RECURSIVE APPROACH OF $H_\infty$ OPTIMAL FILTERING FOR MULTIPARAMETER SINGULARLY PERTURBED SYSTEMS

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Abstract: In this paper, we study the $H_\infty$ optimal filtering for multiparameter singularly perturbed system (MSPS). In order to obtain the solution, we must solve the multiparameter algebraic Riccati equations (MARE) with indefinite sign quadratic term. First, the existence of a unique and bounded solution of such MARE is newly proven. The main results in this paper are to propose a new recursive algorithm for solving the MARE and to find sufficient conditions regarding the convergence of our proposed algorithm. Using the recursive algorithm, we show that the solution of the MARE converges to a positive semi–definite stabilizing solution with the rate of convergence of $O(\|\mu\|^{i+1})$.

Keywords: Multiparameter singularly perturbed system (MSPS), Multiparameter algebraic Riccati equations (MARE), $H_\infty$ optimal filtering, Recursive algorithm

1. INTRODUCTION

Filtering problems for the multiparameter singularly perturbed system (MSPS) have been investigated extensively (see e.g., Coumarbatch and Gajić, 2000 and reference therein). The multi–modeling problems arise in large scale dynamic systems. For example, the multimodel situation in practice is illustrated by the passenger car model (Coumarbatch and Gajić, 2000). In order to obtain the optimal solution to the multimodeling problems, we must solve the multiparameter algebraic Riccati equation (MARE). Various reliable approaches to the theory of the algebraic Riccati equation (ARE) have been well documented in many literatures (see e.g., Laub, 1979). One of the approaches is the invariant subspace approach which is based on the Hamiltonian matrix (Laub, 1979). However, there is no guarantee of symmetry for the computed solution if the ARE is ill-conditioned (Laub, 1979). Note that it is very difficult to solve the MARE due to high dimension and numerical stiffness (Coumarbatch and Gajić, 2000).

A popular approach to deal with the MSPS is the two–time–scale design method (see e.g., Khalil and Kokotović, 1979; Kokotović et al., 1986). However, it is known from Coumarbatch and Gajić (2000) that an $O(\|\mu\|)$ (where $\mu = [\epsilon_1 \epsilon_2]$) accuracy is very often not sufficient. Recently, the exact slow–fast decomposition method for solving the MARE of the MSPS has been proposed (Coumarbatch and Gajić, 2000). However, these results are restricted to the MSPS such that the Hamiltonian matrices for the fast subsystems have no eigenvalues in common (Assumption 5,
of appropriate dimensions. More recently, in Mukaidani et al., (2001), the recursive algorithm (see e.g., Gajić et al., 1990) for the solution of the regulator type MARE which has the positive semidefinite sign quadratic term has been proposed. However, the recursive algorithm for solving the filter type MARE which has the indefinite sign quadratic term appearing in $H_\infty$ filtering problems has not been investigated.

In this paper, we study the $H_\infty$ optimal filtering for the MSPS. The advantage of the $H_\infty$ filter over the standard Kalman filter is that former does not require knowledge of the system and measurement noise intensity matrices. The difficulty encountered with the $H_\infty$ filter for the MSPS is that the MARE contains an indefinite sign quadratic term. Therefore, we first investigate the uniqueness and boundedness of the solution to such MARE and establish its asymptotic structure. The proof of the existence of the solution to the MARE with asymptotic expansion is obtained by an implicit function theorem (Gajić et al., 1990). The main contribution of this paper is to propose a new recursive algorithm for solving the MARE and to find the sufficient conditions regarding the convergence of the recursive algorithm by using the reduced-order ARE. It is important to note that the sufficient conditions derived here are independent of the small perturbation parameter $\mu$. We also prove that the solution of the MARE converges to a positive semi-definite stabilizing solution with the rate of convergence of $O(|\mu|^{i+1})$, where $i$ is the iteration number. As another important feature, we do not assume here that the Hamiltonian matrices $Z_{jj}$, $j = 1, 2$ for the fast fast subsystems have no eigenvalues in common. Thus, our new results are applicable to more realistic MSPS.

2. $H_\infty$ OPTIMAL FILTERING

We consider the linear time-invariant MSPS

\[ \dot{x}_0 = A_{00} x_0 + A_{01} x_1 + A_{02} x_2 + D_{01} w_1 + D_{02} w_2, \]

\[ \epsilon_1 \dot{x}_1 = A_{10} x_0 + A_{11} x_1 + D_{11} w_1, \]

\[ \epsilon_2 \dot{x}_2 = A_{20} x_0 + A_{22} x_2 + D_{22} w_2, \]

with

\[ y_j = C_{jj} x_j + v_j, \quad j = 1, 2, \]

where $x_j \in \mathbb{R}^{n_j}, \quad j = 0, 1, 2$ are state vectors, $y_j \in \mathbb{R}^{q_j}, \quad j = 0, 1, 2$ are system measurements, $w_j \in \mathbb{R}^{r_j}, \quad j = 1, 2$ and $v_j \in \mathbb{R}^{r_j}, \quad j = 1, 2$ are system and measurement disturbances, respectively. All the matrices are constant matrices of appropriate dimensions.

$\epsilon_1$ and $\epsilon_2$ are two small positive singular parameters of the same order of magnitude such that

\[ 0 < k_1 \leq \alpha \equiv \frac{\epsilon_1}{\epsilon_2} \leq k_2 < \infty. \]

That is, we assume that the ratio of $\epsilon_1$ and $\epsilon_2$ is bounded by some positive constants $k_j$, $j = 1, 2$.

In this paper we design a filter to estimate system states $x_j$. The states to be estimated are given by a linear combination

\[ z_j = G_j x_0 + G_j x_j + v_j, \quad j = 1, 2, \]

where $z_j \in \mathbb{R}^{q_j}, \quad j = 1, 2$. The estimation problem is to obtain an estimate $\hat{z}_j$ of $z_j$ using the measurements $y_j$ (Lim and Gajić, 2000). The measure of the infinite horizon estimation problem is defined as a disturbance attenuation function

\[ J = \int_0^\infty \| z - \hat{z} \|^2 dt \cdot \left( \int_0^\infty (\| w \|_q^2 + \| v \|) dt \right)^{-1}, \]

where $z = [z_1^T \ z_2^T]^T$, $\hat{z} = [\hat{z}_1^T \ \hat{z}_2^T]^T$, $w = [w_1^T \ w_2^T]^T$ and $v = [v_1^T \ v_2^T]^T$, and where $R \geq 0$ and $W > 0$ are weighting matrices to be chosen by designer. The $H_\infty$ filter is ensure that the energy gain from the disturbances to estimation errors $z - \hat{z}$ is less that a attenuation level $\gamma^2$. That is,

\[ \sup_{w, v} J < \gamma^2. \]

The $H_\infty$ filter of (1) and (2) is given by (Lim and Gajić, 2000)

\[ \dot{\xi}_0 = A_{00} \xi_0 + A_{01} \xi_1 + A_{02} \xi_2 + F_{01} \eta_1 + F_{02} \eta_2, \]

\[ \epsilon_1 \dot{\xi}_1 = A_{10} \xi_0 + A_{11} \xi_1 + F_{11} \eta_1 + F_{12} \eta_2, \]

\[ \epsilon_2 \dot{\xi}_2 = A_{20} \xi_0 + A_{22} \xi_2 + F_{21} \eta_1 + F_{22} \eta_2, \]

\[ \eta_j = y_j - C_{jj} x_0 - C_{jj} x_j, \quad j = 1, 2, \]

where the filter gain $F_{0j}$ and $F_{jj}$, $j = 1, 2$ are obtained from

\[ F = \begin{bmatrix} F_{01} & F_{02} \\ F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix} = CYc, \]

where $Yc$ satisfies the MARE

\[ A_c Y_c + Y_c A_c^T - Y_c Vc Y_c + U_c = 0, \]

with

\[ A_c = \begin{bmatrix} A_{00} & A_{01} & A_{02} \\ \epsilon_1^{-1} A_{10} & \epsilon_1^{-1} A_{11} & 0 \\ \epsilon_2^{-1} A_{20} & 0 & \epsilon_2^{-1} A_{22} \end{bmatrix}, \]
Lemma 1: The MARE (9) is equivalent to the following generalized multiparameter algebraic Riccati equation (GMARE) (10a)

\[
\mathcal{F}(Y) := AY^T + YA^T - YY^T + U = 0, (10a)
\]

where

\[
Y_e = Y^T \Phi_e^{-1} Y = \Phi_e^{-1} Y, \quad (10b)
\]

where

\[
\Phi_e = \begin{bmatrix} I_n & 0 & 0 & \varepsilon_1 I_n & 0 \\ 0 & \varepsilon_1 I_n & 0 & 0 & \varepsilon_2 I_n \end{bmatrix}, \quad A = \begin{bmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & 0 \\ A_{20} & 0 & A_{22} \end{bmatrix},
\]

\[
U = \begin{bmatrix} U_{00} & U_{01} & U_{02} \\ U_{10} & U_{11} & 0 \\ U_{20} & 0 & U_{22} \end{bmatrix}, \quad Y = \begin{bmatrix} Y_{00} & Y_{10} & Y_{20} \\ \varepsilon_1 Y_{10} & Y_{11} & \sqrt{\alpha} Y_{21} \\ \varepsilon_2 Y_{20} & \sqrt{\alpha} Y_{21} & Y_{22} \end{bmatrix}.
\]

Proof. Firstly, by direct calculation we verify that

\[
Y_e = \Phi_e^{-1} Y_e \quad \text{and} \quad X_e = \Phi_e^{-1} Y \Phi_e^{-1}.
\]

By using the similar calculation, we can immediately rewrite (9) as (10a). \( \square \)

3. THE MARE

Firstly, it is assumed that the limit of \( x_1 \) exists as \( x_2 \rightarrow 0 \) tends to zero (see e.g., Khalil and Kokotović, 1979; Gajić, 1988), that is

\[
\tilde{\alpha} = \lim_{\varepsilon_2 \to +0} \alpha.
\]  (11)

Let \( Y_{00}, Y_{10}, Y_{20}, Y_{11}, Y_{21} \) and \( Y_{22} \) be the limiting solutions of the equations (10) as \( x_1 \to +0 \), \( j = 1, 2 \), then we obtain the following zeroth order equations by partitioning the GMARE (10a).

\[
A_s Y_{00} + Y_{00} A_s^T - Y_{00} V_s Y_{00} + U_s = 0, \quad (12a)
\]

\[
Y_{j0} = Y_{00} S_{0j} - R_{0j}, \quad j = 1, 2, \quad (12b)
\]

\[
A_j Y_{jj} + Y_{jj} A_j^T - Y_{jj} V_j Y_{jj} + U_{jj} = 0, \quad (12c)
\]

where

\[
A_s = A_{00} + A_{01} S_{01}^T + A_{02} S_{02}^T + R_{01} V_{01}^T + R_{02} V_{02}^T,
\]

\[
V_s = V_{00} + S_{01} V_{01}^T + S_{02} V_{02}^T + R_{01} V_{01}^T + R_{02} V_{02}^T,
\]

\[
U_s = U_{00} - R_{01} A_{01}^T - R_{02} A_{02}^T - A_{01} R_{01} - A_{02} R_{02} - R_{01} V_{01}^T - R_{02} V_{02}^T,
\]

\[
S_{0j} = -H_{j0}^T H_{j0}^{-1}, \quad R_{0j} = Q_{0j} H_{j0}^{-T},
\]

\[
Q_{0j} = A_{0j} Y_{jj} + U_{0j}, \quad H_{jj} = A_{jj} - Y_{jj} V_j Y_{jj},
\]

\[
H_{00} = A_{00} - Y_{00} V_{00} - Y_{10} V_{01}^T - Y_{20} V_{02}^T,
\]

\[
H_{j0} = A_{j0} - Y_{jj} V_j Y_{0j}, \quad j = 1, 2,
\]

\[
H_{0j} = A_{0j} - Y_{00} V_{0j} - Y_{j0} V_j.\]

Now, let us define the sets as \( \Gamma_{jj} := \{ \gamma \geq 0 \} \) the ARE (12c) has the positive semidefinite stabilizing solutions, \( j = 1, 2 \). If we choose \( \gamma_{jj} := \inf \{ \gamma \mid \gamma \in \Gamma_{jj} \} < \gamma_j, A_{jj} - Y_{jj} V_j Y_{jj} \) are stable. Hence, the parameter \( \tilde{\alpha} \) does not appear in (12) because \( Y_{21} = 0 \). The matrices \( A_s, V_s \) and \( U_s \) do not depend on \( Y_{11} \) and \( Y_{22} \) because their matrices can be computed by using \( Z_{0p}, p, q = 0, 1, 2 \) which is independent of \( Y_{11} \) and \( Y_{22} \) (Covunaratch and Gajić, 2000), that is

\[
Z_{s} = Z_{00} - Z_{00} Z_{11}^{-1} Z_{10} - Z_{02} Z_{22}^{-1} Z_{20}
\]

\[
= A_{T0}^T V_s, \quad j = 1, 2,
\]

\[
Z_{00} = \begin{bmatrix} A_{00}^T & -V_s \\ -U_s & -A_{00} \end{bmatrix}, \quad Z_{0j} = \begin{bmatrix} A_{j0}^T & -V_{0j} \\ -U_{0j} & -A_{0j} \end{bmatrix}.
\]
\[ Z_{j0} = \begin{bmatrix} A_{j1}^T & -V_{j0}^T \\ -U_{j0}^T & -A_{j0} \end{bmatrix}, \quad Z_{jj} = \begin{bmatrix} A_{j1}^T & -V_{jj} \\ -U_{jj}^T & -A_{jj} \end{bmatrix}. \]

The AREs (12c) will produce the unique positive semi definite stabilizing solution under the following condition if \( \gamma \) is large enough. Moreover, let us define the set as \( \Gamma := \{ \gamma > 0 \} \) the ARE (12a) has a positive semi definite stabilizing solution, \( \gamma_s := \inf \{ \gamma | \gamma \in \Gamma \} \). As the results, for every \( \gamma > \gamma_s = \max \{ \gamma_s, \gamma_1, \gamma_2 \} \), the MARE (9) has the positive semi definite stabilizing solution if \( \varepsilon_1 > 0 \) and \( \varepsilon_2 > 0 \) are small enough. Then, we have the following result.

**Theorem 1**: If we select a parameter \( \gamma > \gamma_s = \max \{ \gamma_s, \gamma_1, \gamma_2 \} \), then there exist small \( \varepsilon_1 \) and \( \varepsilon_2 \) such that for all \( \varepsilon_1 \in (0, \varepsilon_1) \) and \( \varepsilon_2 \in (0, \varepsilon_2) \), the MARE (9) admits a solution such that \( Y_c \) is the symmetric positive semi definite stabilizing solution, which can be written as (13).

\[
Y_c = \begin{bmatrix} Y_{00} + O(\|\mu\|) & Y_{10}^T + O(\|\mu\|) \\ Y_{10} + O(\|\mu\|) & \varepsilon_1^{-1}Y_{11} + O(\|\mu\|) \\ Y_{20} + O(\|\mu\|) & \sqrt{\varepsilon_1 \varepsilon_2}^{-1}O(\|\mu\|) \\ \varepsilon_2^{-1}Y_{20} + O(\|\mu\|) & \sqrt{\varepsilon_1 \varepsilon_2}^{-1}O(\|\mu\|) \\ \varepsilon_2^{-1}(Y_{22} + O(\|\mu\|)) \end{bmatrix}, \tag{13}
\]

where \( \mu = [\varepsilon_1 \varepsilon_2] \).

**Proof**: We apply the implicit function theorem (Gajić et al., 1990; Gajić, 1988) to (10a). To do so, it is enough to show that the corresponding Jacobian is nonsingular at \( \varepsilon_j = 0 \), \( j = 1, 2 \). It can be shown, after some algebra, that the Jacobian of (10a) in the limit as \( \mu \rightarrow \mu = [0 0] \) is given by

\[
J_Y = \begin{bmatrix}
J_{00} & J_{01} & J_{02} & 0 & 0 & 0 \\
J_{10} & J_{11} & 0 & J_{12} & J_{14} & 0 \\
J_{20} & J_{22} & 0 & J_{24} & J_{25} & 0 \\
0 & 0 & 0 & J_{33} & 0 & 0 \\
0 & 0 & 0 & 0 & J_{44} & 0 \\
0 & 0 & 0 & 0 & 0 & J_{55}
\end{bmatrix}, \tag{14}
\]

and

\[
J_{00} = (I_{n_0} \otimes H_{00})U_{n_0,n_0} + H_{00} \otimes I_{n_0},
J_{01} = (I_{n_0} \otimes H_{01})U_{n_0,n_1} + H_{01} \otimes I_{n_0},
J_{02} = H_{10} \otimes I_{n_0},
J_{10} = H_{20} \otimes I_{n_0},
J_{11} = I_{n_1} \otimes H_{01},
J_{12} = \frac{1}{\sqrt{\alpha}}(I_{n_2} \otimes H_{01}),
J_{20} = I_{n_2} \otimes H_{20},
J_{22} = \frac{1}{\sqrt{\alpha}}(I_{n_2} \otimes H_{00}),
J_{24} = \frac{1}{\sqrt{\alpha}}(I_{n_2} \otimes H_{01}),
J_{33} = (I_{n_1} \otimes H_{11})U_{n_1,n_1} + H_{11} \otimes I_{n_1},
J_{44} = \frac{1}{\sqrt{\alpha}}(I_{n_2} \otimes H_{11}),
J_{55} = (I_{n_2} \otimes H_{22})U_{n_2,n_2} + H_{22} \otimes I_{n_2},
\]

where \( \otimes \) denotes Kronecker products and \( U_{n_i,n_j} \) is the permutation matrix in the Kronecker matrix sense. The Jacobian (14) can be expressed as

\[
\det J_Y = \det J_{11} \cdot \det J_{22} \cdot \det J_{33} \cdot \det J_{44} \cdot \det J_{55},
\]

\[
\det[I_{n_0} \otimes H_0 U_{n_0,n_0} + H_0 \otimes I_{n_0}], \tag{15}
\]

where \( H_0 \equiv H_{00} - H_{01}H_{11}^{-1}H_{10} - H_{02}H_{22}^{-1}H_{20} \).

Obviously, \( J_{jj}, \ j = 1, \ldots, 5 \) are nonsingular because the matrices \( H_{jj} = A_{jj} - Y_{jj}V_{jj}, \ j = 1, 2 \) are stable. After some straightforward but tedious algebra, we see that \( A_s - \bar{Y}_{00}V_s = H_{00} - H_{01}H_{11}^{-1}H_{10} - H_{02}H_{22}^{-1}H_{20} = H_0 \). Therefore, the matrix \( H_0 \) is stable if \( \gamma \) is sufficiently large. Thus, \( \det J_Y \neq 0 \). The conclusion of Theorem 1 is obtained directly by using the implicit function theorem. The remainder of the proof is to show that \( Y_c \) is the positive semi definite stabilizing solution. However, the proof is omitted since it is similar to that of the reference Mukaidani et al., (2001).

\[ \square \]

### 4. THE RECURSIVE ALGORITHM

Now, let us define \( [\mu] := E = \sqrt{\alpha} \epsilon \). By making use of the zeroth order solutions (12), the solution (13) can be changed as follows.

\[
Y_{pq} = \bar{Y}_{pq} + E F_{pq}, \ pq = 00, 10, 20, 11, 21, 22, \tag{16}
\]

where \( F_{00} = F_{00}^T, F_{11} = F_{11}^T, F_{22} = F_{22}^T, Y_{21} = 0 \).

Substituting (16) into (10a) and subtracting (12) from (10a), we arrive at the following error equations (17).

\[
H_{00}F_{00} + F_{00}H_{00}^T + H_{10}F_{10} + F_{10}H_{10}^T + \frac{1}{2} \epsilon^2 H_{20}F_{20} + F_{20}H_{20}^T - \epsilon(F_{00}V_{00}F_{00} + F_{10}V_{10}F_{00})
+ F_{00}V_{01}F_{10} + F_{20}V_{20}F_{20} + F_{00}V_{02}F_{20} + F_{10}V_{12}F_{20} + F_{20}V_{22}F_{22} = 0, \tag{17a}
\]

\[
H_{00}F_{00} + F_{00}H_{00}^T + H_{10}F_{10} + F_{10}H_{10}^T + \frac{1}{2} \epsilon^2 H_{20}F_{20} + F_{20}H_{20}^T - \epsilon(F_{00}V_{00}F_{00} + F_{10}V_{10}F_{00})
+ F_{00}V_{01}F_{10} + F_{20}V_{20}F_{20} + F_{00}V_{02}F_{20} + F_{10}V_{12}F_{20} + F_{20}V_{22}F_{22} = 0, \tag{17b}
\]

\[
H_{00}F_{00} + F_{00}H_{00}^T + H_{10}F_{10} + F_{10}H_{10}^T + \frac{1}{2} \epsilon^2 H_{20}F_{20} + F_{20}H_{20}^T - \epsilon(F_{00}V_{00}F_{00} + F_{10}V_{10}F_{00})
+ F_{00}V_{01}F_{10} + F_{20}V_{20}F_{20} + F_{00}V_{02}F_{20} + F_{10}V_{12}F_{20} + F_{20}V_{22}F_{22} = 0, \tag{17c}
\]

\[
H_{11}F_{11} + F_{11}H_{11}^T + \frac{1}{2} \epsilon^2 (H_{10}Y_{10}^T + Y_{10}H_{10}^T)
+ \epsilon(H_{10}F_{10}^T + F_{10}H_{10}^T)
- \frac{1}{2} \epsilon^2 Y_{10}V_{00}Y_{10}^T - \epsilon(F_{11}V_{01}Y_{10}^T + Y_{10}V_{01}F_{11})
\]
Taking into consideration the fact that \(J_{y}\) is nonsingular at \(|\mu| = 0\), \(J_{F}\) is also nonsingular. Therefore, there exists a unique and bounded solution of the error equations (17). Secondly, the proof of (18) uses mathematical induction. However, in order to respect pages limitations, the proof is omitted since it is similar to that of the reference Mukaidani et al., (2001).
5. NUMERICAL EXAMPLE

In order to demonstrate the efficiency of our proposed algorithm, we have run a numerical example. The system matrix is given by

\[
A_{00} = \begin{bmatrix} 0 & 0.45 & 0 & 1 \\ 0.0 & 0 & 4.5 & -1 \\ 0.0 & -0.05 & 0 & -0.1 \\ 0.0 & 0 & -0.05 & 0.1 \end{bmatrix},
A_{jj} = \begin{bmatrix} -0.05 & 0.05 \\ 0 & 0.1 \end{bmatrix},
A_{01} = \begin{bmatrix} 0_2 \times 2 \\ 0_2 \times 2 \\ 0_2 \times 2 \\ 0_2 \times 2 \end{bmatrix},
A_{0q} = \begin{bmatrix} 0_3 \times 2 \\ 0_3 \times 2 \\ 0_3 \times 2 \\ 0_3 \times 2 \end{bmatrix},
A_p = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}^T.
\]

The small parameters are chosen as \( \varepsilon_1 = \varepsilon_2 = 0.01 \). Note that we cannot apply the technique proposed in Comarbatch and Gajić, (2000) to the MSPS since the Hamiltonian matrices \( Z_{jj}, j = 1, 2 \) have eigenvalues in common. We give a solution of the GMARE (10a) in Table 1. We find that the solution of the GMARE (10a) converges to the exact solution with accuracy of \( \|\mathbf{F}(Y^{(i)})\| < \epsilon \) after 22 iterative iterations, where \( \epsilon = x^{-2} \) for different values of \( \varepsilon_1 \) and \( \varepsilon_2 \). To verify the exactness of the solution, the errors (i.e. \( \|\mathbf{F}(Y^{(i)})\| \)) and the necessary iteration numbers of the algorithm (18) are given by Table 2. From Table 2, since for sufficiently small perturbation parameters the convergence speed is quite good, the resulting algorithm of this paper is very useful.

Table 1.

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6. CONCLUSION

In this paper, we have proposed a new recursive algorithm for solving the MARE which has the indefinite sign quadratic term. We have proven that the solution of the MARE converges to a positive semi-definite stabilizing solution with the rate of convergence of \( O(|\mu|^{1+}) \). As another important feature, since we do not assume that the Hamiltonian matrices \( Z_{jj}, j = 1, 2 \) for the fast fast subsystems have no eigenvalues in common compared with Comarbatch and Gajić, (2000), our new results are applicable to more realistic MSPS.

References


