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Lepton Number violation in a unified framework , Revised

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1 Introduction and Motivation

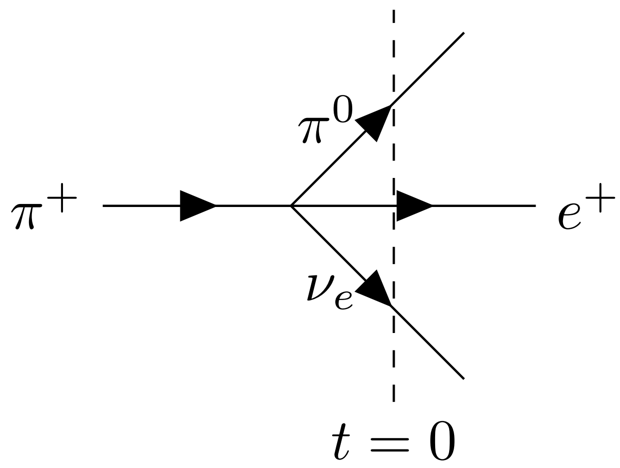
13th, December, last year, I have given a talk at IITB with the same title. However some of the results turned out to be wrong. (arXiv:2004.07664v3, PTEP 2020, 093B07 (under retraction)). So purpose of my talk is to present the corrected results.

Motivation of our work is to investigate the time evolution of lepton (family) number at low energy. CBnu (Cosmic background neutrino) has the temperature as low as 2K$\sim 2 \times 10^{-4}$ (eV). Neutrinos with its rest mass of order 10^{-3} (eV) are non-relativistic. In our work, we derive the formula for time evolution of lepton (family) number carried by Majorana neutrinos which is valid both for relativistic and non-relativistic case.

2 Contents of my talk

- We studied the time evolution of the lepton number operator under the presence of Majorana mass terms.
- By introduction of the Majorana mass term at $t = 0$, the creation and annihilation operators were related between massless neutrinos with a definite lepton number and massive Majorana neutrinos.
- Their time evolution under the presence of Majorana mass terms was investigated with the help of the operators with mass eigenstates.
- We also derived the time evolution of the expectation value of the operators with the initial neutrino of a specific family number (electron-neutrino, muon-neutrino, tau-neutrino). They were studied numerically and the dependencies on the momentum carried by the neutrinos were investigated.

To investigate the Lepton Number under the presence of Majorana mass term, we consider the situation that the Majorana mass term is turned on at $t = 0$.



The situation corresponds to the following Lagrangian:

$$\mathcal{L} = \overline{\nu_L} i\gamma^\mu \partial_\mu \nu_L - \theta(t)\frac{m}{2}(\overline{(\nu_L)^c}\nu_L + h.c.).$$
$$= \frac{1}{2}\overline{\psi_M} i\gamma^\mu \partial_\mu \psi_M - \theta(t)\frac{m}{2}(\overline{\psi_M}\psi_M).$$

where the second line is written in terms of Majorana field, $\psi_M(x)^c = \psi_M(x)$,

$$\psi_M(x) = \nu_L(x) + (\nu_L(x))^c.$$

3 Continuity Condition

For $t > 0$ it is massive Majorana particle and for $t < 0$ it is massless chiral particle. Since equation of motion of fermion is the first order differential equation with respect to time, the field operator is continuous at $t = 0$.

$$L\psi_M(t = 0_+ > 0, \mathbf{x}) = \nu_L(t = 0_- < 0, \mathbf{x})$$

Massless left-handed neutrino is expanded as;

$$\nu_L(\mathbf{x}) = \int' \frac{d^3\mathbf{p}}{(2\pi)^3 2|\mathbf{p}|}(a(\mathbf{p})e^{+i\mathbf{p}\cdot\mathbf{x}}u_L(\mathbf{p}) + b^\dagger(\mathbf{p})e^{-i\mathbf{p}\cdot\mathbf{x}}v_L(\mathbf{p}))$$

With creation and annihilation operators for neutrinos $(a(\mathbf{p}))$ and anti-neutrinos $(b(\mathbf{p}))$, lepton number is given;

$$L = \int \frac{d^3\mathbf{p}}{(2\pi)^3 2|\mathbf{p}|}[a^\dagger(\mathbf{p})a(\mathbf{p}) - b^\dagger(\mathbf{p})b(\mathbf{p})]$$

$$\psi_M(\mathbf{x}) = \int' \frac{d^3\mathbf{p}}{(2\pi)^3 2E}\sum_{\lambda=\pm 1}(a_M(\mathbf{p},\lambda)e^{+i\mathbf{p}\cdot\mathbf{x}}u(\mathbf{p},\lambda) + a_M^\dagger(\mathbf{p},\lambda)e^{-i\mathbf{p}\cdot\mathbf{x}}v(\mathbf{p},\lambda)).$$

$a_M(\mathbf{p},\lambda=\pm 1)$ denotes the annihilation operators for massive Majorana particles with momentum $\mathbf{p}$ and helicities $\Sigma\cdot\frac{\mathbf{p}}{|\mathbf{p}|} = \pm 1$.

4 Relation between operators

From the continuity condition, one obtains

$$\frac{1}{\sqrt{2|\mathbf{p}|}}\begin{pmatrix} a(\mathbf{p}) \\ a^{\dagger}(-\mathbf{p}) \end{pmatrix} = \frac{\sqrt{N_p}}{2E_p}\begin{pmatrix} 1 & \frac{\mathbf{i}m}{E_p+m} \\ \frac{\mathbf{i}m}{E_p+m} & 1 \end{pmatrix}\begin{pmatrix} a_M(\mathbf{p},-) \\ a_M^{\dagger}(-\mathbf{p},-) \end{pmatrix}$$

$N_p = E_p + |\mathbf{p}|, E_p = \sqrt{|\mathbf{p}|^2 + m^2}$. $\{\mathbf{p} \neq 0, \mathbf{p} \in A, -\mathbf{p} \in \bar{A}\}$. In the original paper, we wrote it as,

$$\frac{1}{\sqrt{2|\mathbf{p}|}}a(\mathbf{p}) = \frac{\sqrt{N_p}}{2E_p}(a_M(\mathbf{p},-) + \frac{m}{E_p+m}a_M^{\dagger}(-\mathbf{p},-)).$$

$\mathbf{i}$ was missing compared to the right formula.

5 Time evolution of operators

$$\frac{1}{\sqrt{2|\mathbf{p}|}}\begin{pmatrix} a(\mathbf{p},t) \\ a^\dagger(-\mathbf{p},t) \end{pmatrix}$$

$$= \frac{\sqrt{N_p}}{2E_p}\begin{pmatrix} 1 & \frac{im}{E_p+m} \\ \frac{im}{E_p+m} & 1 \end{pmatrix}\begin{pmatrix} e^{-iE_pt} & 0 \\ 0 & e^{iE_pt} \end{pmatrix}\begin{pmatrix} a_M(\mathbf{p},-) \\ a_M^\dagger(-\mathbf{p},-) \end{pmatrix}$$

6 Time evolution of operators

$$a(\mathbf{p},t) = (\cos E_p t - i\frac{|\mathbf{p}|}{E_p}\sin E_p t)a(\mathbf{p}) - \frac{m}{E_p}a^\dagger(-\mathbf{p})\sin E_p t, \tag{1}$$

$$a^\dagger(-\mathbf{p},t) = (\cos E_p t + i\frac{|\mathbf{p}|}{E_p}\sin E_p t)a^\dagger(-\mathbf{p}) + \frac{m}{E_p}a(\mathbf{p})\sin E_p t.$$

For anti-neutrino operators $b(\mathbf{p},t)$, the equation by replacing a for b holds. In the original paper, we wrote it as,

$$a(\mathbf{p},t) = (\cos E_p t - i\frac{E_p}{|\mathbf{p}|}\sin E_p t)a(\mathbf{p}) + i\frac{m}{|\mathbf{p}|}a^\dagger(-\mathbf{p})\sin E_p t. \tag{2}$$

One can not take $|\mathbf{p}| \to 0$ limit in Eq.(2) while the limit exists for Eq.(1).

7 Lepton Number (LN) (one flavor)

$$
\begin{aligned}
L(t) &= L(0) \\
&- \int' \frac{d^3p}{(2\pi)^3 2|\mathbf{p}|} \frac{2m^2 \sin^2 E_p t}{E_p^2} (a^\dagger(\mathbf{p})a(\mathbf{p}) - b^\dagger(\mathbf{p})b(\mathbf{p})) \\
&- \int'_{\mathbf{p}\in A} \frac{m \sin 2E_p t}{E_p} (a(-\mathbf{p})a(\mathbf{p}) + h.c. - (a \to b)) \\
&+ \int'_{\mathbf{p}\in A} i \frac{2m|\mathbf{p}| \sin^2 E_p t}{E_p^2} (a(-\mathbf{p})a(\mathbf{p}) - h.c. - (a \to b))
\end{aligned} \tag{3}
$$

$\int'_{\mathbf{p}\in A} = \int'_{\mathbf{p}\in A} \frac{d^3p}{(2\pi)^3 2|\mathbf{p}|}$ and $L(0)$ is

$$
L(0) = \int' \frac{d^3p}{(2\pi)^3 2|\mathbf{p}|} (a^\dagger(\mathbf{p})a(\mathbf{p}) - b^\dagger(\mathbf{p})b(\mathbf{p})). \tag{4}
$$

In the original paper, we obtained the following for Eq.(3),

$$
L(t) = L(0). \tag{5}
$$

8 Expectation value of LN

The expectation value with a state with a definite lepton number.

$$|\mathbf{q}\rangle = n_q a^\dagger(\mathbf{q})|0\rangle$$

(n_q normalization const. $\langle \mathbf{q}|\mathbf{q}\rangle = 1$.)

$$\langle \mathbf{q}|L(t)|\mathbf{q}\rangle = \cos^2 E_q t + \frac{q^2 - m^2}{q^2 + m^2} \sin^2 E_q t$$

where $q = |\mathbf{q}|$. With the formula, one can derive,

$$\frac{q^2 - m^2}{q^2 + m^2} \leq \langle \mathbf{q}|L(t)|\mathbf{q}\rangle \leq 1.$$

The lower bound can be negative for $q^2 < m^2$ and reaches to -1 for $\mathbf{q} = 0$.

9 $L_{\alpha=e,\mu,\tau}$ Lepton Family Number (LFN)

$$\langle \mathbf{q},\sigma|L_\alpha(t)|\mathbf{q},\sigma\rangle = \sum_{i=1}^{3}|V_{\alpha i}|^2|V_{\sigma i}|^2\frac{q^2+m_i^2\cos 2E_i t}{E_i^2}$$

$$+\sum_{(i,j)cyclic}\mathrm{Re}(V_{\alpha i}^*V_{\sigma i}V_{\alpha j}V_{\sigma j}^*)\{\cos(E_i-E_j)t\times$$

$$(1+\frac{q^2-m_im_j\mathrm{Re}(\frac{V_{\sigma i}^*V_{\sigma j}}{V_{\sigma i}V_{\sigma j}^*})}{E_iE_j})+\cos(E_i+E_j)t\times$$

$$(1-\frac{q^2-m_im_j\mathrm{Re}(\frac{V_{\sigma i}^*V_{\sigma j}}{V_{\sigma i}V_{\sigma j}^*})}{E_iE_j})\}+\mathrm{Im}(V_{\alpha 1}^*V_{\sigma 1}V_{\alpha 2}V_{\sigma 2}^*)\times$$

$$\sum_{(i,j)cyclic}\{(\cos(E_i-E_j)t-\cos(E_i+E_j)t)\frac{m_im_j}{E_iE_j}\mathrm{Im}(\frac{V_{\sigma i}^*V_{\sigma j}}{V_{\sigma i}V_{\sigma j}^*})$$

$$-((\frac{q}{E_i}-\frac{q}{E_j})\sin(E_i+E_j)t+(\frac{q}{E_i}+\frac{q}{E_j})\sin(E_i-E_j)t)\}$$

10 Various limit (I) $q^2 \gg m_i m_j$

$E_i - E_j \to \frac{\Delta m_{ij}^2}{2q}$.

$$\langle \mathbf{q}, \sigma | L_\alpha(t) | \mathbf{q}, \sigma \rangle \to P_{\sigma \to \alpha}(t)$$

$$P_{\sigma \to \alpha}(t) = \delta_{\sigma\alpha} - 4 \sum_{(i,j) cyclic} \mathrm{Re.}(V_{\alpha i}^* V_{\sigma i} V_{\alpha j} V_{\sigma j}^*) \sin^2 \frac{\Delta m_{ij}^2 t}{4q}$$

$$-2 \mathrm{Im}(V_{\alpha 1}^* V_{\sigma 1} V_{\alpha 2} V_{\sigma 2}^*) \sum_{(i,j) cyclic} \sin \frac{\Delta m_{ij}^2 t}{2q}$$

11 zero momentum limit $q \to 0$

$$\langle \mathbf{q}, \sigma | L_\alpha(t) | \mathbf{q}, \sigma \rangle = \sum_{i=1}^{3} |V_{\alpha i}|^2 |V_{\sigma i}|^2 m_i^2 \cos 2m_i t$$

$$+ \sum_{(i,j) cyclic} \mathrm{Re}(V_{\alpha i}^* V_{\sigma i} V_{\alpha j} V_{\sigma j}^*) \{ (\cos(m_i - m_j)t)(1 - \mathrm{Re}(\frac{V_{\sigma i}^* V_{\sigma j}}{V_{\sigma i} V_{\sigma j}^*}))$$

$$+ (\cos(m_i + m_j)t)(1 + \mathrm{Re}(\frac{V_{\sigma i}^* V_{\sigma j}}{V_{\sigma i} V_{\sigma j}^*})) \} + \mathrm{Im}(V_{\alpha 1}^* V_{\sigma 1} V_{\alpha 2} V_{\sigma 2}^*)$$

$$\times \sum_{(i,j) cyclic} (\cos(m_i - m_j)t - \cos(m_i + m_j)t) \mathrm{Im}(\frac{V_{\sigma i}^* V_{\sigma j}}{V_{\sigma i} V_{\sigma j}^*})$$

12 Total Lepton Number

$$\sum_{\alpha=e,\mu,\tau} \langle \mathbf{q}, \sigma | L_\alpha(t) | \mathbf{q}, \sigma \rangle = \sum_{i=1}^{3} |V_{\alpha i}|^2 (\frac{q^2 + m_i^2 \cos 2E_i t}{E_i^2}) \tag{6}$$

In the original paper, it was conserved as,

$$\sum_{\alpha=e,\mu,\tau} L_\alpha(t) = \sum_{\alpha=e,\mu,\tau} L_\alpha(0). \tag{7}$$

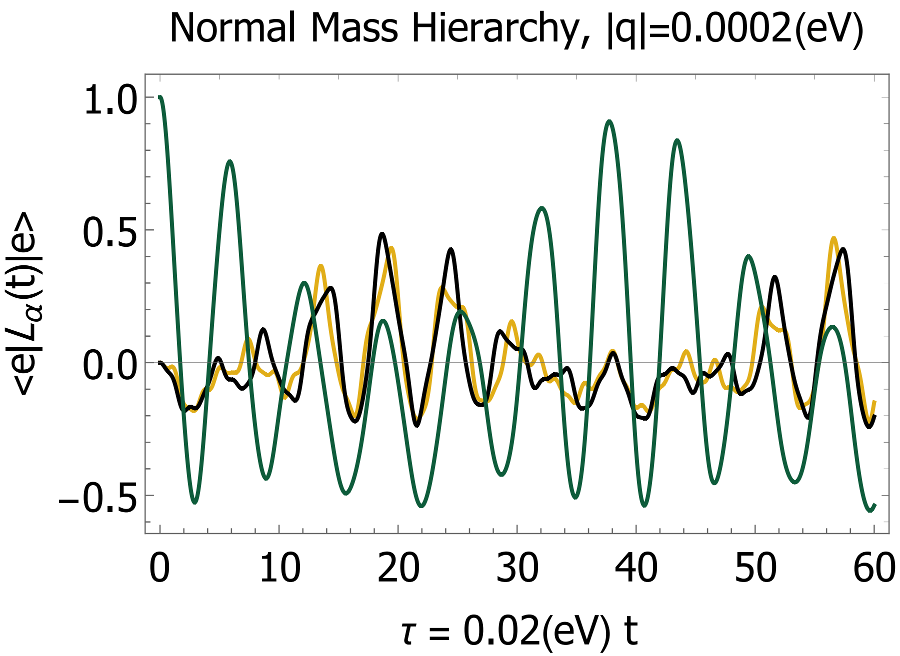


Figure1 Time dependence of lepton family numbers. The momentum of the neutrino is $|\mathbf{q}| = 0.0002$ (eV) $m_1 = 0.01$(eV) Majorana phases are $(\frac{\pi}{2}, \frac{\pi}{3})$.

13 Numerical calculation

14 Summary and conclusions

- We investigated the time evolution of lepton (family) number under the presence of Majorana mass terms.
- Even one flavor case, the lepton number is not conserved. The total lepton number is not conserved for three family case. (In the original paper, they were conserved.)
- By taking momentum carried by a neutrino zero limit, lepton (family) numbers have finite value. (It diverges in the limit in the original paper).