Effective Action for two Higgs doublet model with small Dirac Neutrino mass

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Outline

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Introduction

- Standard Model (SM)
  - SM includes 6 quarks, 6 leptons, 4 gauge bosons and 1 Higgs boson.
  - The SM is known as one of the most successful models because of its excellent agreement with most experiments.
  - In SM, neutrinos are strictly massless, but neutrino oscillations were observed and it was found that they have a small mass.
- Two Higgs model
  - The SM has the one Higgs particle. However, there is no reason why there is only one Higgs particle.
  - Two Higgs model has the two Higgs particle. One is the standard model Higgs and the other is the heavier Higgs particle.
  - There is one model that explains the small mass of neutrinos[1].

[1] S. M. Davidson and H. E. Logan, Phys. Rev. D 80 (2009).

Introduction

- Motivation

  - Today, the energy reached by the accelerator experiment is over 10 TeV.
  - But heavy Higgs have not been observed yet.

    $\longrightarrow$ Is Heavy Higgs more heavy?

- What to do?

  - Increase the energy of the accelerator (Build a big accelerator)

    $\longrightarrow$ It costs a lot of money. We cannot raise energy infinitely.

  - It is desirable to be able to indirectly observe the effect of heavy particles in the current energy region of the accelerator.

    $\longrightarrow$ Effective Field Thory

- Effective Field Theory
  - If you know the theory on the high energy scale (μ), you can construct an effective theory in the low energy region by integrating heavy particles that are not observed on the low scale (μ_0). ($\mu_0 \gg \mu$)
- Example of EFT (β decay)

β deacy at the quark level : $d \rightarrow u + e^- + \bar{\nu}_e$

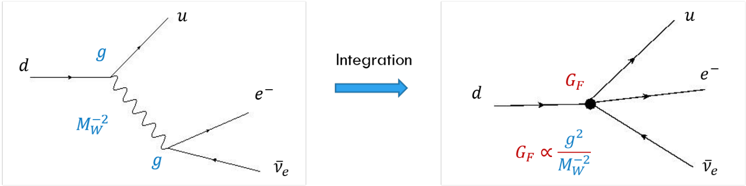


Figure: EFT in the β deacy

Introduction

- Purpose
  - The purpose of our research is constructing EFT of Davidson and Lorgan model.
  - In this research, we construct EFT of the Toy model including two Higgs particles (two real scalars) and neutrinos for simplicity.
  - (Integrate out heavy particles and look for observable effects in low energy experiments.)
  - In this research, we discuss up to One loop level.

Model

- Action
  - We present a simple toy model which leads to a tiny Dirac neutrino mass due to small VEV of the second scalar.
  - The Action is written in terms of bare fields ρ_{0i} and bare masses m_{0i} and bare couplings λ_{0i}.

$$S = \int d^d x \left(-\frac{1}{2} \sum_{i=1}^{2} \rho_{0i} \left(\Box + m_{0i}^2 + \frac{\lambda_{0i}}{2} \rho_{0i}^2 \right) \rho_{0i} - \frac{\lambda_{03}}{4} \rho_{01}^2 \rho_{02}^2 - (y \bar{n} n + m_{012}^2 \rho_{01}) \rho_{02} \right)$$

  - ρ_{01} and ρ_{02} are the two neutral scalar (corespond to Higgs particles), n is neutrino.
  - m_{012}^2 is the bare mixing mass and y is the yukawa coupling of the neutrino and the second Higgs.

Model

- Action
  - The model is renormalizable by imposing the following two Z_2 symmetries (Z_2, Z_2').
  - The cubic interactions of scalars are forbidden by the exact Z_2 symmetry.
  - By considering Z_2 and Z_2', only heavy Higgs couples to neutrino.
  - In order to forbid the Majorana mass terms, one imposes the symmetry which transforms the neutrino n to in.

Table: The charge assignment under Z_2 and Z_2' symmetries

Symmetry	ρ_{01}	ρ_{02}	n_L	n_R
Z_2	$-$	$-$	$+$	$-$
Z_2'	$-$	$+$	$+$	$+$

Model

- Action with renormalized quantities

The relations between bare quantity and renomalized one is given as follows;

$$
\begin{aligned}
\rho_{0i} &= \sqrt{Z_i}\rho_i, \\
m_{0i}^2 Z_i &= \sum_{j=1}^{2} Z_{mij} m_j^2, \\
m_{012}^2 \sqrt{Z_1 Z_2} &= m_{12}^2 Z_{12}, \\
\lambda_{0i} Z_i^2 &= \sum_{l=1}^{3} Z_{\lambda_{il}} \lambda_l \mu^{2\eta}, \\
\lambda_{03} Z_1 Z_2 &= \sum_{l=1}^{3} Z_{\lambda_{3l}} \lambda_l \mu^{2\eta},
\end{aligned}
$$

where, $i = 1, 2$. μ is the renormalization scale and $\eta = 2 - \frac{d}{2}$.

Model

- Action with renormalized quantities

The action in terms of renormalized quantities is given as follows;

$$S[\rho_1, \Delta_2, n] = -\frac{1}{2}\int d^d x \sum_{i=1}^{2}\left(Z_i\rho_i\Box\rho_i + \rho_i^2 Z_{mij} m_j^2 + \frac{\mu^{2\eta}}{2}\sum_{I=1}^{3}\left(\rho_i^4 Z_{\lambda_{iI}}\lambda_I + \rho_1^2\rho_2^2 Z_{\lambda_3 I}\lambda_I\right)\right)$$
$$- \int d^d x \left(\sqrt{Z_2}y\bar{n}n + Z_{12}m_{12}^2\rho_1\right)\rho_2 .$$

Effective Action

- Effective Action

We construct the effective action for the light scalar by integrating the heavy scalar. The effective action is constructed with Legendre transformation. Introduce source J_1 only for ρ_1 and define the following functional.

$$e^{iW[J_1,n]} = \int d\rho_1 \int d\Delta_2 e^{iS[\rho_1,\Delta_2,n]+i\int \rho_1 J_1 d^4x}$$

We define $\bar{\rho}_1$ as the following formula.

$$\bar{\rho}_1|_{J_1} = \frac{\delta W[J_1,n]}{\delta J_1} = \frac{\int d\rho_1 \int d\Delta_2 \rho_1 e^{iS[\rho_1,\Delta_2,n]+i\int \rho_1 J_1 d^4x}}{\int d\rho_1 \int d\Delta_2 e^{iS[\rho_1,\Delta_2]+i\int \rho_1 J_1 d^4x}}$$

- Effective Action

The effective action Γ_{eff} is obtained by Legendre transformation of $W[J_1, n]$.

$$\Gamma_{\text{eff}}[\bar{\rho}_1, n] = W[J_1, n] - \int J_1 \bar{\rho}_1 d^4x$$

The following equation can be easily shown.

$$\begin{aligned}\frac{\delta \Gamma_{\text{eff}}[\bar{\rho}_1, n]}{\delta \bar{\rho}_1(x)} &= \int d^4y \frac{\delta J_1(y)}{\delta \bar{\rho}_1(x)} \left(\frac{\delta W[J_1, n]}{\delta J_1(y)} - \bar{\rho}_1(y) \right) - J_1(x) \\ &= -J_1(x)\end{aligned}$$

- Effective Action

Using above relation, Γ_{eff} can obtain as follows;

$$\Gamma_{\text{eff}}[\bar{\rho}_1, n] = -\, i \log \int d\Delta_1 \int d\Delta_2 e^{iS[\bar{\rho}_1+\Delta_1,\Delta_2,n]-i\int \Delta_1 \frac{\delta\Gamma_{\text{eff}}[\bar{\rho}_1,n]}{\delta\bar{\rho}_1(x)} d^4x}$$

where,we introduce $\rho_1 \equiv \bar{\rho}_1 + \Delta_1$.

Effective Action

- What kind of contribution is calculated?

The following equation can be easily shown.

$$\int d\Delta_1 \Delta_1 \int d\Delta_2 e^{iS[\bar{\rho}_1+\Delta_1,\Delta_2,n]-i\int \Delta_1 \frac{\delta\Gamma_{\rm eff}[\bar{\rho}_1,n]}{\delta\bar{\rho}_1(x)}d^4x} = 0$$

- This formula means that there is no contribution of the tadpole diagram of Δ_1

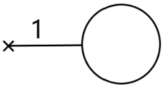


Figure: Example of tadpole diagram of Δ_1

Effective Action

- What kind of contribution is calculated?

Γ_{eff} is written as follows;

$$\Gamma_{\mathrm{eff}}[\bar{\rho}_1, n] = -i \log \int d\Delta_1 \int d\Delta_2 e^{iS[\bar{\rho}_1+\Delta_1,\Delta_2,n]-i\int \Delta_1 \frac{\delta\Gamma_{\mathrm{eff}}[\bar{\rho}_1,n]}{\delta\bar{\rho}_1(x)} d^4x}$$

- The effect of the term proportional to $\frac{\delta\Gamma_{\mathrm{eff}}[\bar{\rho}_1,n]}{\delta\bar{\rho}_1(x)}$ cancels the contribution of the one-particle reducible diagram with respect to Δ_1.

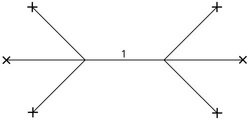
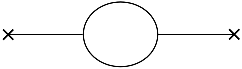


Figure: Example of a diagram with one particle reducible for Δ_1

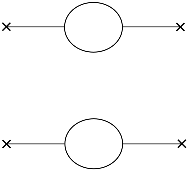
Effective Action

- What kind of contribution is calculated?

Effecive action is only a connected diagram contribution. In other words, it is not necessary to consider the contribution of disconnected diagram.



(a)Example of connected diagram.



(b) Example of disconnected diagram

Effective Action

- Effective action

By integrating for Δ_1 and Δ_2, we obtain as follows;

$$S_{\text{eff}} = \int d^d x \Bigg[\frac{1}{2}\partial^\mu \bar{\rho}_1 \partial_\mu \bar{\rho}_1 - \frac{1}{2} \left\{ \begin{array}{c} m_1^2 \left(1 - \frac{3\lambda_1}{16\pi^2}\left(\frac{3}{2} - \log \frac{m_1^2 + 3\lambda_1 \bar{\rho}_1^2}{\mu^2}\right)\right) - \frac{\lambda_3 m_2^2}{32\pi^2}\left(1 - \log\frac{m_2^2}{\mu^2}\right) \\ -\epsilon m_{12}^2 \left(1 + \frac{(6\lambda_2 - 4\lambda_3)\left(1 - \log\frac{m_2^2}{\mu^2}\right) - \lambda_3}{32\pi^2}\right) \end{array} \right\} \bar{\rho}_1^2$$

$$- \frac{\bar{\rho}_1^4}{4} \left\{ \begin{array}{c} \lambda_1 \left(1 - \frac{3\lambda_1}{16\pi^2}\left(\frac{3}{2} - \log\frac{m_1^2 + 3\lambda_1\bar{\rho}_1^2}{\mu^2}\right)\right) + \frac{\lambda_3^2}{64\pi^2}\log\frac{m_2^2}{\mu^2} \\ +\epsilon^2 \lambda_3 \left(1 - \frac{9\lambda_3 + 12\left\{-\lambda_2\left(2 - \log\frac{m_2^2}{\mu^2}\right) + \lambda_1\left(3\log r - \log\frac{m_2^2}{\mu^2}\right)\right\}}{64\pi^2}\right) \end{array} \right\}$$

$$+ y\epsilon \bar{n} n(x) \bar{\rho}_1 \left\{1 + \frac{3\lambda_2}{16\pi^2}\left(1 - \log\frac{m_2^2}{\mu^2}\right)\right\} + \frac{1}{64\pi^2} m_1^4 \left(\frac{3}{2} - \log\frac{m_1^2 + 3\lambda_1\bar{\rho}_1^2}{\mu^2}\right)$$

$$+ \left(\frac{3}{2} - \log\frac{m_2^2}{\mu^2}\right) m_2^4 + \frac{m_{12}^4}{32\pi^2}\left(1 - \log\frac{m_2^2}{\mu^2}\right) + \frac{1}{2m_2^2}\left(y\bar{n}n(x) - \frac{1}{2}\epsilon\bar{\rho}_1^3\lambda_3\right)^2 \Bigg]$$

where, $\epsilon = \frac{m_{12}^2}{m_2^2}$, $r = \frac{m_1^2}{m_2^2}$.

Neutrino mass

•Neutrino Mass

Finally we focus on the neutrino mass term. The neutrino mass term of the effective action is given as follows;

$$\text{Neutrino mass term}$$
$$= \int d^d x \left[y\epsilon\bar{n}n(x)\bar{\rho}_1 \left\{ 1 + \frac{3\lambda_2}{16\pi^2} \left(1 - \log \frac{m_2^2}{\mu^2} \right) \right\} \right]$$

Therefore, the effective Yukawa coupling constant of the neutrino mass term is $\frac{ym_{12}^2}{m_2^2}$, and it is shown that it is naturally suppressed by the heavy mass Higgs m_2. $(m_2^2 \gg m_{12}^2)$

Summary

Summary

- Low energy effective theory was constructed by integrating heavy Higgs particles from the Two Higgs doublet model.
- By integrating with respect to the heavy Higgs boson in the model of Yukawa coupling of order 1 only with the heavy Higgs boson ρ_2, the effective Yukawa coupling constant of the neutrino mass term is $\frac{ym_{12}^2}{m_2^2}$, and it is shown that it is naturally suppressed by the mass of heavy Higgs m_2^2.
- We derived the effective coupling constant of the quartic interaction of the Higgs boson and the quartic interaction term of the neutrino with the operator dimension of 6.

Future Work

- Applying this method, we obtain the effective theory in the low energy region of the Davidson and Lorgan model. In addition, we will discuss the effects that can be verified by experiments such as ILC using the obtained theory.

Thank you for your listening.