§ 2 : Main results

The purpose of this course is to study "Kobayashi's properness criterion".

Setting : G : a linear reductive Lie group

Γ, H : closed subgroups in G

Theorem [Kobayashi's properness criterion]

The following two conditions on (G, H, Γ) are equivalent:

(i) $\Gamma \curvearrowright G/H$: proper

(ii) $\mathfrak{a}(\Gamma) \pitchfork \mathfrak{a}(H)$ in $\mathfrak{a}$ ← "computable" in some situations.

↑ stated in terms of "Lie algebras"

References

- T. Kobayashi, "Proper action on a homogeneous space of reductive type", Math. Ann. 285 : 249-263 (1989)
- T. Kobayashi, "Criterion for proper actions on homogeneous space of reductive type", J. Lie Theory, 6 : 147 - 163 (1996)
- Y. Benoist, "Actions propres sur les espaces homogènes réductifs" Ann. of Math. 144 : 315 - 347 (1996)

Goal of this course :

For the case where

a pseudo-Riem. mfd.

- $X = G/H$ is of reductive type, and
- Γ is a (torsion-free) discrete subgroup in G,

we give a translation of Kobayashi's properness criterion within the framework of Riemannian geometry.

The main results of this course:

Setting: G : a linear reductive Lie group without compact factors

$X = G/H$: a homogeneous space of reductive type — a pseudo-Riem. mfd.

Γ : torsion-free discrete subgroup

K : a maximal compact subgroup of G

$\rightsquigarrow M = G/K$ is a non-compact Riemannian symmetric spaces

Main Theorem :

The following two conditions on (G, H, Γ) are equivalent

(i) $\Gamma \curvearrowright X = G/H$ is properly - discontinuous.

(ii) For each $p \in \Gamma\backslash M = \Gamma\backslash G/K$ $\left(\begin{array}{c}\text{locally} \\ \text{Riem. symm. sp}\end{array}\right)$

$\nexists$ a family of "X - type" geodesics in $\Gamma\backslash M$ through p "coiling around p"

(Terminologies will be defined later)

A geodesic coiling around p