The intersection of two real forms in Hermitian symmetric spaces of compact type II

Makiko Sumi Tanaka
Hiroyuki Tasaki

September 2012

M : a Hermitian symmetric space of compact type

The fixed point set of an involutive anti-holomorphic isometry of M

= a real form

a connected totally geodesic Lagrangian submanifold

$I(M)$: the group of isom. of M

$A(M)$: — of hol. isom. of M

$I_0(M) = A_0(M)$

: the id. comp. of $I(M), A(M)$

Prop.1 M : irreducible

congruent class of real forms with respect to $A(M)$

$=$ — with respect to $A_0(M)$

Lem.2 $\tau : M \to M$ **an anti-hol. isometry**

$(x, y) \mapsto (\tau^{-1}(y), \tau(x))$

an inv. anti-hol. isom. of $M \times M$

Its fixed point set is

$$D_\tau(M) = \{(x, \tau(x)) \mid x \in M\}.$$

Def.3

$D_\tau(M)$: **diagonal real form**

Prop.4 $\boldsymbol{M}$ **: irreducible**

$\boldsymbol{I(M) - A(M)}$ **: anti-hol.**

$$\boldsymbol{I(M) - A(M) \ni \tau \mapsto D_{\tau}(M)}$$

bijective correspondence between

conn. comp. of $\boldsymbol{I(M) - A(M)}$

and

cong. class of diag. real forms

in $\boldsymbol{M \times M}$

Thm.5 $M = M_1 \times \cdots \times M_n$

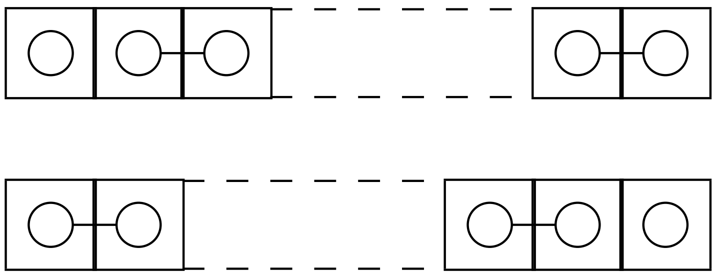
M_i : irreducible

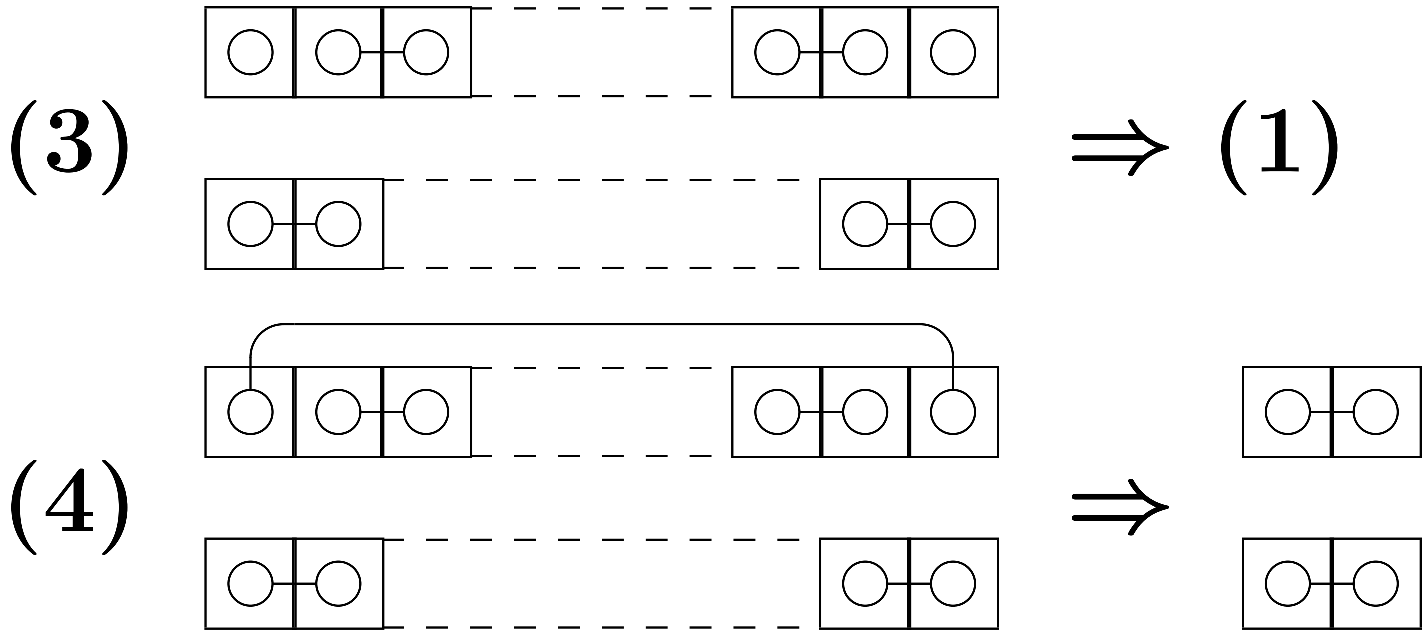
L_1, L_2 : real forms in M

$\Rightarrow L_a = L_{a,1} \times \cdots \times L_{a,m}$

each pair of $L_{1,k}, L_{2,k}$ is

(1) real forms in some M_i

(2)  $\Rightarrow$ (1)


(3)
⇒ (1)
(4)
⇒

$M \supset S$: antipodal

$\Leftrightarrow s_x y = y$ for all $x, y \in S$

$\#_2 M = \max\{\#S : \text{antipodal}\}$

S : great $\Leftrightarrow \#S = \#_2 M$

Thm.(T.-T.)

L_1, L_2 : real forms in M,

$L_1 \cap L_2$: discrete

$\Rightarrow L_1 \cap L_2$: antipodal

Thm.6 M : irreducible

τ_1, τ_2 : anti-hol. isom. of M

$D_{\tau_1}(M) \cap D_{{\tau_2}^{-1}}(M)$: discrete

(1) $M = Q_{2m}(\mathbb{C}), G_m(\mathbb{C}^{2m})$ $(m \geq 2)$

$\tau_2\tau_1 \notin A_0(M) \Rightarrow$

$\#(D_{\tau_1}(M) \cap D_{{\tau_2}^{-1}}(M)) < \#_2 M$

(2) Otherwise

$\#(D_{\tau_1}(M) \cap D_{{\tau_2}^{-1}}(M)) = \#_2 M,$

$D_{\tau_1}(M) \cap D_{{\tau_2}^{-1}}(M)$: great