集中講義「等質空間の不連続群」

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Intensive course

"Discontinuous groups for homogeneous spaces"

Takayuki Okuda (Hiroshima Univ.)

Section 1 : Introduction to discontinuous groups

Reference : T. Kobayashi, "Discontinuous groups for non-Riemannian homogeneous spaces" Mathematical Unlimited — 2001 and beyond. 723-747. Springer-Verlag (2001)

日本語版（内容増）

小林俊行（手塚勝貴，真野元，吉野太郎 訳）

"非リーマン等質空間の不連続群"

「数学の最先端 21世紀への挑戦」

18-72 シュプリンガー・フェアラーク東京 (2002)

Problem :

Find manifolds equipped with

- interesting topologies, and (ex : compact)
- good geometric structures

(ex. Riem. metrics,
pseudo - Riem metrics,
symplectic forms
complex structures)

Global

vs

Local

Idea :

① Homogeneous spaces may admits good geometric structures

② "Good quotients" of homogeneous spaces has good geometric structures

Take $p, q \in \mathbb{Z}_{\geq 0}$.

Ex. Put

$$X(p,q) := \{ x \in \mathbb{R}^{p+q+1} \mid x_1^2 + \cdots + x_{p+1}^2 - x_{p+2}^2 - \cdots - x_{p+q+1}^2 = 1 \} \subset \mathbb{R}^{p+q+1}.$$

$X(p,q)$ is a homogeneous $O(p+1,q)$-space

a semisimple Lie group

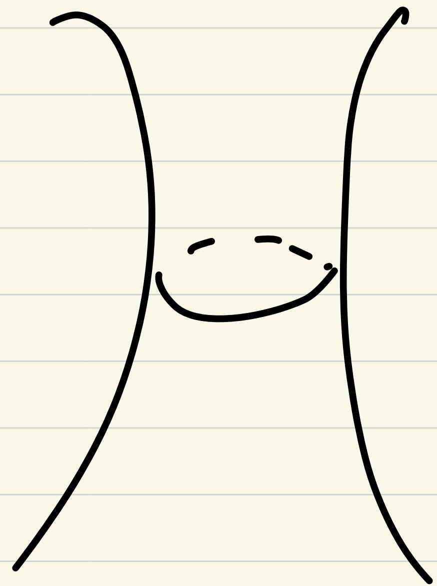
and admits a

$O(p+1, q)$-invariant pseudo-Riemannian metric of signature (p,q)

$X(2,0)$

2-Sphere

$X(1,1)$



1-sheeted hyperboloid

$X(0,2)$

2-sheeted hyperboloid.

The metric on $X(p,q)$ has the following property ⊛ :

⊛ Sectional curvatures are constant

on non-degenerate 2-subspaces

in tangent spaces

(a good geometric structure)

Let Γ be a discrete subgroup of $O(p+1, q)$

If the Γ-action on $X(p,q)$ is

properly - discontinuous

a good group action

Then the quotient space $\Gamma \backslash X(p,q)$

a good quotient.

is a manifold and

admits a pseudo-Riemannian metric

with the property ⊛

(Keyword : The space form problem)

Ex :

Realizations of $\pi_1(\Sigma_g) \subset O(1,2)$

$\wr$

$\pi_1(\Sigma_g) \backslash \underset{X(0,2)}{\text{a conn comp of}} \cong \Sigma_g$

$\wr$

" Hyperbolic structures " on Σ_g

Σ_g

Ex : No infinite discrete subgroup $\Gamma \subset O(2,1)$ acts on $X(1,1)$ properly-discontinuously.

(An example of Calabi - Markus phenomenon)

Let X be a homogeneous G-space.

a Lie group

Problem : Find discrete subgroups $\Gamma < G$

s.t. $\Gamma \curvearrowright X$ is properly-discontinous.

Such Γ is called discontinuous group for X

Sub problem : Fix a discrete subgroup Γ of G

How to check whether

$\Gamma \curvearrowright X$ is properly-discontinuous.

Main topic of this course