Linear correction for four fermi interaction model in Magnetic field

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IITB-Hiroshima Workshop on Neutrino Physics @ ZOOM
30. 10. 2020

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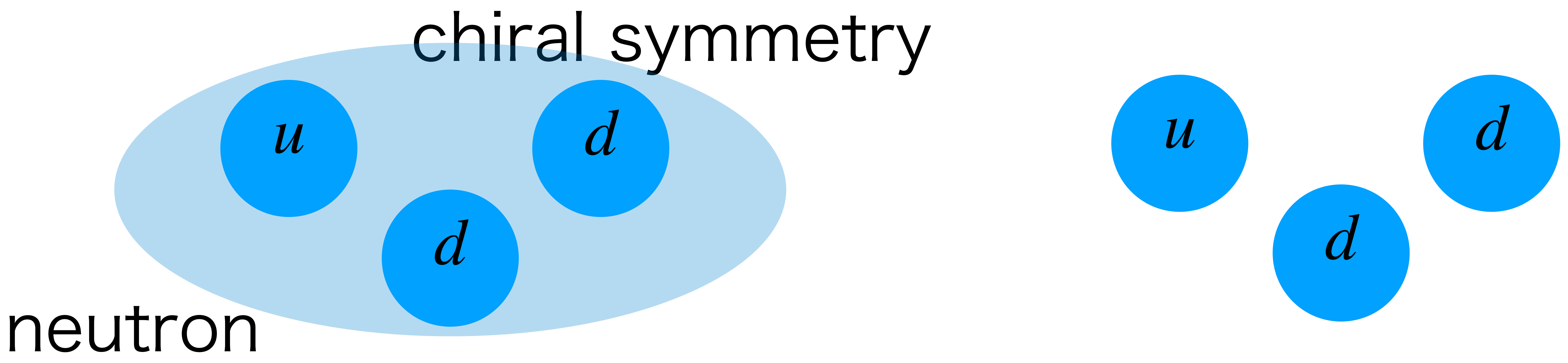
Introduction

chiral symmetry breaking



$m_n = 940$ (MeV) $\quad m_u \simeq 3$ (MeV), $m_d \simeq 5$ (MeV)

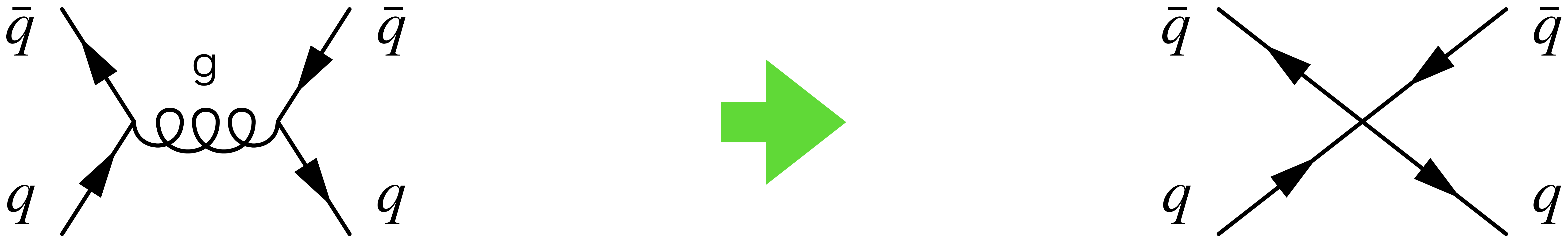
$\Delta m \simeq 900$ (MeV)

chiral symmetry breaking is induced by the magnetic field

(Magnetic Catalysis)

Numbu-Jona-Lasinio(NJL) model

・replace quark-gluon interaction to four-fermi interaction



$$\mathscr{L} = \bar{\psi}\left(i\ \partial\!\!\!/ - m\right)\psi + \frac{\lambda_0}{2N}\left[(\bar{\psi}\psi)^2\right]$$

four-fermi interaction

λ_0 : coupling

N : number of species

invariant under $\psi \to \gamma^5 \psi$: $\to Z_2$ chiral symmetry

induce axially field σ

$$\mathscr{L} = \bar{\psi}\left(i\ \partial\!\!\!/ - m - \sigma\right)\psi + \frac{N}{2\lambda_0}\sigma^2$$

$< \sigma >$: order parameter for chiral symmetry

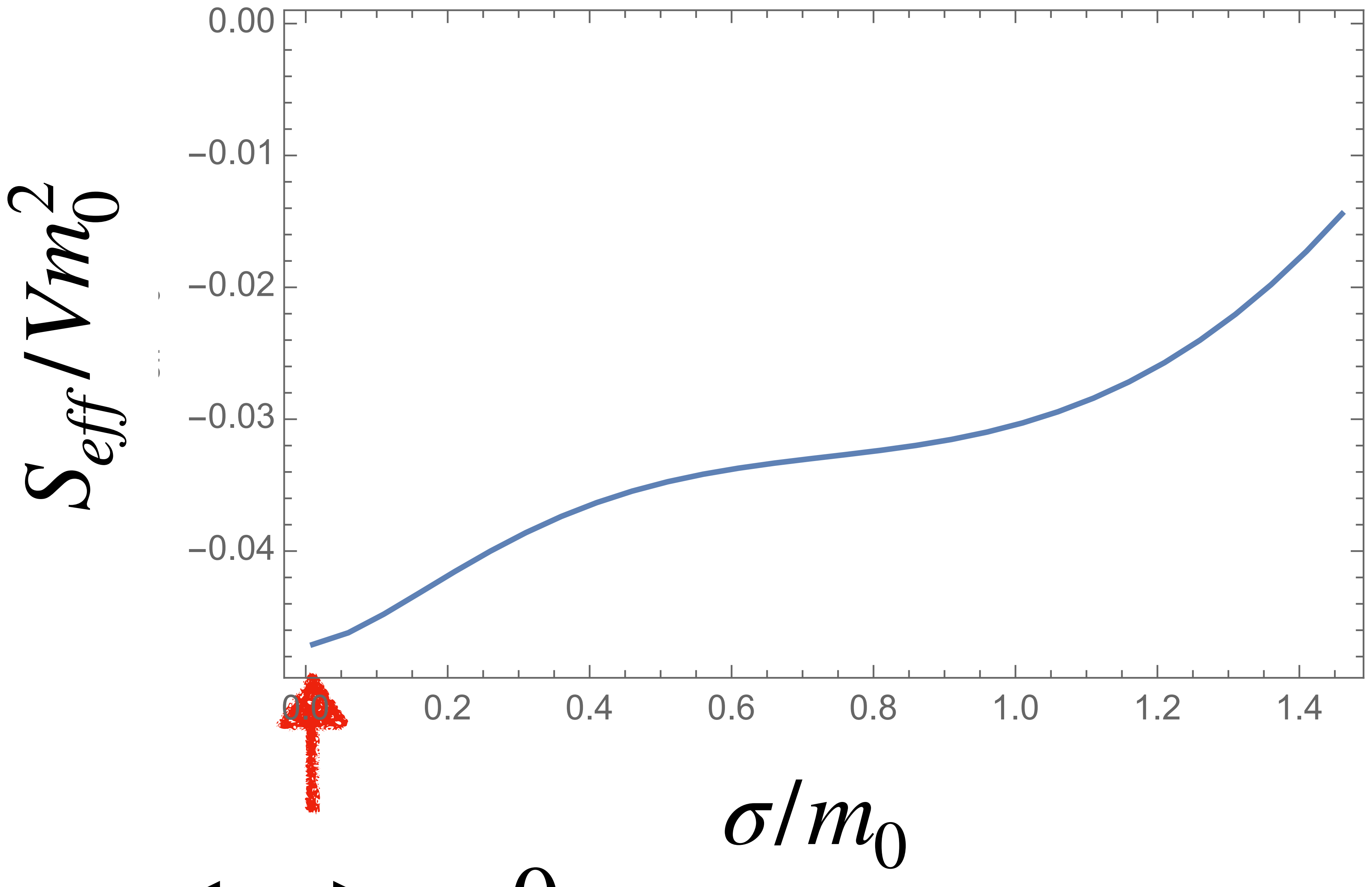
sample) 1+1 dimension and thermal effect

$$\frac{S_{eff}(\sigma)}{Vm_0^2} = -\frac{1}{4\pi^2}\left(1 - \ln\frac{\sigma^2}{m_0^2}\right)\left(\frac{\sigma}{m_0}\right)^2 + \frac{2T\sigma}{\pi m_0^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}K_1\left(n\frac{\sigma}{T}\right)$$

m_0 : expectation value of σ at T=0

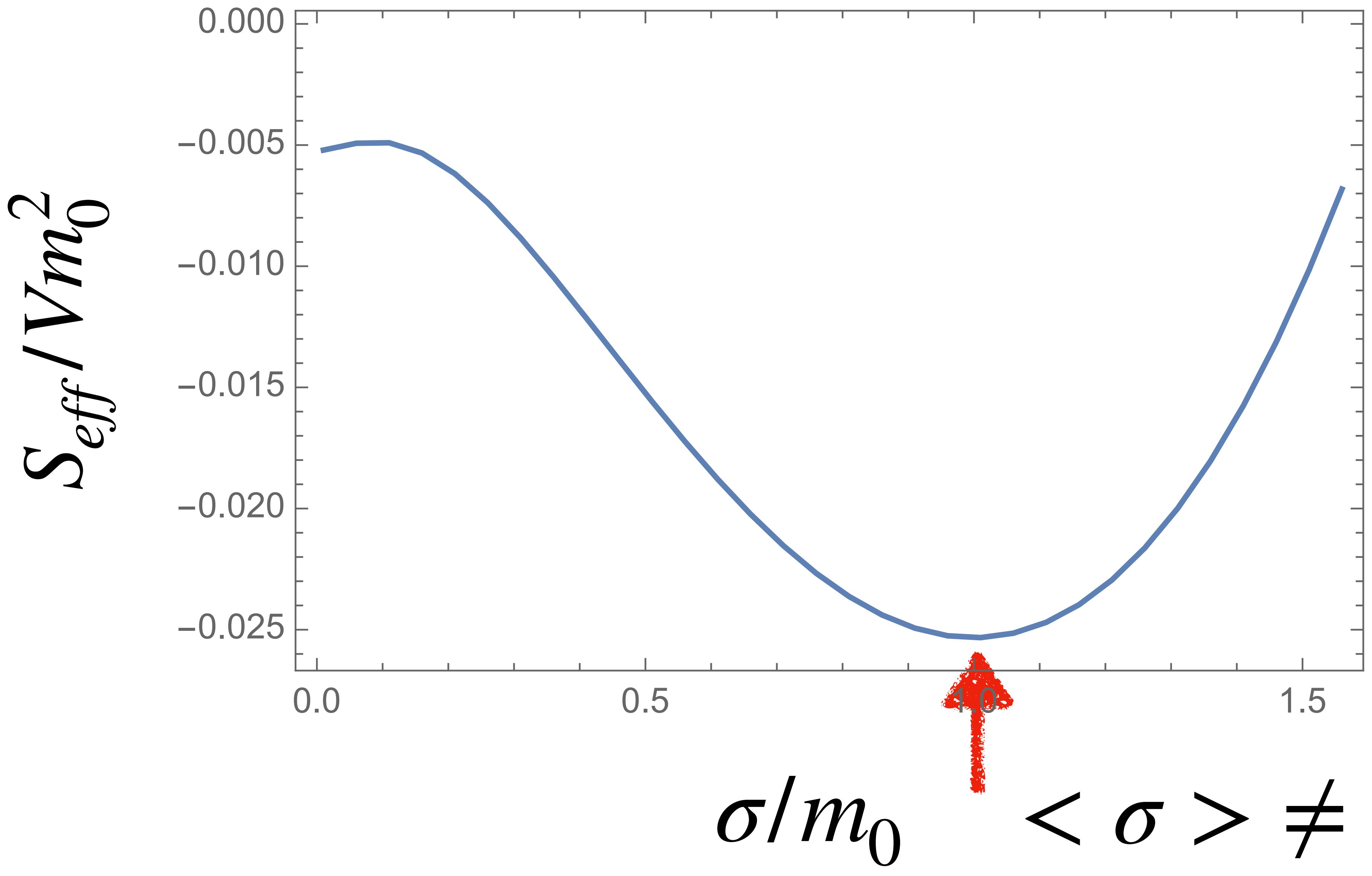
T : temperature

high temperature $T/m_0 = 0.3$



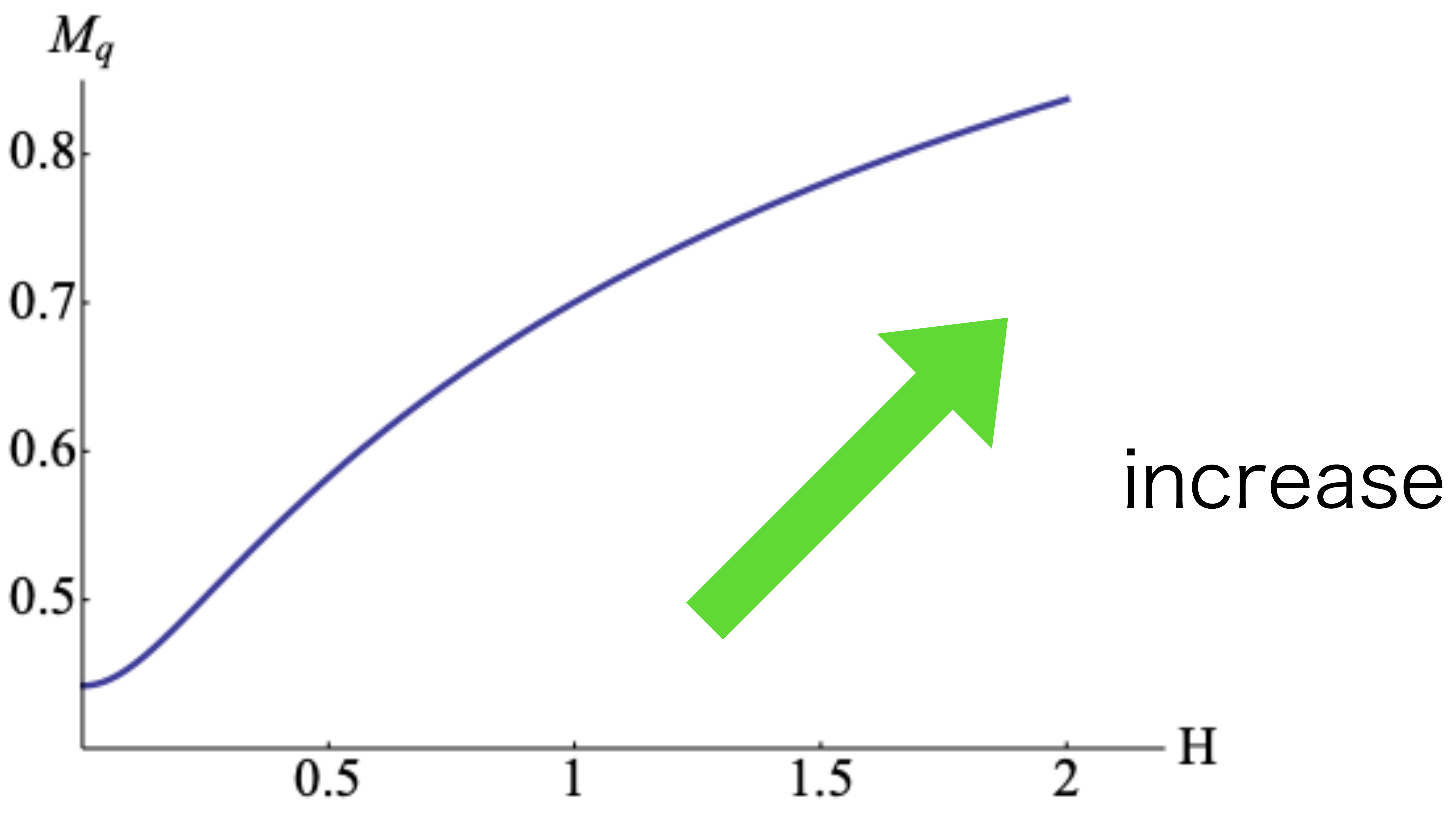
$< \sigma > = 0$

low temperature $T/m_0 = 0.1$



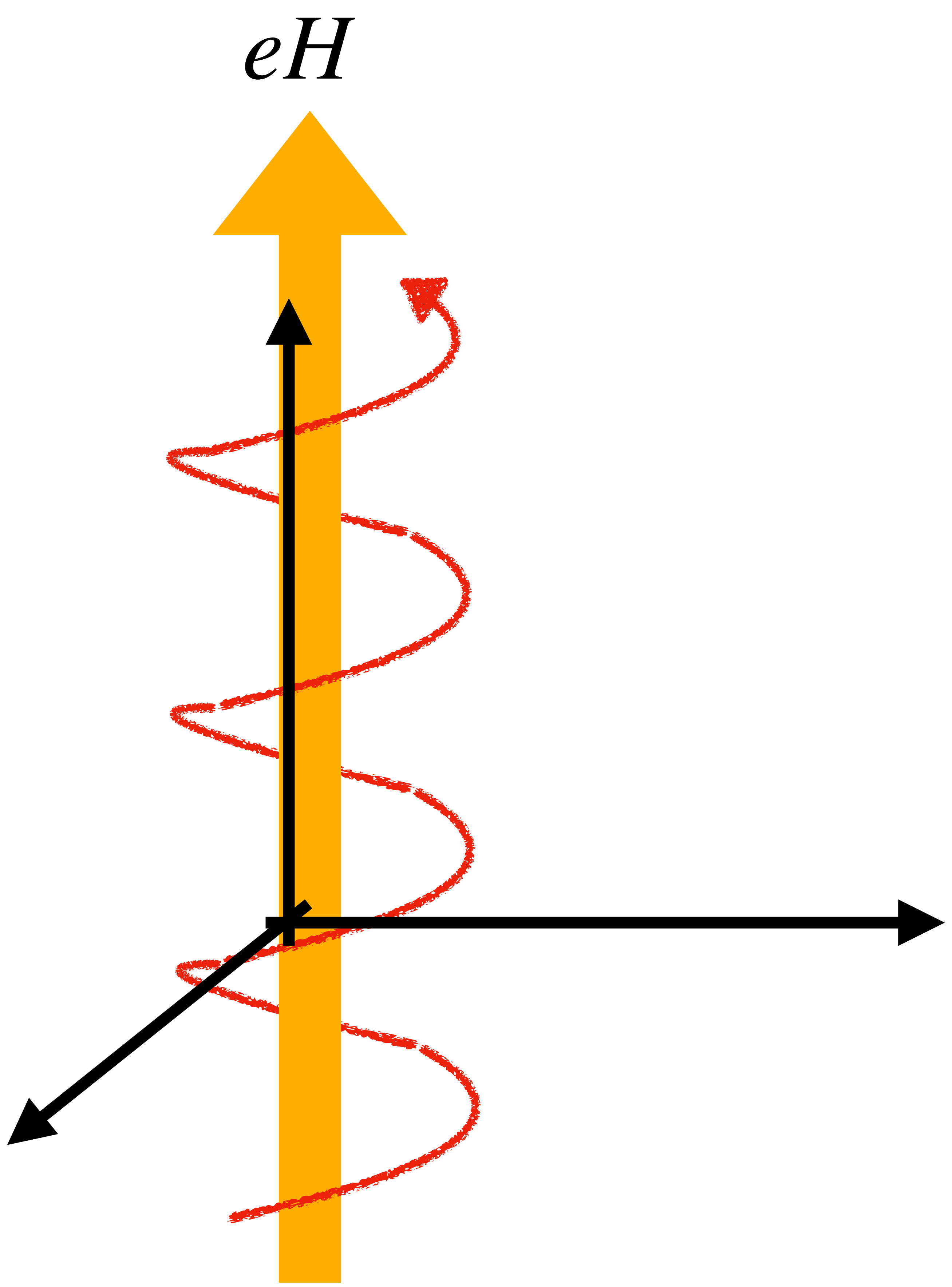
$< \sigma > \neq = 0$

chiral symmetry is enhanced by the magnetic field

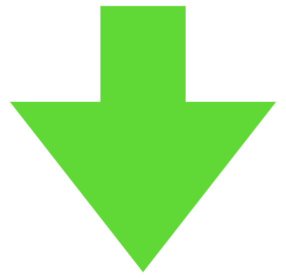


Clifford V. Johnson, Arnab Kundu (2008)

Magnetic Catalysis



the motion on the axis which is orthogonal to magnetic field is suppressed

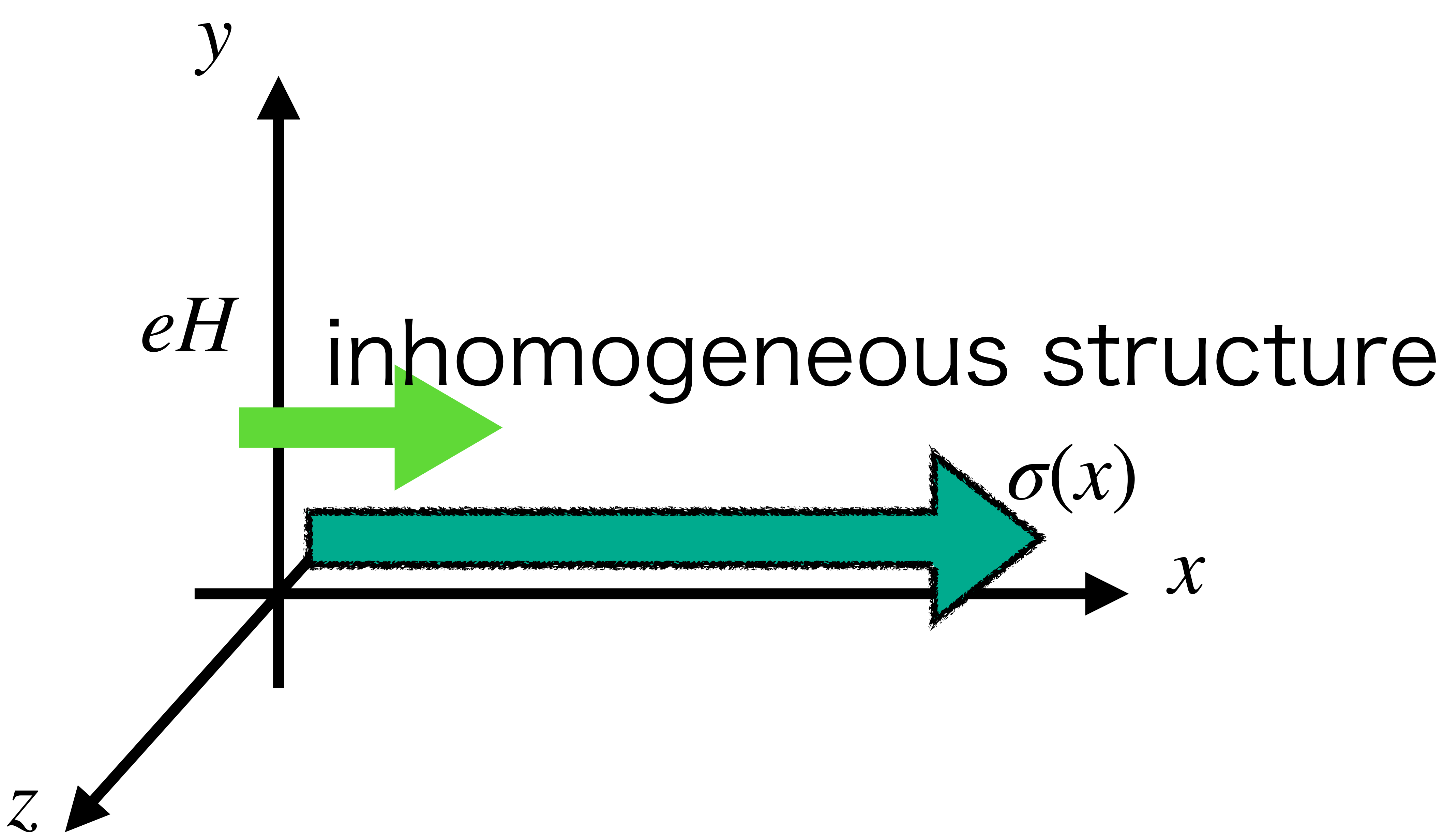


D →(D-2) dimension : dimensional reduction

Question :

How to the inhomogeneous correction (such as quantum correction) affect to Magnetic Catalysis ?

Set up

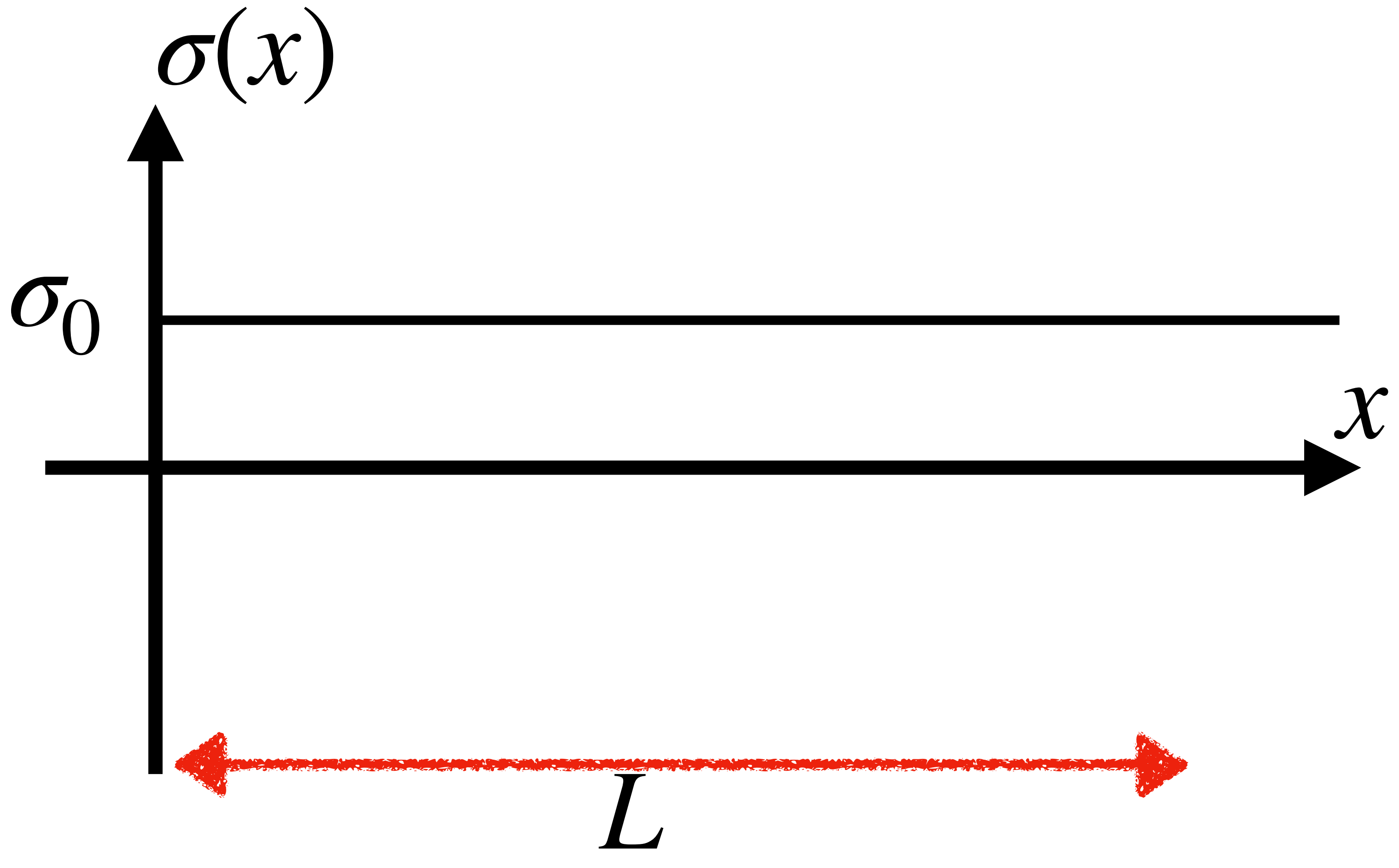


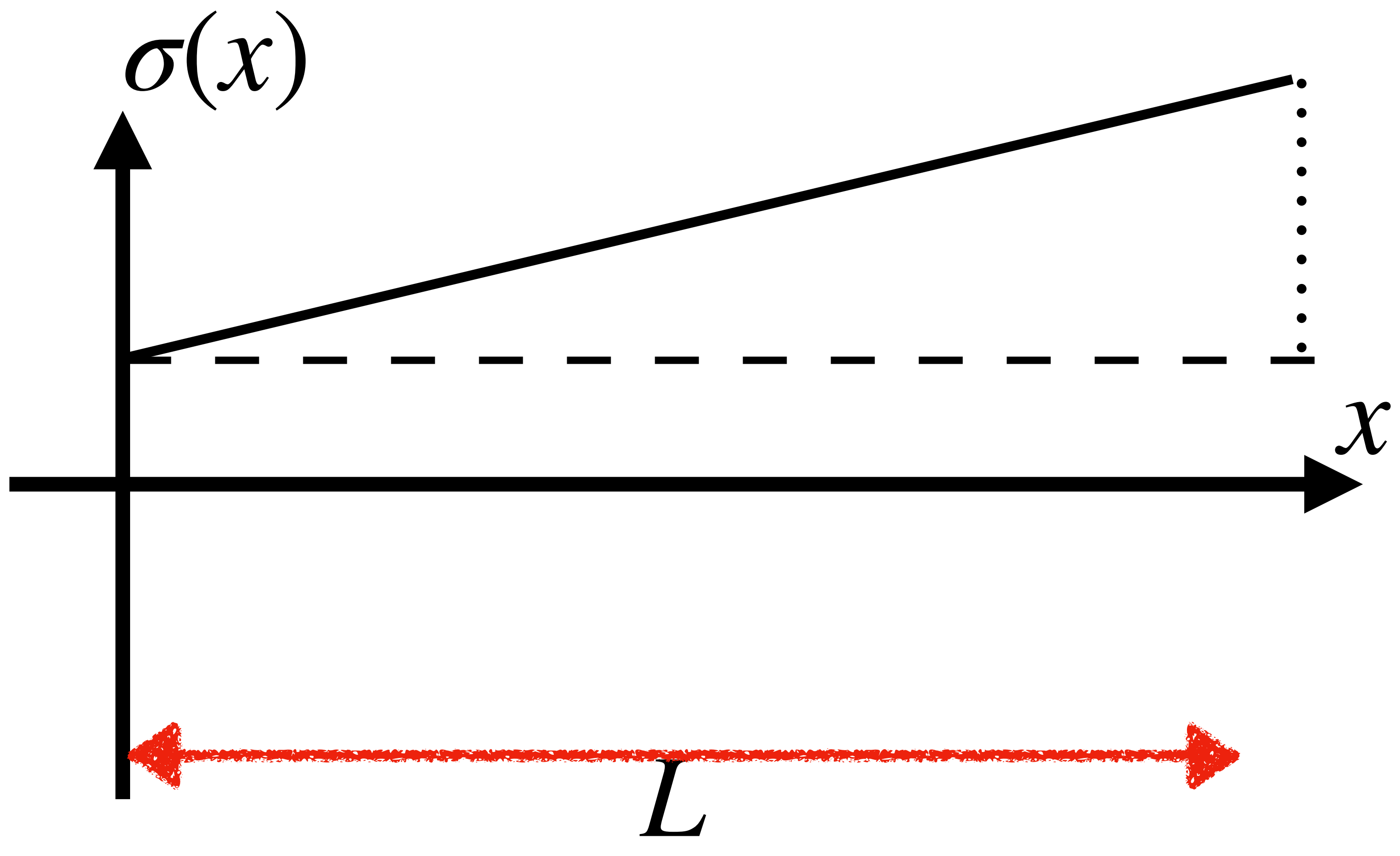
x direction :

MG

and

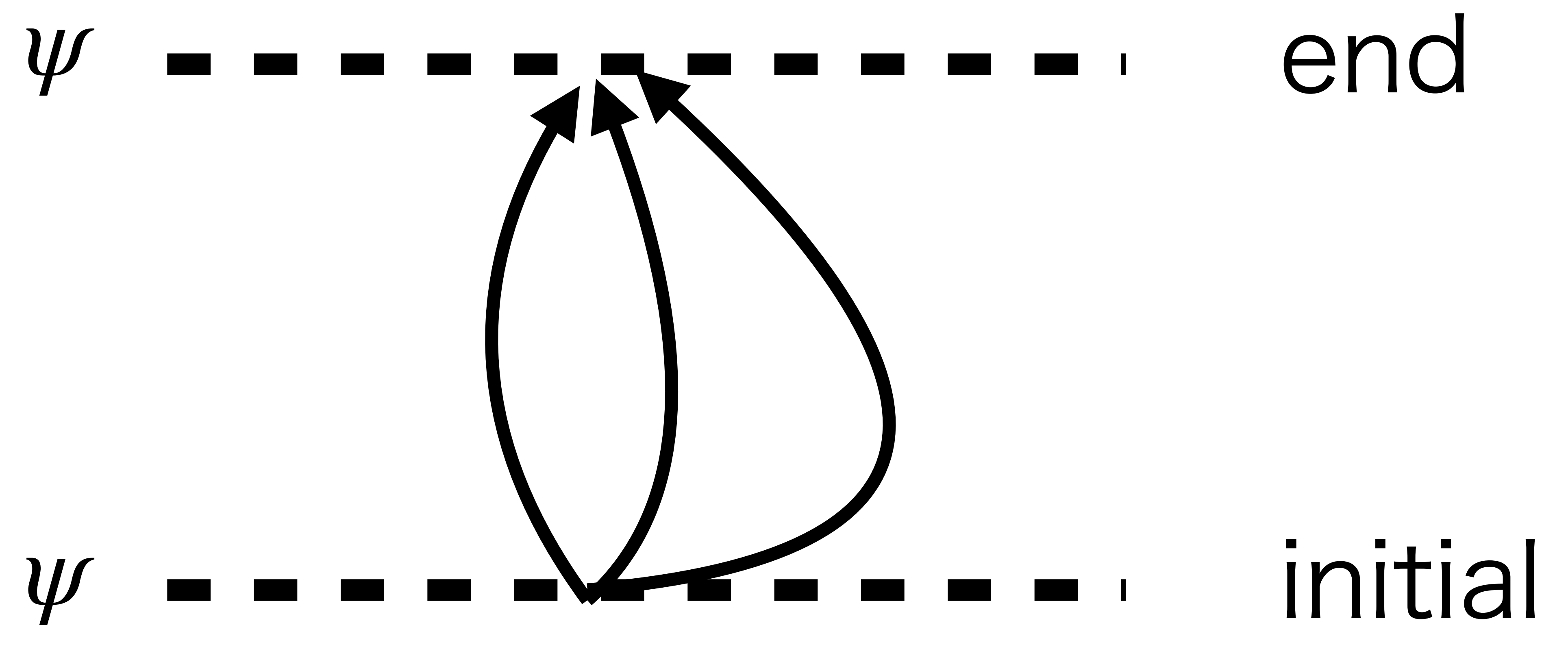
inhomogeneous structure of σ





$$\sigma(x) = \sigma_0 + \sigma' x$$

path integral



quantum states fluctuate

Particles fluctuate in quantum mechanism.

$$Z = \int \mathscr{D}\bar{\psi}\mathscr{D}\psi \exp\left(i\int d^4x\mathscr{L}\right) = e^{-S_{eff}}$$

Z : partiton function

S_{eff} : effective action

The ground states are found by solving $\delta S_{eff} = 0$.

gauged 4-fermi interaction model

$$\mathscr{L} = \bar{\psi}(i \not\partial + e \not A)\psi + \frac{\lambda_0}{2N}\left(\bar{\psi}\psi\right)^2 \qquad A_\mu = (0,0,0,eHy)$$

performing path integral

$$S_{eff} = \int d^4x \frac{N\sigma^2}{2\lambda_0} - \mathrm{tr}\left\langle x \left| \ln\left(\frac{i \not\partial + e \not A - \sigma_0 - \sigma' x}{\omega}\right) \right| x \right\rangle \qquad N \to 1$$

tr : trace on spinor and integrate over space

zeta function regularization :

$$\ln A = -\frac{d}{ds}\frac{1}{\Gamma(s)}\int_0^\infty dt t^{s-1} e^{-tA}\Big|_{s=0}, \qquad \left(\int_0^\infty dt t^{s-1} e^{-tA} = \Gamma(s) A^{-s},\right)$$

renormalize condition $\displaystyle \frac{1}{V}\frac{d^2 S_{eff}}{d\sigma^2}\Big|_{\sigma=\omega} = \frac{\omega^2}{\lambda_r}$ $\qquad \displaystyle \frac{1}{\lambda_0} = \frac{1}{\lambda_r} - \frac{1}{4\pi^2}$

We have to calculate these functions,

$$\exp\left[t\left(\partial_y - k_z - eHy\right)\left(\partial_y + k_z + eHy\right)\right]e^{-ik_y y}$$

$$\exp\left[t\left(i\gamma^1\partial_x - \sigma_0 - \sigma' x\right)\left(i\gamma^1\partial_x + \sigma_0 + \sigma' x\right)\right]e^{-ik_x x}$$

use following formula,

$$F(t,y) = \exp\left[t\left(\partial_y - a - by\right)\left(\partial_y + a + by\right)\right]e^{-ik_y y}$$

$$= \sqrt{\frac{1}{\cosh(2tb)}}\exp\left[bt - \frac{k_y^2}{2b}\tanh(2tb) + \frac{iak_y}{b} - \frac{i(a+by)k_y}{b\cosh(2tb)} - \frac{\tanh(2tb)}{2b}(a+by)^2\right]$$

Above function obeys the following equations,

$$\partial_t F(t,y) = \left(\partial_y - a - by\right)\left(\partial_y + a + by\right)F(t,y), \quad F(0,y) = e^{-ik_y y}.$$

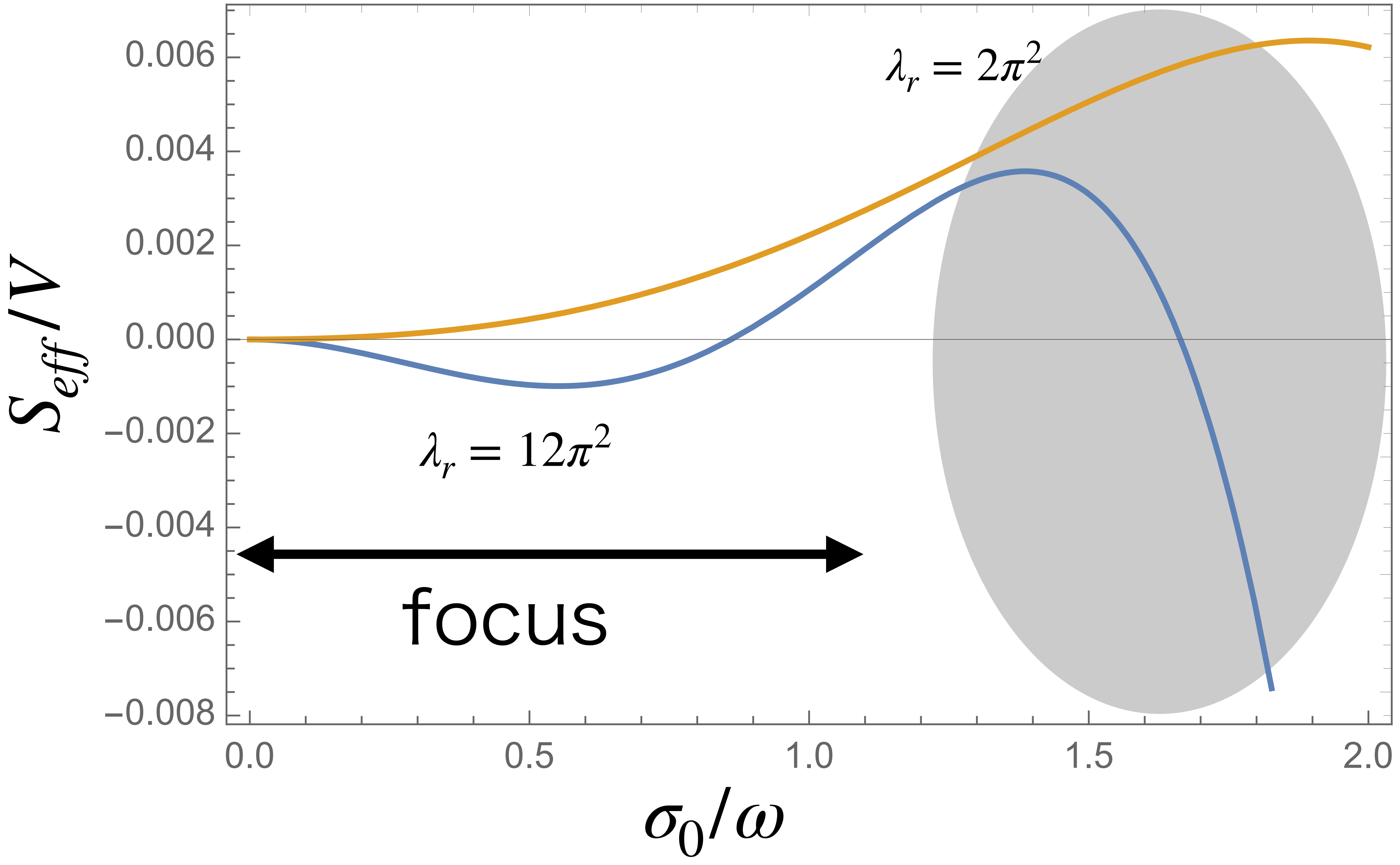
$$
\frac{S_{eff}(\sigma)}{V\omega^4} = \left(\frac{1}{2\lambda_r} - \frac{1}{8\pi^2}\right)\left(\frac{\sigma_0 + \sigma' x}{\omega}\right)^2
$$

$$
+\frac{1}{8\pi^2}\left(\frac{\left(\sigma_0 + \sigma' x\right)^4}{4\omega^4}\left(3 - 2\ln\left(\frac{\sigma_0 + \sigma' x}{\omega}\right)^2\right) + \frac{1}{12}\left(\frac{2e^2H^2 + \sigma'^2}{\omega^4}\right)\ln\left(\frac{\sigma_0 + \sigma' x}{\omega}\right)^2\right)
$$

$$
+\frac{1}{8\pi^2}\int_0^\infty dtt^{-3}\left(\frac{teH}{\sinh(teH)}\sqrt{\frac{t\sigma'}{\sinh(t\sigma')}}\exp\left[-\frac{\tanh(t\sigma')}{\sigma'}\left(\sigma_0 + \sigma' x\right)^2\right] - \left(1 - \frac{1}{12}\left(2e^2H^2 + \sigma'^2\right)t^2\right)e^{-t\left(\sigma_0 + \sigma' x\right)^2}\right)
$$

$\sigma' = 0, eH = 0$

$$\frac{S_{eff}(\sigma)}{V} = \left(\frac{1}{2\lambda_r} - \frac{1}{8\pi^2}\right)\left(\frac{\sigma_0}{\omega}\right)^2 + \frac{1}{8\pi^2}\left[\frac{(\sigma_0/\omega)^4}{4}\left(3 - 2\ln\left(\frac{\sigma_0}{\omega}\right)^2\right)\right].$$

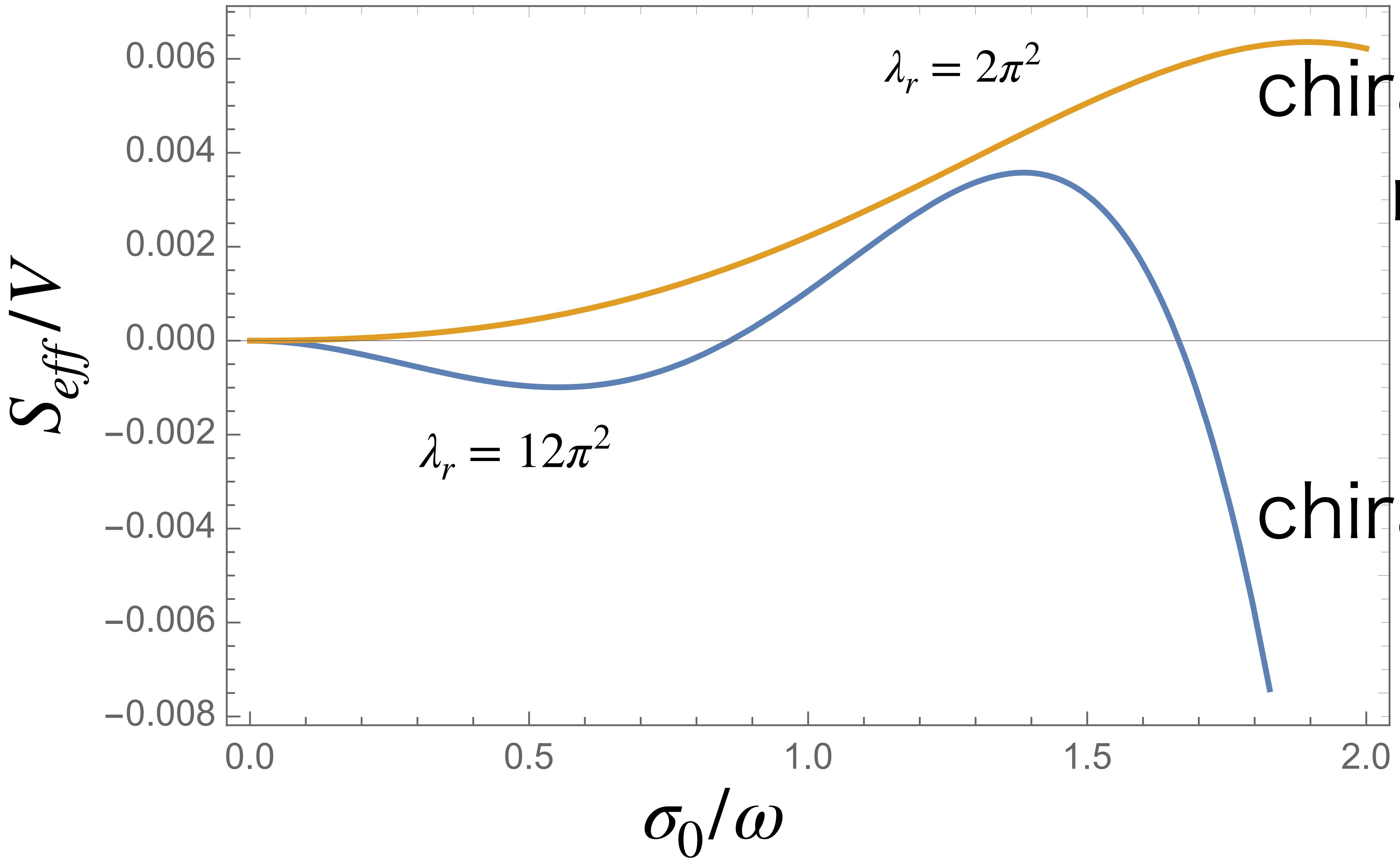


the potential is unstable

focus on small σ_0 region only

$\sigma' = 0, eH = 0$

$$\frac{S_{eff}(\sigma)}{V} = \left(\frac{1}{2\lambda_r} - \frac{1}{8\pi^2}\right)\left(\frac{\sigma_0}{\omega}\right)^2 + \frac{1}{8\pi^2}\left[\frac{(\sigma_0/\omega)^4}{4}\left(3 - 2\ln\left(\frac{\sigma_0}{\omega}\right)^2\right)\right].$$



chiral symmetry is not broken

chiral symmetry is broken

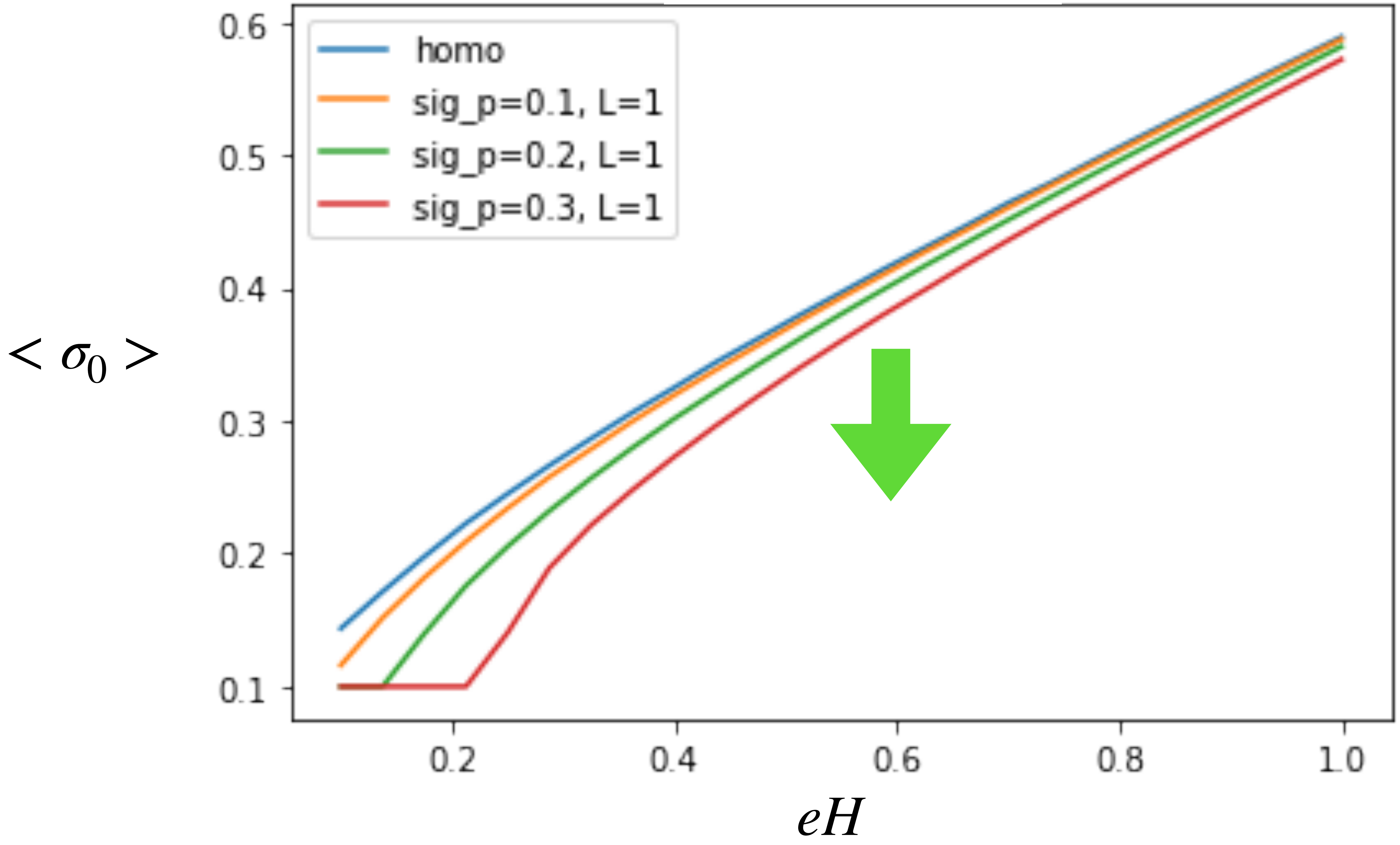
$$\frac{1}{2\lambda_r} - \frac{1}{8\pi^2} \rightarrow$$

positive with $\lambda_r < 4\pi^2$

negative with $\lambda_r > 4\pi^2$

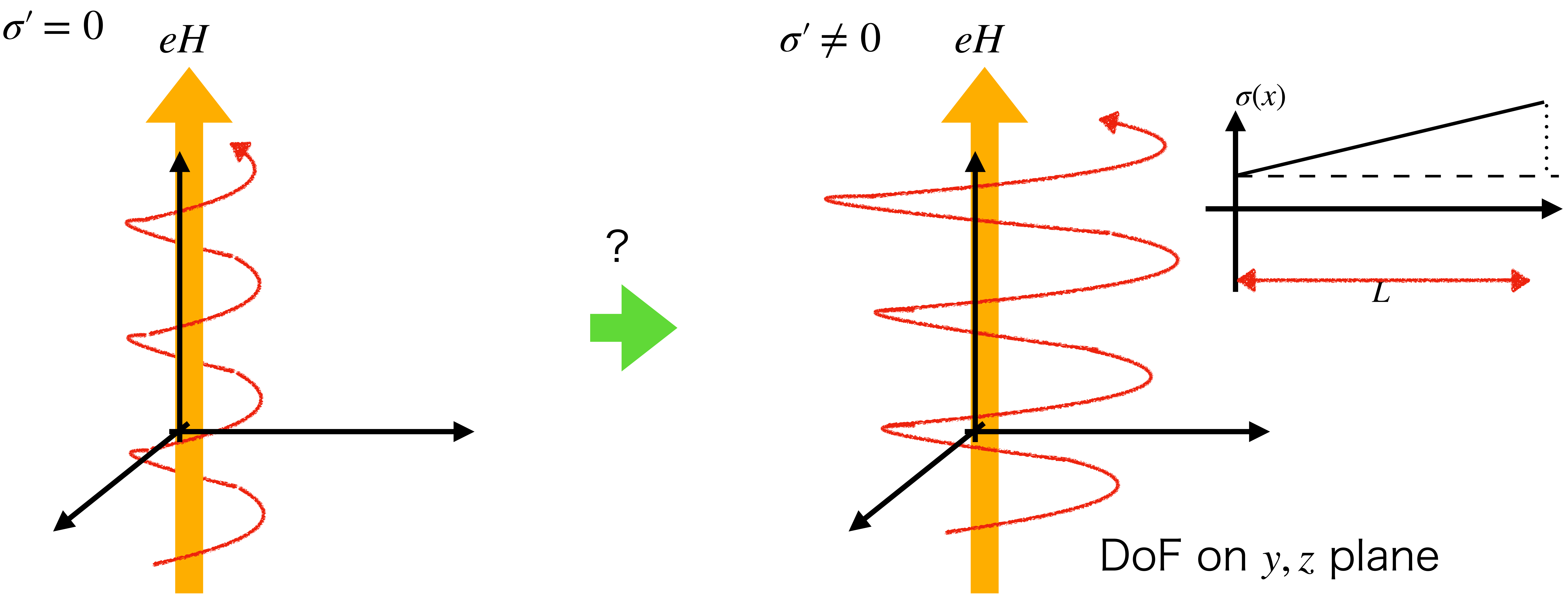
$<\sigma_0>$ - eH graph with different σ'

$\lambda_r = 4\pi^2$



magnetic catalysis is surpassed by large σ'

Discussion



dimensional reduction is suppressed because of σ'

Summary

We calculate the effective potential of four-fermi interaction model with linear correction σ' on the Magnetic field by zeta function regularization.

We analyze the behavior of the Magnetic Catalysis on $\sigma' \neq 0$.

⇒ Magnetic Catalysis is suppressed by $\sigma' \neq 0$.

future work

thermal effect, more ordinary correcton

Thank you !!

4-fermi interaction model

$$\mathscr{L} = \bar{\psi}(i \not{\partial} + e \not{A})\psi + \frac{\lambda_0}{2N}\left(\bar{\psi}\psi\right)^2$$

After path integration, the effective action is given as following.

$$S_{eff} = \int d^4x \frac{N\sigma^2}{2\lambda_0} - \mathrm{tr}\left\langle \mathrm{x} \left| \ln\left(\mathrm{i} \not{\partial} + \mathrm{e} \not{\mathrm{A}} - \sigma\right) \right| \mathrm{x} \right\rangle \qquad N \to 1$$

tr : trace over spinor and integrate with respect to space

$$S_{eff} = \int d^4x \frac{\sigma^2}{2\lambda_0} - \mathrm{tr}\int \frac{\mathrm{d^4k}}{(2\pi)^4} \mathrm{e}^{\mathrm{ikx}} \ln\left(\mathrm{i} \not{\partial} + \mathrm{e} \not{\mathrm{A}} - \sigma\right) \mathrm{e}^{-\mathrm{ikx}}$$

$$\ln[i\hat{\partial} + eA - \sigma(x)] = \frac{1}{2}\ln\left[(i\partial + eA)^2 - \frac{ie}{2}\gamma^{\mu\nu}F_{\mu\nu} - i \not{\partial}\sigma + \sigma^2\right]$$

The analyze this function is difficult because there are derivative operator in logarithmic function.

zeta function regularization

$$\ln A = -\frac{d}{ds}\frac{1}{\Gamma(s)}\int_0^\infty dt\,t^{s-1}e^{-tA}\bigg|_{s=0}, \qquad \left(\int_0^\infty dt\,t^{s-1}e^{-tA} = \Gamma(s)A^{-s},\right)$$

We can rewrite logarithmic function as exp function.

$$S_{eff} = \int d^4x\frac{\sigma^2}{2\lambda_0} + \mathrm{tr}\int\frac{d^4k}{(2\pi)^4}\frac{d}{ds}\frac{1}{\Gamma(s)}\int_0^\infty dt\,t^{s-1}e^{ikx}\exp\left(-t\left((i\partial + eA)^2 - \frac{ie}{2}\gamma^{\mu\nu}F_{\mu\nu} - i\,\partial\!\!\!/\sigma + \sigma^2\right)\right)e^{-ikx}\bigg|_{s=0}$$

When we induce $\sigma = \sigma(x), \quad A_\mu = (0,0,0,eH)$, the exp part of above equation transform as,

$$\exp\left(-t\left((i\partial + eA)^2 - \frac{ie}{2}\gamma^{\mu\nu}F_{\mu\nu} - i\,\partial\!\!\!/\sigma + \sigma^2\right)\right)$$

$$\to e^{-tk_0^2}\exp\left[tieH\gamma^2\gamma^3\right]\exp\left[t\left(\partial_y - k_z - eHy\right)\left(\partial_y + k_z + eHy\right) - teH\right]\exp\left[-t\left(\gamma^5\left(i\gamma^1\partial_x + \sigma\right)\right)^2\right].$$

We have to calculate these functions,

$$\exp\left[t\left(\partial_y - k_z - eHy\right)\left(\partial_y + k_z + eHy\right)\right]e^{-ik_y y}$$

$$\exp\left[t\left(i\gamma^1\partial_x - \sigma_0 - \sigma' x\right)\left(i\gamma^1\partial_x + \sigma_0 + \sigma' x\right)\right]e^{-ik_x x}$$

use following formula,

$$F(t,y) = \exp\left[t\left(\partial_y - a - by\right)\left(\partial_y + a + by\right)\right]e^{-ik_y y}$$

$$= \sqrt{\frac{1}{\cosh(2tb)}}\exp\left[bt - \frac{k_y^2}{2b}\tanh(2tb) + \frac{iak_y}{b} - \frac{i(a+by)k_y}{b\cosh(2tb)} - \frac{\tanh(2tb)}{2b}(a+by)^2\right]$$

Above function obeys the following equations,

$$\partial_t F(t,y) = \left(\partial_y - a - by\right)\left(\partial_y + a + by\right)F(t,y), \quad F(0,y) = e^{-ik_y y}.$$

$$S_{eff} = \int d^4x \frac{\sigma^2}{2\lambda_0} + \int d^4x \frac{\mathrm{tr}\, 1}{32\pi^2} \frac{d}{ds} \frac{1}{\Gamma(s)} \int_0^\infty dt t^{s-3} \frac{teH}{\sinh(teH)} \sqrt{\frac{t\sigma'}{\sinh(t\sigma')}} \exp\left[-\frac{\tanh(t\sigma')}{\sigma'} \left(\sigma_0 + \sigma' x\right)^2 \right] \Bigg|_{s=0}$$

$\frac{1}{\Gamma(s)}$ is approximated as s around $s=0$. If other part is finite for $s=0$ limit, we can rewrite $$\frac{d}{ds}\frac{1}{\Gamma(s)} f(s) \Bigg|_{s=0} = \frac{d}{ds}\left(sf(0) + O(s^2)\right)\Bigg|_{s=0} = f(0)$$

Unfortunately, $f(s)$ is divergent around $s=0$ in our case. So we add and subtract following term,

$$\frac{\mathrm{tr}\, 1}{32\pi^2} \frac{d}{ds} \frac{1}{\Gamma(s)} \int_0^\infty dt t^{s-3} \left(1 - \frac{1}{12}\left(2e^2H^2 + \sigma'^2\right) t^2 \right) e^{-t\left(\sigma_0 + \sigma' x\right)^2} \Bigg|_{s=0} .$$

We can perform integration over t and derivative with respect to s,

$$\frac{\mathrm{tr}\,1}{32\pi^2}\frac{d}{ds}\frac{1}{\Gamma(s)}\int_0^\infty dt t^{s-3}\left(1-\frac{1}{12}\left(2e^2H^2+\sigma'^2\right)t^2\right)e^{-t\left(\sigma_0+\sigma' x\right)^2}\Bigg|_{s=0}$$

$$=\frac{1}{8\pi^2}\left(\frac{\left(\sigma_0+\sigma' x\right)^4}{4}\left(3-2\ln\left(\sigma_0+\sigma' x\right)^2\right)+\frac{1}{12}\left(2e^2H^2+\sigma'^2\right)\ln\left(\sigma_0+\sigma' x\right)^2\right)$$

$$S_{eff}(\sigma)=\int d^4x\left(\frac{1}{2\lambda_0}\left(\sigma_0+\sigma' x\right)^2+\frac{1}{8\pi^2}\left(\frac{\left(\sigma_0+\sigma' x\right)^4}{4}\left(3-2\ln\left(\sigma_0+\sigma' x\right)^2\right)+\frac{1}{12}\left(2e^2H^2+\sigma'^2\right)\ln\left(\sigma_0+\sigma' x\right)^2\right)\right.$$

$$\left.+\frac{1}{8\pi^2}\frac{1}{\Gamma(s)}\int_0^\infty dt t^{s-3}\left(\frac{teH}{\sinh(teH)}\sqrt{\frac{t\sigma'}{\sinh(t\sigma')}}\exp\left[-\frac{\tanh(t\sigma')}{\sigma'}\left(\sigma_0+\sigma' x\right)^2\right]-\left(1-\frac{1}{12}\left(2e^2H^2+\sigma'^2\right)t^2\right)e^{-t\left(\sigma_0+\sigma' x\right)^2}\right)\Bigg|_{s=0}\right)$$

finite by integration

Back up

$$\mathcal{L} = \bar{\psi}\left(i \not{\partial} - m\right)\psi - \frac{\lambda_0}{2N}\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma^5\psi)^2\right]$$

$$\psi \to e^{i\gamma^5\theta}\psi$$
$$\bar{\psi} \to \psi^\dagger e^{-i\gamma^5\theta}\gamma^0 = \bar{\psi}e^{i\gamma^5\theta}$$

$$\bar{\psi}\psi \to \bar{\psi}\psi\cos(2\theta) + \bar{\psi}i\gamma^5\psi\sin(2\theta)$$
$$\bar{\psi}i\gamma^5\psi \to -\bar{\psi}\psi\sin(2\theta) + \bar{\psi}i\gamma^5\psi\cos(2\theta)$$

$$\left(\bar{\psi}\psi\right)^2 \to \cos^2(2\theta)\left(\bar{\psi}\psi\right)^2 + \sin^2(2\theta)\left(\bar{\psi}i\gamma^5\psi\right)^2 + \sin(4\theta)\left(\bar{\psi}\psi\right)\left(\bar{\psi}i\gamma^5\psi\right)$$

$$\left(\bar{\psi}i\gamma^5\psi\right)^2 \to \sin^2(2\theta)\left(\bar{\psi}\psi\right)^2 + \cos^2(2\theta)\left(\bar{\psi}i\gamma^5\psi\right)^2 - \sin(4\theta)\left(\bar{\psi}\psi\right)\left(\bar{\psi}i\gamma^5\psi\right)$$

$$\ln[i\hat{\partial} + eA - \sigma(x)] = \frac{1}{2}\ln\left[(i\not\partial + eA - \sigma(x))\gamma^5(i\not\partial + eA - \sigma(x))\gamma^5\right]$$
$$= \frac{1}{2}\ln\left[-(i\not\partial + eA)^2 - i\not\partial\sigma + \sigma^2\right]$$
$$= \frac{1}{2}\ln\left[-(i\partial + eA)^2 - \frac{ie}{2}\gamma^{\mu\nu}F_{\mu\nu} - i\not\partial\sigma + \sigma^2\right]$$

At first we define vector field as,

$$A^\mu = \begin{pmatrix} 0 \\ 0 \\ 0 \\ Hy \end{pmatrix}$$

and assume σ depend only one direction, x. Then above log function transform as

$$\frac{1}{2}\ln\left[\partial_t^2 - \partial_x^2 - \partial_y^2 + \left(i\partial_z + eHy\right)^2 - ieH\gamma^2\gamma^3 - i\not\partial\sigma + \sigma^2\right]$$
$$= -\frac{1}{2}\frac{d}{ds}\frac{1}{\Gamma(s)}\int_0^\infty dt t^{s-1}\exp\left[-t\left(\partial_t^2 - \partial_x^2 - \partial_y^2 + \left(i\partial_z + eHy\right)^2 - ieH\gamma^2\gamma^3 - i\not\partial\sigma + \sigma^2\right)\right]$$

The exp part is rewritten as following,

$$e^{-t\omega_n^2}\exp\left[tieH\gamma^2\gamma^3\right]\exp\left[t\left(\partial_y - k_z - eHy\right)\left(\partial_y + k_z + eHy\right) - teH\right]\exp\left[-t\left(\gamma^5\left(i\gamma^1\partial_x + \sigma\right)\right)^2\right]$$