SYNTHESIS OF A STATIC ANTI-WINDUP COMPENSATOR FOR SYSTEMS WITH MAGNITUDE AND RATE LIMITED ACTUATORS

Nobutaka Wada * **and Masami Saeki** **

* *Graduate School of Engineering, Hiroshima University, Higashi-Hiroshima, 739-8527, Japan*
E-mail: nwada@ipc.hiroshima-u.ac.jp
** *Faculty of Engineering, Hiroshima University, Higashi-Hiroshima, 739-8527, Japan*
E-mail: saeki@ipc.hiroshima-u.ac.jp

Abstract: In this paper, we propose a design method of a static anti-windup compensator for systems with input magnitude and rate saturations. First, we present an anti-windup controller for systems with input magnitude and rate saturations. Then, we show that the design problem of the anti-windup compensator that guarantees the local stability of the closed-loop system against saturation nonlinearities and optimizes the robust control performance during the saturation period can be reduced to a linear matrix inequality (LMI) problem.

Keywords: saturation, robust stability, windup, multivariable, feedback control

1. INTRODUCTION

Many practical control systems involve magnitude saturation on control input, which causes severe performance degradation called windup phenomena in the case where a controller has slow dynamics. An anti-windup scheme is one way to deal with the windup phenomena, and it is usually performed by adding an anti-windup compensator to the controller which gives acceptable performance in the unsaturated region. If the compensator is a constant gain matrix, it is referred to as a static anti-windup compensator in this paper. The general structure of the static anti-windup compensator for the system with input magnitude saturation is presented in Kothare, Campo, Morari and Nett (1994). Recently, in Wada and Saeki (1999), it is shown, for the first time, that the design problem of the static anti-windup compensator which guarantees robust stability against saturation functions can be represented as an equivalent LMI problem, and an optimal solution can be obtained efficiently. Since the design method of Wada *et al.* (1999) treats a nominal performance control problem, it is extended to the robust performance problem in Saeki and Wada (2000).

In many practical control systems, there exist not only magnitude saturations but also rate saturations. It is pointed out in Berg, Hammett, Schwartz and Banda (1996) that the phase-lag associated with rate saturation has a destabilizing effect. Several design methods of a controller for the system with both magnitude and rate saturations have been proposed. For example, a dynamic output feedback is used in Tyan and Bernstein (1997),Lin (1997),Kapila and Haddad (1998); the error governor is constructed in Kapasouris and Athans (1990). However, as far as we know, the static anti-windup scheme has not been applied to the problem of input magnitude and rate saturations.

In this paper, we extend the result of Wada *et al.* (1999) to the input magnitude and rate saturation problem. First, a new structure of the anti-windup controller is derived for systems with input magnitude and rate saturations. As a rate saturation model, a position-feedback-type system with saturation inside the closed-loop system is utilized, and the closed-loop system is represented as a linear system with two saturation functions. Secondly, a closed-loop stability condition is derived based on the multivariable circle theorem, and the design problem is reduced to an LMI problem. Finally, a numerical example is given.

2. ANTI-WINDUP COMPENSATION FOR SYSTEMS WITH INPUT MAGNITUDE AND RATE SATURATIONS

A controller with magnitude and rate saturations is shown in Fig.1(a). The state equation of $K(s)$ is given by

$$\begin{cases} \dot{x}_c &= Ax_c + By \\ \tilde{u} &= Cx_c + Dy \end{cases} \tag{1}$$

where $x_c \in \mathbf{R}^{n_c}$, $\tilde{u} \in \mathbf{R}^{n_p}$, $y \in \mathbf{R}^{n_q}$. $\phi_m(\cdot)$ denotes the magnitude saturation, and is defined by

$$u = \phi_m(u_m) \tag{2}$$

where

$$\begin{aligned} \phi_m(u_m) &= (\,\phi_{m1}(u_{m1}), \cdots, \phi_{mn_p}(u_{mn_p})\,)^T \\ \phi_{mi}(u_{mi}) &= \begin{cases} \mathrm{sgn}(u_{mi})u_{mi,max}, & |u_{mi}| > u_{mi,max} \\ u_{mi}, & |u_{mi}| \leq u_{mi,max} \end{cases} \end{aligned}$$

and $u \in \mathbf{R}^{n_p}$, $u_m \in \mathbf{R}^{n_p}$. The rate saturation on control input is defined by

$$\begin{cases} \dot{u}_m &= \phi_r(\tilde{u}_r) \\ \tilde{u}_r &= R(\tilde{u} - u_m) \end{cases} \tag{3}$$

where

$$\begin{aligned} \phi_r(\tilde{u}_r) &= (\phi_{r1}(\tilde{u}_{r1}), \cdots, \phi_{rn_p}(\tilde{u}_{rn_p})) \\ \phi_{ri}(\tilde{u}_{ri}) &= \begin{cases} \mathrm{sgn}(\tilde{u}_{ri})u_{ri,max}, & |\tilde{u}_{ri}| > u_{ri,max} \\ \tilde{u}_{ri}, & |\tilde{u}_{ri}| \leq u_{ri,max} \end{cases} \\ R &= \mathrm{diag}(r_1, \cdots, r_{n_p}), r_i > 0, \forall i \end{aligned}$$

and $\tilde{u}_r \in \mathbf{R}^{n_p}$. By using this model, the rate of u_m (i.e. $\dot{u}_m$) is limited by ϕ_r. This rate saturation model was proposed by Tyan *et al.* (1997). When $R = I$, it is equivalent to a model used by Kapasouris *et al.* (1990).

We add anti-windup compensators to the above controller as shown in Fig.1(b). The state equation of $\tilde{K}(s)$ is defined by

$$\begin{cases} \dot{x}_c &= Ax_c + By + f_m + f_r \\ \tilde{u} &= Cx_c + Dy \\ f_m &= H_m(u - u_m) \\ f_r &= H_r(u_r - \tilde{u}_r) \end{cases} \tag{4}$$

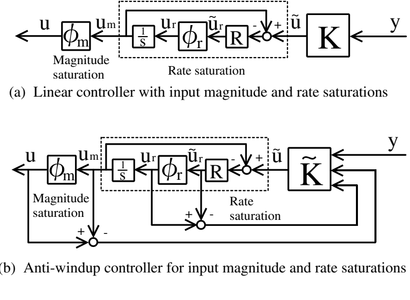

Fig. 1. Anti-windup controller for input magnitude and rate saturations

where $H_m \in \mathbf{R}^{n_c \times n_p}$, $H_r \in \mathbf{R}^{n_c \times n_p}$ and $u_r = \dot{u}_m$. The controller shown in Fig.1(b) can be considered the observer based anti-windup controller (see, Åström and Wittenmark, 1984) with a compensator $f_r = H_r(u_r - \tilde{u}_r)$ that works when the rate of u_m exceeds its maximum value. We suppose that an original linear time-invariant (LTI) controller $K(s)$ has been already designed to meet some design specifications (e.g., a closed-loop stability, time domain criterion) when there is no input saturation. Therefore, the matrices H_m and H_r are only design parameters.

Next, let us consider a closed-loop system shown in Fig.2. $G(s)$ denotes a generalized plant, and its state equation is given by

$$\begin{cases} \dot{x}_G &= A_G x_G + B_{G1} w_1 + B_{G2} u \\ z_1 &= C_{G1} x_G + D_{G11} w_1 + D_{G12} u \\ y &= C_{G2} x_G + D_{G21} w_1 \end{cases} \tag{5}$$

where $x_G \in \mathbf{R}^{n_g}$, $z_1 \in \mathbf{R}^{n_w}$, $w_1 \in \mathbf{R}^{n_w}$. This system can be represented as a system shown in Fig.3. It is described by

$$\begin{cases} \dot{x}_M &= A_M x_M + B_{M1} w_1 + B_{M2}\xi + B_{M3} f \\ z_1 &= C_{M1} x_M + D_{M11} w_1 + D_{M12}\xi + D_{M13} f \\ \tilde{\xi} &= C_{M2} x_M + D_{M21} w_1 + D_{M22}\xi + D_{M23} f \end{cases} \tag{6}$$

$$f = H(\xi - \tilde{\xi}) \tag{7}$$

$$\xi = \phi(\tilde{\xi}) \tag{8}$$

where

$$A_M = \begin{bmatrix} A_G & 0 & 0 \\ BC_{G2} & A & 0 \\ 0 & 0 & 0 \end{bmatrix}, B_{M1} = \begin{bmatrix} B_{G1} \\ BD_{G21} \\ 0 \end{bmatrix}$$

$$B_{M2} = \begin{bmatrix} B_{G2} & 0 \\ 0 & 0 \\ 0 & I \end{bmatrix}, B_{M3} = \begin{bmatrix} 0 \\ I \\ 0 \end{bmatrix}$$

$$C_{M1} = \begin{bmatrix} C_{G1} & 0 & 0 \end{bmatrix}, C_{M2} = \begin{bmatrix} 0 & 0 & I \\ RDC_{G2} & RC & -R \end{bmatrix}$$

$$D_{M11} = D_{G11}, D_{M12} = [D_{G12}, 0], D_{M13} = 0$$

$$D_{M21} = \begin{bmatrix} 0 \\ RDD_{G21} \end{bmatrix}, D_{M22} = 0, D_{M23} = 0$$

$$H = [H_m, H_r], \phi(\cdot) = \begin{bmatrix} \phi_m(\cdot) \\ \phi_r(\cdot) \end{bmatrix}$$

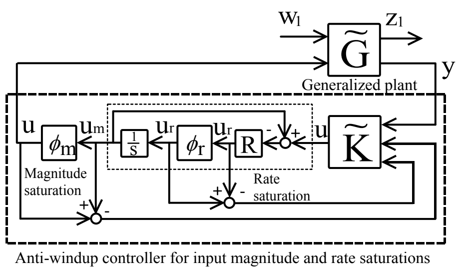

Fig. 2. Closed-loop system with an anti-windup controller

and $\xi = [u^T, u_r^T]^T$, $\tilde{\xi} = [u_m^T, \tilde{u}_r^T]^T$, $f = f_m + f_r$, $x_M = [x_G^T, x_c^T, u_m^T]^T$. The static matrix H is the design parameter of the closed-loop system (6)-(8). We refer to H as a static anti-windup compensator. In the next section, we consider a design method of H.

Note 1. Kothare *et al.* (1994) proposed the following anti-windup controller.

$$\begin{cases} \dot{x}_c &= Ax_c + By + f_1 \\ \tilde{u} &= Cx_c + Dy + f_2 \\ f_1 &= \Lambda_1(u - \tilde{u}) \\ f_2 &= \Lambda_2(u - \tilde{u}) \\ u &= \phi(\tilde{u}) \end{cases} \tag{9}$$

where Λ_1 and Λ_2 are constant matrices. The closed-loop system consisted of (5) and (9) is included in the anti-windup structure shown in Fig.3 as a special case. Actually, the closed-loop system (5) and (9) can be written in the form of (6)-(8) with $\xi = u$, $\tilde{\xi} = \tilde{u}$, $f = [f_1^T, f_2^T]^T$, $x_M = [x_G^T, x_c^T]^T$ and

$$\begin{aligned} A_M &= \begin{bmatrix} A_G & 0 \\ BC_{G2} & A \end{bmatrix}, B_{M1} = \begin{bmatrix} B_{G1} \\ BD_{G21} \end{bmatrix} \\ B_{M2} &= \begin{bmatrix} B_{G2} \\ 0 \end{bmatrix}, B_{M3} = \begin{bmatrix} 0 & 0 \\ I & 0 \end{bmatrix} \\ C_{M1} &= \begin{bmatrix} C_{G1} & 0 \end{bmatrix}, C_{M2} = \begin{bmatrix} DC_{G2} & C \end{bmatrix} \\ D_{M11} &= D_{G11}, D_{M12} = D_{G12}, D_{M13} = 0 \\ D_{M21} &= DD_{G21}, D_{M22} = 0, D_{M23} = [0, I] \\ H &= \begin{bmatrix} \Lambda_1 \\ \Lambda_2 \end{bmatrix} \end{aligned}$$

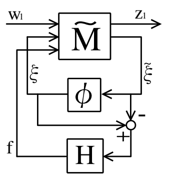

Fig. 3. General anti-windup structure

3. SYNTHESIS OF AN ANTI-WINDUP COMPENSATOR

3.1 *Problem setting*

In order to consider the robust control performance during the saturation period, we add an imaginary uncertainty:

$$w_1 = \Delta_p \delta z_1 \tag{10}$$

to the system of Fig.3, where $\Delta_p \in \{\text{block-diag}\ (\Delta_{p1}, \cdots, \Delta_{pm}) | \Delta_{pi} \in [-1, 1], \forall i\}$ and δ is a positive scalar. We study a design method of H that satisfies following design specifications.

(1) To locally stabilize the closed-loop system against the saturation nonlinearity ϕ.
(2) To make δ as large as possible in order to optimize the robust control performance during the saturation period.

For simplicity, we assume $D_{M13} = 0$ and $D_{M23} = 0$ in the following discussion.

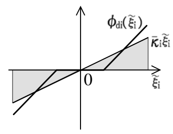

Fig. 4. Dead-zone nonlinearity

3.2 *Robust stability condition*

In this section, we derive a robust stability condition of the system of Fig.3 with (10), and reduce the design problem of H which satisfies this condition to an LMI problem.

The saturation function $\phi_i(\cdot)$ can be represented as $\phi_i(\tilde{\xi}_i) = \tilde{\xi}_i - \phi_{di}(\tilde{\xi}_i)$ by using a dead zone function $\phi_{di}(\cdot)$. We model $\phi_{di}(\cdot)$ as the sector bounded nonlinearity that lies inside the sector $[0, \bar{\kappa}_i]$, i.e. $\phi_{di}(\cdot) \in [0, \bar{\kappa}_i]$, as shown in Fig.4. Namely, we model $\phi_{di}(\cdot)$ as $\phi_{di}(\tilde{\xi}_i) = \tilde{\Delta}_{\phi i}\bar{\kappa}_i\tilde{\xi}_i$, where $\tilde{\Delta}_{\phi i} \in [0, 1]$. When $\bar{\kappa}_i = 1$, the sector entirely covers $\phi_{di}(\cdot)$, and when $\bar{\kappa}_i < 1$, the sector covers a finite region with the center $\tilde{\xi}_i = 0$. By using this model, we represent the saturation function $\phi(\cdot)$ as

$$\begin{cases} \xi &= \tilde{\xi} - \tilde{w}_2 \\ \tilde{z}_2 &= \bar{\kappa}\tilde{\xi} \\ \tilde{w}_2 &= \tilde{\Delta}_\phi \tilde{z}_2 \end{cases} \tag{11}$$

where $\tilde{w}_2 \in \mathbf{R}^{n_b}$, $\tilde{z}_2 \in \mathbf{R}^{n_b}$, $\bar{\kappa} = \text{diag}(\bar{\kappa}_1, \cdots, \bar{\kappa}_{n_b})$, $\tilde{\Delta}_\phi = \text{diag}(\tilde{\Delta}_{\phi 1}, \cdots, \tilde{\Delta}_{\phi n_b})$. Substitution of (11) into (7) gives

$$\begin{aligned} f &= H(\xi - \tilde{\xi}) \\ &= H\{(\tilde{\xi} - \tilde{w}_2) - \tilde{\xi}\} \\ &= -H\tilde{w}_2 \end{aligned} \tag{12}$$

From (12), we can obtain an interpretation that the static anti-windup compensator estimates the disturbance $\tilde{w}_2$, and makes use of it as a feedforward signal.

The transformation $\tilde{\Delta}_{pi} = (\Delta_{pi} + 1)/2$ bijectively maps $\Delta_{pi} \in [-1, 1]$ into $\tilde{\Delta}_{pi} \in [0, 1]$ (e.g. Safonov, Jonckheere, Verma and Limebeer, 1987). By using this relation, (10) can be represented as

$$\begin{cases} w_1 &= 2\tilde{w}_1 - \tilde{z}_1 \\ \tilde{z}_1 &= \delta z_1 \\ \tilde{w}_1 &= \tilde{\Delta}_p \tilde{z}_1 \end{cases} \tag{13}$$

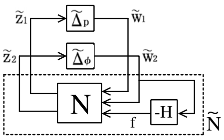

Fig. 5. Closed-loop system for the circle theorem

where $\tilde{z}_1 = \delta z_1$, $\tilde{\Delta}_p = \text{block-diag}(\tilde{\Delta}_{p1}, \cdots, \tilde{\Delta}_{pm})$. The closed-loop system shown in Fig.3 with (11)-(13) can be represented as the system shown in Fig.5, and it is described by

$$\begin{cases} \dot{x}_M &= A_N x_M + B_{N1}\tilde{w}_1 + (B_{N2} - B_{N3}H)\tilde{w}_2 \\ \tilde{z}_1 &= C_{N1}x_M + D_{N11}\tilde{w}_1 + D_{N12}\tilde{w}_2 \\ \tilde{z}_2 &= C_{N2}x_M + D_{N21}\tilde{w}_1 + D_{N22}\tilde{w}_2 \end{cases} \quad (14)$$

$$\begin{bmatrix} \tilde{w}_1 \\ \tilde{w}_2 \end{bmatrix} = \begin{bmatrix} \tilde{\Delta}_p & 0 \\ 0 & \tilde{\Delta}_\phi \end{bmatrix} \begin{bmatrix} \tilde{z}_1 \\ \tilde{z}_2 \end{bmatrix} \quad (15)$$

where

$$\begin{cases} A_N &= \tilde{A} + \tilde{B}_2(I - \tilde{D}_{22})^{-1}\tilde{C}_2 \\ B_{N1} &= \tilde{B}_1 + \tilde{B}_2(I - \tilde{D}_{22})^{-1}\tilde{D}_{21} \\ B_{N2} &= -\tilde{B}_2(I - \tilde{D}_{22})^{-1} \\ B_{N3} &= B_{M3} \\ C_{N1} &= \tilde{C}_1 + \tilde{D}_{12}(I - \tilde{D}_{22})^{-1}\tilde{C}_2 \\ C_{N2} &= \bar{\kappa}(I - \tilde{D}_{22})^{-1}\tilde{C}_2 \\ D_{N11} &= \tilde{D}_{11} + \tilde{D}_{12}(I - \tilde{D}_{22})^{-1}\tilde{D}_{21} \\ D_{N12} &= -\tilde{D}_{12}(I - \tilde{D}_{22})^{-1} \\ D_{N21} &= \bar{\kappa}(I - \tilde{D}_{22})^{-1}\tilde{D}_{21} \\ D_{N22} &= -\bar{\kappa}\tilde{D}_{22}(I - \tilde{D}_{22})^{-1} \end{cases}$$

$$\begin{cases} \tilde{A} &= A_M - B_{M1}(I + \delta D_{M11})^{-1}\delta C_{M1} \\ \tilde{B}_1 &= 2B_{M1}(I + \delta D_{M11})^{-1} \\ \tilde{B}_2 &= B_{M2} - B_{M1}(I + \delta D_{M11})^{-1}\delta D_{M12} \\ \tilde{C}_1 &= (I + \delta D_{M11})^{-1}\delta C_{M1} \\ \tilde{C}_2 &= C_{M2} - D_{M21}(I + \delta D_{M11})^{-1}\delta C_{M1} \\ \tilde{D}_{11} &= 2(I + \delta D_{M11})^{-1}\delta D_{M11} \\ \tilde{D}_{12} &= (I + \delta D_{M11})^{-1}\delta D_{M12} \\ \tilde{D}_{21} &= 2D_{M21}(I + \delta D_{M11})^{-1} \\ \tilde{D}_{22} &= D_{M22} - D_{M21}(I + \delta D_{M11})^{-1}\delta D_{M12} \end{cases}$$

Next, by applying the multivariable circle theorem (see, Rosenbrock, 1973) to the system (14),(15), the following robust stability condition is derived.

$$(I - \tilde{N}(j\omega)) + (I - \tilde{N}(j\omega))^* > 0, \forall\omega \quad (16)$$

where

$$\begin{aligned} \tilde{N}(s) &= \tilde{C}_N(sI - A_N)^{-1}\tilde{B}_N + \tilde{D}_N \in RH_\infty \\ \tilde{B}_N &= [B_{N1} \quad B_{N2} - B_{N3}H] \\ \tilde{C}_N &= \begin{bmatrix} C_{N1} \\ C_{N2} \end{bmatrix}, \tilde{D}_N = \begin{bmatrix} D_{N11} & D_{N12} \\ D_{N21} & D_{N22} \end{bmatrix} \end{aligned}$$

Further, this robust stability condition can be represented as a matrix inequality condition by using the following lemma.

Lemma 1. (see e.g., Boyd, Ghaoui, Feron and Balakrishnan, 1994) For a given transfer function $G(s) := C(sI - A)^{-1}B + D \in RH_\infty$, the following statements are equivalent:

(1) $G(j\omega) + G(j\omega)^* > 0, \forall\omega$
(2) There exists $Q = Q^T > 0$ such that

$$\begin{bmatrix} QA^T + AQ & B - QC^T \\ B^T - CQ & -(D + D^T) \end{bmatrix} < 0 \quad (17)$$

From Lemma 1, the condition (16) holds if and only if there exist $Q = Q^T > 0$ and H such that

$$F(Q, H) = \begin{bmatrix} QA_N^T + A_N Q & \tilde{B}_N + Q\tilde{C}_N^T \\ \tilde{B}_N^T + \tilde{C}_N Q & \tilde{D}_N + \tilde{D}_N^T - 2I \end{bmatrix} < 0 \quad (18)$$

Since only $\tilde{B}_N$ includes H, (18) is LMI. Therefore, the design problem of H that satisfies $Q = Q^T > 0$ and (18) can be solved by the numerical optimization method efficiently (see e.g., Gahinet, Nemirovski, Laub and Chilali, 1995).

Note 2. When the above LMI problem is solved with $\bar{\kappa} = I$, the global stability is assured by using the obtained H. When the above LMI problem can not be solved with $\bar{\kappa} = I$, the only local stability is assured. We do not discuss the domain of attraction (see e.g., Hindi and Boyd, 1998) directly in this paper. We simply make the sector larger in order to enlarge the maximum value of $\tilde{\xi}$ that satisfies the sector condition.

3.3 *Introduction of a constant scaling matrix*

The conservativeness of the design result can be reduced by using the following scaled passivity condition instead of (16).

$$(I - \tilde{N}(j\omega))W + W(I - \tilde{N}(j\omega))^* > 0, \forall\omega \quad (19)$$

where $W \in \{\text{block-diag}(w_1 I_1, \cdots, w_m I_m, w_{m+1}, \cdots, w_{m+n_b-1}, 1) | w_i > 0, \forall i\}$. From Lemma 1, the condition (19) holds if and only if there exist $Q = Q^T > 0$, W and H such that

$$F_W(Q, H, W) = \begin{bmatrix} QA_N^T + A_N Q & \tilde{B}_N W + Q\tilde{C}_N^T \\ W\tilde{B}_N^T + \tilde{C}_N Q & \tilde{D}_N W + W\tilde{D}_N^T - 2W \end{bmatrix} < 0 \quad (20)$$

Unfortunately, (20) is BMI with respect to W and H, thus it is difficult to obtain an optimal solution in general. Nevertheless, the conservativeness of the design result can be reduced as compared with the case without a scaling matrix by using the following alternating minimization algorithm.

step1: Let $i = 0$ and fix $W_0 = I$.

step2: Fix $W = W_i$.
minimize σ subject to $F_W(Q, H, W) < \sigma I$ and $Q = Q^T > 0$. Let $H_i = H$.

step3: Fix $H = H_i$.
minimize σ subject to $F_W(Q, H, W) < \sigma I$ and $Q = Q^T > 0$. Let $W_{i+1} = W$ and $\sigma_{i+1} = \sigma$.

step4: If $\sigma_{i+1} < 0$ or $|\sigma_{i+1} - \sigma_i| < \epsilon$ for sufficiently small $\epsilon > 0$, then stop.
Otherwise, let $i \to i + 1$ and go to step2.

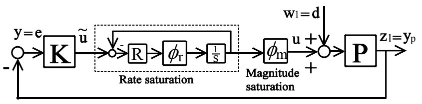

Fig. 6. Control system

4. NUMERICAL EXAMPLE

We consider a closed-loop system shown in Fig.6. $P(s) = C_p(sI - A_p)^{-1}B_p$ is a plant, and its coefficient matrices are given by

$$A_p = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.1 \end{bmatrix}, B_p = \begin{bmatrix} 0.5 & 0.4 \\ 0.4 & 0.3 \end{bmatrix}, C_p = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$K(s) = C(sI - A)^{-1}B + D$ is a controller, and its coefficient matrices are given by

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, D = \begin{bmatrix} 10 & 0 \\ 0 & -10 \end{bmatrix}$$

The above $P(s)$ and $K(s)$ are considered in Campo *et al.* (1990). Although this problem looks simple, the closed-loop system becomes unstable because of the change of the direction of control inputs even in the case where there exists only input magnitude saturation. Thus this problem is appropriate for evaluating the effectiveness of our design method.

We solve the matrix inequality (20) by using the alternating minimization algorithm with $R = 100I$, $\bar{\kappa} = 0.9I$, $\delta = 0.03$. After three iterations, a feasible solution is obtained as

$$H = \begin{bmatrix} 5.090 & 3.548 & 0.629 & -0.363 \\ -4.040 & -2.586 & -0.560 & 0.339 \end{bmatrix} \quad (21)$$

We conduct numerical simulations for a step disturbance $d(t) = [1, 2]^T, (t \geq 0)$ for three cases shown in Table1. Fig.7 and Fig.8 show the results of case1 and case2. The solid lines show the responses of the system with the proposed anti-windup compensator, and the dashed lines show the responses of the system without anti-windup compensation. The closed-loop stability is assured by using the anti-windup compensator. Fig.9 and Fig.10 show the results of case3. Although simulation conditions are severer than case2, the closed-loop stability is assured, and the step disturbance is attenuated in these figures.

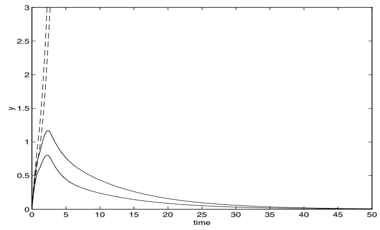

Fig. 7. $y_p(t)$ (dashed:case1, solid:case2)

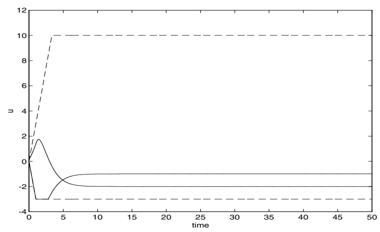

Fig. 8. $u(t)$ (dashed:case1, solid:case2)

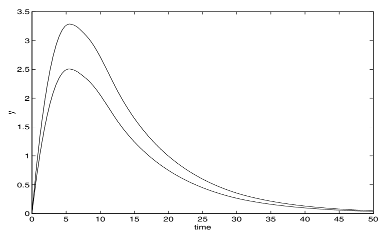

Fig. 9. $y_p(t)$ (case3)

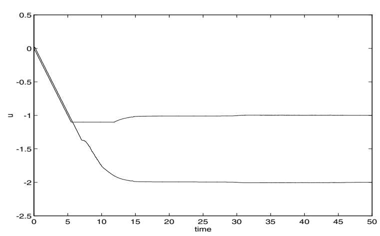

Fig. 10. $u(t)$ (case3)

Table 1. Simulation conditions

case1	w/o AWC	$u_{m1,max} = 3$	$u_{r1,max} = 3$
		$u_{m2,max} = 10$	$u_{r2,max} = 3$
case2	with AWC	$u_{m1,max} = 3$	$u_{r1,max} = 3$
		$u_{m2,max} = 10$	$u_{r2,max} = 3$
case3	with AWC	$u_{m1,max} = 1.1$	$u_{r1,max} = 0.2$
		$u_{m2,max} = 2.5$	$u_{r2,max} = 0.2$

5. CONCLUSION

We propose a design method of an anti-windup compensator which guarantees the local stability of the closed-loop system with input rate and magnitude saturations. The proposed design method has following useful properties.

- The design problem is reduced to an LMI problem. Thus, the solution can be obtained efficiently by a numerical optimization method.
- The proposed design method is applicable to the design of the anti-windup compensator for any class of LTI controllers (e.g., centralized controllers, decentralized controllers, PID controllers).

6. REFERENCES

Åström, K.J. and B.Wittenmark (1984). *Computer Controlled Systems Theory and Design.* Prentice-Hall, Englewood Cliffs, NJ.

Berg,J.M., K.D.Hammett, C.A.Schwartz and S.S.Banda (1996). An analysis of the destabilizing effect of daisy chained rate-limited actuators. *IEEE Trans. Control Systems Technology*, **4** (2), pp.171-176.

Bernstein,D.S. and A.N.Michel (1995). A chronological bibliography on saturating actuators. *Int. J. of Robust and Nonlinear control*, **5**, pp.375-380

Boyd,S.,L.E.Ghaoui,E.Feron and V.Balakrishnan (1994). Linear Matrix Inequalities in System and Control Theory. SIAM.

Campo, P.J. and M.Morari (1990). Robust Control of Process subject to Saturation Nonlinearities. *Computers and Chemical Engineering*, **14** (4/5), pp.343-358.

Doyle,J.C., R.S.Smith and D.F.Enns (1987). Control of plants with input saturation nonlinearities. *Proc. of ACC*, pp.2147-2152.

Gahinet, P., A.Nemirovski, A.J.Laub and M.Chilali (1995). LMI Control Toolbox; The Math Works Inc.

Hanus, R., M.Kinnaert and J.L.Henrotte (1987). Conditioning technique, a general antiwindup and bumpless transfer method. *Automatica*, **23** (6), pp.729-739.

Hindi, H. and S.Boyd (1998). Analysis of Linear Systems with Saturation using Convex Optimization. *Proc. of IEEE CDC*, pp.903-908.

Kapasouris, P. and M.Athans (1990). Control Systems with Rate and Magnitude Saturation for Neutrally Stable Open Loop Systems. *Proc. of IEEE CDC*, pp.3404-3409.

Kapila, V. and W.M.Haddad (1998). Fixed-Structure Controller Design for Systems with Actuator Amplitude and Rate Nonlinearities. *Proc. of IEEE CDC*, pp.909-914.

Kapoor, N., A.R.Teel and P.Daoutidis (1998). An Anti-Windup Design for Linear Systems with Input Saturation. *Automatica*, **34** (5), pp.559-574.

Kothare, M.V., P.J.Campo, M.Morari and C.N.Nett (1994). A Unified Framework for the Study of Anti-Windup Designs. *Automatica*, **30** (12), pp.1869-1883.

Kothare, M.V. and M.Morari (1997). Multivariable Anti-Windup Controller Synthesis using Multi-Objective Optimization. *Proc. of ACC*, pp.3093-3097.

Lin, Z. (1997). Semi-Global stabilization of linear systems with position and rate-limited actuators. *Systems & Control Letters*, **30**, pp.1-11

Marcopoli, V.R. and S.M.Phillips (1996). Analysis and synthesis tools for a class of actuator-limited multivariable control systems: A linear matrix inequality approach. *Int. J. Robust and Nonlinear control*, **6**, pp.1045-1063.

Rosenbrock, H.H. (1973). *Multivariable Circle Theorems, Recent Math. Developments in Control*; edited by D.J.Bell, Academic Press, London, pp.345-365.

Saeki, M. and N.Wada (1996). Anti-Windup controller design based on Matrix Inequalities. *Proc. of IEEE CDC*, pp.261-262.

Saeki, M. and N.Wada (2000). Synthesis of a static anti-windup compensator via linear matrix inequalities. *IFAC Symposium on Robust Control Design.*

Safonov, M.G., E.A.Jonckheere, M.Verma and D.J.N.Limebeer (1987). Synthesis of positive real multivariable feedback systems. *Int. J. of Control*, **45** (3), pp.817-842.

Tyan, F. and D.S.Bernstein (1997). Dynamic output feedback compensation for linear systems with independent amplitude and rate saturations. *Int. J. of Control*, **67** (1), pp.89-116.

Wada, N. and M.Saeki (1998). Robust controller design for the system with input saturation. *Proc. of SICE Dynamical Systems Theory Symposium*, pp.221-226, (In Japanese).

Wada, N. and M.Saeki (1999). Design of a static anti-windup compensator which guarantees robust stability. *Trans. of the Institute of Systems, Control and Information Engineers*, **12** (11), pp.664-670, (In Japanese).