Research on Quantum Entanglement of the Vacuum of Fields

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Abstract

The quantum entanglement is deeply related to the Unruh effect, which predicts the thermal excitation of the Minkowski vacuum on a curved coordinate. It is well known that the Minkowski vacuum is expressed as an entangled state between the left and right Rindler regions when it is constructed on the Rindler coordinate. We clarify the expression of the Minkowski vacuum extended from the ordinary left and right regions to the entire Minkowski space-time, including the Kasner expanding universe and Kasner shrinking universe. We also investigate the two-dimensional case, and clarify the structure of the quantum entanglement.

1 Introduction

Unruh and Wald pointed that the Minkowski vacuum state is expressed as an entangled state of right Rindler state and left Rindler state[1]. The fact indicates that the entanglement is important to understand the Unruh effect. But the description derived by Unruh and Wald covers only a half of the Minkowski spacetime. Therefore, the description which covers entire Minkowski spacetime is necessary to consider the entanglement of the Minkowski vacuum. This research provide the description.

2 Discussion

We quantize a massless scalar field in each region which is described by Rindler coordinates and Kasner coordinates. The action of massless scalar filed is given by:

$$S = \frac{1}{2}\int d^4x\sqrt{-g}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi. \tag{1}$$

We used following coordinates to describe entire Minkowski spacetime.

$$\begin{aligned}
&\text{R region}: \ t=\frac{1}{a}e^{a\xi}\sinh a\tau,\ z=\frac{1}{a}e^{a\xi}\cosh a\tau,\ (-\infty<\tau,\xi<\infty)\\
&\text{L region}: \ t=\frac{1}{a}e^{a\tilde{\xi}}\sinh a\tilde{\tau},\ z=-\frac{1}{a}e^{a\tilde{\xi}}\cosh a\tilde{\tau},\ (-\infty<\tilde{\tau},\tilde{\xi}<\infty)\\
&\text{F region}: \ t=\frac{1}{a}e^{a\eta}\cosh a\zeta,\ z=\frac{1}{a}e^{a\eta}\sinh a\zeta,\ (-\infty<\eta,\zeta<\infty)\\
&\text{P region}: \ t=-\frac{1}{a}e^{-a\tilde{\eta}}\cosh a\tilde{\zeta},\ z=\frac{1}{a}e^{-a\tilde{\eta}}\sinh a\tilde{\zeta},\ (-\infty<\tilde{\eta},\tilde{\zeta}<\infty)
\end{aligned}$$

Each coordinates describes a quarter of entire Minkowski spacetime. By constructing the quantized field on each coordinates, we obtain mode functions. We conducted analytic continuation of modes and revealed the relation:

$$v^{\rm I}_{\omega,\boldsymbol{k}_\perp}(x)=\begin{cases} v^{\rm F,s}_{\omega,\boldsymbol{k}_\perp} & \text{F}\\ v^{\rm R}_{\omega,\boldsymbol{k}_\perp} & \text{R}\\ 0 & \text{L}\\ v^{\rm P,d}_{\omega,\boldsymbol{k}_\perp} & \text{P}\end{cases},\qquad v^{\rm II}_{\omega,\boldsymbol{k}_\perp}(x)=\begin{cases} v^{\rm F,d}_{\omega,\boldsymbol{k}_\perp} & \text{F}\\ 0 & \text{R}\\ v^{\rm L}_{\omega,\boldsymbol{k}_\perp} & \text{L}\\ v^{\rm P,s}_{\omega,\boldsymbol{k}_\perp} & \text{P}\end{cases}. \tag{2}$$

Here, the indices F,R,L,P express regions where the mode are. And indices s and d express right moving and left moving. From the relation (2), we can express the mode expansion of scalar field as:

$$\phi(x)=\sum_{\sigma={\rm I,II}}\int_0^\infty d\omega\int_{-\infty}^\infty d^2k_\perp\left(\hat{a}^\sigma_{\omega,\boldsymbol{k}_\perp}v^\sigma_{\omega,\boldsymbol{k}_\perp}(x)+\text{h.c.}\right). \tag{3}$$

Therefore, the description of Minkowski vacuum reduces to

$$|0,{\rm M}\rangle=\prod_j\Big[N_j\sum_{n_j=0}^\infty e^{-\pi n_j\omega/a}|n_j,{\rm I}\rangle\otimes|n_j,{\rm II}\rangle\Big], \tag{4}$$

where $N_j=\sqrt{1-e^{-2\pi\omega/a}}$, and $j=(\omega,\boldsymbol{k}_\perp)$. The form of the Minkowski vacuum is same as the description derived Unruh and Wald. Although, the state of right hand side in eq.(4) is defined in the entire Minkowski spacetime.

We can consider the 2 dimensional case by the almost same procedure. In 2 dimensional case, Minkowski vacuum is consisted of 4 type of mode. The 2 dimensional Minkowski vacuum is expressed as

$$|0,{\rm M}\rangle=\prod_\omega\Big[N_\omega\sum_{n_\omega=0}^\infty e^{-\pi n_\omega\omega/a}|n_\omega,{\rm I}\rangle\otimes|n_\omega,{\rm III}\rangle\Big]\otimes\prod_{\omega'}\Big[N_{\omega'}\sum_{n_{\omega'}=0}^\infty e^{-\pi n_{\omega'}\omega'/a}|n_{\omega'},{\rm II}\rangle\otimes|n_{\omega'},{\rm IV}\rangle\Big] \tag{5}$$

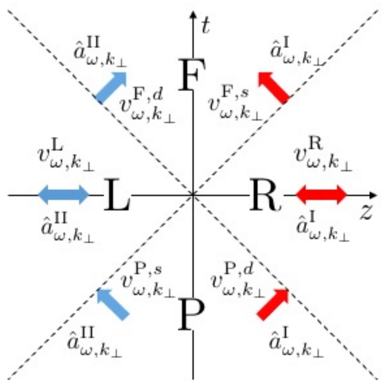

Figure 1: Mode functions (4-D)

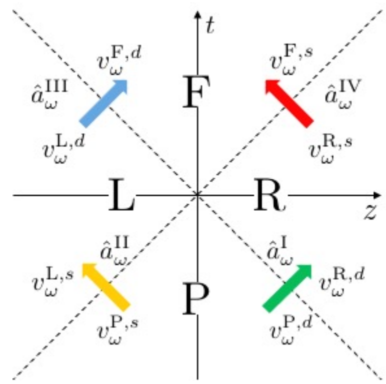

Figure 2: Mode functions (2-D)

The definition of I,II,III,IV in the equation (5) is explained in Figure 2. Figure 1 indicates that the mode of 4 dimensional massless scalar field propagates to timelike direction just like massive wave, which comes from the momentum perpendicular to the direction of acceleration ($\boldsymbol{k}_\perp \neq 0$). On the other hand, the mode of 2 dimensional massless scalar field propagates along with the lightcone. This facts indicates that the wavenumber which corresponds to spacial axis perpendicular to the direction of acceleration plays the role of mass.

Acknowledgement

This talk is based on collaboration with Atsushi Higuchi, Satoshi Iso, Rumi Tatsukawa and Kazuhiro Yamamoto [3, 4]. I express my gratitude to all person who helped me to contribute to this research.

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