Non-Abelian Discrete Groups and Neutrino Flavor Symmetry

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Contents

1 Introduction

C, P, T are well known fundamental symmetries
in particle physics: Abelian Discrete Symmetry

On the other hand,
recent experimental data of neutrino flavor mixing
suggest Non-Abelian Discrete Symmetry for flavors.

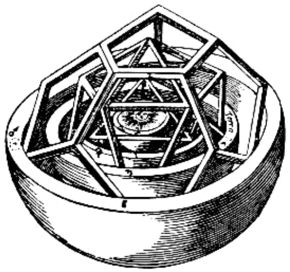
The discrete symmetries are described by

Finite Groups.

The classification of the finite groups has been completed in 2004,
(Gorenstein announced in 1981 that the finite simple groups had all been classified.)
about 100 years later than the case of the continuous groups.

Thompson, Gorenstein, Aschbacher ……

Johannes Kepler



The Cosmographic Mystery

More than 400 years ago,
Kepler tried to understand
cosmological structure by the
symmenty of five Platonic solids.

Scientists like symmetries !

Finite groups are used to classify crystal structures, regular polyhedra, and the symmetries of molecules.
The assigned point groups can then be used to determine physical properties, spectroscopic properties and to construct molecular orbitals.

Symmetry is a powerful approach if the dynamics is unknown.
We investigate the flavor structure of leptons by Discrete Symmetry.

2 Examples of Finite Groups

Ishimori, Kobayashi, Ohki, Shimizu, Okada, M.T, PTP supprement, 183,2010,arXiv1003.3552,
Lect. Notes Physics (Springer) 858,2012

Finite group G

consists of a finite number of element of G.

- The number of elements in G is called **order**.
- The group G is called Abelian
  if all elements are commutable each other,i.e. $ab = ba$.
- The group G is called non-Abelian
  if all elements do not satisfy the commutativity.

Familiar non-Abelian finite groups

			order
S_n :	$S_2 = Z_2$, S_3, S_4 …	Symmetric group	N !
A_n:	$A_3 = Z_3$, A_4 =T , A_5 …	Alternating group	(N !)/2
D_n:	$D_3 = S_3$, D_4, D_5 …	Dihedral group	2N
$Q_{N(even)}$:	Q_4, Q_6 ….	Binary dihedral group	2N
$\Sigma(2N^2)$:	$\Sigma(2) = Z_2$, $\Sigma(18)$, $\Sigma(32)$, $\Sigma(50)$ …		$2N^2$
$\Delta(3N^2)$:	$\Delta(12) = A_4$, $\Delta(27)$ …		$3N^2$
$T_{N(\text{prime number})}$ $\simeq Z_N \rtimes Z_3$:	T_7, T_{13}, T_{19}, T_{31}, T_{43}, T_{49}		3N
$\Sigma(3N^3)$:	$\Sigma(24)=Z_2 \times (12)$, $\Sigma(81)$ …		$3N^3$
$\Delta(6N^2)$:	$\Delta(6)=S_3$, $\Delta(24)=S_4$, $\Delta(54)$ …		$6N^2$
T' :	double covering group of A_4 = T		24

For flavor physics, we are interested in finite groups with **triplet representations**.

S_3 has two singlets and one doublet: $1, 1', 2,$ no triplet representation.

Some examples of non-Abelian Finite groups with triplet representation, which are often used in Flavor symmetry

$S_4, \quad A_4, \quad A_5 \quad \ldots\ldots$

Elements of G are classified by Conjugacy Class

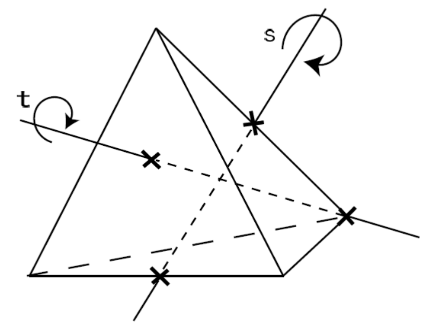
The number of irreducible representations is equal to the number of conjugacy classes. Schur's lemma

The elements $g^{-1}ag$ for $g \in G$ are called elements **conjugate** to the element a.

The set including all elements to conjugate to an element a of G, $\{g^{-1}ag, \forall g \in G\}$, is called a **conjugacy class**.

When $a^h = e$ for an element $a \in G$, the number h is called the order of a.
All of elements in a conjugacy class have the same order.

A_4 group



Symmetry of tetrahedron

Even permutation group of four objects (1234)
12 elements (order 12) are generated by
S and T: $S^2=T^3=(ST)^3=1$: S=(14)(23), T=(123)

4 conjugacy classes

C1: 1 h=1
C3: S, T^2ST, TST^2 h=2
C4: T, ST, TS, STS h=3
C4': T^2, ST^2, T^2S, ST^2S h=3

	h	χ_1	$\chi_{1'}$	$\chi_{1''}$	χ_3
C_1	1	1	1	1	3
C_3	2	1	1	1	-1
C_4	3	1	ω	ω^2	0
$C_{4'}$	3	1	ω^2	ω	0

Irreducible representations: 1, 1', 1", 3
The minimum group containing triplet without doublet.

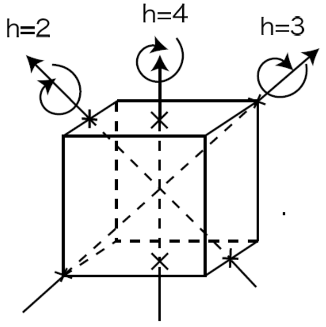
For triplet $S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \quad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}; \quad \omega = e^{2\pi i/3}$

S_4 group

All permutations among four objects, 4 ! =24 elements

24 elements are generated by S, T and U:

$S^2=T^3=U^2=1$, $ST^3 = (SU)^2 = (TU)^2 = (STU)^4 = 1$



Symmetry of a cube

5 conjugacy classes

- C1: 1 h=1
- C3: S, T^2ST, TST^2 h=2
- C6: U, TU, SU, T^2U, STSU, ST^2SU h=2
- C6': STU, TSU, T^2SU, ST^2U, TST^2U, T^2STU h=4
- C8: T, ST, TS, STS, T^2, ST^2, T^2S, ST^2S h=3

	h	χ_1	$\chi_{1'}$	χ_2	χ_3	$\chi_{3'}$
C_1	1	1	1	2	3	3
C_3	2	1	1	2	-1	-1
C_6	2	1	-1	0	1	-1
$C_{6'}$	4	1	-1	0	-1	1
C_8	3	1	1	-1	0	0

Irreducible representations:

1, 1', 2, 3, 3'

For triplet 3 and 3'

$$U = \mp\begin{pmatrix}1 & 0 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\end{pmatrix} \quad S = \frac{1}{3}\begin{pmatrix}-1 & 2 & 2\\ 2 & -1 & 2\\ 2 & 2 & -1\end{pmatrix}, \quad T = \begin{pmatrix}1 & 0 & 0\\ 0 & \omega^2 & 0\\ 0 & 0 & \omega\end{pmatrix}; \quad \omega = e^{2\pi i/3}$$

3 Flavor Symmetry with non-Abelian Finite Group

3.1 Towards non-Abelian Discrete Flavor Symmetry

In Quark sector

There was no information of lepton flavor mixing before 1998.

Discrete Symmetry and Cabibbo Angle,
Phys. Lett. 73B (1978) 61, S.Pakvasa and H.Sugawara

S_3 symmetry is assumed for the Higgs interaction with the quarks and the leptons for the self-coupling of the Higgs bosons.

2 generations

S_3 doublet S_3 singlets S_3 doublet

$$\left\{\begin{pmatrix} p_1 \\ n_1 \end{pmatrix}_L, \begin{pmatrix} p_2 \\ n_2 \end{pmatrix}_L\right\} \Big| \; \{p_{1R}\}, \{p_{2R}\}, \{n_{1R}, n_{2R}\} \quad \Rightarrow \quad \tan\theta_c = m_d/m_s$$

one S_3 singlet $\{\phi_0\}$ and one S_3 doublet $\{\phi_1, \phi_2\}$

Top quark was discovered in 1995

A Geometry of the generations, 3 generations
Phys. Rev. Lett. 75 (1995) 3985, L.J.Hall and H.Murayama

1st and 2nd generations are <u>2</u> of S_3, 3rd one is $\underline{1}_A$ of S_3

$(S(3))^3$ flavor symmetry for quarks Q, U, D

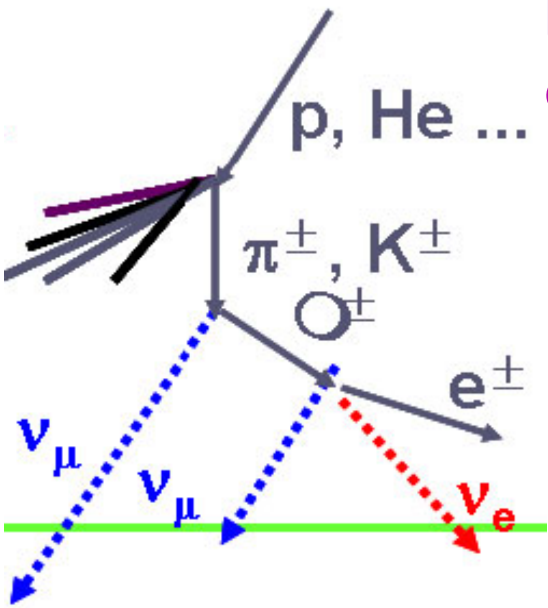
$(S(3))^3$ flavor symmetry and p ---> K^0 e^+, (SUSY version)
Phys. Rev.D 53 (1996) 6282, C.D.Carone, L.J.Hall and H.Murayama

fundamental sources of flavor symmetry breaking are gauge singlet fields ϕ :flavons
Incorporating the lepton flavor based on the discrete flavor group $(S_3)^3$.

1998 Revolution in Neutrinos !

Atmospheric neutrinos brought us informations of neutrino masses and flavor mixing.

$$P_{\nu_\mu \to \nu_\mu} = 1 - 4\left|U_{\mu 3}\right|^2 \left(1 - \left|U_{\mu 3}\right|^2\right) \sin^2 \frac{\Delta_{13}}{2} + 2\left|U_{\mu 2}\right|^2 \left|U_{\mu 3}\right|^2 \Delta_{12} \sin \Delta_{13} + \mathcal{O}(\Delta_{12}^2)$$



First clear evidence of neutrino oscillation was discovered in 1998

$$R = \frac{(\nu_\mu + \overline{\nu}_\mu)/(\nu_e + \overline{\nu}_e)\,|_{DATA}}{(\nu_\mu + \overline{\nu}_\mu)/(\nu_e + \overline{\nu}_e)\,|_{MC}} = 0.65 \pm 0.05 \pm 0.08$$

Multi-GeV

MC $(\nu_\mu + \overline{\nu}_\mu)/(\nu_e + \overline{\nu}_e)\,|_{MC} \approx 2$

Before 2012 (no data for θ_{13})

Neutrino Data presented $\sin^2\theta_{12} \sim 1/3$, $\sin^2\theta_{23} \sim 1/2$

Harrison, Perkins, Scott (2002) proposed

Tri-bimaximal Mixing of Neutrino flavors.

$$\sin^2\theta_{12} = 1/3,\ \sin^2\theta_{23} = 1/2,\ \sin^2\theta_{13} = 0,$$

$$U_{\text{tri-bimaximal}} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}$$

PDG

$$U_{\text{PMNS}} \equiv \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{pmatrix}$$

Tri-bimaximal Mixing of Neutrinos motivates to consider Non-Abelian Discrete Flavor Symmetry.

Tri-bimaximal Mixing (TBM) is realized by the mass matrix

$$m_{TBM} = \frac{m_1+m_3}{2}\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix} + \frac{m_2-m_1}{3}\begin{pmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{pmatrix} + \frac{m_1-m_3}{2}\begin{pmatrix}1 & 0 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\end{pmatrix}$$

A_4 symmetric

in the diagonal basis of charged leptons.

Mixing angles are independent of neutrino masses.

Integer (inter-family related) matrix elements suggest Non-Abelian Discrete Flavor Symmetry.

A_4 symmetry E. Ma and G. Rajasekaran, PRD64(2001)113012

In 2012

θ_{13} was measured by Daya Bay, RENO, T2K, MINOS, Double Chooz

Tri-bimaximal mixing was ruled out !

$$\theta_{13} \simeq 9^\circ \simeq \theta_c/\sqrt{2}$$

Rather large θ_{13} suggests to search for CP violation !

$$J_{CP} = s_{23}c_{23}s_{12}c_{12}s_{13}c_{13}^2 \sin\delta_{CP} \simeq \ 0.0327 \sin\delta$$

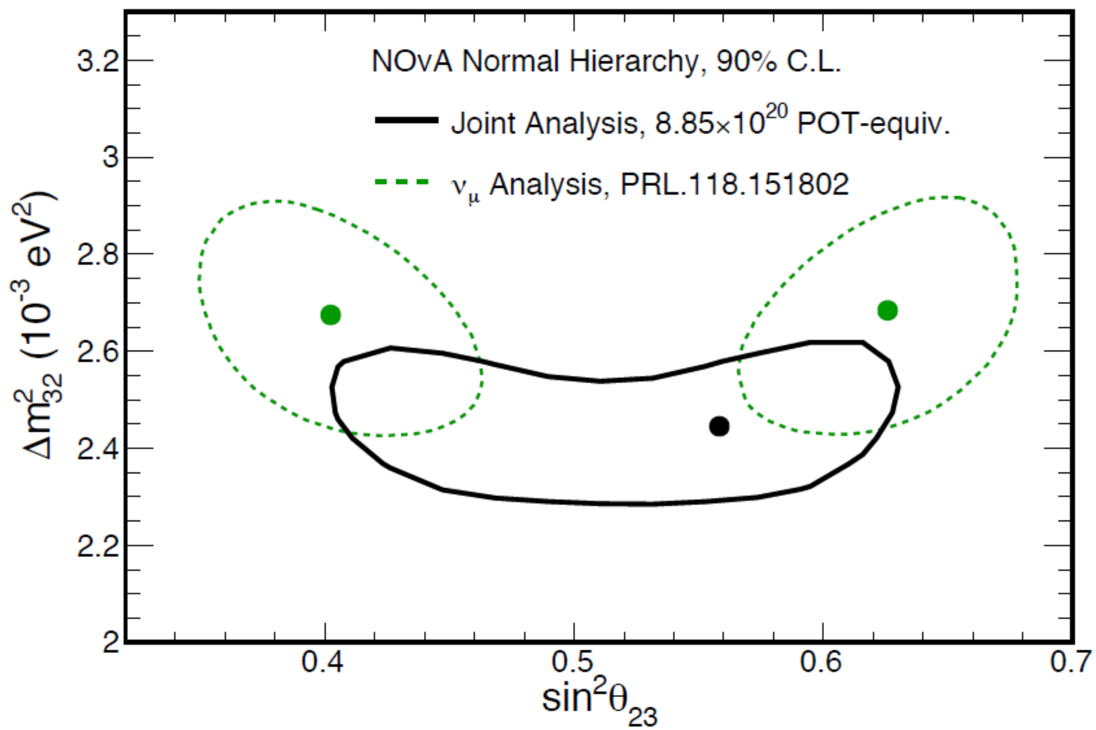
J_{CP}(quark)$\sim 3\times10^{-5}$

Challenge for flavor and CP symmetries for leptons

ν_μ Result

- Full joint fit with appearance analysis. Feldman Cousins corrections in 2D & 1D limits.
- All systematics, oscillation pull terms shared.
- Constrain θ_{13} using world average from PDG, $\sin^2 2\theta_{13} = 0.082$

NOvA Preliminary



Best fit:

$\Delta m^2_{32} =$
$2.444^{+0.079}_{-0.077} \times 10^{-3}\ \mathrm{eV}^2$

UO preferred at 0.2σ

$\sin^2\theta_{23} =$
UO: $0.558^{+0.041}_{-0.033}$
LO: $0.475^{+0.036}_{-0.044}$

Atmospheric Mixing and World Constraints

- Consistent with world expectation.
- Competitive measurement of Δm^2_{32}.

Best fit:

$\Delta m^2_{32} =$

$2.444^{+0.079}_{-0.077} \times 10^{-3}\ eV^2$

UO preferred at 0.2σ

$\sin^2\theta_{23} =$

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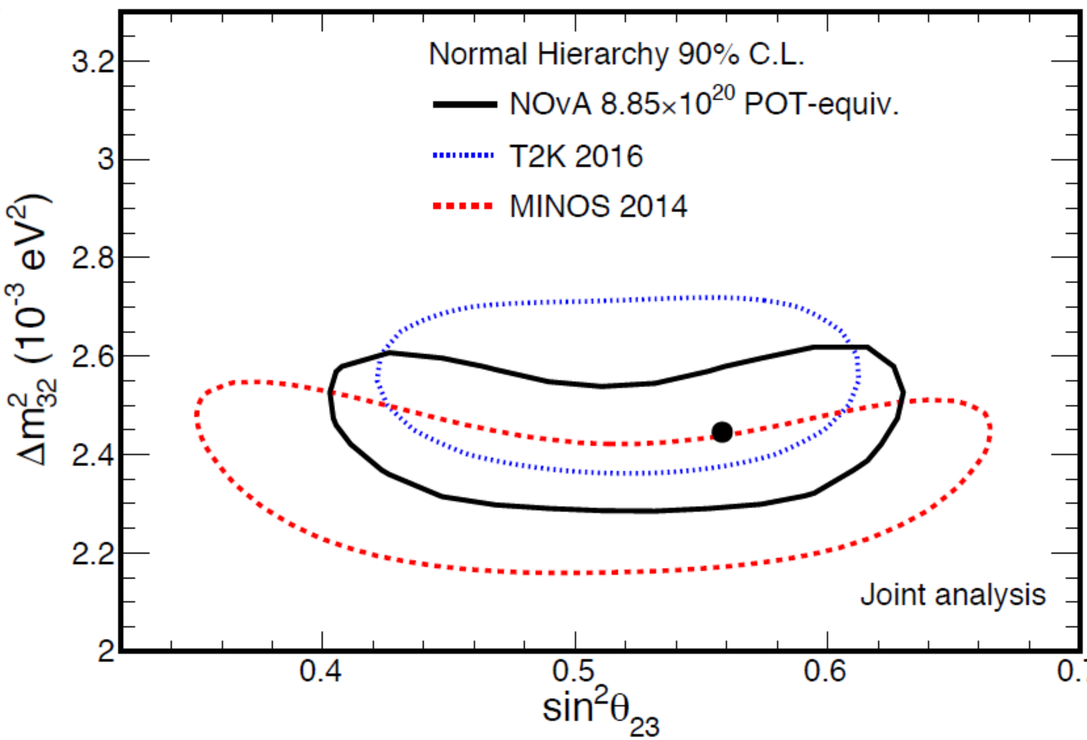
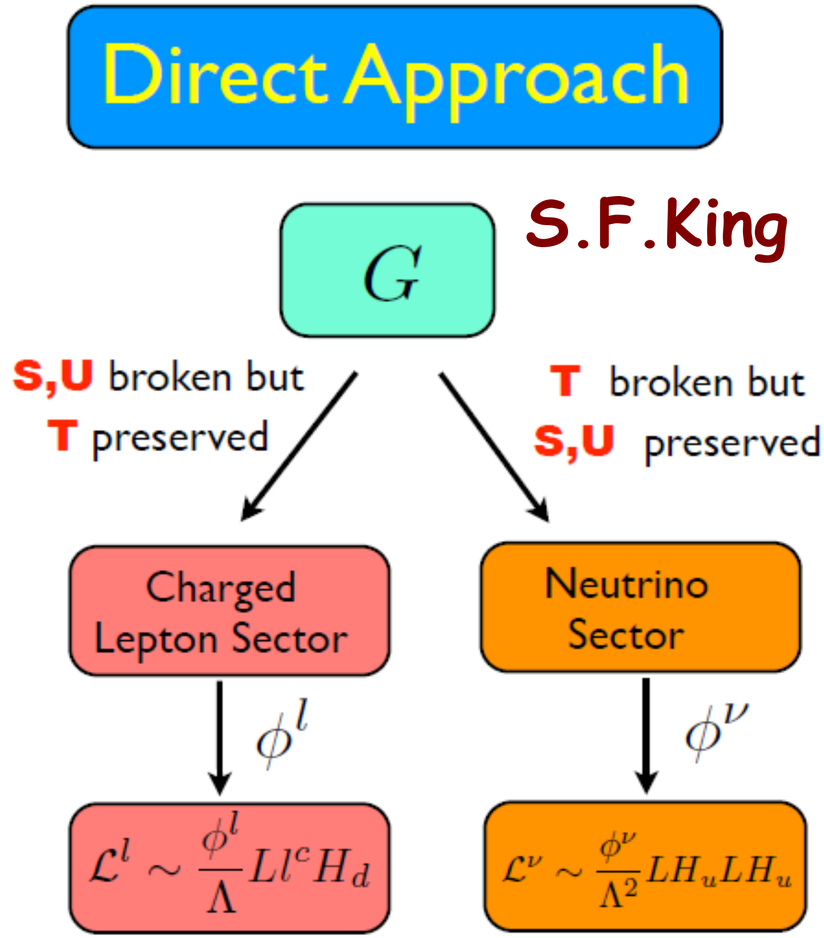
NOvA Preliminary



3.2 Direct approach of Flavor Symmetry

Suppose Flavor Symmetry Group G at high energy

G breaks down to subgroups of G, which are different in Yukawa sectors of Neutrinos and Charged leptons, respectively.



S, T, U are generators of Finite groups

arXiv: 1402.4271 King, Merle, Morisi, Simizu, M.T

Consider S_4 flavor symmetry:

24 elements are generated by S, T and U:

$S^2=T^3=U^2=1$, $ST^3 = (SU)^2 = (TU)^2 = (STU)^4 = 1$

Irreducible representations: 1, 1', 2, 3, 3'

It has subgroups, nine Z_2, four Z_3, three Z_4, four $Z_2 \times Z_2$ (K_4)

Suppose S_4 is spontaneously broken to one of subgroups:

Neutrino sector preserves (1,S,U,SU) (K_4)

Charged lepton sector preserves (1,T,T^2) (Z_3)

For 3 and 3'

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \quad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}; \quad \omega = e^{2\pi i/3}$$

$$U = \mp \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Neutrino and charged lepton mass matrices respect S, U and T generators, respectively:

$$S^T m_{LL}^{\nu} S = m_{LL}^{\nu}, \quad U^T m_{LL}^{\nu} U = m_{LL}^{\nu}, \quad T^{\dagger} Y_e Y_e^{\dagger} T = Y_e Y_e^{\dagger}$$

⇓

$$[S, m_{LL}^{\nu}] = 0, \quad [U, m_{LL}^{\nu}] = 0, \quad [T, Y_e Y_e^{\dagger}] = 0$$

Mixing matrices diagonalize mass matrices also diagonalize S,U, and T, respectively !

The charged lepton mass matrix is diagonal because T is diagonal matrix.

$$V_{\nu} = \begin{pmatrix} 2/\sqrt{6} & /1\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

Tri-bimaximal mixing $\theta_{13}=0$

C.S.Lam, PRD98(2008)
arXiv:0809.1185

which digonalizes both S and U.

Independent of mass eigenvalues !

If S_4 is spontaneously broken to another subgroups,

Neutrino sector preserves (1,SU) (Z_2)

Charged lepton sector preserves (1,T,T^2) (Z_3),

mixing matrix is changed !

$$(SU)^T m_{LL}^{\nu} SU = m_{LL}^{\nu}, \quad T^{\dagger} Y_e Y_e^{\dagger} T = Y_e Y_e^{\dagger}$$

$$[SU, m_{LL}^{\nu}] = 0, \quad [T, Y_e Y_e^{\dagger}] = 0$$

$$V_{\nu} = \begin{pmatrix} 2/\sqrt{6} & c/\sqrt{3} & s/\sqrt{3} \\ -1/\sqrt{6} & c/\sqrt{3} - s/\sqrt{2} & -s/\sqrt{3} - c/\sqrt{2} \\ -1/\sqrt{6} & c/\sqrt{3} + s/\sqrt{2} & -s/\sqrt{3} + c/\sqrt{2} \end{pmatrix}$$

Tri-maximal mixing TM$_1$

$c = \cos\theta, \quad s = \sin\theta$ includes CP phase.

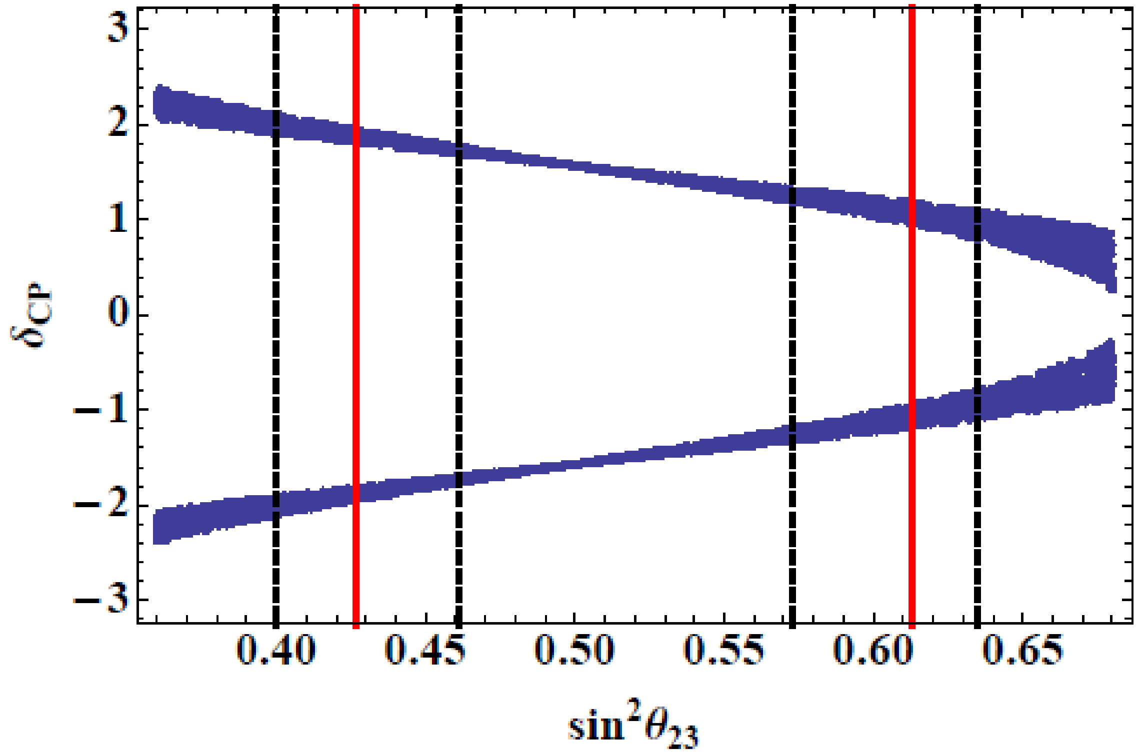
Θ is not fixed by the flavor symmetry.

Mixing sum rules

$$\sin^2\theta_{12} = 1 - \frac{2}{3}\frac{1}{\cos^2\theta_{13}} \leq \frac{1}{3}, \qquad \cos\delta_{CP}\tan 2\theta_{23} \simeq -\frac{1}{2\sqrt{2}\sin\theta_{13}}\left(1 - \frac{7}{2}\sin^2\theta_{13}\right)$$

Inputting the experimental data of 3 mixing angles, mixing sum rules predict δ_{CP}

Shimizu, Tanimoto, Yamamoto, arXiv:1405.1521



The sign of δ_{CP} is not fixed.

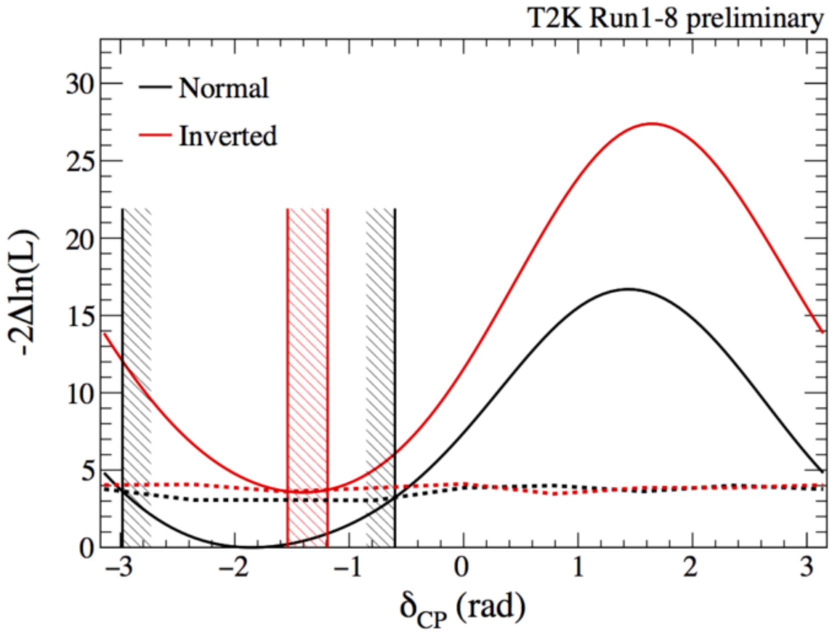
$$\sin^2\theta_{12} = 1 - \frac{2}{3}\frac{1}{\cos^2\theta_{13}} \leq \frac{1}{3}\ , \qquad \cos\delta_{CP}\tan 2\theta_{23} \simeq -\frac{1}{2\sqrt{2}\sin\theta_{13}}\left(1 - \frac{7}{2}\sin^2\theta_{13}\right)$$

3.3 CP symmetry in neutrinos

Exciting Era of Observation of CP violating phase @T2K and NOvA

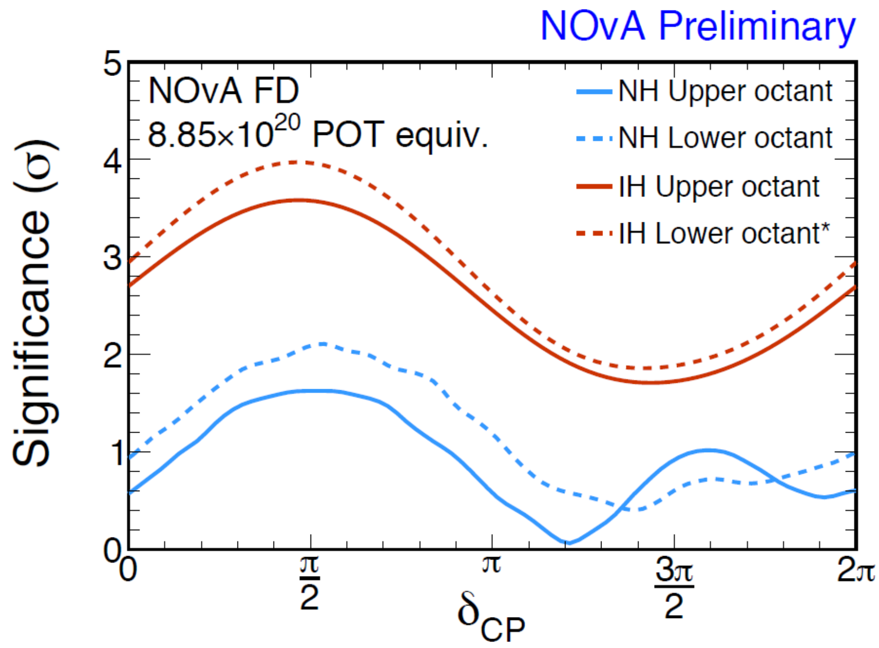
T2K reported the constraint on δ_{CP} data.

August 4, 2017



Nova reported the preliminary

January, 2018



Consistency of CP symmetry and Flavor Symmetry

Example: Impose A_4 symmetry for leptons

$$|\,3\,\rangle = \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix} \overset{A_4\ \text{transformation}}{\Rightarrow} T \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}, \quad S \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}$$

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \quad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}; \quad \omega = e^{2\pi i/3}$$

$$\begin{pmatrix} e^* \\ \mu^* \\ \tau^* \end{pmatrix} \Rightarrow S \begin{pmatrix} e^* \\ \mu^* \\ \tau^* \end{pmatrix} \ ; \quad \begin{pmatrix} e^* \\ \mu^* \\ \tau^* \end{pmatrix} \Rightarrow T^* \begin{pmatrix} e^* \\ \mu^* \\ \tau^* \end{pmatrix} = T^2 \begin{pmatrix} e^* \\ \mu^* \\ \tau^* \end{pmatrix}$$

CP transformation: $(S, T) \Rightarrow (S, T^2)$

Out $(A_4)=Z_2$ outer automorphism

4 conjugacy classes

C1: 1	h=1
C3: S, T^2ST, TST^2	h=2
C4: T, ST, TS, STS	h=3
C4': T^2, ST^2, T^2S, ST^2S	h=3

G.Ecker, W.Grimus and W.Konetschny, Nucl. Phys. B 191 (1981) 465
G.Ecker, W.Grimus and H.Neufeld, Nucl.Phys.B 229(1983) 421

Generalized CP Symmetry in the flavor space

CP Symmetry $\varphi(x) \xrightarrow{\text{CP}} X_{\mathbf{r}}\varphi^*(x'), \quad x' = (t, -\mathbf{x})$

Flavor Symmetry $\varphi(x) \xrightarrow{\text{g}} \rho_{\mathbf{r}}(g)\varphi(x), \quad g \in G_f$

Is CP symmetry consistent with Flavor symmetry ?

Finite groups inconsistent with CP symmetry

$\Delta(27)$, T_7 ……. **Explicit CP violation** Chen, et al: arXiv 1402.0507

Investigation in Finite group theory
class-inverting automorphism

Finite groups consistent with CP symmetry

A_4 , A_5 , T' …… **non-trivial CP symmetry**

S_3 , S_4 , ……. **trivial CP symmetry**

Spontaneous CP violation

CP can be predicted

Suppose a symmetry including FLASY and CP symmetry at high energy:

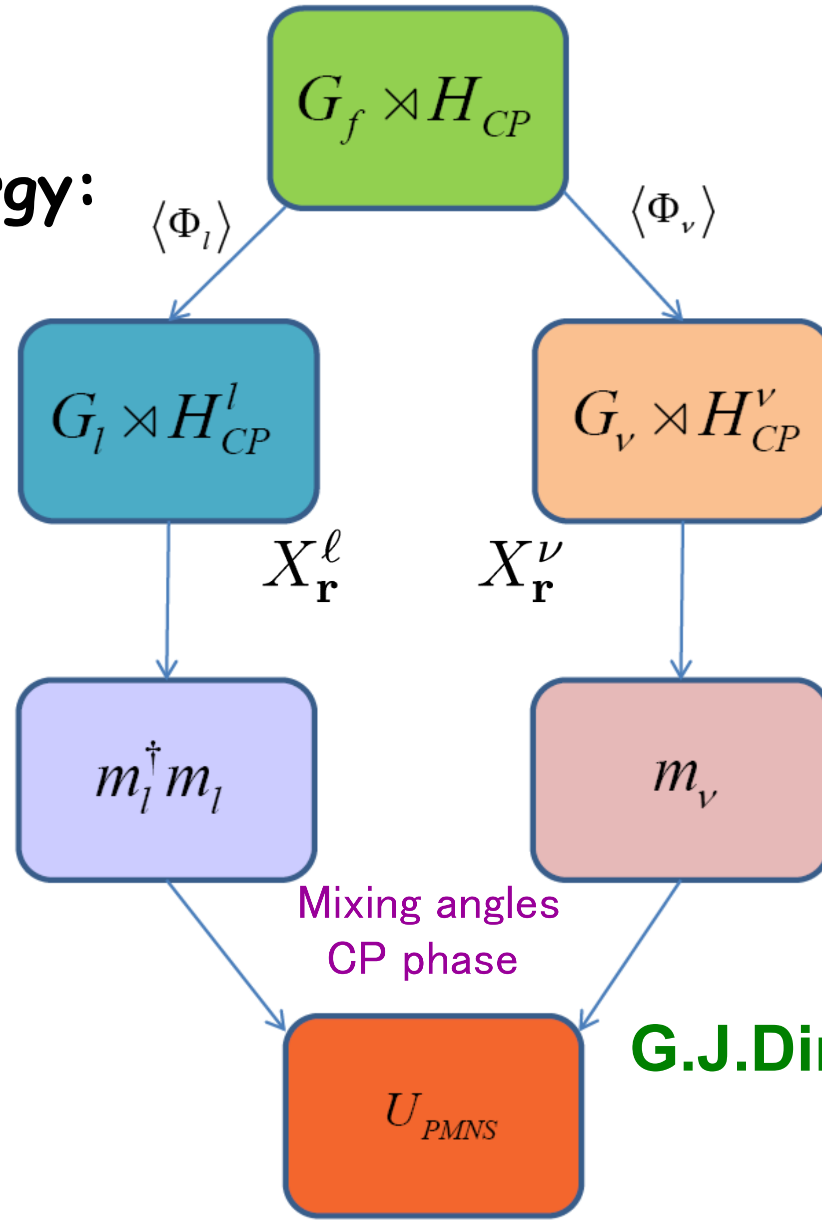
$$G_{CP} = G_f \rtimes H_{CP}$$

is broken to the subgroups in neutrino sector and charged lepton sector, differently.

CP symmetry gives

$$X_{\mathbf{r}}^{\nu T} m_{\nu LL} X_{\mathbf{r}}^{\nu} = m_{\nu LL}^{*}$$

$$X_{\mathbf{r}}^{\ell\dagger} (m_\ell^\dagger m_\ell) X_{\mathbf{r}}^{\ell} = (m_\ell^\dagger m_\ell)^*$$



G.J.Ding

Generalized CP Symmetry

G.Ecker, W.Grimus and W.Konetschny, Nucl. Phys. B 191 (1981) 465
G.Ecker, W.Grimus and H.Neufeld, Nucl.Phys.B 229(1983) 421

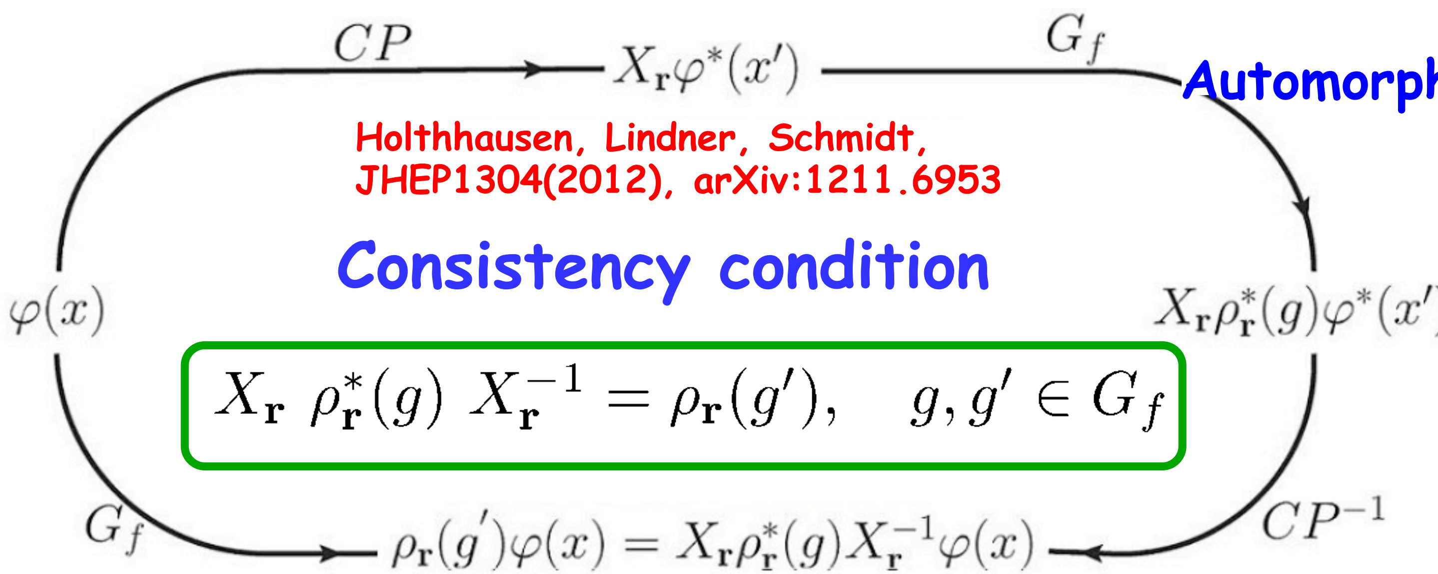
CP Symmetry $\varphi(x) \xrightarrow{\text{CP}} X_{\mathbf{r}}\varphi^*(x'), \quad x' = (t, -\mathbf{x})$

Flavour Symmetry $\varphi(x) \xrightarrow{\text{g}} \rho_{\mathbf{r}}(g)\varphi(x), \quad g \in G_f$

$$X_{\mathbf{r}}^{\nu T} m_{\nu LL} X_{\mathbf{r}}^{\nu} = m_{\nu LL}^*$$

$$X_{\mathbf{r}}^{\ell \dagger}(m_\ell^\dagger m_\ell) X_{\mathbf{r}}^{\ell} = (m_\ell^\dagger m_\ell)^*$$

X_r must be consistent with Flavor Symmetry $\rho_{\mathbf{r}}(g)$



Automorphism

Holthhausen, Lindner, Schmidt, JHEP1304(2012), arXiv:1211.6953

Consistency condition

$$X_{\mathbf{r}}\ \rho_{\mathbf{r}}^*(g)\ X_{\mathbf{r}}^{-1} = \rho_{\mathbf{r}}(g'), \quad g, g' \in G_f$$

Mu-Chun Chen, Fallbacher, Mahanthappa, Ratz, Trautner, Nucl.Phys. B883 (2014) 267-305

An example of S_4 model

Ding, King, Luhn, Stuart, JHEP1305, arXiv:1303.6180

One example of S_4: $G_\nu=\{1,S\}$ **(Z_2)** and $X_3^\nu=U$, $X_3^l=1$

satisfy the consistency condition

$$X_{\mathbf{r}}\ \rho^*_{\mathbf{r}}(g)\ X_{\mathbf{r}}^{-1} = \rho_{\mathbf{r}}(g'), \quad g, g' \in G_f$$

$$m_{\nu LL} = \alpha \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} + \beta \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + \gamma \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} + \epsilon \begin{pmatrix} 0 & 1 & -1 \\ 1 & -1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$$

Commutative with S

Impose CP symmetry on $m_{\nu LL}$ $\quad X_{\mathbf{r}}^{\nu T} m_{\nu LL} X_{\mathbf{r}}^{\nu} = m^*_{\nu LL}$

⇓

α, β, γ are real, ε is imaginary.

$$V_\nu = \begin{pmatrix} 2c/\sqrt{6} & 1/\sqrt{3} & 2s/\sqrt{6} \\ -c/\sqrt{6} + is/\sqrt{2} & 1/\sqrt{3} & -s/\sqrt{6} - ic/\sqrt{2} \\ -c/\sqrt{6} + is/\sqrt{2} & 1/\sqrt{3} & -s/\sqrt{6} + ic/\sqrt{2} \end{pmatrix}$$

$$c = \cos\theta, \quad s = \sin\theta$$

⬇

$$\sin^2\theta_{13} = \tfrac{2}{3}\sin^2\theta, \quad \sin^2\theta_{12} = \tfrac{1}{2+\cos 2\theta}, \quad \sin^2\theta_{23} = \tfrac{1}{2}$$

$$|\sin\delta_{CP}| = 1, \quad \sin\alpha_{21} = \sin\alpha_{31} = 0$$

$$\delta_{CP} = \pm\pi/2$$

The predicton of CP phase depends on the residual generators of FLASY and CP symmetry. Typically, it is simple value, 0, π, ±π/2 .

A_4, A_5, $\Delta(6N^2)$ …

Direct Approach

☆ Flavor Structure of Yukawa Interactions are directly related with the Generators of Finite groups. Predictions are clear.

★ One cannot discuss the related phenomena without Lagrangian.
Leptogenesis, Quark CP violation, Lepton flavor violation

Go to Indirect Approach !

☆ Introduce flavons (gauge singlet scalars) to discuss dynamics of flavors, so write down Lagrangian.
Flavor symmetry is broken spontaneously.
Also investigate the vacuum structure in the broken symmetry.

★ The number of parameters of Yukawa interactions increases.
Predictivity of models is less than the Direct approach.

3.4 Indirect approach of Flavor Symmetry

Model building by flavons

Flavor symmetry G is broken by flavon (SU_2 singlet scalors) VEV's.
Flavor symmetry controls Yukaw couplings
among leptons and flavons with special vacuum alignments.

Consider an example : A_4 model

	Leptons	flavons	
A_4 triplets	(L_e, L_μ, L_τ)	$\phi_\nu(\phi_{\nu 1}, \phi_{\nu 2}, \phi_{\nu 3})$	couple to neutrino sector
		$\phi_E(\phi_{E1}, \phi_{E2}, \phi_{E3})$	couple to charged lepton sector
A_4 singlets	$e_R : \mathbf{1} \quad \mu_R : \mathbf{1}'' \quad \tau_R : \mathbf{1}'$		

Mass matrices are given by A_4 invariant couplings with flavons

$$\mathbf{3}_L \times \mathbf{3}_L \times \mathbf{3}_{flavon} \rightarrow \mathbf{1}\ , \qquad \mathbf{3}_L \times \mathbf{1}_R^{(')('')} \times \mathbf{3}_{flavon} \rightarrow \mathbf{1}$$

G. Altarelli, F. Feruglio, Nucl.Phys. B720 (2005) 64

Flavor symmetry G is broken by VEV of flavons

$3_L \times 3_L \times 3_{flavon} \to 1$ $\qquad$ $3_L \times 1_R (1_R', 1_R'') \times 3_{flavon} \to 1$

$$m_{\nu LL} \sim y \begin{pmatrix} 2\langle\phi_{\nu 1}\rangle & -\langle\phi_{\nu 3}\rangle & -\langle\phi_{\nu 2}\rangle \\ -\langle\phi_{\nu 3}\rangle & 2\langle\phi_{\nu 2}\rangle & -\langle\phi_{\nu 1}\rangle \\ -\langle\phi_{\nu 2}\rangle & -\langle\phi_{\nu 1}\rangle & 2\langle\phi_{\nu 3}\rangle \end{pmatrix} \qquad m_E \sim \begin{pmatrix} y_e\langle\phi_{E1}\rangle & y_e\langle\phi_{E3}\rangle & y_e\langle\phi_{E2}\rangle \\ y_\mu\langle\phi_{E2}\rangle & y_\mu\langle\phi_{E1}\rangle & y_\mu\langle\phi_{E3}\rangle \\ y_\tau\langle\phi_{E3}\rangle & y_\tau\langle\phi_{E2}\rangle & y_\tau\langle\phi_{E1}\rangle \end{pmatrix}$$

Suppose specific Vacuum Alingnments, which preserve S or T generator.

Take $\langle\phi_{\nu 1}\rangle = \langle\phi_{\nu 2}\rangle = \langle\phi_{\nu 3}\rangle$ and $\langle\phi_{E2}\rangle = \langle\phi_{E3}\rangle = 0$

$$\Rightarrow \quad \langle\phi_\nu\rangle \sim (1,1,1)^T , \qquad \langle\phi_E\rangle \sim (1,0,0)^T$$

$$S\begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}1\\1\\1\end{pmatrix}, \quad T\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\end{pmatrix}$$

Then, $\langle\phi_\nu\rangle$ preserves S and $\langle\phi_E\rangle$ preserves T.

m_E is a diagonal matrix, on the other hand, $m_{\nu LL}$ is

$$m_{\nu LL} \sim 3y \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - y \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

Rank 2

two generated masses and one massless neutrinos !

(0, 3y, 3y)

Flavor mixing is not fixed !

Adding A_4 singlet $\xi : \mathbf{1}$ in order to fix flavor mixing matrix.

$$\mathbf{3_L} \times \mathbf{3_L} \times \mathbf{1_{flavon}} \to \mathbf{1}$$

$$m_{\nu LL} \sim y_1 \begin{pmatrix} 2\langle\phi_{\nu 1}\rangle & -\langle\phi_{\nu 3}\rangle & -\langle\phi_{\nu 2}\rangle \\ -\langle\phi_{\nu 3}\rangle & 2\langle\phi_{\nu 2}\rangle & -\langle\phi_{\nu 1}\rangle \\ -\langle\phi_{\nu 2}\rangle & -\langle\phi_{\nu 1}\rangle & 2\langle\phi_{\nu 3}\rangle \end{pmatrix} + y_2\langle\xi\rangle \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$\langle\phi_{\nu 1}\rangle = \langle\phi_{\nu 2}\rangle = \langle\phi_{\nu 3}\rangle$, which preserves S symmetry.

$$m_{\nu LL} = 3a \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - a \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + b \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Flavor mixing is determined: Tri-bimaximal mixing. $\theta_{13}=0$

$$m_\nu = 3a + b,\ b,\ 3a - b \ \Rightarrow m_{\nu_1} - m_{\nu_3} = 2m_{\nu_2}$$

There appears a Neutrino Mass Sum Rule.

This is a minimal framework of A_4 symmetry predicting mixing angles and masses.

A_4 model easily realizes non-vanishing θ_{13} .

Y. Simizu, M. Tanimoto, A. Watanabe, PTP 126, 81(2011)

$$\mathbf{3} \times \mathbf{3} \Rightarrow \mathbf{1} \quad = a_1 * b_1 + a_2 * b_3 + a_3 * b_2$$
$$\mathbf{3} \times \mathbf{3} \Rightarrow \mathbf{1}' \quad = a_1 * b_2 + a_2 * b_1 + a_3 * b_3$$
$$\mathbf{3} \times \mathbf{3} \Rightarrow \mathbf{1}'' \quad = a_1 * b_3 + a_2 * b_2 + a_3 * b_1$$

ξ $\qquad$ ξ'

$$\mathbf{1} \times \mathbf{1} \Rightarrow \mathbf{1} \quad , \qquad \mathbf{1}'' \times \mathbf{1}' \Rightarrow \mathbf{1}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

Additional Matrix

$$M_\nu = a\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} + b\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix} + c\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix} + d\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix}$$

$$a = \frac{y^\nu_{\phi_\nu}\alpha_\nu v_u^2}{\Lambda}, \quad b = -\frac{y^\nu_{\phi_\nu}\alpha_\nu v_u^2}{3\Lambda}, \quad c = \frac{y^\nu_{\xi}\alpha_\xi v_u^2}{\Lambda}, \quad d = \frac{y^\nu_{\xi'}\alpha_{\xi'} v_u^2}{\Lambda} \qquad a = -3b$$

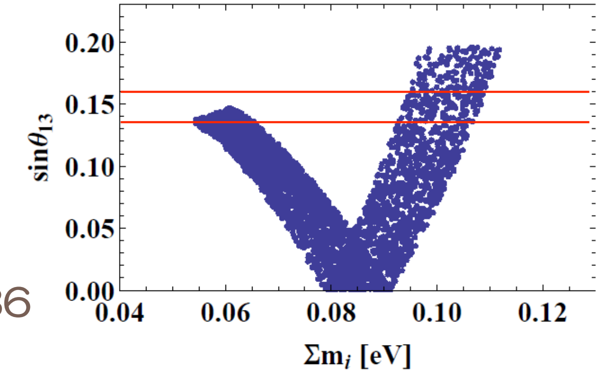
Both normal and inverted mass hierarchies are possible.

$$M_\nu = V_{\text{tri-bi}}\begin{pmatrix}a+c-\frac{d}{2} & 0 & \frac{\sqrt{3}}{2}d\\ 0 & a+3b+c+d & 0\\ \frac{\sqrt{3}}{2}d & 0 & a-c+\frac{d}{2}\end{pmatrix}V^T_{\text{tri-bi}}$$

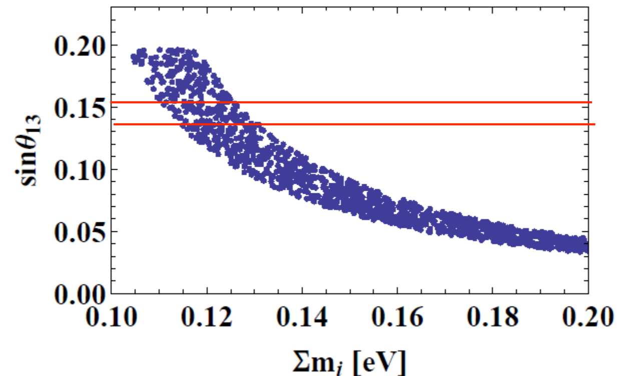
Tri-maximal mixing: TM_2

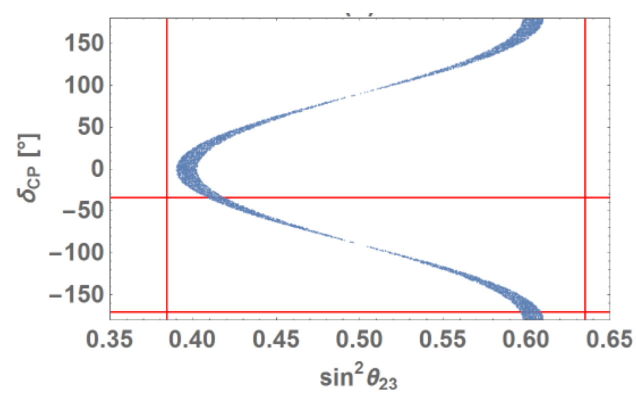
$$\Delta m^2_{31} = -4a\sqrt{c^2+d^2-cd}\,, \qquad \Delta m^2_{21} = (a+3b+c+d)^2 - (a+\sqrt{c^2+d^2-cd})^2$$

Normal hierarchy



Inverted hierarchy





4 Prospect

Quark Sector ?

☆How can Quarks and Leptons become reconciled ?

T', S_4, A_5 and $\Delta(96)$ — SU(5)
S_3, S_4, $\Delta(27)$ and $\Delta(96)$ can be embeded in SO(10) GUT.
A_4 and S_4 — PS

For example: See references S.F. King, 1701.0441
quark sector (2, 1) for SU(5) 10
lepton sector (3) for SU(5) 5

Different flavor structures of quarks and leptons appear !

Cooper, King, Luhn (2010,2012), Callen,Volkas (2012), Meroni, Petcov, Spinrath (2012)
Antusch, King, Spinrath (2013), Gehrlein, Oppermann, Schaefer, Spinrath (2014)
Gehrlein, Petcov,Spinrath (2015), Bjoreroth, Anda, Medeiros Varzielas, King (2015) …

Origin of Cabibbo angle ?

☆ Flavour symmetry in Higgs sector ?

Does a Finite group control Higgs sector ?
2HDM, 3HDM …
an interesting question since Pakvasa and Sugawara 1978

☆ How to test Flavor Symmetry ?

* Mixing angle sum rules

Example: TM_1

$$\sin^2\theta_{12} = 1 - \frac{2}{3}\frac{1}{\cos^2\theta_{13}} \le \frac{1}{3}\,, \qquad \cos\delta_{CP}\tan 2\theta_{23} \simeq -\frac{1}{2\sqrt{2}\sin\theta_{13}}\left(1 - \frac{7}{2}\sin^2\theta_{13}\right)$$

* Neutrino mass sum rules in FLASY⇔ neutrinoless double beta decays

* Prediction of CP violating phase up to sign Takagi's talk

Backup slides

3.2 Origin of Flavor symmetry

Is it possible to realize such discrete symmetres in string theory?
Answer is yes !

Superstring theory on a certain type of six dimensional compact space leads to stringy selection rules for allowed couplings among matter fields in four-dimensional effective field theory.

Such stringy selection rules and geometrical symmetries result in discrete flavor symmetries in superstring theory.

- Heterotic orbifold models (Kobayashi, Nilles, Ploger, Raby, Ratz, 07)
- Magnetized/Intersecting D-brane Model
  (Kitazawa, Higaki, Kobayashi,Takahashi, 06)
  (Abe, Choi, Kobayashi, HO, 09, 10)

Stringy origin of non-Abelian discrete flavor symmetries

T. Kobayashi, H. Niles, F. PloegerS, S. Raby, M. Ratz, hep-ph/0611020

D_4, $\Delta(54)$

Non-Abelian Discrete Flavor Symmetries from Magnetized/Intersecting Brane Models

H. Abe, K-S. Choi, T. Kobayashi, H. Ohki, 0904.2631

D_4, $\Delta(27)$, $\Delta(54)$

Non-Abelian Discrete Flavor Symmetry from T^2/Z_N Orbifolds

A.Adulpravitchai, A. Blum, M. Lindner, 0906.0468

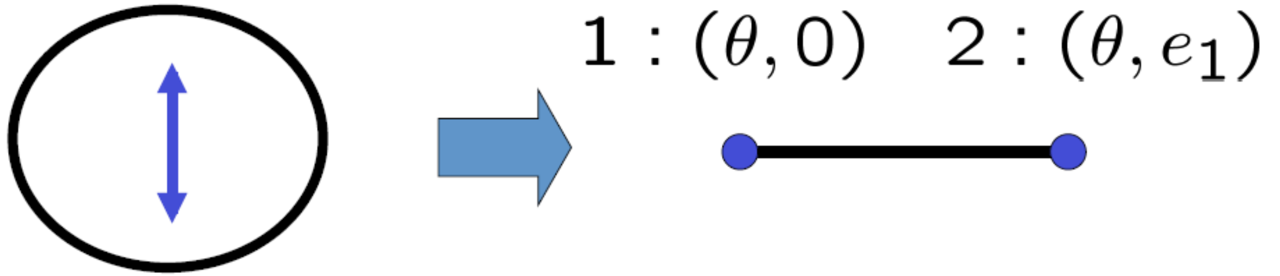
A_4, S_4, D_3, D_4, D_6

Non-Abelian Discrete Flavor Symmetries of 10D SYM theory with Magnetized extra dimensions

H. Abe, T. Kobayashi, H. Ohki, K.Sumita, Y. Tatsuta 1404.0137

S_3, $\Delta(27)$, $\Delta(54)$

S^1/Z_2 **orbifold** (Kobayashi, Nilles, Ploger, Raby, Ratz, 07)



There are two fixed point under the orbifold twist

These two fixed points can be represented by space group elements which act (θ, v)

$$(\theta, v)\alpha = \theta\alpha + v$$

e_1 : shift vector in one torus $(y \sim y + e_1)$

charge assignment of Z_2 : $\begin{pmatrix} 1 \\ 2 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

(stringy selection rule: Coupling is only allowed in matching of the string boundary conditions)

Discrete flavor symmetry from orbifold $S^1/\mathbf{Z}_2$

This effective Lagrangian also have permutation symmetry of these two fixed point (orbifold geometry).

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} \to \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

Closed algebra of these transformations $\left\{ \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right\}$

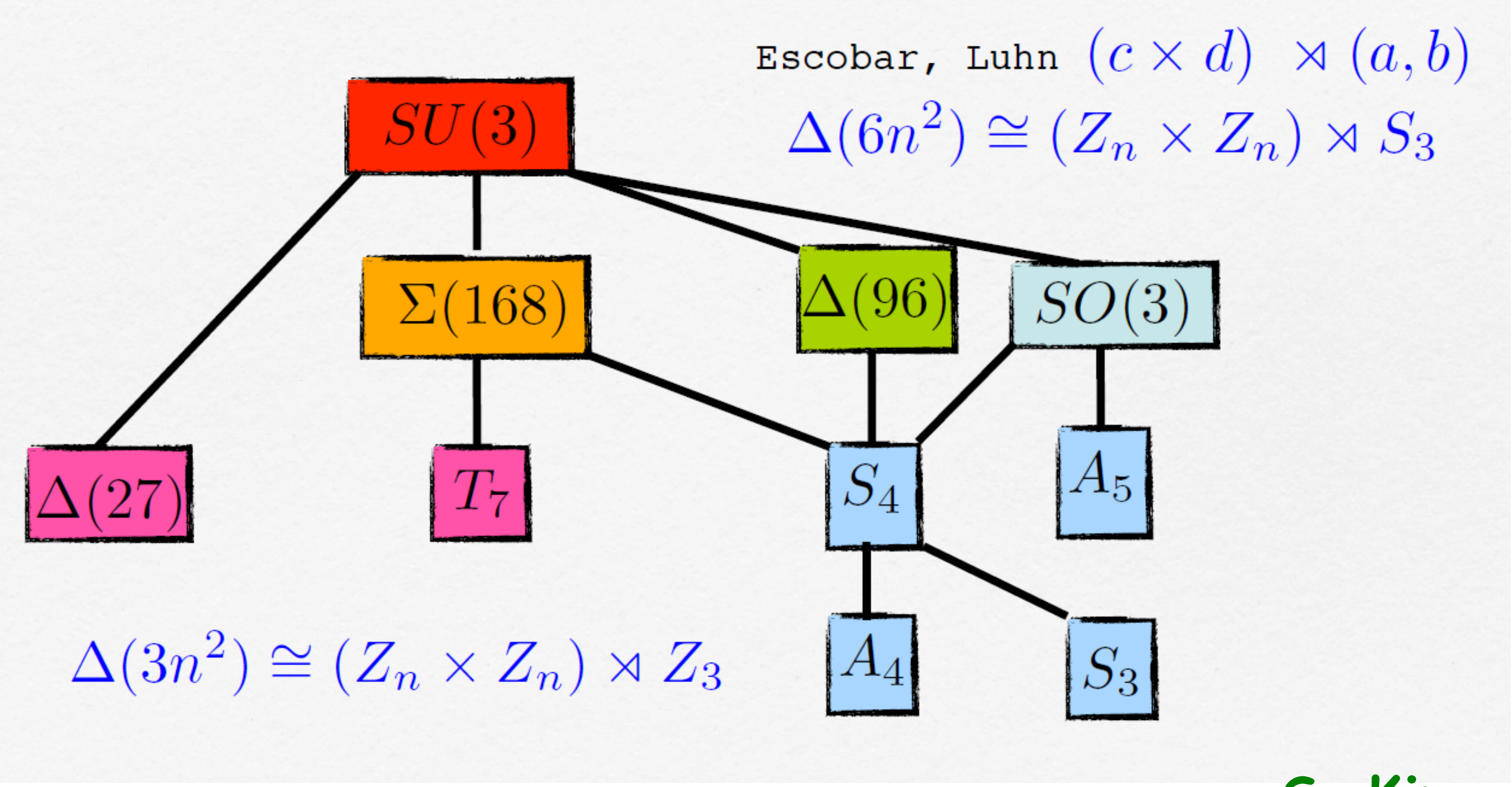
$$\Rightarrow \quad D_4 \sim S^2 \cup (\mathbf{Z}_2 \times \mathbf{Z}_2)$$

Two field localized at two fixed points : doublet of D4 $\mathbf{2}$

Bulk mode (untwisted mode) : singlet of D4 $\mathbf{1}$

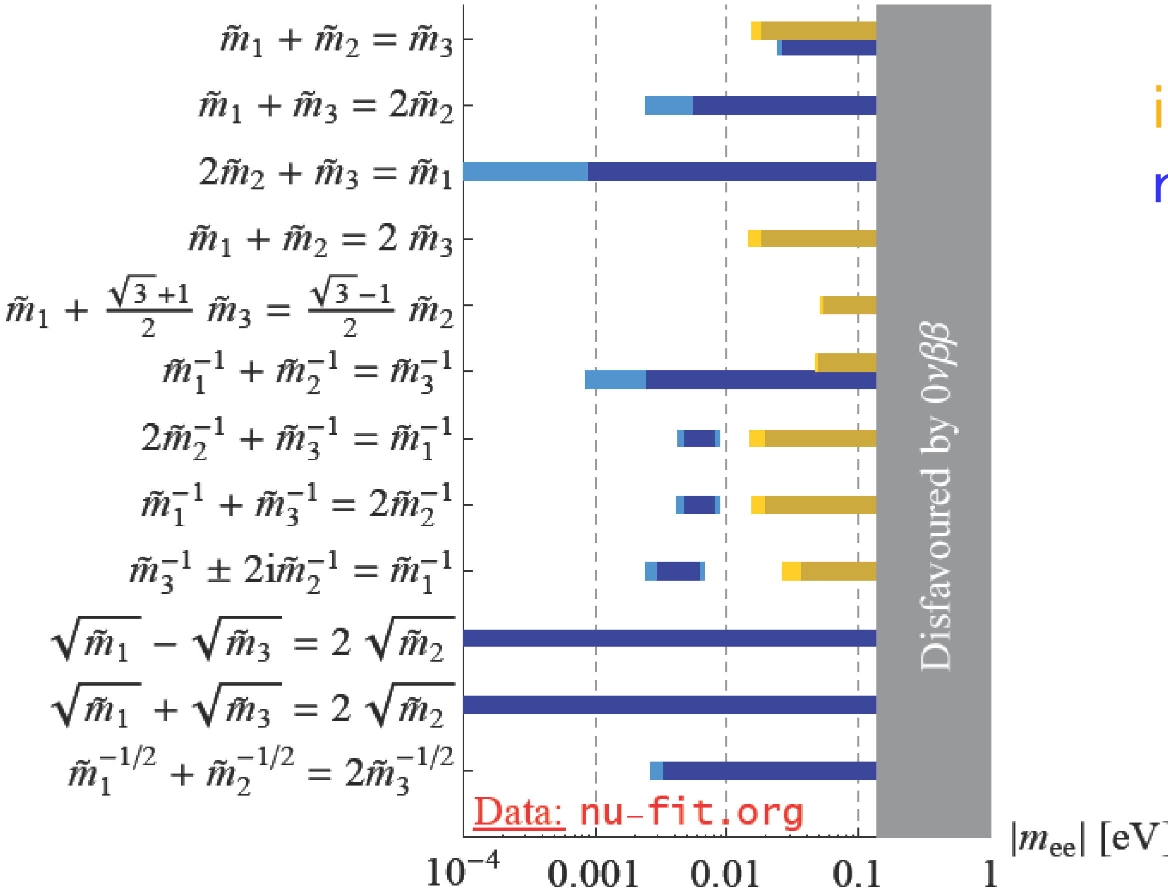
Thus full symmetry is larger than geometric symmetry

Alternatively, discrete flavor symmetries may be originated from continuous symmetries



S. King

Restrictions by mass sum rules on $|m_{ee}|$



inverted
normal

King, Merle, Stuart, JHEP 2013, arXiv:1307.2901

Mass sum rules in A_4, T', S_4, A_5, $\Delta(96)$ …

(Talk of Spinrath) Barry, Rodejohann, NPB842(2011) arXiv:1007.5217

Different types of neutrino mass spectra correspond to the neutrino mass generation mechanism.

$$\chi\tilde{m}_2 + \xi\tilde{m}_3 = \tilde{m}_1$$ (Χ=2, ξ=1) (Χ=-1, ξ=1)

$$\frac{\chi}{\tilde{m}_2} + \frac{\xi}{\tilde{m}_3} = \frac{1}{\tilde{m}_1}$$ **M_R structre in See-saw**

$$\chi\sqrt{\tilde{m}_2} + \xi\sqrt{\tilde{m}_3} = \sqrt{\tilde{m}_1}$$ **M_D structre in See-saw**

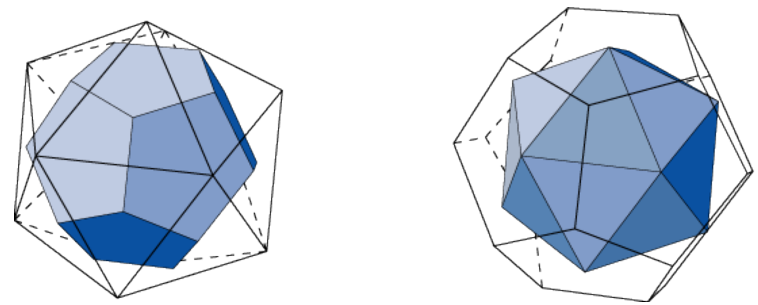
$$\frac{\chi}{\sqrt{\tilde{m}_2}} + \frac{\xi}{\sqrt{\tilde{m}_3}} = \frac{1}{\sqrt{\tilde{m}_1}}$$ **M_R in inverse See-saw**

Χ and ξ are model specific complex parameters

King, Merle, Stuart, JHEP 2013, arXiv:1307.2901
King, Merle, Morisi, Simizu, M.T, arXiv: 1402.4271

A_5 group (simple group)

The A_5 group is isomorphic to the symmetry of a regular icosahedron and a regular dodecahedron.



60 elements are generated S and T .

$S^2 = (ST)^3 = 1$ and $T^5 = 1$

5 conjugacy classes

Irreducible representations:
1, 3, 3', 4, 5

	h	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3'}$	$\mathbf{4}$	$\mathbf{5}$
C_1	1	1	3	3	4	5
C_{15}	2	1	-1	-1	0	1
C_{20}	3	1	0	0	1	-1
C_{12}	5	1	ϕ	$1-\phi$	-1	0
$C_{12'}$	5	1	$1-\phi$	ϕ	-1	0

For triplet 3

$$S = \frac{1}{\sqrt{5}}\begin{pmatrix} 1 & \sqrt{2} & \sqrt{2} \\ \sqrt{2} & -\phi & \frac{1}{\phi} \\ \sqrt{2} & \frac{1}{\phi} & -\phi \end{pmatrix} \quad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{\frac{2\pi i}{5}} & 0 \\ 0 & 0 & e^{\frac{8\pi i}{5}} \end{pmatrix}$$

$$\phi = \frac{1+\sqrt{5}}{2}$$ Golden Ratio

Mixing pattern in A_5 flavor symmetry

It has subgroups, ten Z_3, six Z_5, five $Z_2 \times Z_2$ (K_4) .

Suppose A_5 is spontaneously broken to one of subgroups:
Neutrino sector preserves S and U (K_4)
Charged lepton sector preserves T (Z_5)

$$S^T m^\nu_{LL} S = m^\nu_{LL}, \quad U^T m^\nu_{LL} U = m^\nu_{LL}, \quad T^\dagger Y_e Y_e^\dagger T = Y_e Y_e^\dagger$$

⇓

$$[S, m^\nu_{LL}] = 0, \quad [U, m^\nu_{LL}] = 0, \quad [T, Y_e Y_e^\dagger] = 0$$

$$S = \frac{1}{\sqrt{5}} \begin{pmatrix} 1 & \sqrt{2} & \sqrt{2} \\ \sqrt{2} & -\phi & \frac{1}{\phi} \\ \sqrt{2} & \frac{1}{\phi} & -\phi \end{pmatrix} \qquad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{\frac{2\pi i}{5}} & 0 \\ 0 & 0 & e^{\frac{8\pi i}{5}} \end{pmatrix} \qquad U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

F. Feruglio and Paris, JHEP 1103(2011) 101 arXiv:1101.0393

$$U_{GR} = \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ \frac{\sin\theta_{12}}{\sqrt{2}} & -\frac{\cos\theta_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{\sin\theta_{12}}{\sqrt{2}} & -\frac{\cos\theta_{12}}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$\theta_{13}=0$

$\tan\theta_{12} = 1/\phi$: $\phi = \frac{1+\sqrt{5}}{2}$ **Golden Ratio**

Neutrino mass matrix has μ-τ symmetry.

$$m_\nu = \begin{pmatrix} x & y & y \\ y & z & w \\ y & w & z \end{pmatrix}$$ with $z + w = x - \sqrt{2}y$

$\sin^2\theta_{12} = 2/(5+\sqrt{5}) = 0.2763\ldots$
which is rather smaller than the experimental data.

$$\sin^2\theta_{12} = 0.306 \pm 0.012$$