A Study of Renormalization Group Effects on the Mass of the Lightest Neutrino

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with
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Based on: arXiv:2020.XXXXX

Outline

- Introduction
  - Massive Neutrino Limits
  - Type-I Seesaw Mechanism
- Renormalization Group Flow
  - (3,3) Type Model
  - (3,2) “Missing Partner” Type Model
- Conclusion

Introduction: Massive Neutrino Limits

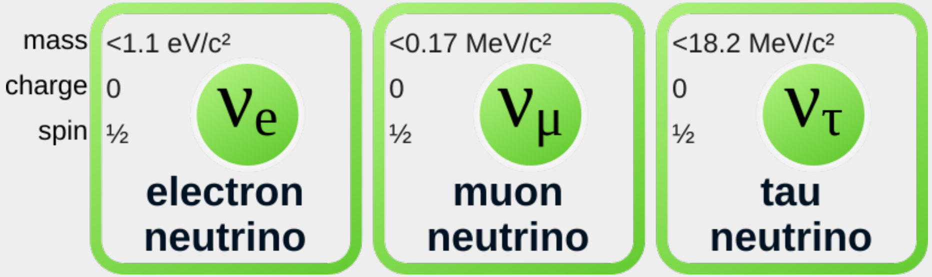
- Observation of Neutrino Flavor Oscillations (1998)
  - Implied Neutrinos have a non-zero mass
- Currently, well established in experiments

Y. Fukuda et al. (Super-Kamiokande Collaboration)
Phys. Rev. Lett. 81, 1562
DOI: 10.1103/PhysRevLett.81.1562

Introduction: Massive Neutrino Limits

- KATRIN Experiment results (Nov. 2019)
  - Tritium Decay Experiment
  - Upper limit of lightest Neutrino absolute mass
- Limit of 1.1 eV (90% confidence level)
  - No lower limit

lightest mass of 0 is allowed



M. Aker et al. (KATRIN Collaboration)
Phys. Rev. Lett. 123, 221802
DOI: 10.1103/PhysRevLett.123.221802

Introduction

- Fermion masses in Standard Model are arbitrary
  - Must be determined experimentally
- However, neutrino masses are tiny compared to all others
  - A goal of theory is to understand why

- To understand the Standard Model must be extended

Introduction

- **An effective nonrenormalizable D=5 operator can lead to small neutrino masses (Weinberg, 1979)**

$$-\mathcal{L}_{\text{new}} = \frac{1}{2}\left(\bar{l}_L H\right)\kappa\left(H^T l_L^c\right) + h.c$$

electroweak symmetry breaking

$$v = 246 GeV$$

$$m_\nu \equiv \kappa \frac{v^2}{2}$$

m_ν neutrino mass

H SM Higgs doublet

Introduction

- **An effective nonrenormalizable D=5 operator can lead to small neutrino masses (Weinberg, 1979)**

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electroweak symmetry breaking

$v = 246GeV$

m_ν neutrino mass

H SM Higgs doublet

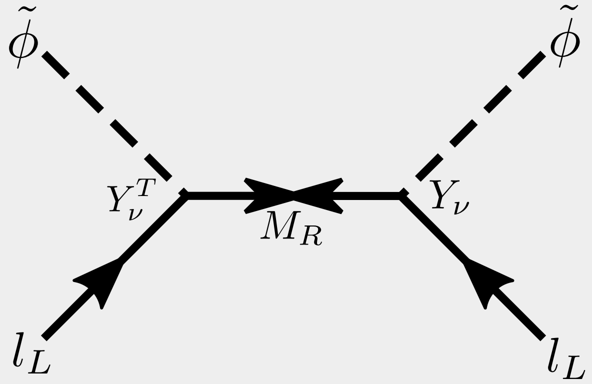
what is kappa?

$$m_\nu \equiv \kappa\frac{v^2}{2}$$

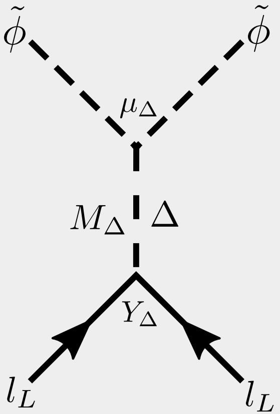
Introduction

- Type-I Seesaw
  - Right-Handed Singlets

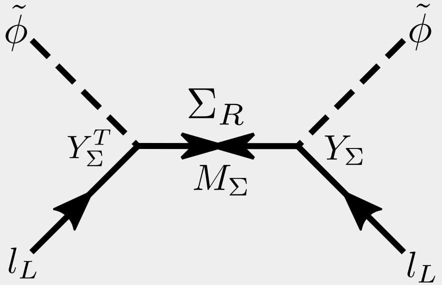
$$m_\nu = -Y_\nu^T \frac{v}{\sqrt{2}} \frac{1}{M_R} \frac{v}{\sqrt{2}} Y_\nu$$



- Type-II Seesaw
  - Scalar Triplet



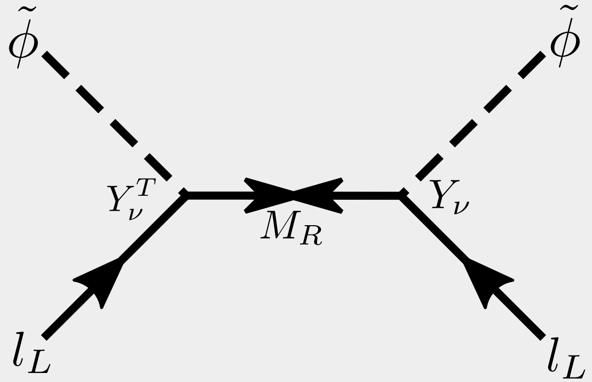
- Type-III Seesaw
  - Fermion Triplet



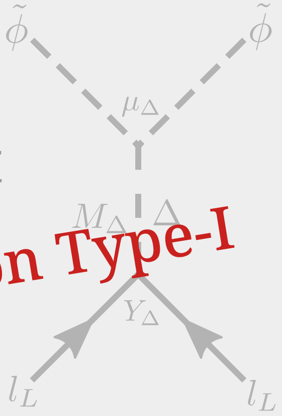
Introduction

- **Type-I Seesaw**
  - Right-Handed Singlets

$$m_\nu = -Y_\nu^T \frac{\nu}{\sqrt{2}} \frac{1}{M_R} \frac{\nu}{\sqrt{2}} Y_\nu$$

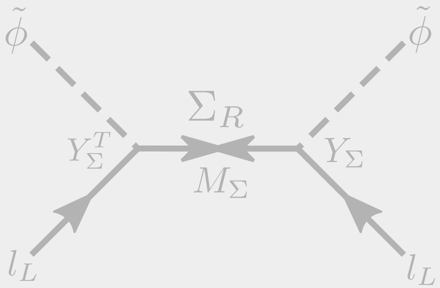


- Type-II Seesaw
  - Scalar Triplet



we focus only on Type-I

- Type-III Seesaw
  - Fermion Triplet



Introduction

3 Possible Type-I Seesaws

- (3,3) Model
  - 3 Heavy Right-handed singlets

$$M_3, M_2, M_1$$

- (3,2) Model
  - 2 heavy right-handed singlets
  - “Missing partner”

$$M_2, M_1$$

- (3,1) Model
  - 1 heavy right-handed singlet
  - “Minimal Type-I Seesaw”

$$M_1$$

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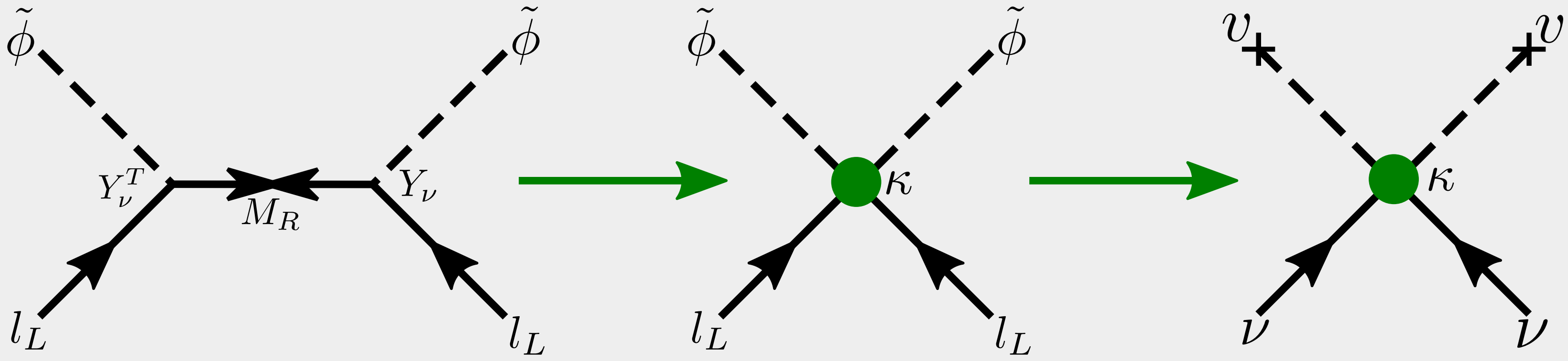
Renormalization Group Flow

- Higher energies can effect lower energy physics
  - Method pioneered by Wilson (late 1900's)
- Energies higher than a cut-off are integrated out
  - Effects of the integration are carried into the lower energy region



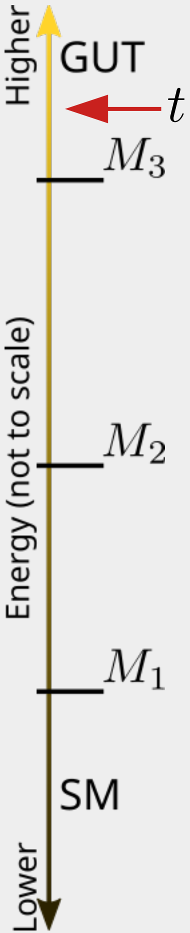
$$m_\nu \equiv \kappa \frac{v^2}{2}$$

Renormalization Group Flow

- Our application is to see if a Type-I seesaw places a lower limit on the lightest neutrino mass
  - Focus on (3,3) model and (3,2) model
  - One-loop renormalization is considered

Detailed information on how the equations are determined for different models:
Stefan Antusch et al JHEP03(2005)024
https://doi.org/10.1088/1126-6708/2005/03/024

RGE (3,3) Seesaw Model



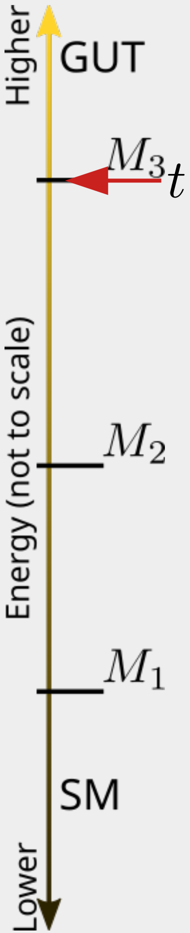
- First we calculate the effect in a (3,3) model
  - We constrain our model with texture analysis to a Rank 2 Dirac mass matrix
  - Or we can say the determinant of the Dirac mass matrix is zero

$$m_D^0 = \begin{pmatrix} m_{D1}^0 & m_{D2}^0 & m_{D3}^0 \end{pmatrix} \qquad m_{D3}^0 = a m_{D1}^0 + b m_{D2}^0$$

- We start with Beta function equations

$$16\pi^2 \beta_\kappa \qquad 16\pi^2 \beta_{m_{eff}} \qquad 16\pi^2 \beta_y$$

RGE (3,3) Seesaw Model



- To determine the modified neutrino mass term we must match the scales
  - This is done by considering at the energy equal to M_3

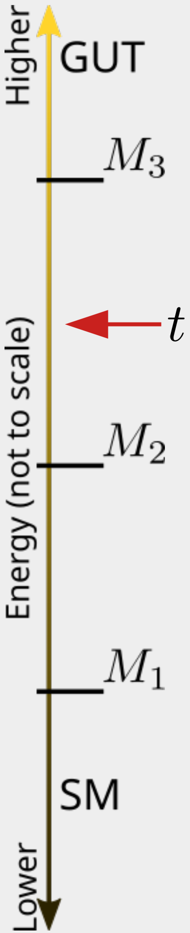
$$m_T(0_+) = m_T(0_-) = \kappa^{(3)}(0) + m_{eff}^{(3)}(0)$$

Where,

$$\kappa^{(3)}(0) = -m_{D3}^{0} \frac{1}{M_3} m_{D3}^{0T}$$

$$m_{eff}^{(3)}(0) = -m_D^{(3)}(0) \frac{1}{M^{(3)}(0)} m_D^{(3)T}(0)$$

RGE (3,3) Seesaw Model



- Next we integrate out the heaviest mass term
  - Effectively lowering the energy region

$$16\pi^2 \frac{d\kappa^{(3)}(t)}{dt} = (P^{(3)}(t))^T \kappa^{(3)}(t) + \kappa^{(3)}(t) P^{(3)}(t) + \alpha_\kappa^{(3)}(t)\kappa^{(3)}(t)$$

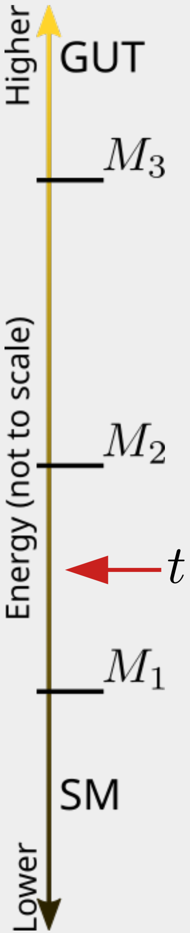
$$16\pi^2 \frac{dm_{eff}^{(3)}(t)}{dt} = (P^{(3)}(t))^T m_{eff}^{(3)}(t) + m_{eff}^{(3)}(t) P^{(3)}(t) + \alpha_{eff}^{(3)}(t) m_{eff}^{(3)}(t)$$

- Which results in the modified neutrino mass term

$$m_T(t_2) = \kappa^{(3)}(t_2) + m_{eff}^{(3)}(t_2)$$

Preliminary

RGE (3,3) Seesaw Model



- Then we integrate out the next mass term with
  - Similar method as before

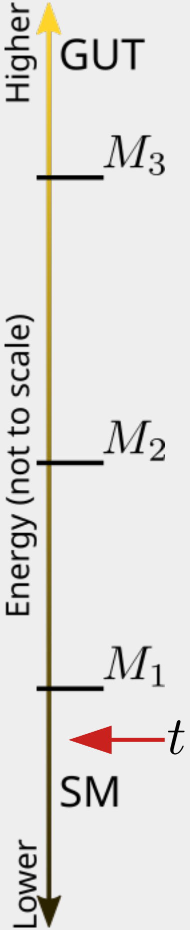
$$16\pi^2 \frac{d\kappa^{(2)}(t)}{dt} = (P^{(2)}(t))^T \kappa^{(2)}(t) + \kappa^{(2)}(t) P^{(2)}(t) + \alpha_\kappa^{(2)}(t)\kappa^{(2)}(t)$$

$$16\pi^2 \frac{dm_{eff}^{(2)}(t)}{dt} = (P^{(2)}(t))^T m_{eff}^{(2)}(t) + m_{eff}^{(2)}(t) P^{(2)}(t) + \alpha_{eff}^{(2)}(t) m_{eff}^{(2)}(t)$$

- Which results results in a lower scale mass

$$m_T(t_1) = \kappa^{(2)}(t_1) + m_{eff}^{(2)}(t_1)$$

RGE (3,3) Seesaw Model



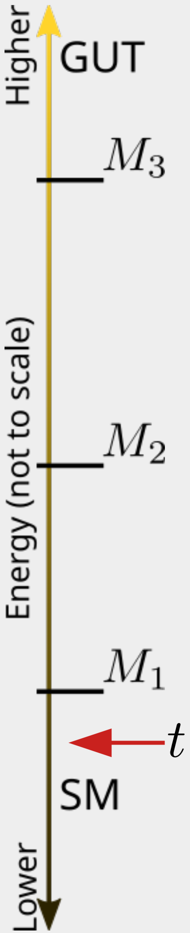
- Last we integrate out the lightest mass term
  - Back to SM level energies

$$16\pi^2 \frac{d\kappa^{(1)}(t)}{dt} = (P^{(1)}(t))^T \kappa^{(1)}(t) + \kappa^{(1)}(t) P^{(1)}(t) + \alpha_\kappa^{(1)}(t) \kappa^{(1)}(t)$$

- Which results in the neutrino mass matrix

$$m_T(t_z) = \kappa^{(1)}(t_z)$$

RGE (3,3) Seesaw Model



- Now we can investigate the low energy mass matrix
  - Asking the question: Is a zero mass still allowed?

$$\kappa^{(1)}(t_1) = k^{(3)}(t_1 - t_2) + m^{(3)}_{eff}(t_1 - t_2) + m^{(2)}_{eff}(t_1 - t_2) - m^{(1)}_{eff}(t_1)$$

- If a texture is required for the heavy mass matrix
  - This results in a low energy matrix of,

$$\det \kappa^{(1)}(t_1) \neq 0$$

All low energy neutrinos are massive

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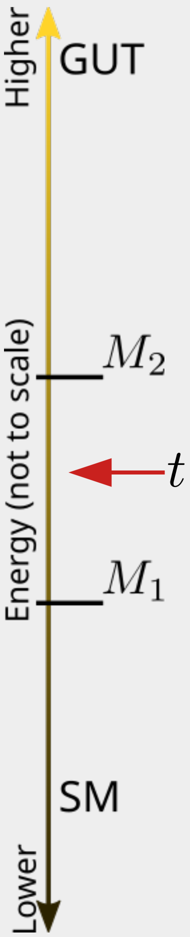
RGE (3,2) Seesaw Model



- Similar to before we start at the highest energy scale
- We then integrate out the heaviest neutrino
  - Again we use Beta equations

$$16\pi^2 \frac{\kappa^{(2)}(t)}{dt} = (P^{(2)}(t))^T \kappa^{(2)}(t) + \kappa^{(2)}(t) P^{(2)}(t) + \alpha_\kappa^{(2)}(t)\kappa^{(2)}(t)$$

$$16\pi^2 \frac{m_{eff}^{(2)}(t)}{dt} = (P^{(2)}(t))^T m_{eff}^{(2)}(t) + m_{eff}^{(2)}(t) P^{(2)}(t) + \alpha_{eff}^{(2)}(t) m_{eff}^{(2)}(t)$$

- This results in a neutrino mass matrix

$$m_T(t_1) = \kappa^{(2)}(t_1) + m_{eff}^{(2)}(t_1)$$

RGE (3,2) Seesaw Model

- Lastly, we integrate out the last heavy neutrino
  - Resulting mass equation is

$$m_T(t_1) = \kappa^{(1)}(t_1) = m^{(2)}_{eff}(t_1) + \kappa^{(2)}(t_1)$$

- Now we can investigate the low energy mass matrix
  - Here we have the following complete equation,

$$k^{(1)}(t_1) = -\frac{1}{R^{(2)}M_1} r(t_1) m^0_{D1} m^{0T}_{D1} r^T(t_1) - \frac{e^{\frac{1}{16\pi^2}\int_0^{t_1} ds\,\alpha_\kappa^{(2)}(s)}}{M_2} U^{(2)}(t_1) m^0_{D2} m^{0T}_{D2} U^{T(2)}(t_1)$$

RGE (3,2) Seesaw Model

- **To determine the rank of Kappa we note that**

$$r(t_1)m^0_{D1} = \begin{pmatrix} m^0_{D1} & m^0_{D2} & m^{ext}_D \end{pmatrix} \begin{pmatrix} \alpha_{11} \\ \alpha_{21} \\ \alpha_{31} \end{pmatrix} \qquad U^{(2)}(t_1)m^0_{D2} = \begin{pmatrix} m^0_{D1} & m^0_{D2} & m^{ext}_D \end{pmatrix} \begin{pmatrix} \alpha_{12} \\ \alpha_{22} \\ \alpha_{32} \end{pmatrix}$$

- **This results in the following**

$$k^{(3)}(t_1) = -\frac{1}{M_1(t_1)} m_D^{(3)} \begin{pmatrix} \alpha_{11}^2 & \alpha_{11}\alpha_{21} & \alpha_{11}\alpha_{31} \\ \alpha_{11}\alpha_{21} & \alpha_{21}^2 & \alpha_{21}\alpha_{31} \\ \alpha_{11}\alpha_{31} & \alpha_{21}\alpha_{31} & \alpha_{31}^2 \end{pmatrix} m_D^{T(3)} - \frac{1}{M_2(t_1)} m_D^{(3)} \begin{pmatrix} \alpha_{12}^2 & \alpha_{12}\alpha_{22} & \alpha_{12}\alpha_{32} \\ \alpha_{12}\alpha_{22} & \alpha_{22}^2 & \alpha_{22}\alpha_{32} \\ \alpha_{12}\alpha_{32} & \alpha_{22}\alpha_{32} & \alpha_{32}^2 \end{pmatrix} m_D^{T(3)}$$

lowest neutrino maintains massless-ness

$$\det \kappa^{(3)}(t_1) = 0$$

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Conclusion

- We have calculated two the 1-loop effects on the lower mass neutrinos
  - Specifically we considered the case of a Dirac mass matrix with rank 2
  - We also only considered the Type-I seesaw
- Our results show
  - (3,3) with an initial Dirac mass matrix of rank 2 can be increased in rank due to renormalization effects
  - (3,2) the Dirac mass matrix does not change rank after the renormalization effects

Thank you!

Questions?