Research on Quantum Entanglement of the Vacuum of Fields

Reference
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flow of this talk

①Abstract

②Introduction

Why is the entanglement of vacuum important ?

How this talk is related to the theme of PPAP ?

③Motivation and Purpose

④Procedure and Calculations

⑤Results and Discussions

⑥Summary & Future work

Details of our research

Abstract

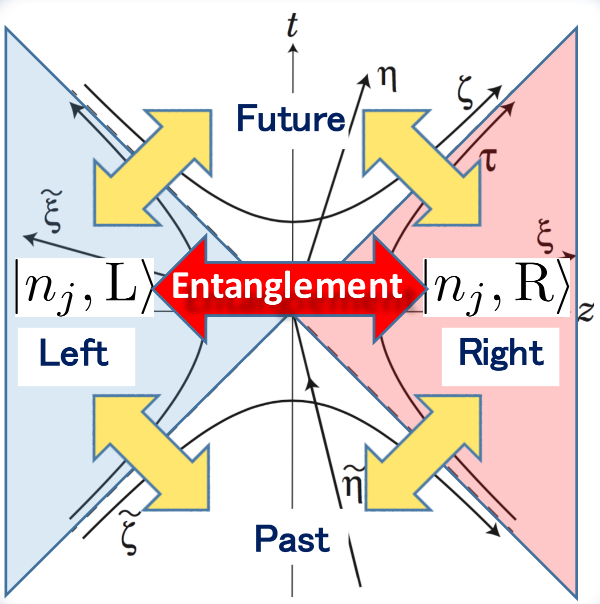
Unruh effect

A uniformly accelerating observer sees the Minkowski vacuum as a thermal state

Entangled state between the left and right Rindler wedges

$$|0, \mathrm{M}\rangle = \prod_j \left[N_j \sum_{n_j=0}^{\infty} e^{-\pi n_j \omega_j / a} \boxed{|n_j, \mathrm{L}\rangle} \otimes \boxed{|n_j, \mathrm{R}\rangle} \right]$$

:Extension



Extend the description of the Minkowski vacuum state to the entire Minkowski spacetime

Clarify the structure of the entanglements of the states in these spacetimes

Introduction

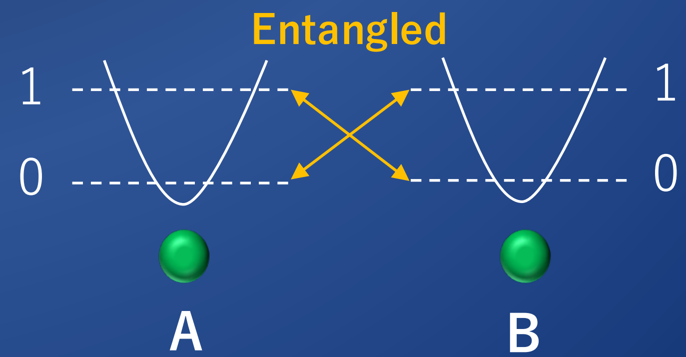
Quantum entanglement

A state which can't be expressed by a direct product

Example: A state of complex system of particle A and B

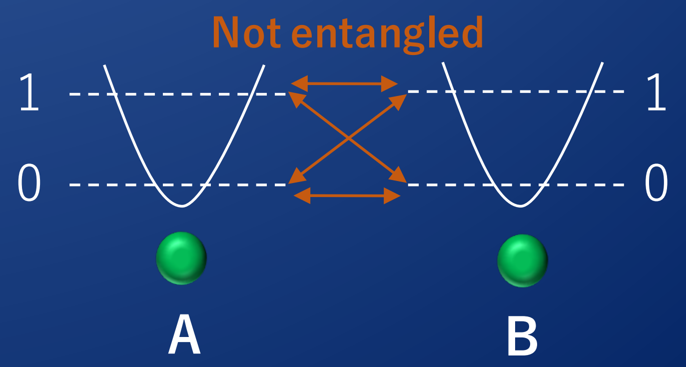
$$|\mathrm{AB}\rangle = \frac{1}{\sqrt{2}}(|0\rangle_{\mathrm{A}} \otimes |1\rangle_{\mathrm{B}} + |1\rangle_{\mathrm{A}} \otimes |0\rangle_{\mathrm{B}})$$

→ A and B are entangled



$$|\mathrm{AB}\rangle = \frac{1}{2}(|0\rangle_{\mathrm{A}} + |1\rangle_{\mathrm{A}}) \otimes (|0\rangle_{\mathrm{B}} + |1\rangle_{\mathrm{B}})$$

→ A and B aren't entangled



Introduction

What's the Unruh effect

$$T_{\mathrm{U}} = \frac{a}{2\pi}$$

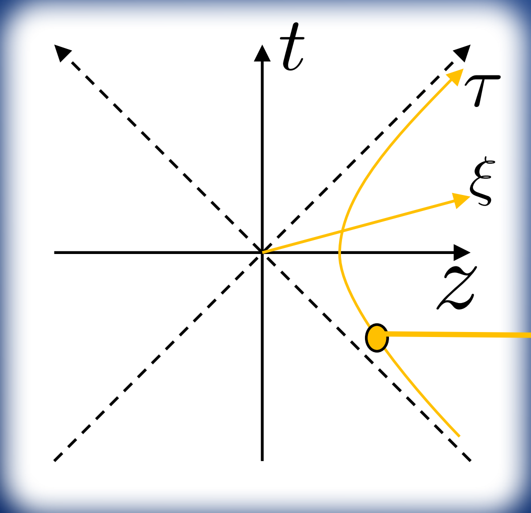
Unruh temperature

Unruh effect

The phenomenon that an uniformly accelerated observer sees the Minkowski vacuum as a thermal state.

The temperature is proportional to the acceleration of the observer.

Orbit of an uniformly accelerating observer



R Rindler coordinate (R-region)

$$t = \frac{1}{a} e^{a\xi} \sinh a\tau \qquad z = \frac{1}{a} e^{a\xi} \cosh a\tau$$

$$\boxed{t = \frac{1}{a} \sinh a\tau \qquad z = \frac{1}{a} \cosh a\tau}$$

$$\xi = \mathrm{const} = 0$$

Introduction

Minkowski vacuum state

$$\hat{b}_{k_z \boldsymbol{k}_\perp} |0, \mathrm{M}\rangle = 0$$

Bogoliubov transformation

Rindler vacuum state

$$\hat{a}^{\mathrm{R}}_{\omega,\boldsymbol{k}_\perp} |0, \mathrm{R}\rangle = 0$$

$$\hat{a}^{\mathrm{R}}_{\omega,\boldsymbol{k}_\perp} |0, \mathrm{M}\rangle \neq 0$$

The vacuum state which is naturally defined on the Rindler coordinate is different from the vacuum which is defined on the Minkowski coordinate.

The expectation value of number operator of the Rindler observer in non-inertial frame is not zero

Unruh effect

Introduction

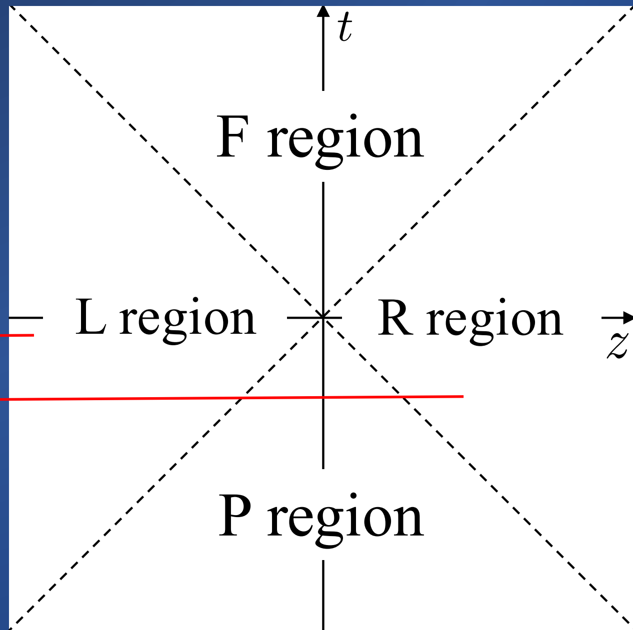
The model of the field consisting of two harmonic oscillators

simplified Minkowski vacuum

$$|0,\mathrm{M}\rangle = N_j \sum_{n_j=0}^{\infty} e^{-\pi n_j \omega_j / a} |n_j\rangle_{\mathrm{R}} \otimes |n_j\rangle_{\mathrm{L}}$$

density operator

$$\hat{\rho}_{\mathrm{R}} = \mathrm{Tr}_{\mathrm{L}}[|0,\mathrm{M}\rangle\langle 0,\mathrm{M}|]$$



Expectation value of number operator

$$\mathrm{Tr}_{\mathrm{R}}[\hat{\rho}_R a^\dagger a] = \frac{1}{e^{2\pi n\omega/a} - 1}$$

: Bose distribution function

$$\mathrm{T}_{\mathrm{U}} = \frac{a}{2\pi}$$

Unruh temperature

Therefore…

The description of the Minkowski vacuum state is important for the understanding of the Unruh effect.

Motivation and Purpose

Significance of the description of vacuum

The description of Minkowski vacuum is important for the Unruh effect.

The Unruh effect is basic prediction of QFT in curved spacetime.

The Unruh effect is related to the Hawking effect.

It's necessary to understand the difference of expectation value of number operator on each coordinates.

Motivation and Purpose

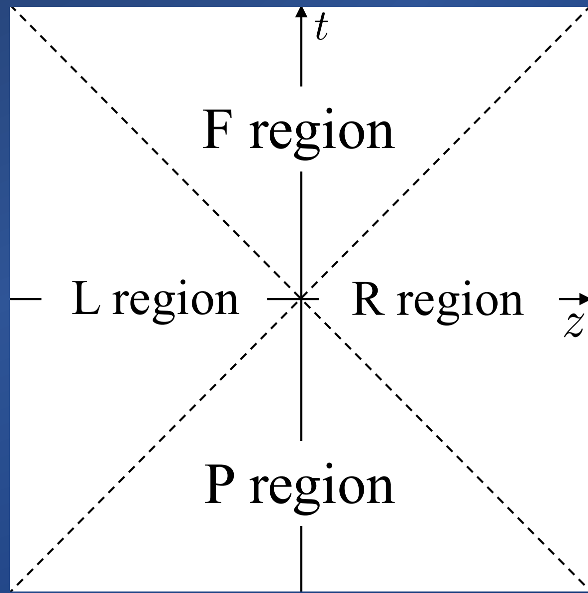
Historical back ground of the study of
the description of Minkowski vacuum with the entanglement

Unruh, Wald (1984)

Minkowski vacuum (4-dimensional case)

$$|0,\mathrm{M}\rangle \propto \prod_j \Big[\sum_{n_j=0}^{\infty} e^{-\pi n_j \omega_j / a} |n_j\rangle_{\mathrm{R}} \otimes |n_j\rangle_{\mathrm{L}}\Big]$$

The entanglement of R region and L region is important to understand the Unruh effect



Olson, Ralph (2010)

There is entanglement between F region and P region (2-dimensional case)

Obtain the description which explains the entanglement structure of the entire Minkowski spacetime in 2,4-dimensional case

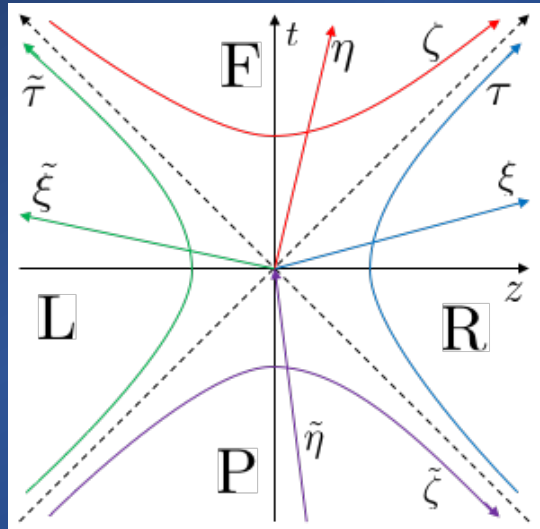
Procedure and Calculations

(1). **Obtain the mode expansion of scalar field in each (F,R,P,L) region**

(2). **Connect the mode functions of each region**

(3). **Describe the Minkowski vacuum state on the curved space-time**

(4). **Verify the obtained description by calculating 2 point correlation function**



Procedure and Calculations

Coordinates that we used in this research

R region（R Rindler coordinate）

$$t = \frac{1}{a} e^{a\xi} \sinh a\tau \qquad z = \frac{1}{a} e^{a\xi} \cosh a\tau$$

L region（L Rindler coordinate）

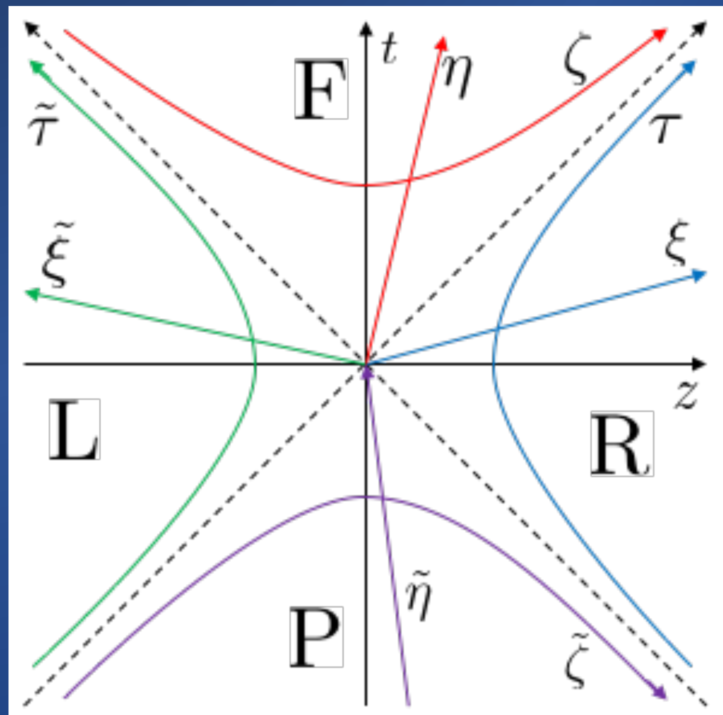
$$t = \frac{1}{a} e^{a\tilde{\xi}} \sinh a\tilde{\tau} \qquad z = -\frac{1}{a} e^{a\tilde{\xi}} \cosh a\tilde{\tau}$$

F region（F expanding degenerate Kasner

$$t = \frac{1}{a} e^{a\eta} \cosh a\zeta \qquad z = \frac{1}{a} e^{a\eta} \sinh a\zeta$$

P region（Past shrinking degenerate Kasner universe）

$$t = -\frac{1}{a} e^{-a\tilde{\eta}} \cosh a\tilde{\zeta} \quad z = \frac{1}{a} e^{-a\tilde{\eta}} \sinh a\tilde{\zeta}$$



Procedure and Calculations

Action of massless scalar field

$$S = \frac{1}{2}\int d^4x\sqrt{-g}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

$$ds^2 = dt^2 - dz^2 - d\boldsymbol{x}_\perp^2$$

$$\delta \mathrm{S} = 0$$

Eq. of motion

$$\left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial z^2} - \frac{\partial^2}{\partial \boldsymbol{x}_\perp^2}\right)\phi = 0$$

Commutation relation

$$[\hat{\phi}(t,\boldsymbol{x}), \hat{\pi}(t,\boldsymbol{y})] = i\delta_D(\boldsymbol{x}-\boldsymbol{y})$$

Mode expansion

$$\phi = \int_{-\infty}^{\infty} \frac{dk_z d^2k_\perp}{(2\pi)^{3/2}\sqrt{2k_0}}\left(\hat{b}_{k_z\boldsymbol{k}_\perp} e^{-ik_0t+ik_zz+i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp} + \mathrm{h.c.}\right)$$

Minkowski vacuum

$$\hat{b}_{k_z\boldsymbol{k}_\perp}|0,\mathrm{M}\rangle = 0$$

Creation and annihilation operator

$$[\hat{b}_{k_z\boldsymbol{k}_\perp}, \hat{b}^\dagger_{k'_z,\boldsymbol{k}'_\perp}] = \delta_D(k_z-k'_z)\delta_D^{(2)}(\boldsymbol{k}_\perp - \boldsymbol{k}'_\perp)$$

$$[\hat{b}_{k_z\boldsymbol{k}_\perp}, \hat{b}_{k'_z,\boldsymbol{k}'_\perp}] = [\hat{b}^\dagger_{k_z\boldsymbol{k}_\perp}, \hat{b}^\dagger_{k'_z,\boldsymbol{k}'_\perp}] = 0$$

Basically, the mode expansion on other coordinates are derived by the same procedure

Procedure and Calculations

Action of massless scalar field

$$S=\frac{1}{2}\int d^4x\sqrt{-g}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

$$ds^2=e^{2a\xi}(d\tau^2-d\xi^2)-d\boldsymbol{x}_\perp^2$$

$$\delta\mathrm{S}=0$$

Eq. of motion

$$\left(\frac{\partial^2}{\partial\tau^2}-\frac{\partial^2}{\partial\xi^2}-e^{2a\xi}\frac{\partial^2}{\partial\boldsymbol{x}_\perp^2}\right)\phi=0$$

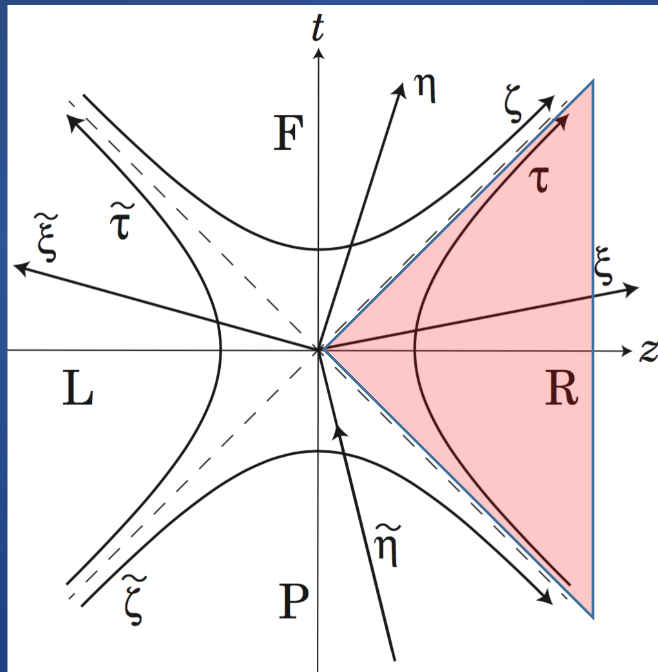
$$[\hat{\phi}(t,\boldsymbol{x}),\hat{\pi}(t,\boldsymbol{y})]=i\delta_D(\boldsymbol{x}-\boldsymbol{y})$$

$$\phi(x)=\int_0^\infty d\omega\int_{-\infty}^\infty d^2k_\perp\left(\hat{a}^{\mathrm{I}}_{\omega,\boldsymbol{k}_\perp}v^{\mathrm{R}}_{\omega,\boldsymbol{k}_\perp}(x_{\mathrm{R}})+\mathrm{h.c.}\right)$$

$$v^{\mathrm{R}}_{\omega,\boldsymbol{k}_\perp}(x_{\mathrm{R}})=\sqrt{\frac{\sinh\pi\omega/a}{4\pi^4a}}K_{i\omega/a}\left(\frac{\kappa e^{a\xi}}{a}\right)e^{i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp-i\omega\tau}$$

$$\hat{a}^{\mathrm{R}}_{\omega,\boldsymbol{k}_\perp}|0,\mathrm{R}\rangle=0$$

$$|n_j,\mathrm{R}\rangle=\frac{1}{\sqrt{n_j!}}(\hat{a}^{\mathrm{R}\dagger}_j)^{n_j}|0,\mathrm{R}\rangle\qquad j=(\omega,\boldsymbol{k}_\perp)$$



R Rindler corrdinate(R region: z>|t|)

$$t=\frac{1}{a}e^{a\xi}\sinh a\tau\quad z=\frac{1}{a}e^{a\xi}\cosh a\tau$$

Procedure and Calculations

Action of massless scalar field

$$S=\frac{1}{2}\int d^4x\sqrt{-g}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

$$ds^2=e^{2a\tilde{\xi}}(d\tilde{\tau}^2-d\tilde{\xi}^2)-d\boldsymbol{x}_\perp^2$$

$$\delta \mathrm{S}=0$$

Eq. of motion

$$\left(\frac{\partial^2}{\partial\tilde{\tau}^2}-\frac{\partial^2}{\partial\tilde{\xi}^2}-e^{2a\tilde{\xi}}\frac{\partial^2}{\partial\boldsymbol{x}_\perp^2}\right)\phi=0$$

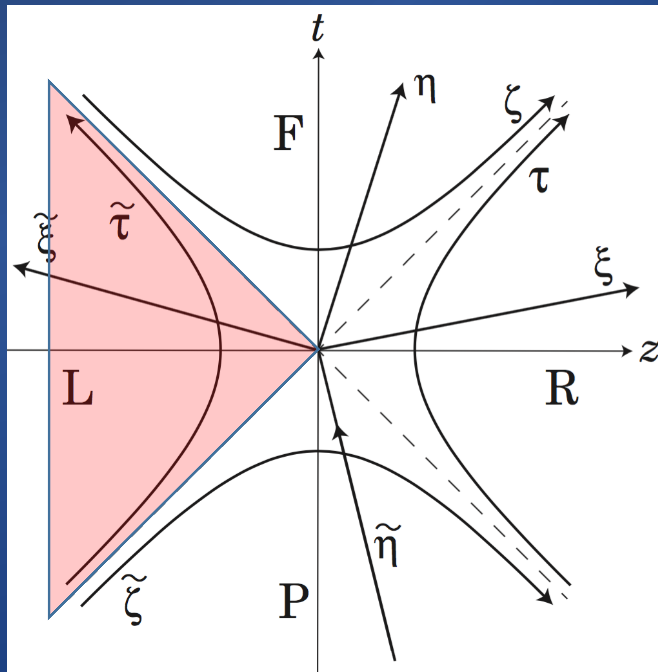
$$[\hat{\phi}(t,\boldsymbol{x}),\hat{\pi}(t,\boldsymbol{y})]=i\delta_D(\boldsymbol{x}-\boldsymbol{y})$$

$$\phi(x)=\int_0^\infty d\omega\int_{-\infty}^{\infty}d^2k_\perp\left(\hat{a}^{\mathrm{L}}_{\omega,\boldsymbol{k}_\perp}v^{\mathrm{L}}_{\omega,\boldsymbol{k}_\perp}(x_{\mathrm{L}})+\mathrm{h.c.}\right)$$

$$v^{\mathrm{L}}_{\omega,\boldsymbol{k}_\perp}(x_{\mathrm{L}})=\sqrt{\frac{\sinh\pi\omega/a}{4\pi^4 a}}K_{i\omega/a}\left(\frac{\kappa e^{a\tilde{\xi}}}{a}\right)e^{-i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp-i\omega\tilde{\tau}}$$

$$\hat{a}^{\mathrm{L}}_{\omega,\boldsymbol{k}_\perp}|0,\mathrm{L}\rangle=0$$

$$|n_j,\mathrm{L}\rangle=\frac{1}{\sqrt{n_j!}}(\hat{a}_j^{\mathrm{L}\dagger})^{n_j}|0,\mathrm{L}\rangle\qquad j=(\omega,\boldsymbol{k}_\perp)$$



L Rindler corrdinate(L region: -z>|t|)

$$t=\frac{1}{a}e^{a\tilde{\xi}}\sinh a\tilde{\tau}\qquad z=-\frac{1}{a}e^{a\tilde{\xi}}\cosh a\tilde{\tau}$$

Procedure and Calculations

Action of massless scalar field

$$S = \frac{1}{2}\int d^4x\sqrt{-g}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

$$ds^2 = e^{2a\eta}(d\eta^2 - d\zeta^2) - d\boldsymbol{x}_\perp^2$$

$$\delta\mathrm{S} = 0$$

Eq. of motion

$$\left(\frac{\partial^2}{\partial\eta^2} - \frac{\partial^2}{\partial\zeta^2} - e^{2a\eta}\frac{\partial^2}{\partial\boldsymbol{x}_\perp^2}\right)\phi = 0$$

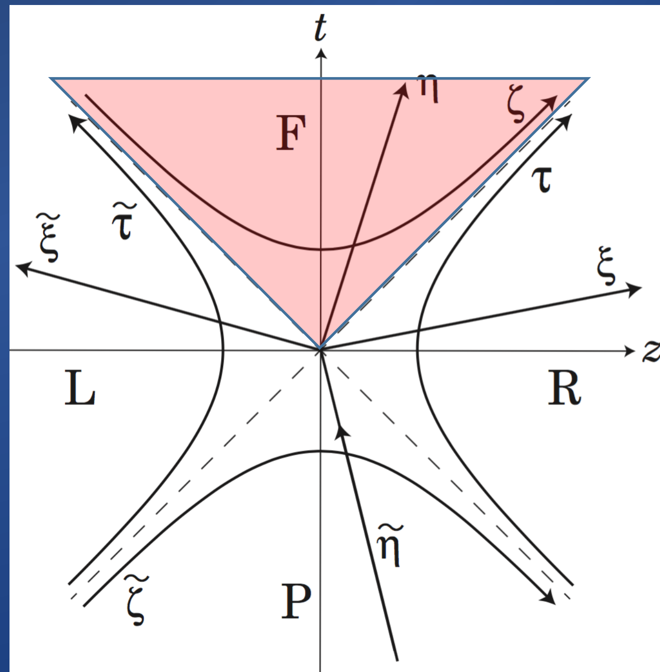
$$[\hat{\phi}(t,\boldsymbol{x}), \hat{\pi}(t,\boldsymbol{y})] = i\delta_D(\boldsymbol{x}-\boldsymbol{y})$$

$$\phi(x) = \int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} d^2k_\perp \left(\hat{a}^{\mathrm{F}}_{\omega,\boldsymbol{k}_\perp} v^{\mathrm{F}}_{\omega,\boldsymbol{k}_\perp}(x_{\mathrm{F}}) + \mathrm{h.c.}\right)$$

$$v^{\mathrm{F}}_{\omega,\boldsymbol{k}_\perp}(x_{\mathrm{F}}) = \frac{-ie^{i\omega\zeta}}{2\pi\sqrt{4a\sinh(\pi|\omega|/a)}} J_{-i|\omega|/a}\left(\frac{\kappa e^{a\eta}}{a}\right) e^{i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp}$$

$$\hat{a}^{\mathrm{F}}_{\omega,\boldsymbol{k}_\perp}|0,\mathrm{F}\rangle = 0$$

$$|n_j,\mathrm{F}\rangle = \frac{1}{\sqrt{n_j!}}(\hat{a}_j^{\mathrm{F}\dagger})^{n_j}|0,\mathrm{F}\rangle \qquad j = (\omega, \boldsymbol{k}_\perp)$$



F Kasner corrdinate(F region: t>|z|)

$$t = \frac{1}{a}e^{a\eta}\cosh a\zeta \qquad z = \frac{1}{a}e^{a\eta}\sinh a\zeta$$

Procedure and Calculations

Future (expanding) degenerate Kasner coordinate(F region: t>|z|)

$$\phi(x) = \phi^{\mathrm{F,s}}(x) + \phi^{\mathrm{F,d}}(x)$$

$$\phi^{\mathrm{F,s}}(x) = \int_0^\infty d\omega \int_{-\infty}^{\infty} d^2k_\perp \left(\hat{a}^{\mathrm{F,s}}_{\omega,\boldsymbol{k}_\perp} v^{\mathrm{F,s}}_{\omega,\boldsymbol{k}_\perp}(x) + \mathrm{h.c.}\right)$$

$$\phi^{\mathrm{F,d}}(x) = \int_0^\infty d\omega \int_{-\infty}^{\infty} d^2k_\perp \left(\hat{a}^{\mathrm{F,d}}_{\omega,\boldsymbol{k}_\perp} v^{\mathrm{F,d}}_{\omega,\boldsymbol{k}_\perp}(x) + \mathrm{h.c.}\right)$$

$$v^{\mathrm{F,s}}_{\omega,\boldsymbol{k}_\perp}(x) := v^{\mathrm{F}}_{-\omega,\boldsymbol{k}_\perp}(x) = \frac{-ie^{-i\omega\zeta}}{2\pi\sqrt{4a\sinh(\pi\omega/a)}} J_{-i\omega/a}\left(\frac{\kappa e^{a\eta}}{a}\right) e^{i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp}$$

$$v^{\mathrm{F,d}}_{\omega,\boldsymbol{k}_\perp}(x) := v^{\mathrm{F}}_{\omega,-\boldsymbol{k}_\perp}(x) = \frac{-ie^{i\omega\zeta}}{2\pi\sqrt{4a\sinh(\pi\omega/a)}} J_{-i\omega/a}\left(\frac{\kappa e^{a\eta}}{a}\right) e^{-i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp}$$

$$\hat{a}^{\mathrm{F,s}}_{\omega,\boldsymbol{k}_\perp} = \hat{a}^{\mathrm{F}}_{-\omega,\boldsymbol{k}_\perp} \qquad \hat{a}^{\mathrm{F,d}}_{\omega,\boldsymbol{k}_\perp} = \hat{a}^{\mathrm{F}}_{\omega,-\boldsymbol{k}_\perp}$$

We decompose the solution of scalar field into two parts.

Procedure and Calculations

Action of massless scalar field

$$S=\frac{1}{2}\int d^4x\sqrt{-g}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

$$ds^2=e^{-2a\tilde{\eta}}(d\tilde{\eta}^2-d\tilde{\zeta}^2)-d\boldsymbol{x}_\perp^2$$

$$\delta \mathrm{S}=0$$

Eq. of motion

$$\left(\frac{\partial^2}{\partial\tilde{\eta}^2}-\frac{\partial^2}{\partial\tilde{\zeta}^2}-e^{-2a\tilde{\eta}}\frac{\partial^2}{\partial\boldsymbol{x}_\perp^2}\right)\phi=0$$

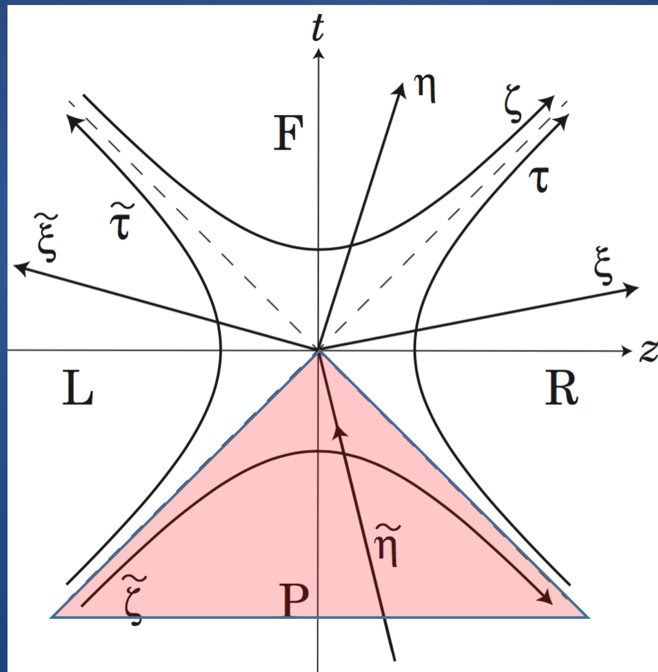
$$[\hat{\phi}(t,\boldsymbol{x}),\hat{\pi}(t,\boldsymbol{y})]=i\delta_D(\boldsymbol{x}-\boldsymbol{y})$$

$$\phi(x)=\int_{-\infty}^{\infty}d\omega\int_{-\infty}^{\infty}d^2k_\perp\left(\hat{a}^{\mathrm{P}}_{\omega,\boldsymbol{k}_\perp}v^{\mathrm{P}}_{\omega,\boldsymbol{k}_\perp}(x_{\mathrm{P}})+\mathrm{h.c.}\right)$$

$$v^{\mathrm{P}}_{\omega,\boldsymbol{k}_\perp}(x_{\mathrm{P}})=\frac{ie^{i\omega\tilde{\zeta}}}{2\pi\sqrt{4a\sinh(\pi|\omega|/a)}}J_{i|\omega|/a}\left(\frac{\kappa e^{-a\tilde{\eta}}}{a}\right)e^{i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp}$$

$$\hat{a}^{\mathrm{P}}_{\omega,\boldsymbol{k}_\perp}|0,\mathrm{P}\rangle=0$$

$$|n_j,\mathrm{P}\rangle=\frac{1}{\sqrt{n_j!}}(\hat{a}^{\mathrm{P}\dagger}_j)^{n_j}|0,\mathrm{P}\rangle\qquad j=(\omega,\boldsymbol{k}_\perp)$$



P Kasner corrdinate(P region: -t>|z|)

$$t=-\frac{1}{a}e^{-a\tilde{\eta}}\cosh a\tilde{\zeta}\qquad z=\frac{1}{a}e^{-a\tilde{\eta}}\sinh a\tilde{\zeta}$$

Procedure and Calculations

Past(shrinking) degenerate Kasner coordinate(P region: -t>|z|)

$$\phi(x) = \phi^{\mathrm{P,s}}(x) + \phi^{\mathrm{P,d}}(x)$$

$\tilde{\zeta}$

$$\phi^{\mathrm{P,s}}(x) = \int_0^{\infty} d\omega \int_{-\infty}^{\infty} d^2k_\perp \left(\hat{a}^{\mathrm{P,s}}_{\omega,\boldsymbol{k}_\perp} v^{\mathrm{P,s}}_{\omega,\boldsymbol{k}_\perp}(x) + \mathrm{h.c.}\right)$$

$$\phi^{\mathrm{P,d}}(x) = \int_0^{\infty} d\omega \int_{-\infty}^{\infty} d^2k_\perp \left(\hat{a}^{\mathrm{P,d}}_{\omega,\boldsymbol{k}_\perp} v^{\mathrm{P,d}}_{\omega,\boldsymbol{k}_\perp}(x) + \mathrm{h.c.}\right)$$

$$\begin{aligned} v^{\mathrm{P,s}}_{\omega,\boldsymbol{k}_\perp}(x) &= v^{\mathrm{P}}_{-\omega,-\boldsymbol{k}_\perp}(x) \\ &= \frac{ie^{-i\omega\tilde{\zeta}}}{2\pi\sqrt{4a\sinh(\pi\omega/a)}} J_{i\omega/a}\left(\frac{\kappa e^{-a\tilde{\eta}}}{a}\right) e^{-i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp} \end{aligned}$$

$$\begin{aligned} v^{\mathrm{P,d}}_{\omega,\boldsymbol{k}_\perp}(x) &:= v^{\mathrm{P}}_{\omega,\boldsymbol{k}_\perp}(x) \\ &= \frac{ie^{i\omega\tilde{\zeta}}}{2\pi\sqrt{4a\sinh(\pi\omega/a)}} J_{i\omega/a}\left(\frac{\kappa e^{-a\tilde{\eta}}}{a}\right) e^{i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp} \end{aligned}$$

$$\hat{a}^{\mathrm{P,s}}_{\omega,\boldsymbol{k}_\perp} = \hat{a}^{\mathrm{P}}_{-\omega,-\boldsymbol{k}_\perp} \qquad \hat{a}^{\mathrm{P,d}}_{\omega,\boldsymbol{k}_\perp} = \hat{a}^{\mathrm{P}}_{\omega,\boldsymbol{k}_\perp}$$

We can write the quantum field separating the right-moving wave modes from the left-moving wave modes by decomposing the solution.

Results and discussions

4-dimensional massless scalar field

derived mode functions

L region

$$v^{\mathrm{L}}_{\omega,\boldsymbol{k}_\perp}(x_L) = e^{-i\omega\tilde{\tau}} e^{-i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp}\sqrt{\frac{\sinh\pi\omega/a}{4\pi^4 a}}K_{i\omega/a}\left(\frac{\kappa e^{a\tilde{\xi}}}{a}\right)$$

R region

$$v^{\mathrm{R}}_{\omega,\boldsymbol{k}_\perp}(x_R) = e^{-i\omega\tau} e^{i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp}\sqrt{\frac{\sinh\pi\omega/a}{4\pi^4 a}}K_{i\omega/a}\left(\frac{\kappa e^{a\xi}}{a}\right)$$

F region, right moving mode

$$v^{\mathrm{F,d}}_{\omega,\boldsymbol{k}_\perp}(x) = \frac{-ie^{i\omega\zeta}}{2\pi\sqrt{4a\sinh(\pi\omega/a)}}J_{-i\omega/a}\left(\frac{\kappa e^{a\eta}}{a}\right)e^{-i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp}$$

F region, left moving mode

$$v^{\mathrm{F,s}}_{\omega,\boldsymbol{k}_\perp}(x) = \frac{-ie^{-i\omega\zeta}}{2\pi\sqrt{4a\sinh(\pi\omega/a)}}J_{-i\omega/a}\left(\frac{\kappa e^{a\eta}}{a}\right)e^{i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp}$$

P region, left moving mode

$$v^{\mathrm{P,s}}_{\omega,\boldsymbol{k}_\perp}(x) = \frac{ie^{-i\omega\tilde{\zeta}}}{2\pi\sqrt{4a\sinh(\pi\omega/a)}}J_{i\omega/a}\left(\frac{\kappa e^{-a\tilde{\eta}}}{a}\right)e^{-i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp}$$

P region, right moving mode

$$v^{\mathrm{P,d}}_{\omega,\boldsymbol{k}_\perp}(x) = \frac{ie^{i\omega\tilde{\zeta}}}{2\pi\sqrt{4a\sinh(\pi\omega/a)}}J_{i\omega/a}\left(\frac{\kappa e^{-a\tilde{\eta}}}{a}\right)e^{i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp}$$

Results and discussions

Analytic continuation of exponential term

$$
\begin{array}{lll}
\mathrm{F} \longrightarrow \mathrm{R} & \sqrt{t^2-z^2} \to e^{-\pi i/2}\sqrt{z^2-t^2}, & \sqrt{\dfrac{t+z}{t-z}} \to e^{+\pi i/2}\sqrt{\dfrac{z+t}{z-t}} \\
\mathrm{F} \longrightarrow \mathrm{L} & \sqrt{t^2-z^2} \to e^{-\pi i/2}\sqrt{z^2-t^2}, & \sqrt{\dfrac{t+z}{t-z}} \to e^{-\pi i/2}\sqrt{\dfrac{z+t}{z-t}} \\
\mathrm{P} \longrightarrow \mathrm{R} & \sqrt{t^2-z^2} \to e^{+\pi i/2}\sqrt{z^2-t^2}, & \sqrt{\dfrac{t+z}{t-z}} \to e^{+\pi i/2}\sqrt{\dfrac{z+t}{z-t}} \\
\mathrm{P} \longrightarrow \mathrm{L} & \sqrt{t^2-z^2} \to e^{+\pi i/2}\sqrt{z^2-t^2}, & \sqrt{\dfrac{t+z}{t-z}} \to e^{-\pi i/2}\sqrt{\dfrac{z+t}{z-t}}
\end{array}
$$

Corresponding analytic continuation of variables

$$
\begin{array}{lll}
\mathrm{F} \longrightarrow \mathrm{R} & \tau = \zeta - \dfrac{\pi}{2a}i, & \xi = \eta + \dfrac{\pi}{2a}i \\
\mathrm{F} \longrightarrow \mathrm{L} & \tilde{\tau} = -\zeta - \dfrac{\pi}{2a}i, & \tilde{\xi} = \eta + \dfrac{\pi}{2a}i \\
\mathrm{P} \longrightarrow \mathrm{R} & \tau = -\tilde{\zeta} - \dfrac{\pi}{2a}i, & \xi = -\tilde{\eta} - \dfrac{\pi}{2a}i \\
\mathrm{P} \longrightarrow \mathrm{L} & \tilde{\tau} = \tilde{\zeta} - \dfrac{\pi}{2a}i, & \tilde{\xi} = -\tilde{\eta} - \dfrac{\pi}{2a}i
\end{array}
$$

Results and discussions

Analytic continuation

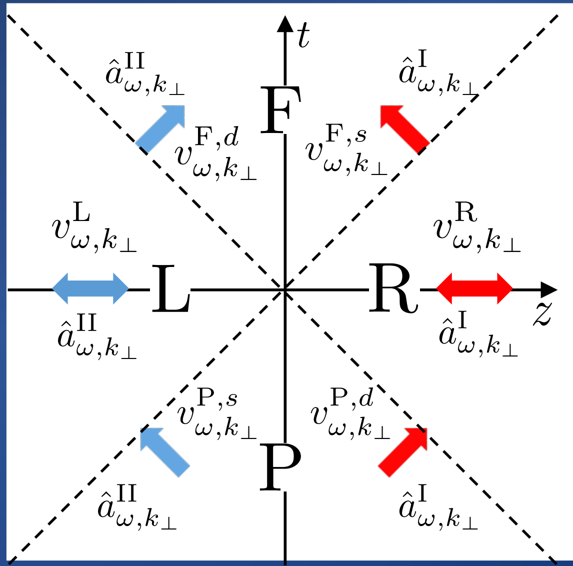
$$v^{\mathrm{F,d}}_{\omega,\boldsymbol{k}_\perp} \longrightarrow v^{\mathrm{L}}_{\omega,\boldsymbol{k}_\perp}$$

$$v^{\mathrm{P,s}}_{\omega,\boldsymbol{k}_\perp} \longrightarrow v^{\mathrm{L}}_{\omega,\boldsymbol{k}_\perp}$$

$$v^{\mathrm{F,s}}_{\omega,\boldsymbol{k}_\perp} \longrightarrow v^{\mathrm{R}}_{\omega,\boldsymbol{k}_\perp}$$

$$v^{\mathrm{P,d}}_{\omega,\boldsymbol{k}_\perp} \longrightarrow v^{\mathrm{R}}_{\omega,\boldsymbol{k}_\perp}$$

4-dimensional massless scalar field



$$v^{\mathrm{II}}_{\omega,\boldsymbol{k}_\perp}(x) = \begin{cases} v^{\mathrm{F,d}}_{\omega,\boldsymbol{k}_\perp} & \mathrm{F} \\ 0 & \mathrm{R} \\ v^{\mathrm{L}}_{\omega,\boldsymbol{k}_\perp} & \mathrm{L} \\ v^{\mathrm{P,s}}_{\omega,\boldsymbol{k}_\perp} & \mathrm{P} \end{cases}$$

$$v^{\mathrm{I}}_{\omega,\boldsymbol{k}_\perp}(x) = \begin{cases} v^{\mathrm{F,s}}_{\omega,\boldsymbol{k}_\perp} & \mathrm{F} \\ v^{\mathrm{R}}_{\omega,\boldsymbol{k}_\perp} & \mathrm{R} \\ 0 & \mathrm{L} \\ v^{\mathrm{P,d}}_{\omega,\boldsymbol{k}_\perp} & \mathrm{P} \end{cases}$$

$$\hat{a}^{\mathrm{II}}_{\omega,\boldsymbol{k}_\perp} := \hat{a}^{\mathrm{L}}_{\omega,\boldsymbol{k}_\perp} = \hat{a}^{\mathrm{P,s}}_{\omega,\boldsymbol{k}_\perp} = \hat{a}^{\mathrm{F,d}}_{\omega,\boldsymbol{k}_\perp}$$

$$\hat{a}^{\mathrm{I}}_{\omega,\boldsymbol{k}_\perp} := \hat{a}^{\mathrm{R}}_{\omega,\boldsymbol{k}_\perp} = \hat{a}^{\mathrm{P,d}}_{\omega,\boldsymbol{k}_\perp} = \hat{a}^{\mathrm{F,s}}_{\omega,\boldsymbol{k}_\perp}$$

Scalar field :

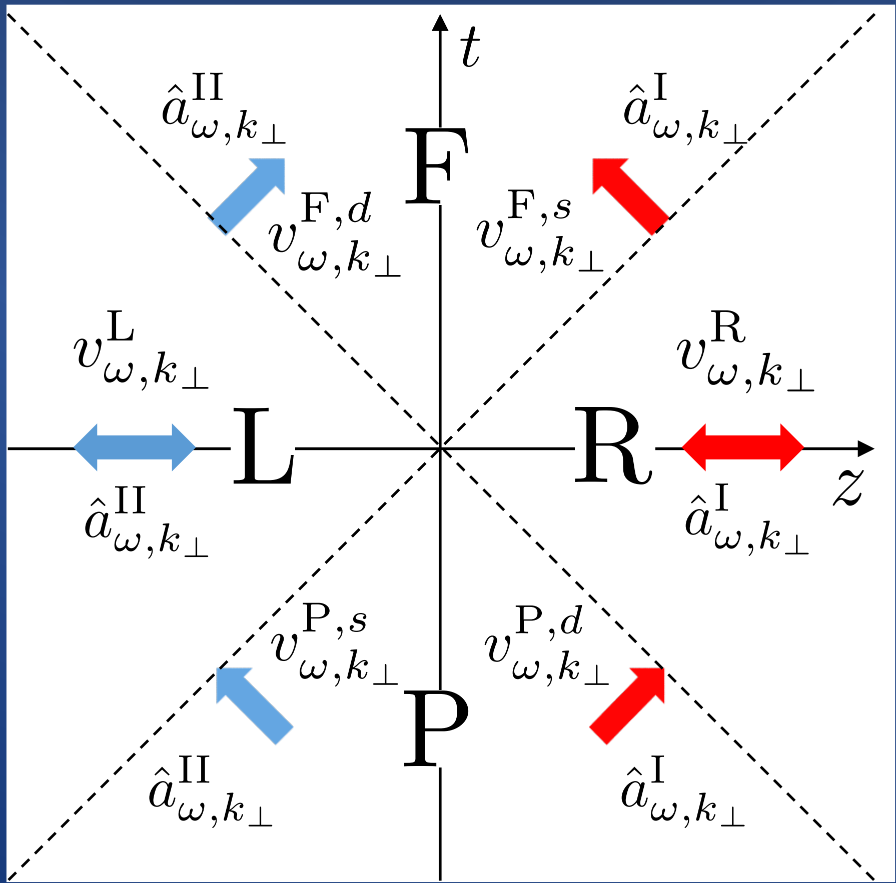
$$\phi(x) = \sum_{\sigma=\mathrm{I,II}} \int_0^\infty d\omega \int_{-\infty}^{\infty} d^2k_\perp \left(\hat{a}^\sigma_{\omega,\boldsymbol{k}_\perp} v^\sigma_{\omega,\boldsymbol{k}_\perp}(x) + \mathrm{h.c.}\right)$$

Minkowski vacuum :

$$|0,\mathrm{M}\rangle = \prod_j \left[N_j \sum_{n_j=0}^{\infty} e^{-\pi n_j \omega_j / a} |n_j, \mathrm{II}\rangle \otimes |n_j, \mathrm{I}\rangle\right]$$

Results and discussions

4-dimensional massless scalar field



Each mode is propagating like massive wave

Results and discussions

Analytic continuation

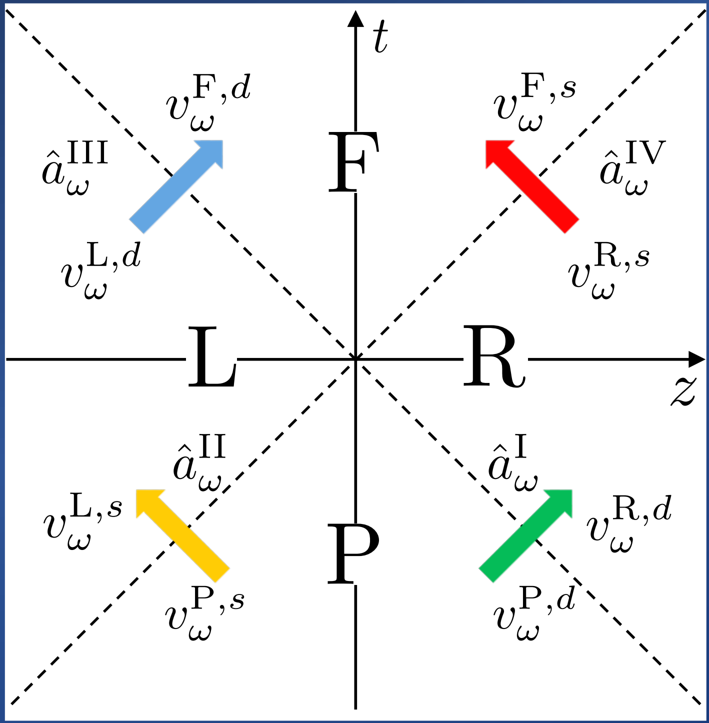
$$
v^{\mathrm{I}}_\omega(x) = \theta(-U)\frac{1}{\sqrt{4\pi\omega}}(-aU)^{i\omega/a}
= \begin{cases} 0 & \mathrm{F} \\ v^{\mathrm{R,d}}_\omega = e^{-i\omega(\tau-\xi)}/\sqrt{4\pi\omega} & \mathrm{R} \\ 0 & \mathrm{L} \\ v^{\mathrm{P,d}}_\omega = e^{-i\omega(\tilde{\eta}-\tilde{\zeta})}/\sqrt{4\pi\omega} & \mathrm{P} \end{cases}
$$

$$
v^{\mathrm{II}}_\omega(x) = \theta(-V)\frac{1}{\sqrt{4\pi\omega}}(-aV)^{i\omega/a}
= \begin{cases} 0 & \mathrm{F} \\ 0 & \mathrm{R} \\ v^{\mathrm{L,s}}_\omega = e^{-i\omega(\tilde{\tau}-\tilde{\xi})}/\sqrt{4\pi\omega} & \mathrm{L} \\ v^{\mathrm{P,s}}_\omega = e^{-i\omega(\tilde{\eta}+\tilde{\zeta})}/\sqrt{4\pi\omega} & \mathrm{P} \end{cases}
$$

$$
v^{\mathrm{III}}_\omega(x) = \theta(U)\frac{1}{\sqrt{4\pi\omega}}(aU)^{-i\omega/a}
= \begin{cases} v^{\mathrm{F,d}}_\omega = e^{-i\omega(\eta-\zeta)}/\sqrt{4\pi\omega} & \mathrm{F} \\ 0 & \mathrm{R} \\ v^{\mathrm{L,d}}_\omega = e^{-i\omega(\tilde{\tau}+\tilde{\xi})}/\sqrt{4\pi\omega} & \mathrm{L} \\ 0 & \mathrm{P} \end{cases}
$$

$$
v^{\mathrm{IV}}_\omega(x) = \theta(V)\frac{1}{\sqrt{4\pi\omega}}(aV)^{-i\omega/a}
= \begin{cases} v^{\mathrm{F,s}}_\omega = e^{-i\omega(\eta+\zeta)}/\sqrt{4\pi\omega} & \mathrm{F} \\ v^{\mathrm{R,s}}_\omega = e^{-i\omega(\tau+\xi)}/\sqrt{4\pi\omega} & \mathrm{R} \\ 0 & \mathrm{L} \\ 0 & \mathrm{P} \end{cases}
$$

2-dimensional massless scalar field

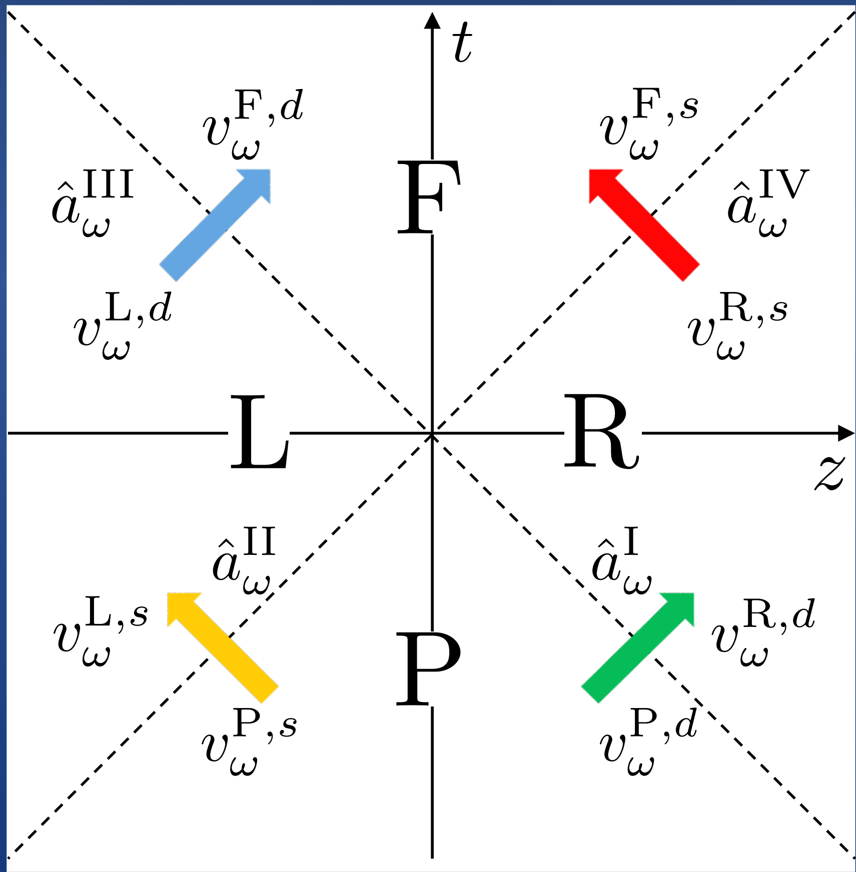


$$
\phi(x) = \sum_{\sigma=\mathrm{I,II,III,IV}} \int_0^\infty d\omega\, \left(\hat{a}^\sigma_\omega v^\sigma_\omega(x) + \mathrm{h.c.}\right)
$$

$$
|0,\mathrm{M}\rangle = \prod_\omega \left[N_\omega \sum_{n_\omega=0}^{\infty} e^{-\pi n_\omega \omega/a} |n_\omega, \mathrm{I}\rangle \otimes |n_\omega, \mathrm{III}\rangle \right]
\otimes \prod_{\omega'} \left[N_{\omega'} \sum_{n_{\omega'}=0}^{\infty} e^{-\pi n_{\omega'} \omega'/a} |n_{\omega'}, \mathrm{II}\rangle \otimes |n_{\omega'}, \mathrm{IV}\rangle \right]
$$

Results and discussions

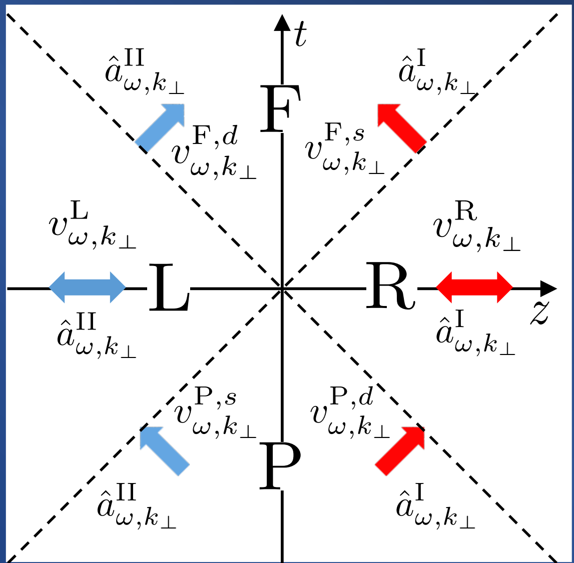
2-dimensional massless scalar field



Each mode is propagating along with the light cone just like massless wave

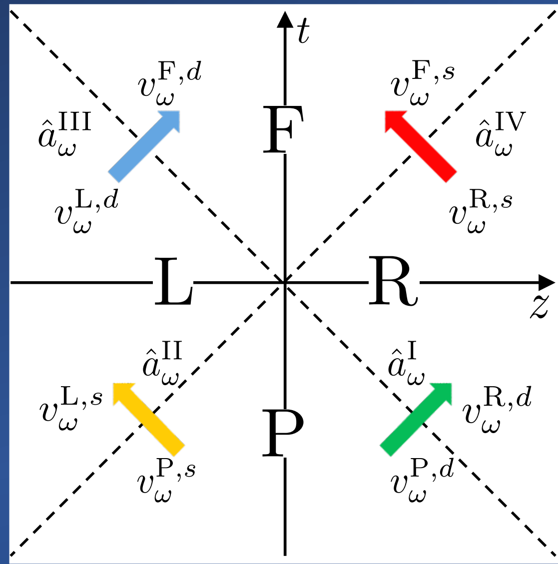
Results and discussions

4-dimensional massless



massive

2-dimensional massless case



massless

The cause of the difference is the existence of wavenumber which corresponds to the spatial axis which is perpendicular to the direction of acceleration.

P region, massive field

$$\phi = \sum_{\omega,\boldsymbol{k}} N e^{i\omega\tilde{\zeta}} f(\tilde{\eta}) e^{i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp}$$

Eq. of motion

$$\left(\frac{\partial^2}{\partial\tilde{\eta}^2} - \frac{\partial^2}{\partial\tilde{\zeta}^2} - e^{-2a\tilde{\eta}}\frac{\partial^2}{\partial\boldsymbol{x}_\perp^2} + e^{-2a\tilde{\eta}}m^2\right)\phi = 0$$

$$\kappa \equiv \sqrt{\boldsymbol{k}_\perp{}^2 + m^2}$$

$$\left(\frac{\partial^2}{\partial\tilde{\eta}^2} + \omega^2 + \kappa^2 e^{-2a\tilde{\eta}}\right) f(\tilde{\eta}) = 0$$

Results and discussions

Calculation of 2-point correlation function

We calculated 2-point function with derived descriptions and compared the result with the 2-point function on the Minkowski coordinate

Verification of derived description

massless scalar field (4–**dim**)

$$\phi(x) = \sum_{\sigma=\mathrm{I,II}} \int_0^\infty d\omega \int_{-\infty}^\infty d^2k_\perp \left(\hat{a}^\sigma_{\omega,\boldsymbol{k}_\perp} v^\sigma_{\omega,\boldsymbol{k}_\perp}(x) + \mathrm{h.c.}\right)$$

(2–**dim**)

$$\phi(x) = \sum_{\sigma=\mathrm{I,II,III,IV}} \int_0^\infty d\omega \left(\hat{a}^\sigma_\omega v^\sigma_\omega(x) + \mathrm{h.c.}\right)$$

Minkowski vacuum (4–**dim**)

$$|0,\mathrm{M}\rangle = \prod_j \left[N_j \sum_{n_j=0}^\infty e^{-\pi n_j \omega_j/a} |n_j, \mathrm{II}\rangle \otimes |n_j, \mathrm{I}\rangle\right]$$

(2–**dim**)

$$|0,\mathrm{M}\rangle = \prod_\omega \left[N_\omega \sum_{n_\omega=0}^\infty e^{-\pi n_\omega \omega/a} |n_\omega, \mathrm{I}\rangle \otimes |n_\omega, \mathrm{III}\rangle\right]$$
$$\otimes \prod_{\omega'} \left[N_{\omega'} \sum_{n_{\omega'}=0}^\infty e^{-\pi n_{\omega'} \omega'/a} |n_{\omega'}, \mathrm{II}\rangle \otimes |n_{\omega'}, \mathrm{IV}\rangle\right]$$

Results and discussions

4-dimension

$$\langle 0,\mathrm{M}|\phi(x)\phi(x')|0,\mathrm{M}\rangle \longrightarrow \int \frac{dk_z d^2k_\perp}{(2\pi)^3 2k_0} e^{-ik_0(t-t'-i\varepsilon)+ik_z(z-z')+i\boldsymbol{k}_\perp\cdot(\boldsymbol{x}_\perp-\boldsymbol{x}'_\perp)}$$

$$|0,\mathrm{M}\rangle = \prod_j \left[N_j \sum_{n_j=0}^{\infty} e^{-\pi n_j\omega_j/a}|n_j,\mathrm{II}\rangle \otimes |n_j,\mathrm{I}\rangle\right]$$

$$\phi(x) = \sum_{\sigma=\mathrm{I,II}} \int_0^\infty d\omega \int_{-\infty}^{\infty} d^2k_\perp \left(\hat{a}^\sigma_{\omega,\boldsymbol{k}_\perp} v^\sigma_{\omega,\boldsymbol{k}_\perp}(x) + \mathrm{h.c.}\right)$$

2-point correlation function on flat spacetime

2-dimension

$$\langle 0,\mathrm{M}|\phi(x)\phi(x')|0,\mathrm{M}\rangle \longrightarrow \int_{-\infty}^{\infty} \frac{dk}{4\pi|k|} e^{-i|k|(t-t')+ik(z-z')}$$

$$|0,\mathrm{M}\rangle = \prod_\omega \left[N_\omega \sum_{n_\omega=0}^{\infty} e^{-\pi n_\omega \omega/a}|n_\omega,\mathrm{I}\rangle \otimes |n_\omega,\mathrm{III}\rangle\right] \otimes \prod_{\omega'} \left[N_{\omega'} \sum_{n_{\omega'}=0}^{\infty} e^{-\pi n_{\omega'}\omega'/a}|n_{\omega'},\mathrm{II}\rangle \otimes |n_{\omega'},\mathrm{IV}\rangle\right]$$

$$\phi(x) = \sum_{\sigma=\mathrm{I,II,III,IV}} \int_0^\infty d\omega \left(\hat{a}^\sigma_\omega v^\sigma_\omega(x) + \mathrm{h.c.}\right)$$

Results and discussions

By using the property of operators and Bogoliubov transformations, we obtained…

(4-**dim**)

$$\langle 0, \mathrm{M}|\phi(x)\phi(x')|0, \mathrm{M}\rangle \longrightarrow \int \frac{dk_z d^2k_\perp}{(2\pi)^3 2k_0} e^{-ik_0(t-t'-i\varepsilon)+ik_z(z-z')+i\boldsymbol{k}_\perp\cdot(\boldsymbol{x}_\perp-\boldsymbol{x}'_\perp)}$$

(2-**dim**)

$$\langle 0, \mathrm{M}|\phi(x)\phi(x')|0, \mathrm{M}\rangle \longrightarrow \int_{-\infty}^{\infty} \frac{dk}{4\pi|k|} e^{-i|k|(t-t')+ik(z-z')}$$

The 2-point function with derived descriptions is equal to 2-point function on the Minkowski coordinate.

The derived description is

- consistent with QFT on the flat spacetime
- reliable to calculate 2-point function
- applicable for calculation of quantum radiation

Summary & Future work

Unruh, Wald(1984)

Minkowski vacuum (4-dimensional case)

$$|0,\mathrm{M}\rangle \propto \prod_j \Big[\sum_{n_j=0}^{\infty} e^{-\pi n_j \omega_j/a} |n_j\rangle_{\mathrm{R}} \otimes |n_j\rangle_{\mathrm{L}}\Big]$$

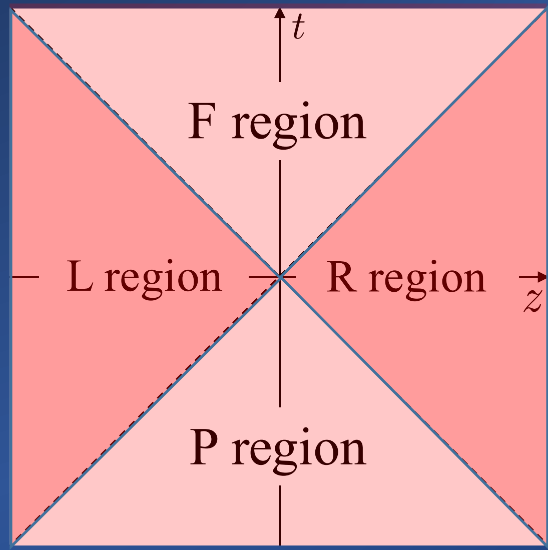
Our group(2017)

Minkowski vacuum (4-dimensional case)

$$|0,\mathrm{M}\rangle \propto \prod_j \Big[\sum_{n_j=0}^{\infty} e^{-\pi n_j \omega_j/a} |n_j,\mathrm{I}\rangle \otimes |n_j,\mathrm{II}\rangle\Big]$$

Minkowski vacuum (2-dimensional case)

$$|0,\mathrm{M}\rangle = \prod_\omega \Big[N_\omega \sum_{n_\omega=0}^{\infty} e^{-\pi n_\omega \omega/a} |n_\omega,\mathrm{I}\rangle \otimes |n_\omega,\mathrm{III}\rangle\Big] \otimes \prod_{\omega'} \Big[N_{\omega'} \sum_{n_{\omega'}=0}^{\infty} e^{-\pi n_{\omega'} \omega'/a} |n_{\omega'},\mathrm{II}\rangle \otimes |n_{\omega'},\mathrm{IV}\rangle\Big]$$



defined on the entire Minkowski spacetime

We extended the description of the Minkowski vacuum state with states of curved spacetime.

Summary & Future work

Summary

We derived the description which explains the entanglement structure of the massless scalar field in the entire Minkowski spacetime in 2,4-dimensional case.

This results is useful to the understanding of the quantum radiation due to the accelerated particle.

Entanglement induced quantum radiation

Summary & Future work

We derived the description which explains the entanglement structure of the massless scalar field in the entire Minkowski spacetime in 2,4-dimensional case.

Future work
#1 the description of Dirac filed with entanglement
#2 the description in other spacetimes
#3 the relation of the QFT on the curved space and Cosmology
etc.

#1
Klein-Gordon eq.
➡Dirac eq.

#2
Minkowski spacetime
➡other spacetimes

END

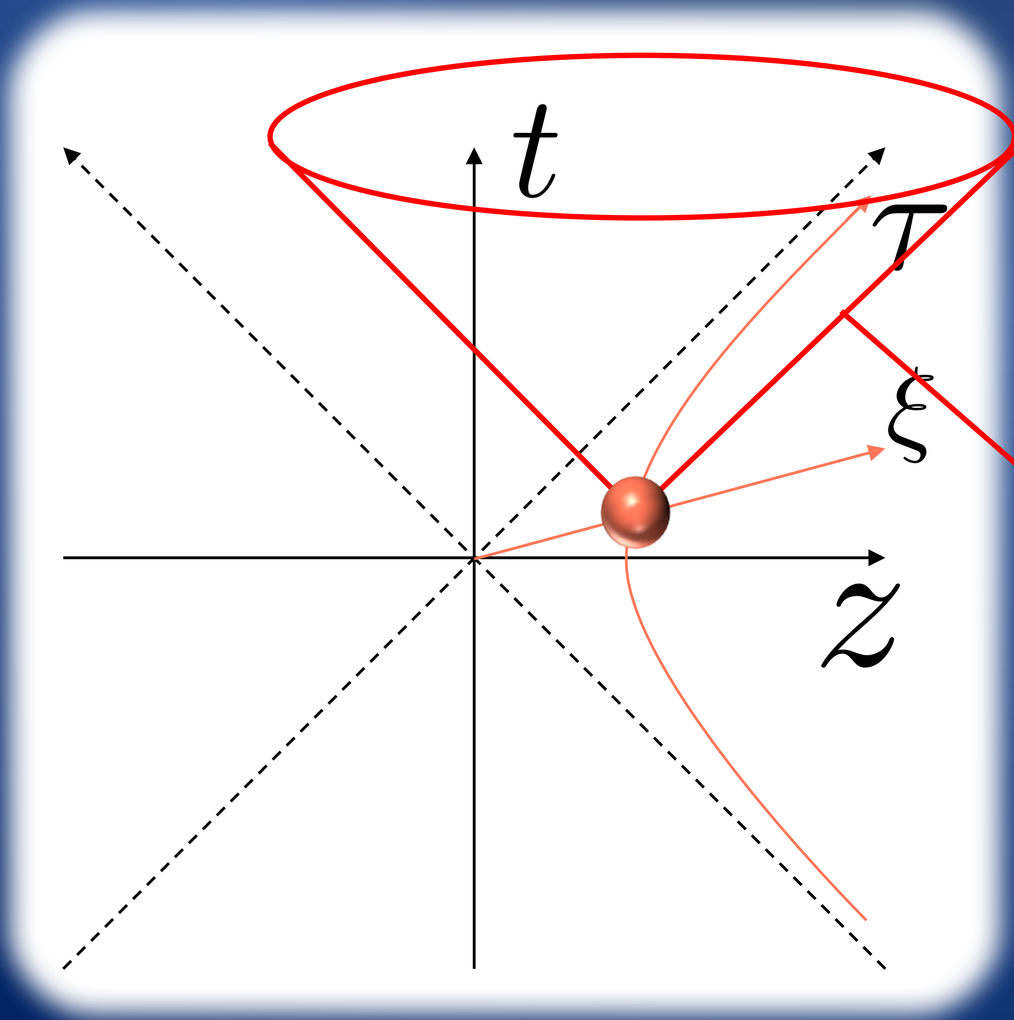
Thank you for listening.

Quantum radiation

$$\frac{d\langle E\rangle}{dt} \sim \frac{\lambda^2}{8\pi^2}\frac{a^3}{\Omega^2}$$

Radiation due to the detector
[Lin and Hu (2006)]

Entanglement-induced



Plankian radiation

$$\langle E\rangle = \frac{a}{2\pi} \equiv T_U$$

Canceled

Energy-momentum tensor

$$T_{0i} = \lim_{y \to x} < \partial_0 \phi(x) \partial_i \phi(y) >$$

Energy flux

$$f = -\sum_{i=1}^{3} T_{0i} \frac{x_i}{r}$$

Energy radiation ratio

$$\frac{dE}{dt} \sim 4\pi r^2 f \sim \frac{a^3 \lambda^2}{8\pi^2 \Omega^2} \propto T_U^3$$

We divide the solution into homogeneous term and inhomogeneous term

$$\phi(x) = \phi_h(x) + \phi_{inh}(x)$$

thermal radiation
(-B)

2-point correlation function

$$< \phi(x)\phi(y) >=< \phi_h(x)\phi_h(y) > + \underline{< \phi_{inh}(x)\phi_h(y) > + < \phi_h(x)\phi_{inh}(y) >} + < \phi_{inh}(x)\phi_{inh}(y) >$$

Vacuum fluctuation
(don't contribute to radiation)

Interference term of detector and field
A+B

Entanglement-induced
quantum radiation

Shown by S.Iso , K.Yamamoto and R.Tatsukawa

END

Thank you for listening.