q)}\right) \cos([E_i + E_j]t) \right)$$

$$- \tfrac{1}{2}\sum_{i,j}^{3} \Im\left(V^*_{\alpha i} V_{\sigma i} V_{\alpha j} V^*_{\sigma j}\right) \left(\left(\frac{q}{E_i(q)} - \frac{q}{E_j(q)}\right) \sin([E_i + E_j]t) \right.$$

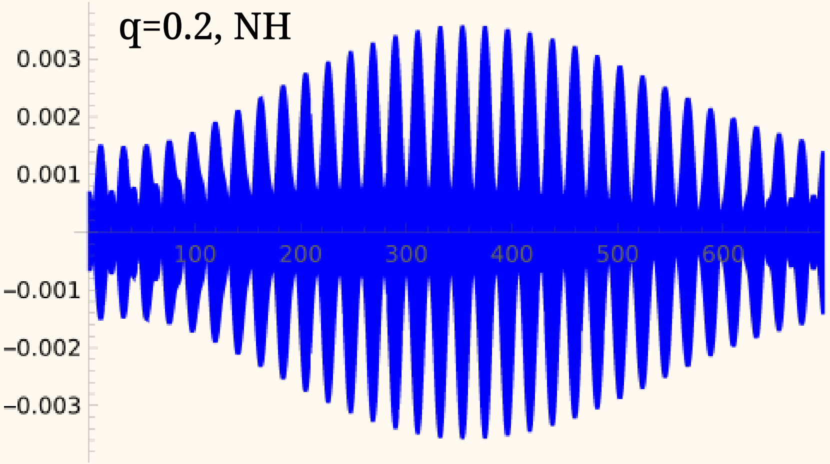
$$\left. + \left(\frac{q}{E_i(q)} + \frac{q}{E_j(q)}\right) \sin([E_i - E_j]t) \right)$$

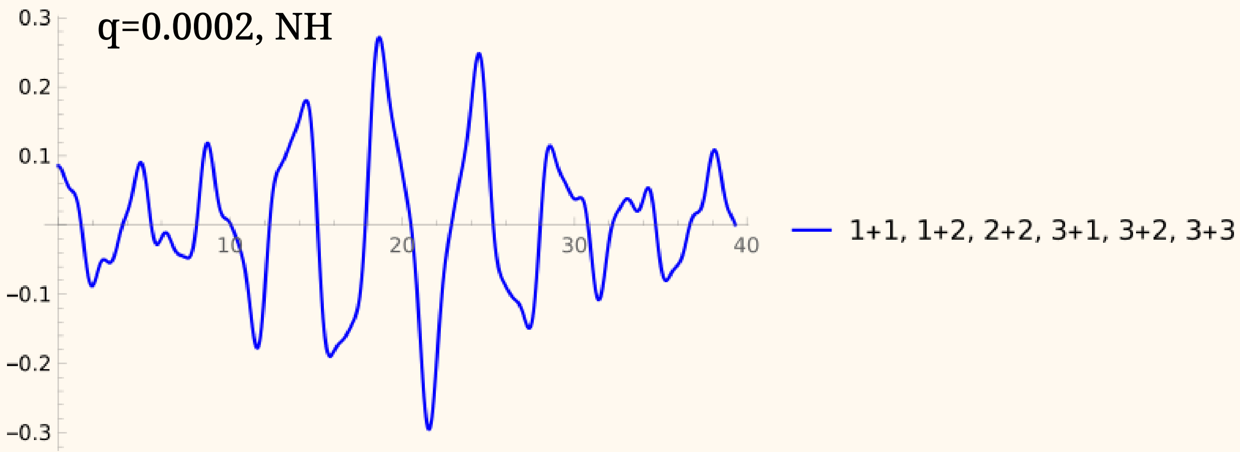
depressed closer to 0

$T_{+,11} = \frac{2\pi}{[E_1+E_1]}$	Smaller
$T_{+,12} = \frac{2\pi}{[E_1+E_2]}$	
$T_{+,22} = \frac{2\pi}{[E_2+E_2]}$	
$T_{+,31} = \frac{2\pi}{[E_3+E_1]}$	
$T_{+,23} = \frac{2\pi}{[E_2+E_3]}$	
$T_{+,33} = \frac{2\pi}{[E_3+E_3]}$	Smallest

Numerical Results

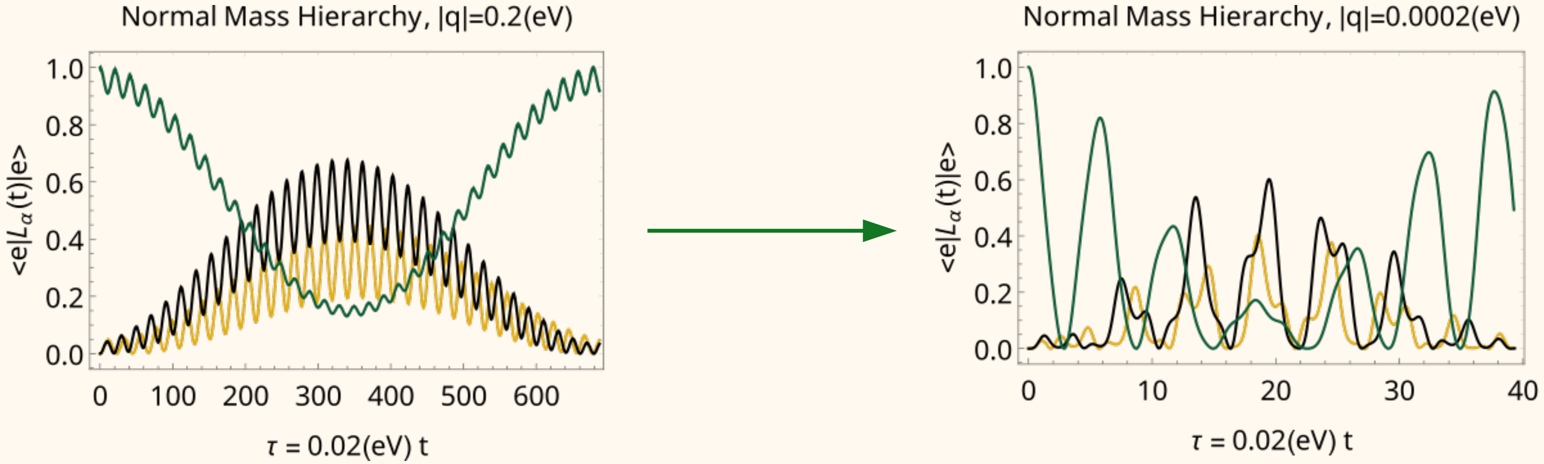
- Graphically this effect is seen as
  - 100x magnitude change in amplitude for smaller momentum value





Numerical Results

- **Now we understand changes in lower momentum**



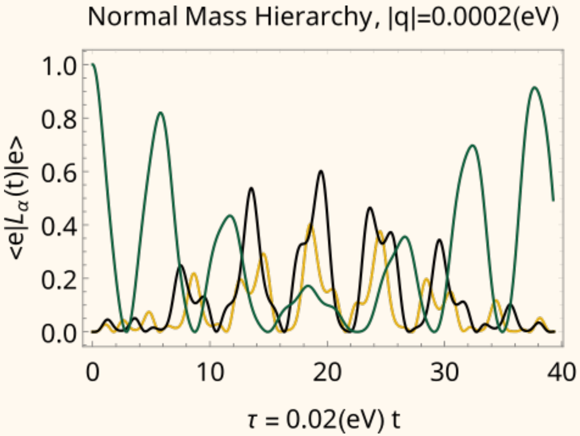
this term is not present in usual formulation

$$\frac{1}{2}\sum_{i,j}^{3}\Re\left(V_{\alpha i}^{*}V_{\sigma i}V_{\alpha j}V_{\sigma j}^{*}\right)\left(1-\frac{q}{E_i(q)}\frac{q}{E_j(q)}\right)\cos([E_i+E_j]t)$$

Numerical Results (q=0.0002 eV)

- So, what is cause of differences between hierarchies?
  - Same part of equation becomes important

$$\frac{1}{2}\sum_{i,j}^{3}\Re\left(V_{\alpha i}^{*}V_{\sigma i}V_{\alpha j}V_{\sigma j}^{*}\right)\left(1-\frac{q}{E_i(q)}\frac{q}{E_j(q)}\right)\cos([E_i+E_j]t)$$



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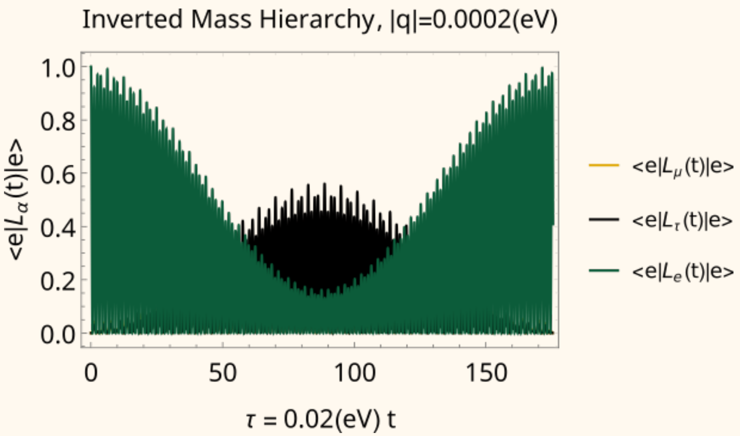
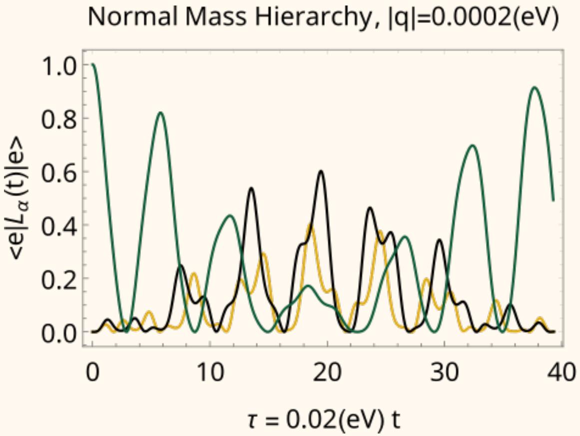


Numerical Results (q=0.0002 eV)

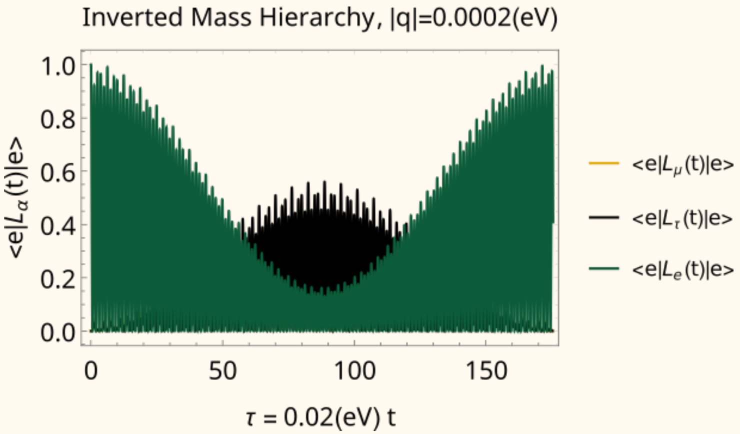
- So, what is cause of differences between hierarchies?
  - Same part of equation becomes important

absolute masses become important

$$\frac{1}{2}\sum_{i,j}^{3}\Re\left(V_{\alpha i}^{*}V_{\sigma i}V_{\alpha j}V_{\sigma j}^{*}\right)\left(1-\frac{q}{E_i(q)}\frac{q}{E_j(q)}\right)\boxed{\cos([E_i+E_j]t)}$$



$\neq$



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Conclusion

- Formulated non-relativistic model for neutrinos
  - Usual model is recovered at large momentum limit
- Numerically explored the model
  - Found clear differences (NH vs IH) in non-relativistic region
- Explained how these differences appear in the model
  - Frequency decomposition analysis
  - Large vs Small momentum limits

Thank you!

Questions?