Towards the minimal seesaw model for the prediction of neutrino CP violation

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7, March, 2018 at Hiroshima Univ.

JHEP **1711** (2017) 201 & Phys. Lett. B **778** (2018) 6

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Outline

- **Introduction**
  --- Background and motivation

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- **Model**
  --- Setup for minimal seesaw model

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- **Prediction of Dirac CP violating phase**

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- **Baryon asymmetry in the Universe (BAU) and CP violation**

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- **Summary and discussions**

Background and motivation

CP violating interaction is necessary
for the Baryon Asymmetry in the Universe (BAU).

Sakharov's three conditions

- Baryon number violation
- C and CP violation
- Interact out of thermal equilibrium era

Kobayashi-Maskawa model :
Mixing among three flavors can violate CP symmetry (quark sector)

$$V_{CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{13}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{CP}} & c_{13}c_{23} \end{pmatrix}$$

The origin of CP violation closely relates to the flavor structure

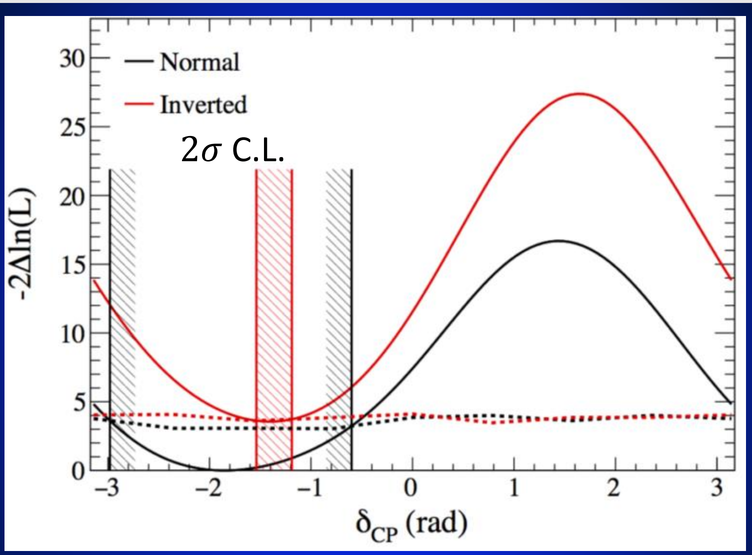
CP violating phase in the lepton sector δ_{CP}:

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{13}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{CP}} & c_{13}c_{23} \end{pmatrix} \begin{pmatrix} 1 & & \\ & e^{i\alpha} & \\ & & e^{i\beta} \end{pmatrix}.$$

CP conservation ($\delta_{CP} = 0, \pm\pi$) is excluded in 2σ C.L.

$\delta_{CP} = -\frac{\pi}{2}$ may be favored ?

Is there something symmetric structure?



from T2K web site (August, 4, 2017)

2σ C.L.: Normal Hierarchy (NH) $[-171°, -34.4°]$
Inverted Hierarchy (IH) $[-88.2°, -68.2°]$

How to predict CP violating phase

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{13}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{CP}} & c_{13}c_{23} \end{pmatrix} \begin{pmatrix} 1 & & \\ & e^{i\alpha} & \\ & & e^{i\beta} \end{pmatrix}.$$

PMNS mixing matrix is derived from neutrino mass matrix.

9 parameters contribute to the PMNS mixing matrix at most

Global experimental data of neutrino oscillation NuFIT 3.2 (2018), JHEP **01** (2018) 087

3σ interval	Normal Hierarchy	Inverted Hierarchy
Δm_{12}^2	$[6.80\,, 8.02] \times 10^{-5} [\mathrm{eV}^2]$	$[6.80\,, 8.02] \times 10^{-5} [\mathrm{eV}^2]$
Δm_{13}^2	$[2.399\,, 2.593] \times 10^{-3} [\mathrm{eV}^2]$	$-[2.369\,, 2.562] \times 10^{-3} [\mathrm{eV}^2]$
$\sin^2\theta_{12}$	$[0.272\,, 0.346]$	$[0.272\,, 0.346]$
$\sin^2\theta_{23}$	$[0.418\,, 0.613]$	$[0.435\,, 0.616]$
$\sin^2\theta_{13}$	$[1.981\,, 2.436] \times 10^{-2}$	$[2.006\,, 2.452] \times 10^{-2}$

5 parameters are available

Approaches to δ_{CP} -- reduce model parameters --

(A). 2 right-handed (RH) Majorana neutrinos

-- The lightest neutrino becomes massless.

(B). Flavor symmetry (A_4, S_4, A_5, etc.)

-- control Yukawa couplings in the Lagrangian.
-- introduce gauge singlet scalars (called as “flavons”).

(C). Texture zeros

-- put zeros in some elements of the neutrino mass matrix.
-- can not construct the Lagrangian.

Our model is a combination of the three methods

(--). First setting (without loss of generality)

-- Diagonal basis of charged lepton mass matrix

$$M_l = \begin{pmatrix} m_e & & \\ & m_\mu & \\ & & m_\tau \end{pmatrix} \qquad U_{PMNS} = U_l^\dagger U_\nu = U_\nu$$

(A). 2 right-handed (RH) Majorana neutrinos

$$M_R = \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix} = M_2 \begin{pmatrix} p^{-1} & 0 \\ 0 & 1 \end{pmatrix} \qquad p = \frac{M_2}{M_1}$$

We can take diagonal basis of M_R in the seesaw mechanism

$$M_\nu = -M_D M_R M_D^T$$

(B). Flavor symmetry (A_4 or S_4 are implied)

--Assume tri-maximal mixing

TM_1 (fix first column)

$$U_{PMNS} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & e^{i\sigma}\sin\phi \\ 0 & -e^{-i\sigma}\sin\phi & \cos\phi \end{pmatrix}$$

TM_2 (fix second column)

$$U_{PMNS} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \cos\phi & 0 & e^{i\sigma}\sin\phi \\ 0 & 1 & 0 \\ -e^{-i\sigma}\sin\phi & 0 & \cos\phi \end{pmatrix}$$

named by W. Rodejohann *et al.*

tri-bimaximal (TBM) mixing

$$V_{TBM} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Harrison, Perkins, Scott
Phys. Lett. B **458**, (1999) 79

→ We focus on TM_1 here

TM_2 will be discussed in numerically…

TM_1 realization

We obtain the following Dirac mass matrix:

$$M_D = v\begin{pmatrix} \frac{b+c}{2} & \frac{e+f}{2} \\ b & e \\ c & f \end{pmatrix}$$

This leads to TM_1 mixing with NH.

- IH pattern will be shown numerically.

$v \sim 174.1$ GeV : Higgs doublet vacuum expectation value

Assume the relative phase between b and c to be 0 or π. → b/c is real.

(C). Texture zeros -- finalize the model minimization --

- impose a 0 in the Dirac mass matrix.

$$M_D = v\begin{pmatrix} 0 & \frac{e+f}{2} \\ b & e \\ -b & f \end{pmatrix}$$

Case I

$$M_D = v\begin{pmatrix} \frac{b}{2} & \frac{e+f}{2} \\ b & e \\ 0 & f \end{pmatrix}$$

Case II

$$M_D = v\begin{pmatrix} \frac{c}{2} & \frac{e+f}{2} \\ 0 & e \\ c & f \end{pmatrix}$$

Case III

Excluded from 3σ interval (off the edge but near)

Symmetry realization by S_4

Dirac mass term : $\mathcal{L}_D = \frac{y_1}{\Lambda}\boldsymbol{\phi_1} L\, H_u\, \nu^c_{R1} + \frac{y_2}{\Lambda}\boldsymbol{\phi_2}\, L\, H_u\, \nu^c_{R2}$

TM_1 with NH

$$M_D = v\begin{pmatrix} \frac{b+c}{2} & \frac{e+f}{2} \\ b & e \\ c & f \end{pmatrix} \qquad \langle\phi_1\rangle \sim \begin{pmatrix} \frac{b+c}{2} \\ c \\ b \end{pmatrix} \qquad \langle\phi_2\rangle \sim \begin{pmatrix} \frac{e+f}{2} \\ f \\ e \end{pmatrix}$$

$$SU^+\langle\phi_1\rangle = \langle\phi_1\rangle$$

$$SU^+\langle\phi_2\rangle = \langle\phi_2\rangle$$

-- residual Z_2 symmetry from S_4

generators of S_4 : $S, T, U^\pm$

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix},\quad T = \begin{pmatrix} 1 & & \\ & \omega^2 & \\ & & \omega \end{pmatrix},\quad U^\mp = \mp\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$-$: for 3

$+$: for $3'$

Profile of case I

The neutrino mass matrix (seesaw mechanism) $M_\nu = -M_D M_R M_D^T$

in the TBM basis:

$$M_\nu^{TBM} \equiv V_{TBM}^T M_\nu V_{TBM} = -\frac{f^2 v^2}{M_2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{3}{4}(k+1)^2 & -\frac{\sqrt{3}}{2\sqrt{2}}(k^2-1) \\ 0 & -\frac{\sqrt{3}}{2\sqrt{2}}(k^2-1) & \left\{2|B|^2 p\, e^{2i\phi_B} + \frac{1}{2}(k-1)^2\right\} \end{pmatrix}$$

$k \equiv e/f$

$|B|e^{i\phi_B} \equiv b/f$

k can be make **real** by freedom of the phase redefinition.

- 3 model parameters in the mixing matrix : $\left\{k,\ |B|'\left(\equiv |B|\sqrt{p}\right),\ \phi_B\right\}$

- Jarlskog invariant :

This factor determines the sign of $\sin\delta_{CP}$.

$$J_{CP} = -\frac{3}{8}\frac{f^{12}}{M_0^6}(|B|\sqrt{p})^6(k+1)^4\boxed{(k^2-1)\sin 2\phi_B}\frac{v^{12}}{(\Delta m_{13}^2 - \Delta m_{12}^2)\Delta m_{13}^2 \Delta m_{12}^2} \propto \sin\delta_{CP}$$

Profile of TM_1 with IH

$$M_\nu^{TBM} = -\frac{v^2}{M_2}\begin{pmatrix} 6b^2p & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \frac{f^2v^2}{M_2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{3}{4}\left(ke^{i\phi_k}+1\right)^2 & -\frac{\sqrt{3}}{2\sqrt{2}}\left(k^2e^{2i\phi_k}-1\right) \\ 0 & -\frac{\sqrt{3}}{2\sqrt{2}}\left(k^2e^{2i\phi_k}-1\right) & \frac{1}{2}\left(ke^{i\phi_k}-1\right)^2 \end{pmatrix}$$

Profile of TM_2 with NH or IH

$$M_\nu^{TBM} = -\frac{v^2}{M_2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 3b^2p & 0 \\ 0 & 0 & 0 \end{pmatrix} - \frac{f^2v^2}{M_2}\begin{pmatrix} \frac{3}{2}\left(ke^{i\phi_k}+1\right)^2 & 0 & \frac{\sqrt{3}}{2}\left(k^2e^{2i\phi_k}-1\right) \\ 0 & 0 & 0 \\ \frac{\sqrt{3}}{2}\left(k^2e^{2i\phi_k}-1\right) & 0 & \frac{1}{2}\left(ke^{i\phi_k}-1\right)^2 \end{pmatrix}$$

- 2 model parameters in the mixing matrix : $\{k, \phi_k\}$

Numerical Results

NuFIT 3.2 (2018), JHEP **01** (2018) 087

3σ interval	Normal Hierarchy	Inverted Hierarchy
Δm^2_{12}	$[6.80\,,8.02]\times 10^{-5}[\mathrm{eV}^2]$	$[6.80\,,8.02]\times 10^{-5}[\mathrm{eV}^2]$
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We show these minimal models

Case I

$$M_D = v\begin{pmatrix} 0 & \frac{e+f}{2} \\ b & e \\ -b & f \end{pmatrix}$$

TM_1 with IH

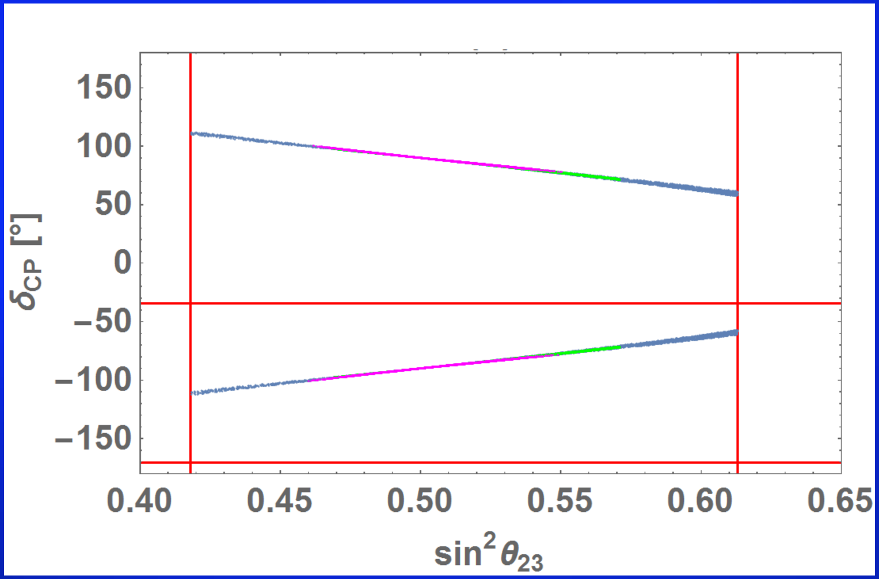
$$M_D = v\begin{pmatrix} -2b & \frac{e+f}{2} \\ b & e \\ b & f \end{pmatrix}$$

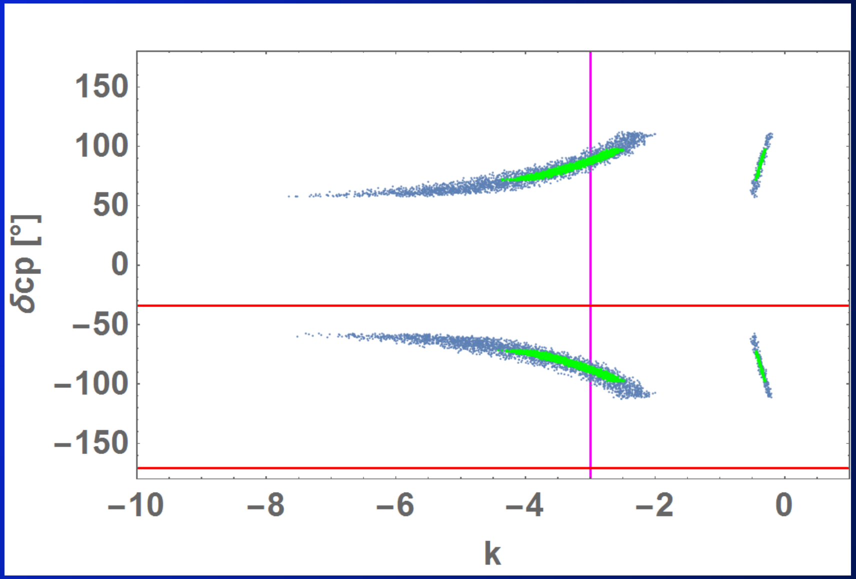
TM_2 (common in NH and IH)

$$M_D = v\begin{pmatrix} b & \frac{e+f}{2} \\ b & e \\ b & f \end{pmatrix}$$

Numerical Results (case I)

$k = e/f$





$\delta_{CP}: \pm[71.4°, 97.9°]\ (1\sigma)$

$: \pm[57.5°, 112°]\ (3\sigma)$

$: \pm[77.8°, 101°]\ (k = -3)$

Blue : 3σ plot

Green : 1σ plot

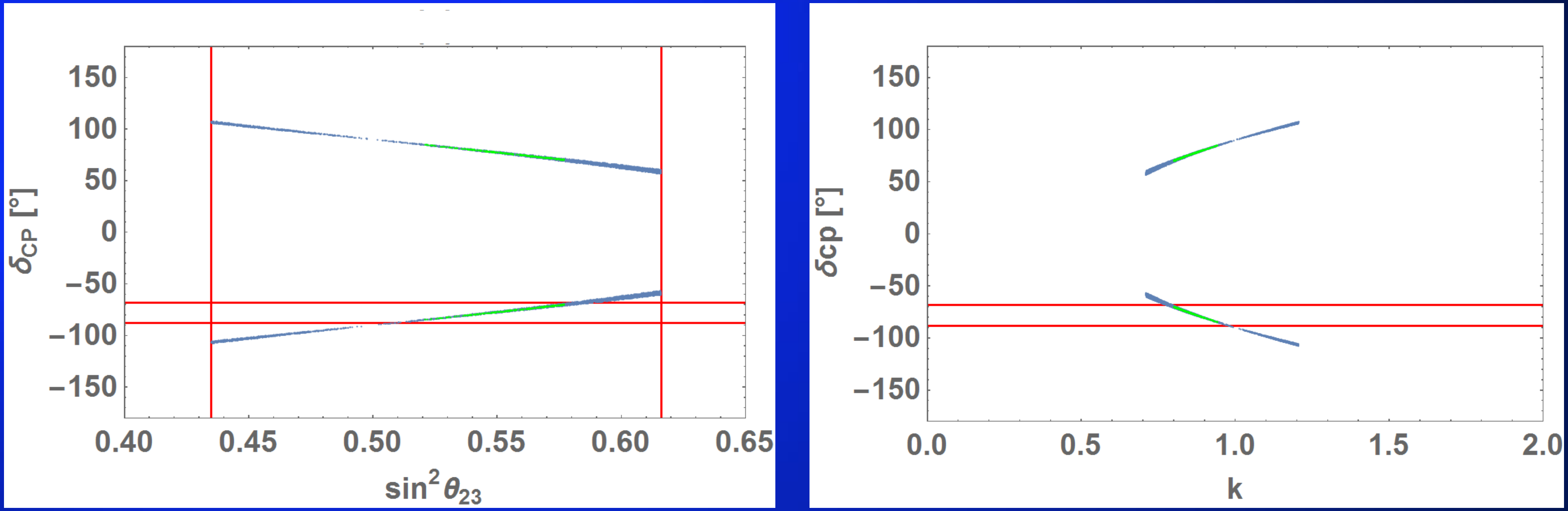
Magenta : 3σ plot with $k = -3$

Red (horizontal) : 2σ interval by T2K $[-171°, -34.4°]$

Red (vertical) : 3σ interval by NuFIT

Numerical Results (TM_1 with IH)

$k = e/f$



$\delta_{CP}: \pm[69.9°, 84.7°]\ (1\sigma)$

$: \pm[56.8°, 107°]\ \ (3\sigma)$

Blue : 3σ plot

Green : 1σ plot

Magenta : 3σ plot with $\mathrm{k} = -3$

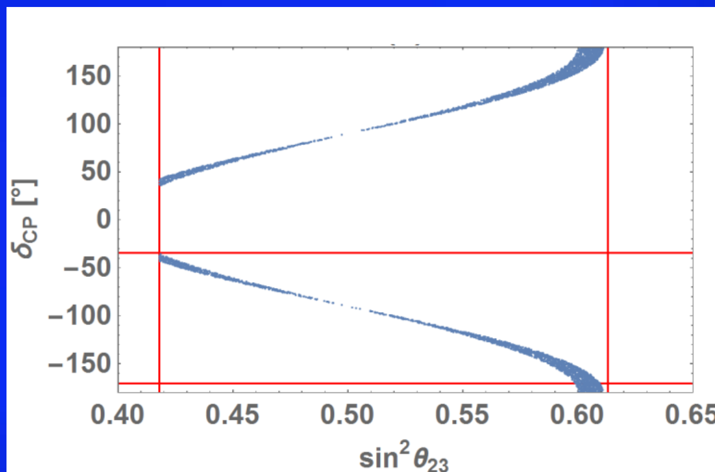
Red (horizontal) : 2σ interval by T2K $[-88.2°, -68.2°]$

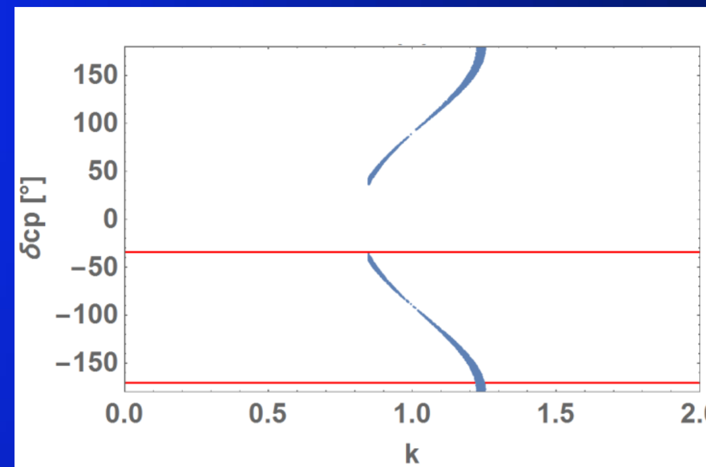
Red (vertical) : 3σ interval by NuFIT

Numerical Results (TM_2)

$k = e/f$

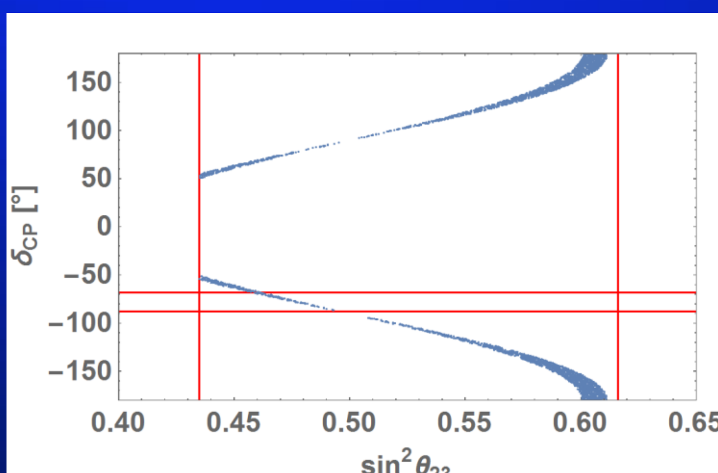
NH

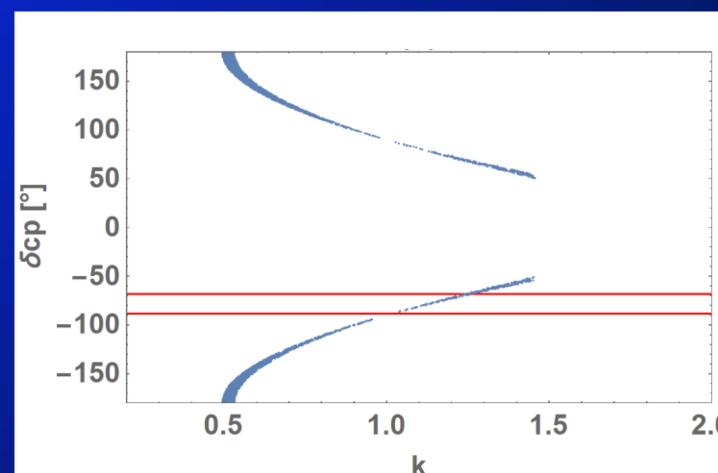




$\delta_{CP}: \pm[36.2°, 180°]\ (3\sigma)$

IH



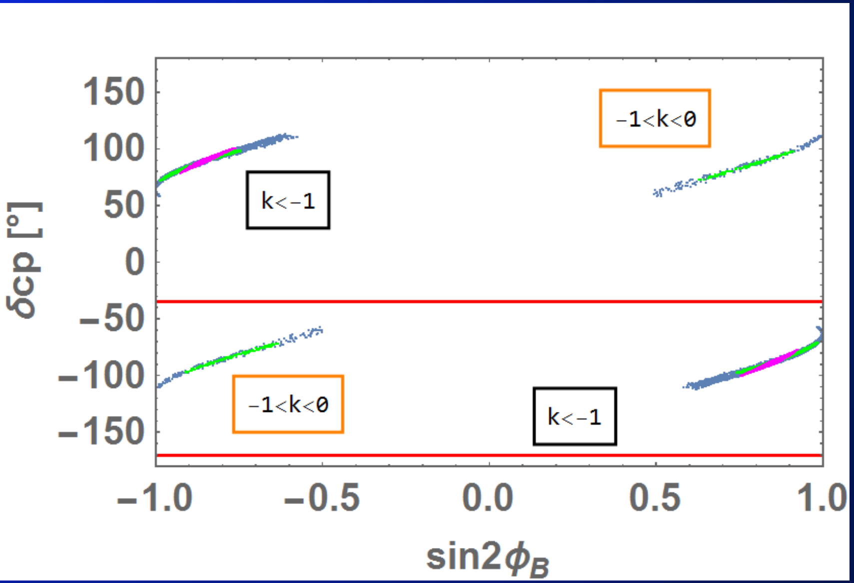


$\delta_{CP}: \pm[51.0\ °, 180°]\ (3\sigma)$

back to case I

The predicted δ_{CP} is sensitive to k.

But the sign of δ_{CP} is not determined…



This result (case I) indicates

$$\mathbf{Sign}[\,J_{CP}\,] = \mathbf{Sign}[\,\delta_{CP}\,]$$

*Recall

$$J_{CP} = -\frac{3}{8}\frac{f^{12}}{M_0^6}(|B|\sqrt{p})^6(k+1)^4(k^2-1)\sin 2\phi_B\frac{v^{12}}{(\Delta m_{13}^2-\Delta m_{12}^2)\Delta m_{13}^2\Delta m_{12}^2}$$

Leptogenesis in our models

B− L asymmetry in the comoving volume $(M_1 \ll M_2)$

$$Y_{B-L} \equiv \frac{n_{B-L}}{s} = -\epsilon_1 \kappa\, Y_{N1}^{eq}(T \gg M_1)$$

is relevant to CP asymmetry of the lighter RH neutrino N_1 decay.

$$\epsilon_1 \sim -\frac{3}{16\pi} \frac{\mathrm{Im}\left[\left(Y_D^\dagger Y_D\right)_{21}^2\right]}{\left(Y_D^\dagger Y_D\right)_{11}} \frac{1}{p} \qquad p = \frac{M_2}{M_1}$$

The heavier RH neutrino decay is relevant at $M_1 \geq 10^{14}$[GeV]

Here, we assume $M_1 \ll 10^{14}$[GeV] for simplicity.

CP asymmetry in 1 loop decay of N_1

$$\epsilon_1 = -\frac{3}{16\pi}\frac{\mathrm{Im}\left[\left(Y_D^\dagger Y_D\right)_{21}^2\right]}{\left(Y_D^\dagger Y_D\right)_{11}}\frac{1}{p}$$

Case I

$$Y_D^\dagger Y_D = \begin{pmatrix} 2|b|^2 & b^*(e-f) \\ \boldsymbol{b(e-f)^*} & \frac{|e+f|^2}{4}+|e|^2+|f|^2 \end{pmatrix} \Rightarrow \epsilon_1 = -\frac{3}{16\pi}\frac{1}{2}|f|^2(k-1)^2\sin 2\phi_B\frac{1}{p}$$

TM_1 with IH

$$Y_D^\dagger Y_D = \begin{pmatrix} 6|b|^2 & 0 \\ \mathbf{0} & \frac{|e+f|^2}{4}+|e|^2+|f|^2 \end{pmatrix} \Rightarrow \epsilon_1 = 0$$

No leptogenesis

TM_2

$$Y_D^\dagger Y_D = \begin{pmatrix} 3|b|^2 & 0 \\ \mathbf{0} & |e+f|^2+|e|^2+|f|^2 \end{pmatrix} \Rightarrow \epsilon_1 = 0$$

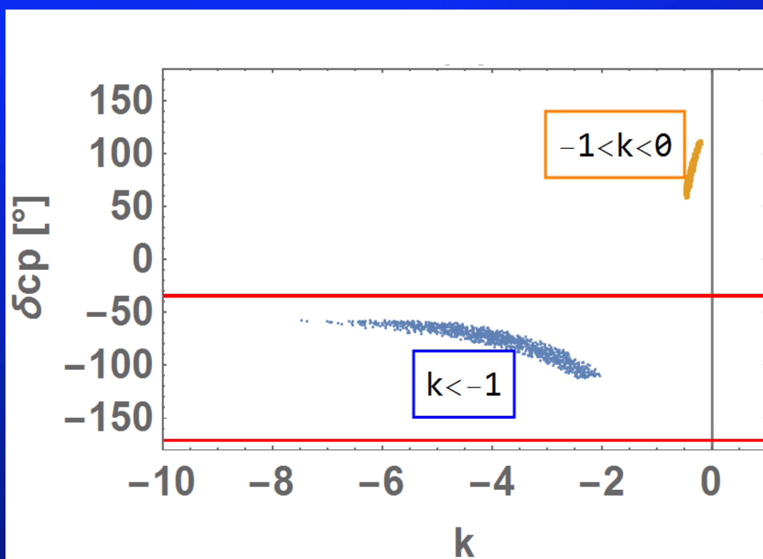
No leptogenesis

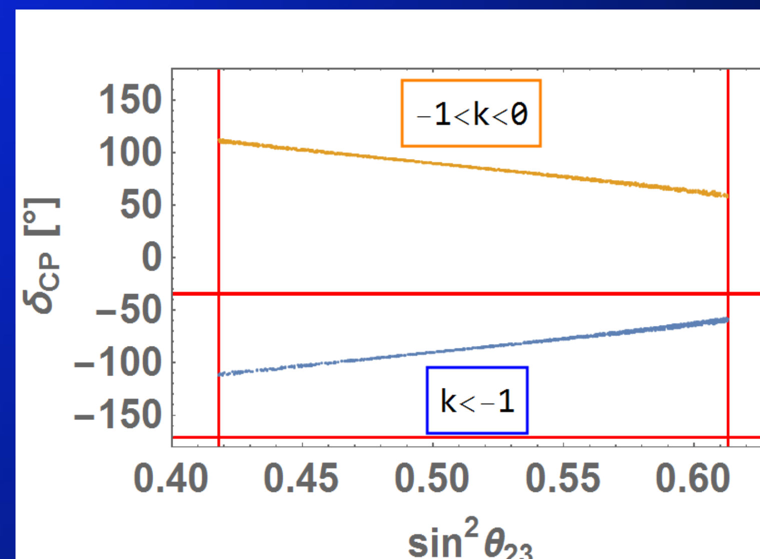
Numerical results (3σ)

Demanded Baryon asymmetry for the nucleosynthesis:

$$\eta_B \equiv \frac{n_B}{n_\gamma} = 7.04 Y_B = [5.8\,, 6.6] \times 10^{-10} \text{ (95\% C.L.)}$$

[PDG] Chin. Phys. C **40** (2016) 10, 10001]





$M_2 = 10^{14}$ [GeV]

The sign of δ_{CP} is split by k

Sign of δ_{CP}

B and $B-L$ asymmetry are related (sphaleron transition at $T > T_{EW} \sim 100$[GeV]):

$$Y_B \equiv \frac{n_B}{s} = \frac{8N_{flavor} + 4N_{Higgs}}{22N_{flavor} + 13N_{Higgs}} Y_{B-L} = \frac{28}{79} Y_{B-L}$$ (Suppose 3 flavors and 1 Higgs)

S.Yu. Khlebnikov, M.E. Shaposhnikov, Nucl. Phys. B **308**, 885 (1998)

Demanded Baryon asymmetry for the nucleosynthesis:

$$\eta_B \equiv \frac{n_B}{n_\gamma} = 7.04 Y_B = [5.8\,, 6.6] \times 10^{-10}$$ (95% C.L.)

[PDG] Chin. Phys. C **40** (2016) 10, 10001]

$$-\epsilon_1 \kappa \, Y_{N1}^{eq} = \frac{1}{7.04}\frac{79}{28}\eta_B > 0$$

For simple analysis

$$\frac{1}{\kappa} \sim \frac{3.3\times 10^{-3}[\mathrm{eV}]}{\widetilde{m_1}} + \left(\frac{\widetilde{m_1}}{0.55\times 10^{-3}[\mathrm{eV}]}\right)^{1.16} > 0$$ valid for $M_1 \ll 10^{14}$[GeV]

where $$\widetilde{m_1} \equiv \frac{v^2 (Y_D Y_D^\dagger)_{11}}{M_1} = 2|f|^2|B|^2 \frac{v^2}{M_2}$$ for case I

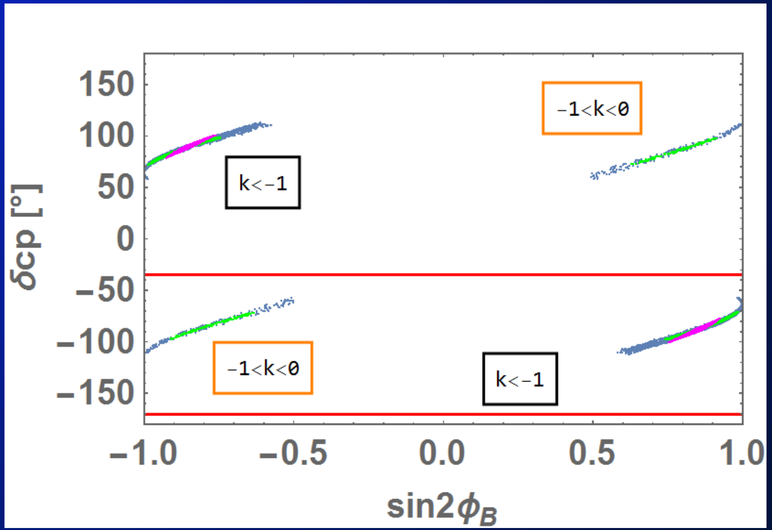
fitted by G.F. Giudice, A.Notari, M. Raidal, A.Riotto and A. Strumia, [Nucl. Phys. B **685** (2004) 89]

The number density of N_1 in thermal equilibrium

$$Y_{N1}^{eq} = \frac{135\zeta(3)}{4\pi^4 g_*} > 0 \qquad g_* = 106.75 \text{ (SM)}$$

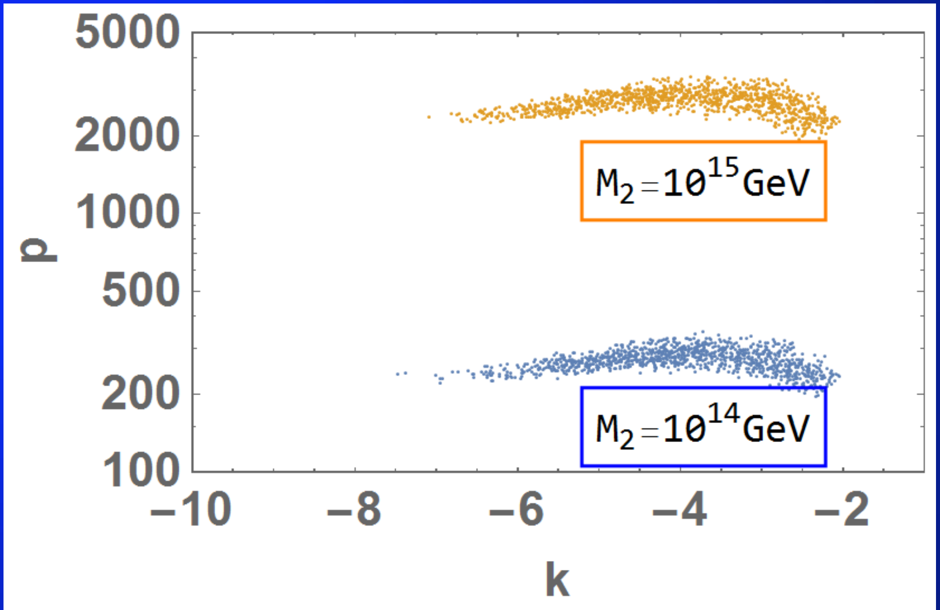
Nucl. Phys. B **685** (2004) 89

$$-\epsilon_1 = \frac{3}{16\pi}\frac{1}{2}|f|^2(k-1)^2 \sin 2\phi_B \frac{1}{p} > 0$$



Mass hierarchy of RH neutrinos

$$p = \frac{M_2}{M_1}$$



$$k < -1$$

Consistent with our assumptions:

$$M_1 \ll M_2$$
$$M_1 \ll 10^{14}[\mathrm{GeV}]$$

Minimal seesaw model with

2 RH neutrinos & tri-maximal mixing TM_1 and TM_2

$$M_D = v\begin{pmatrix} 0 & \frac{e+f}{2} \\ b & e \\ -b & f \end{pmatrix}$$

--- Normal hierarchy
--- distinct the sign of δ_{CP}
--- include $(\theta_{23}, \delta_{CP}) = \left(\frac{\pi}{4}, -\frac{\pi}{2}\right)$

explain neutrino oscillation & BAU

Symmetry realization by S_4

Dirac mass term : $\frac{y_1}{\Lambda}\phi_1 L H_u \nu_{R1}^c + \frac{y_2}{\Lambda}\phi_2 L H_u \nu_{R2}^c$

$$SU\begin{pmatrix} \frac{e+f}{2} \\ e \\ f \end{pmatrix} = \begin{pmatrix} \frac{e+f}{2} \\ e \\ f \end{pmatrix}$$

-- residual Z_2 symmetry from S_4

S, T, U : generators of S_4

Is it consist with quark sector?

$\delta_{CP}^{CKM} \sim +70°$ can be realized with our model?

THANK YOU

Global experimental data of neutrino oscillation

	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 4.14$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.307^{+0.013}_{-0.012}$	$0.272 \to 0.346$	$0.307^{+0.013}_{-0.012}$	$0.272 \to 0.346$
$\theta_{12}/^\circ$	$33.62^{+0.78}_{-0.76}$	$31.42 \to 36.05$	$33.62^{+0.78}_{-0.76}$	$31.43 \to 36.06$
$\sin^2\theta_{23}$	$0.538^{+0.033}_{-0.069}$	$0.418 \to 0.613$	$0.554^{+0.023}_{-0.033}$	$0.435 \to 0.616$
$\theta_{23}/^\circ$	$47.2^{+1.9}_{-3.9}$	$40.3 \to 51.5$	$48.1^{+1.4}_{-1.9}$	$41.3 \to 51.7$
$\sin^2\theta_{13}$	$0.02206^{+0.00075}_{-0.00075}$	$0.01981 \to 0.02436$	$0.02227^{+0.00074}_{-0.00074}$	$0.02006 \to 0.02452$
$\theta_{13}/^\circ$	$8.54^{+0.15}_{-0.15}$	$8.09 \to 8.98$	$8.58^{+0.14}_{-0.14}$	$8.14 \to 9.01$
$\delta_{\mathrm{CP}}/^\circ$	234^{+43}_{-31}	$144 \to 374$	278^{+26}_{-29}	$192 \to 354$
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.40^{+0.21}_{-0.20}$	$6.80 \to 8.02$	$7.40^{+0.21}_{-0.20}$	$6.80 \to 8.02$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.494^{+0.033}_{-0.031}$	$+2.399 \to +2.593$	$-2.465^{+0.032}_{-0.031}$	$-2.562 \to -2.369$

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