Initial Time Renormalization of a Non-Equilibrium Effective Field Theory

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Based on: arXiv:1709.08781 & arXiv:1403.0733

Outline

- Introduction
  - Particle Number Asymmetry (PNA)
- Initial time divergences
  - 1-Loop calculations
- Literature review
- Current application of solution
- Summary

Introduction

- Baryon Asymmetry Problem
  - How did Universe become matter dominated?

$$\eta = \frac{n_B - n_{\bar{B}}}{n_\gamma} \approx 6.09 \times 10^{-10}$$

Source: Planck 2015

n_B is baryon number
$n_{\bar{B}}$ is anti-baryon number
n_γ is photon number

- Standard Model is not enough

$$\eta_{SM} \sim 10^{-18}$$
$$\eta_{SM} \ll \eta \sim 10^{-10}$$

Too Small!

Introduction

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Too Small!

Goal: Design novel effective field theory to explain baryon asymmetry

Introduction

- **Solutions should follow Sakharov Conditions**

Charge-Parity (CP) Violation	Baryon Number (n_B) Violation	Departure from Thermal Equilibrium

- Matter/Antimatter different charges

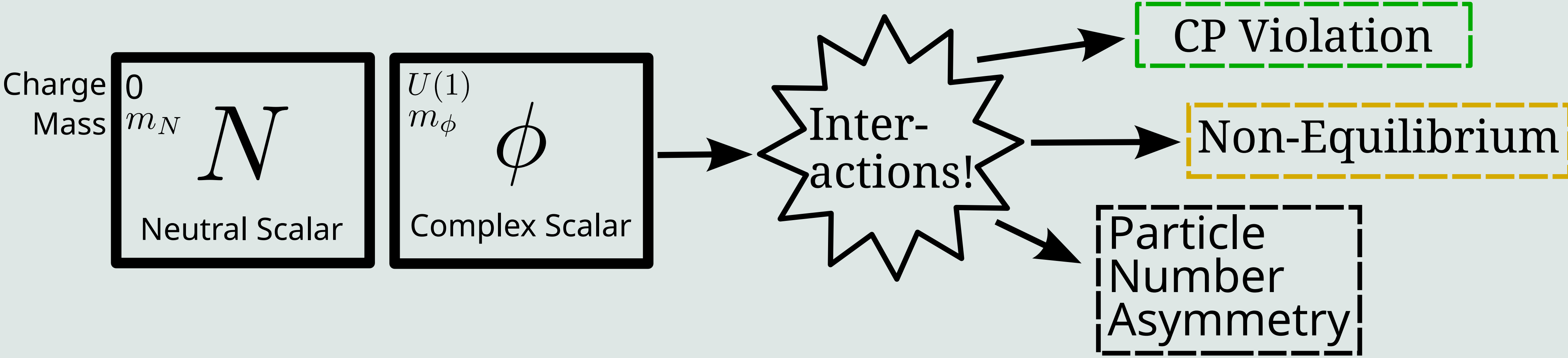
$$n_B > n_{\bar{B}}$$

$$\eta = \frac{n_B - n_{\bar{B}}}{n_\gamma}$$

$$n_B \neq n_{\bar{B}}$$

Introduction

- Possible Solution proposed (2019 Morozumi, Nagao, Adam, Takata)
  - Called Particle Number Asymmetry (PNA)



Introduction

- Important parts of PNA model
  - Global symmetry is explicitly broken by interaction

$$\mathcal{L}_{int} = A\phi^2 N + A^* \phi^{2\dagger} N + A_0 |\phi|^2 N$$

TABLE I. The cubic interactions and their properties

Cubic interaction coupling	Property
$A_{113} = \frac{A_0}{2} + \mathrm{Re.}(A)$	–
$A_{223} = \frac{A_0}{2} - \mathrm{Re.}(A)$	–
$A_{113} - A_{223} = 2\mathrm{Re.}(A)$	U(1) violation
$A_{123} = -\mathrm{Im.}(A)$	U(1), CP violation

“A new mechanism for generating particle number asymmetry through interactions” T. Morozumi, K. I. Nagao, A. S. Adam, H. Takata; Adv.High Energy Phys. 2019 DOI: 10.1155/2019/6825104

Introduction

- Goals for improvement of PNA Model
  - Identify the scalars
  - Translate PNA to Baryon Number Asymmetry
  - Potential stability issues
  - Renormalization questions

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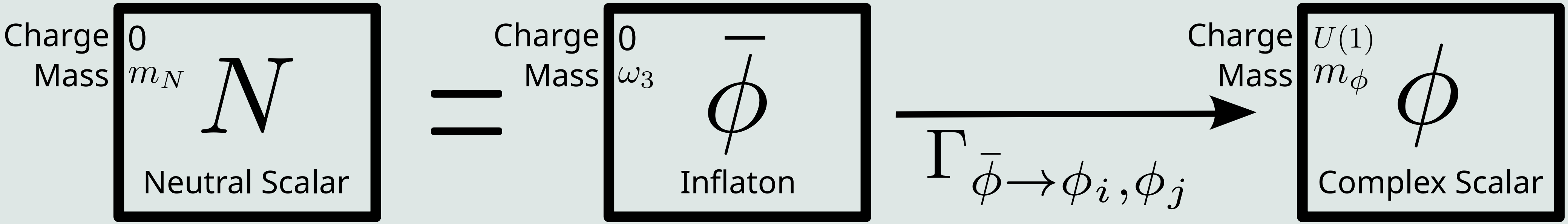
Current Work

- Identified the Neutral Scalar as the Inflaton
  - Calculate decay (interaction) into complex scalar
- Inflaton is proposed field from Cosmology



Current Work

- Goal: Calculate Inflaton evolution with decay
  - Treat decay as friction term

Friction term

Bare Mass

Constant Term

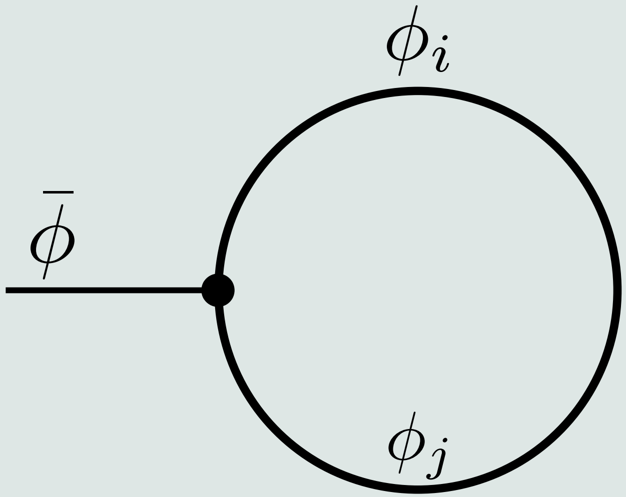
$$\left[\frac{\partial^2}{\partial x^{0^2}} + \left(3H(x^0) + \Gamma^3_{\bar{\phi}}\right)\frac{\partial}{\partial x^0} + \omega^2_{3,0}\right]\bar{\phi}^2_3(x^0) = F$$

Hubble Term

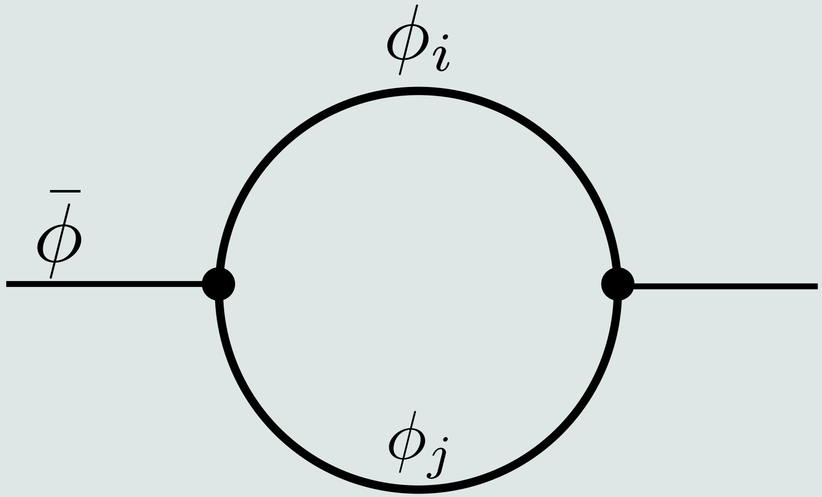
Preliminary

Initial Time Divergence Problem

- In F we find two 1-loop diagrams
  - Modifies mass of Inflaton $\omega_{3,0}^2 \rightarrow \Omega_3^2(T)$



Tadpole Diagram



One-Loop Self Energy

Initial Time Divergence Problem

- After mass modification divergences appear

Bare Mass

Cause of divergences

$$\begin{aligned}\lim_{T\to\infty}\Omega_3^2(T) \equiv\ & \omega_{3,\mathbf{0}}^2 \\ & -\tfrac{1}{2}\sum_{i,j=1}^{2} A_{ij3}^2 \int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{1}{\omega_{i,\mathbf{k}}\omega_{j,\mathbf{k}}}\left(\frac{1-cos(C_{ij,\mathbf{k}}T)}{C_{ij,\mathbf{k}}}+\frac{1-cos(B_{ij,\mathbf{k}}T)}{B_{ij,\mathbf{k}}}\right) \\ =\ & \infty\end{aligned}$$

Quantum Self-Energy Corrections

Not Physically Useful

Initial Time Divergence Problem

- To fix this we follow renormalization
  - Add “bulk counter term” to remove divergences

$$\Omega^2_{3R}(T) = \Omega^2_3(T) + \delta\omega^2_3$$

Renormalized Mass

Bulk counter term

- However this has a problem at initial time

$$\begin{aligned} \Omega^2_{3R}(T=0) \quad &= \Omega^2_3(T=0) + \delta\omega^2_3 \\ &= \omega^2_{3,\mathbf{0}} + \delta\omega^2_3 \\ &= \infty \end{aligned}$$

Bulk counter term is infinite!

Preliminary

Initial Time Divergence Problem

- **Summary of Problem:**

$$\lim_{T\to\infty} \Omega_3^2(T) = \infty$$

Quantum corrected mass is divergent

$$\Omega_{3R}^2(T) = \Omega_3^2(T) + \delta\omega_3^2$$

Renormalized mass

Bulk counter term

$$\Omega_{3R}^2(T=0) = \infty$$

Renormalized mass is divergent!

Preliminary

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Initial Time Renormalization

- We look to Collins, etc. for a possible solution
  - “Renormalization of Initial Conditions and the Trans-Planckian Problem of Inflation” by Collins and Holman (2005)
- Goal: To develop a renormalization scheme for arbitrary initial states

Collins (2005)

Initial Time Renormalization

- Consider simple $\lambda\varphi^4$ non-equilibrium theory:

$$\varphi(t,\vec{x}) = \int \frac{d^3\vec{k}}{(2\pi)^3}\left[U_k(t)e^{i\vec{k}\cdot\vec{x}}a_{\vec{k}} + U_k(t)^* e^{-i\vec{k}\cdot\vec{x}}a^\dagger_{\vec{k}}\right]$$

arbitrary time-dependent modes

- Initial Condition

$$\frac{\partial}{\partial t}U_k(t)\Big|_{t=t_0} = -i\overline{\omega}_k U_k(t_0)$$

Collins (2005)

Initial Time Renormalization

- Identify field as Inflaton in expanding background

$$\varphi(t,x) = \phi(t) + \psi(t,x)$$

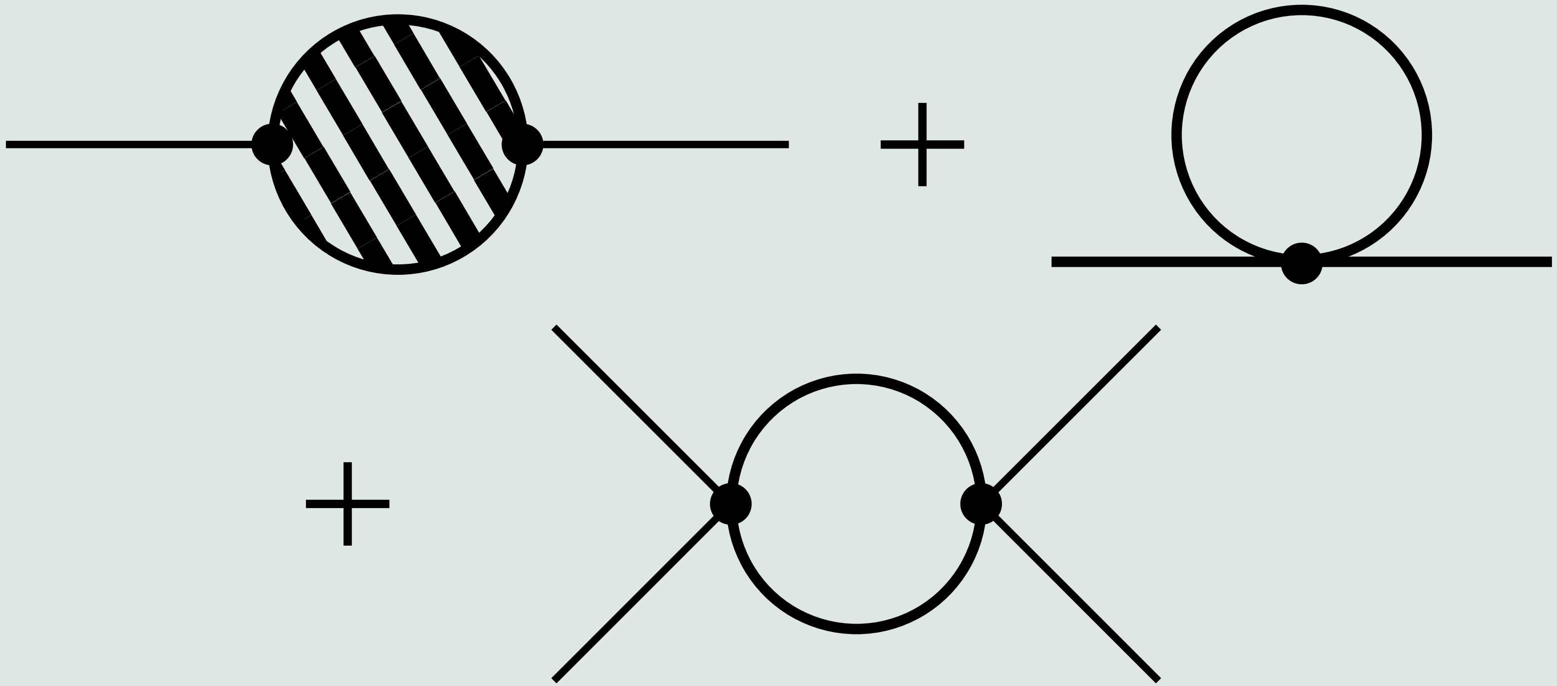
classical mode   quantum fluctuations

- Consider simple renormalization condition
  - Vanishing of the tadpole

$$\langle \alpha_k(t) \left| \psi^+(x) \right| \alpha_k(t) \rangle = 0$$

Initial Time Renormalization

- Leading order correction diagrams (Collins 2005)



Collins (2005)

Initial Time Renormalization

- Separate terms that depend on the initial condition

$$\langle \alpha_k(t) \left| \psi^+(x) \right| \alpha_k(t) \rangle \xrightarrow[\mathcal{O}(H_I^2)]{} \underbrace{A(\omega_k)}_{\text{bulk terms}} + \underbrace{B(\overline{\omega}_k)}_{\text{boundary terms}}$$

- Apply dimensional regularization
  - Classify the type of boundary divergences
  - Linear or logarithmic

Collins (2005)

Initial Time Renormalization

- Lastly, prepare counter terms to cancel divergences
  - Follow minimal subtraction scheme

$$z \propto \left[\frac{1}{\epsilon} - \gamma + \ln \frac{4\pi\mu^2}{m^2} \right]$$

Example of counter term

Collins (2005)

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Application

- Goal: To apply Collins' ideas to our theory
  - However, there are differences

	Collins	Us
Interactions	$\lambda\phi^4$	$A_{ijk}\phi^3$
Initial conditions	$-i\overline{\omega_k}U_k(t_0)$	$\rho(t_0)$

Application

- We update our theory to be 3+4 scalar theory

symmetry breaking cubic terms

$$\mathcal{L} = \frac{1}{2}\sum_{i=1}^{3}\partial_\mu\varphi_i\partial^\mu\varphi_i - \frac{1}{2}e\sum_{ij}\varphi_i m_{ij}^2\varphi_j - \frac{1}{2}m_3^2\varphi_3^2 + \sum_{ijk}\frac{A_{ijk}}{3}\varphi_i\varphi_j\varphi_k$$

$$-\frac{\lambda_1}{4}\left(\sum_i \varphi_i^2\right)^2 - \frac{\lambda_2}{4}\left(\sum_i \varphi_i^2\right)\varphi_3^2 - \frac{\lambda_3}{4!}\varphi_3^4 + \Lambda_3\varphi_3 + \Lambda_4$$

new quartic terms

Preliminary

Application

- To study application we start simple
  - Suppress $A_{ijk}\phi^3$ interactions and up to $\mathcal{O}(H_I)$

- We take the tadpole condition,

$$\langle\alpha_k(t_0)|T_a\Psi_m(x)\left(-i\int dy^0[H_I(\phi_i,\Psi_i^+)-H_I(\phi_i,\Psi_i^-)]\right)|\alpha_k(t_0)\rangle=0$$

Preliminary

Application

- **Terms similar to below appear**

$$\beta_j(t) = \int \frac{d^3k}{2\omega_k^j(2\pi)^3}\left(1 + e^{\alpha_k^j} e^{2i\omega_k^j(t_0 - t)}\right)$$

Field number

boundary indicator

- **After dimensional regularization 2 divergences appear**

$$\left(\frac{1}{\epsilon} + 1 - \gamma + \log 4\pi + \log \frac{\mu^2}{m_i^2}\right)$$

Bulk Divergence

$$\left(\frac{1}{\epsilon} - \gamma + \log 4\pi + \log \frac{\mu^2}{m_i^2}\right)$$

Boundary Divergence

Preliminary

Application

- **Boundary counter term**

$$\mathcal{O}(H_{bnc}) = \frac{1}{2}\left(z_{0m}\frac{d}{dt_0} - z_{1m}m_m\right)\phi_m(t_0)\frac{\sin m_m(t_f - t_0)}{m_m}$$

Thus, for the simple case we can renormalize the boundary!

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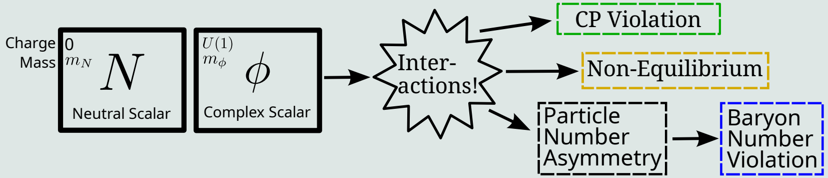
Summary

- Our overall goal is to explain Baryon Asymmetry
  - Started with interacting theory



  - Next, identified N as inflaton and found divergences

$$\Omega_{3R}^2(T) = \Omega_3^2(T) + \delta\omega_3^2$$

  - Studied Collins for incite into a solution
  - Applied Collins to simple case successfully!

Thank you!

Questions?