ウィルソンクォークを用いた N_f=2+1 QCD の熱力学量の研究

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JPS meeting, Okayama Univ., Okayama , 20-23 March 2010

Motivation

QCD Thermodynamics on the lattice

- Phase diagram in (T, μ , m_{ud}, m_s)
- Transition temperature
- Equation of state (e, p, s,...)
- Heavy quarkonium
- Transport coefficients (shear/bulk viscosity)
- Finite chemical potential
- etc...

} quantitative studies (Phase diagram, Transition temperature, Equation of state)

} qualitative studies (Heavy quarkonium, Transport coefficients, Finite chemical potential)

These are important to study

- Quark Gluon Plasma in Heavy Ion Collision exp.
- Early universe
- Neutron star
- etc...

QCD Thermodynamics on the lattice

Most studies done with staggerd-type quarks

- less computational costs
- a part of chiral sym. preserved ...

  → N_f=2+1, almost physical quark mass, $\mu \neq 0$

- 4th-root trick to remove unphysical “tastes”

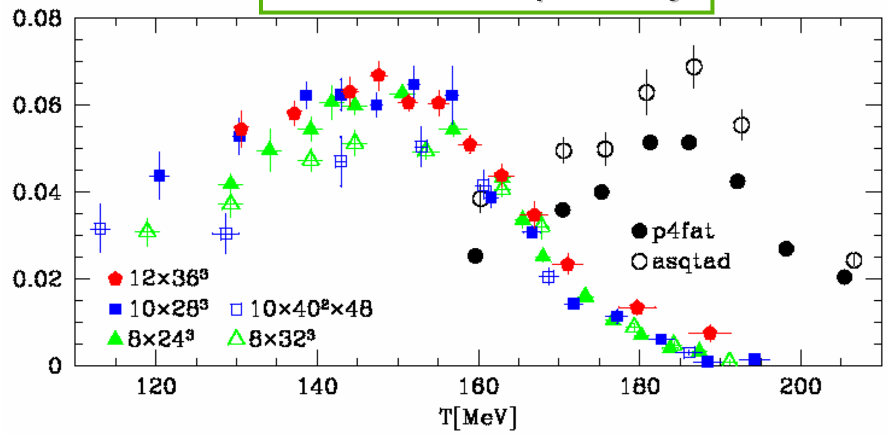
  → non-locality “universality is not guaranteed”

It is important to cross-check with
theoretically sound lattice quarks

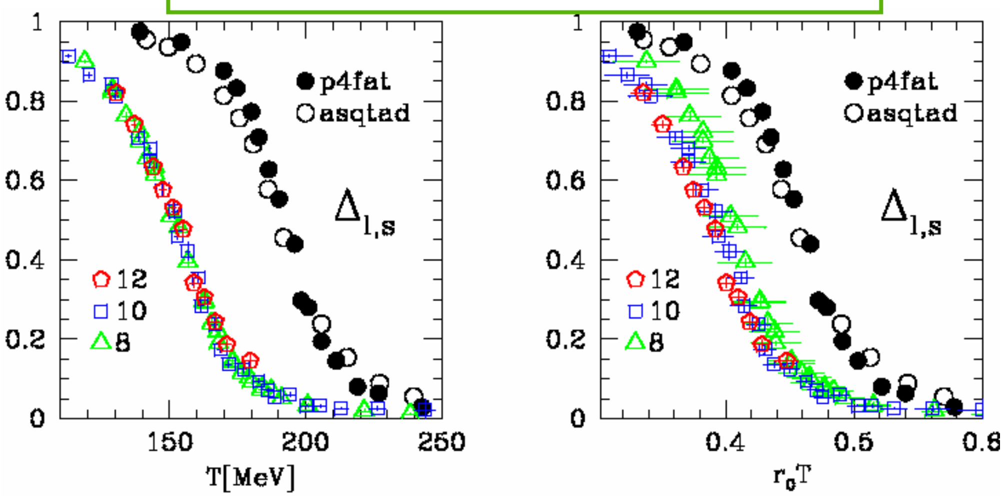
> Our aim is to investigate
> QCD Thermodynamics with Wilson-type quarks

Improved staggered (p4fat vs asqtad)

chiral susceptibility



renormalized chiral condensate



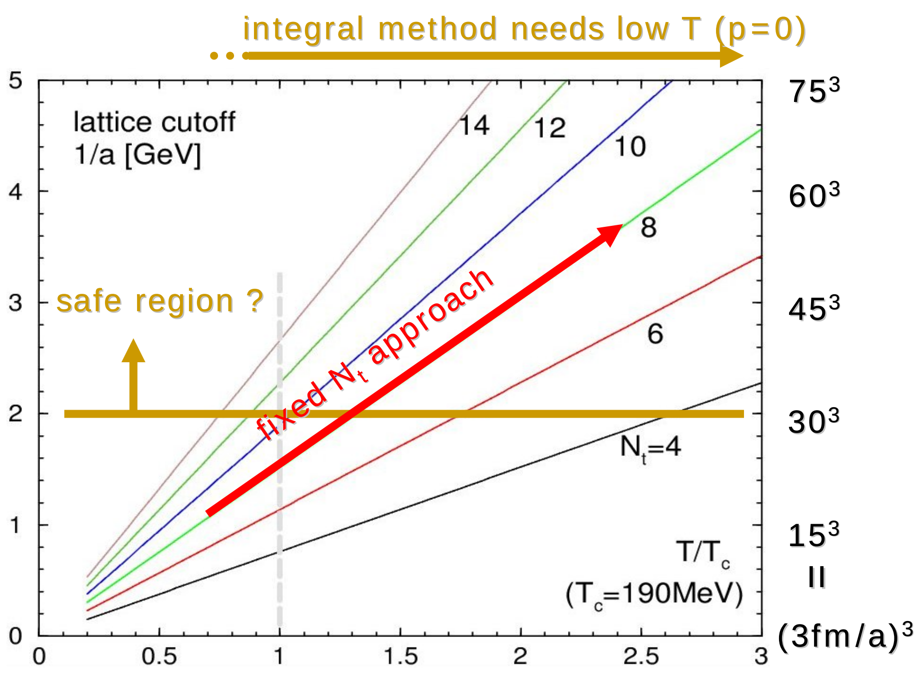
Y.Aoki et al., JHEP06 (2009) 088

(In Sect.4: conclusions, outlooks)

As a final remark we have to mention that the staggered formalism used in this work and all other large scale thermodynamics studies may suffer from theoretical problems. To date it is not proven that the staggered formalism with 2+1 flavors really describes QCD in the continuum limit. Therefore it is desirable to also study QCD thermodynamics with a theoretically firmly established (e.g. Wilson type) fermion discretization.

Conventional approach to study QCD thermodynamics

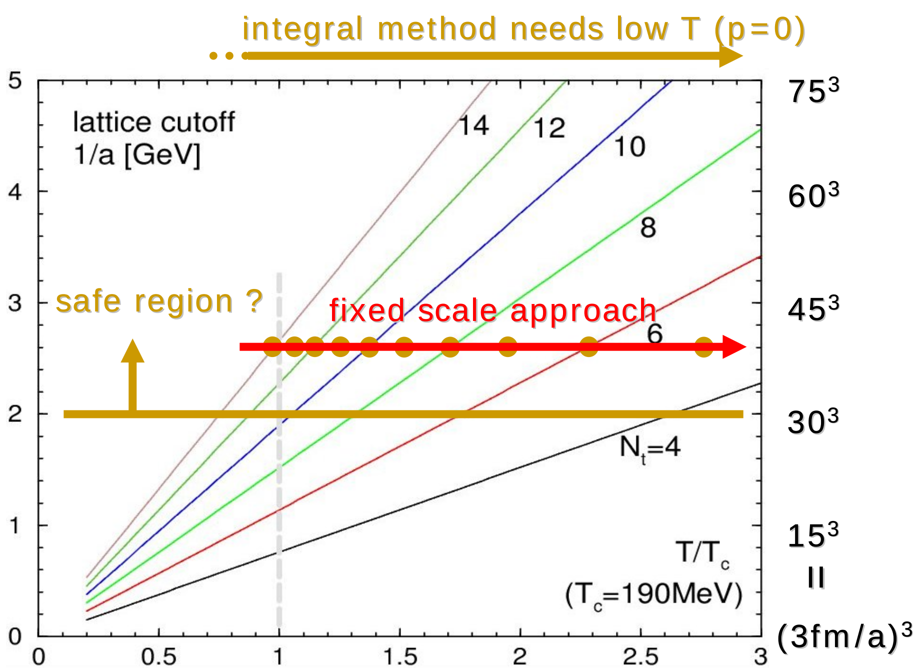
Temperature $T=1/(N_t a)$ is varied by a at fixed N_t



- Disadvantages
  - Line of Constant Physics
  - T=0 subtraction for renorm.
  - small 1/a at low T region
- Advantages
  - T resolution by integer N_t
  - program for odd N_t
  - (1/a) vs T at high T

Fixed scale approach to study QCD thermodynamics

Temperature $T=1/(N_t a)$ is varied by N_t at fixed a



- **Advantages**
  - Line of Constant Physics
  - T=0 subtraction for renorm. (spectrum study at T=0)
  - larger 1/a at whole T region
- **Disadvantages**
  - T resolution by integer N_t
  - program for odd N_t
  - (1/a) vs T at high T

T-integration method to calculate the EOS

We propose a new method ("T-integration method") to calculate the EOS at fixed scales

T.Umeda et al. (WHOT-QCD), Phys.Rev.D79 (2009) 051501(R)

Our method is based on the trace anomaly (interaction measure),

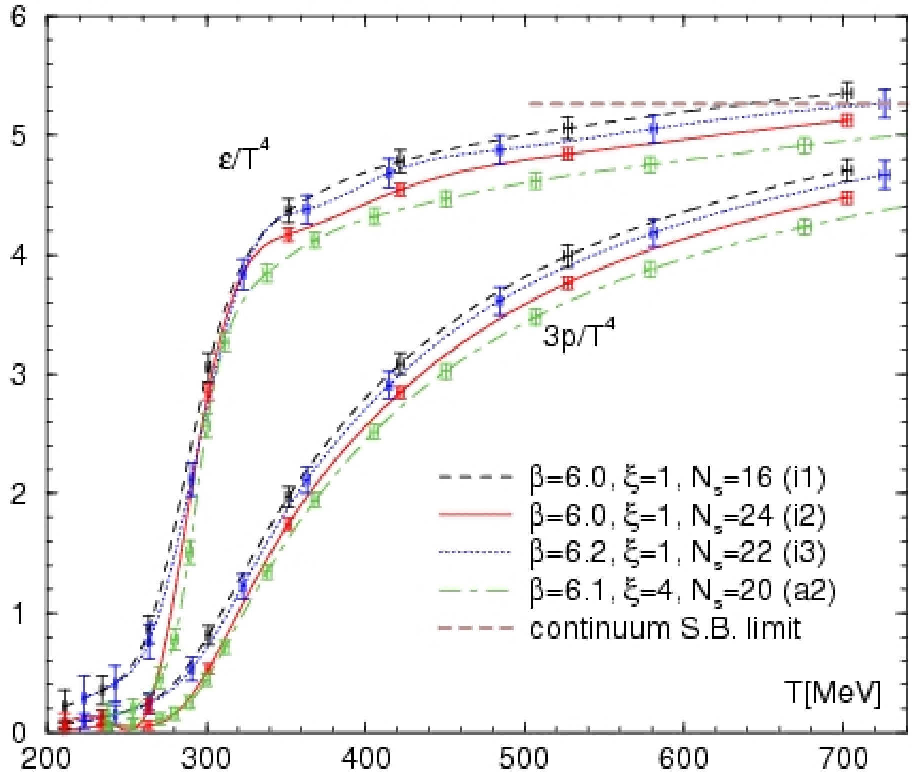
$$\frac{\epsilon - 3p}{T^4} = \left(\frac{N_t^3}{N_s^3}\right) a\frac{d\beta}{da}\left\langle\frac{dS}{d\beta}\right\rangle_{sub}$$

and the thermodynamic relation.

$$\frac{\epsilon - 3p}{T^4} = T\frac{\partial(p/T^4)}{\partial T}$$

$$\Rightarrow \quad \frac{p}{T^4} = \int_0^T dT' \, \frac{\epsilon - 3p}{T'^5}$$

Pressure & Energy density in quenched QCD



- Integration $\left(\frac{p}{T^4} = \int_0^T dT' \frac{\epsilon - 3p}{T'^5}\right)$

  is performed with the cubic spline of $(e-3p)/T^4$

- Cubic spline vs trapezoidal inte. yields small difference ~ 1σ

- Our results are roughly consistent with previous results.

- Unlike the fixed N_τ approach, scale/temp. is not constant.

→ Lattice artifacts increase as temperature increases.

T=0 & T>0 configurations for N_f=2+1 QCD

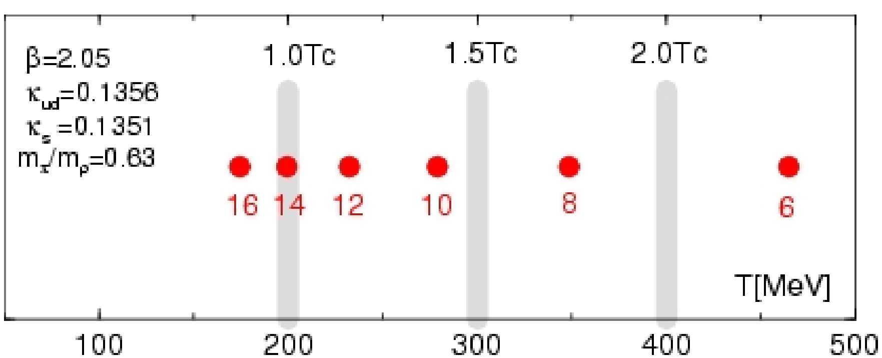
- Basic T=0 simulation:

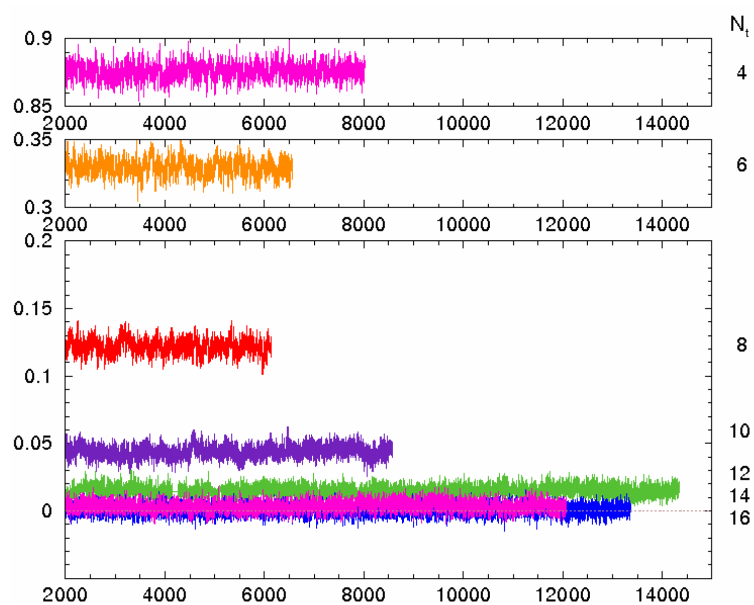
  CP-PACS / JLQCD Collab. N_f=2+1 study *Phys. Rev. D78 (2008) 011502.*

  - RG-improved Iwasaki glue + NP clover-improved Wilson quarks
  - $(2\ \mathrm{fm})^3$ lattice, a=0.07, 0.1, 0.12 fm
  - configurations available on the ILDG

- T>0 simulations: on 32^3 x N_t (N_t=4, 6, ..., 14, 16) lattices

Nt's correspond to T~170—700MeV





Beta-functions from CP-PACS/JLQCD results

Beta-functions to calculate the EOS of Nf=2+1 QCD

$$\frac{\epsilon - 3p}{T^4} = \frac{N_t^3}{N_s^3}\left(a\frac{\partial \beta}{\partial a}\left\langle \frac{\partial S}{\partial \beta}\right\rangle_{sub} + a\frac{\partial \kappa_{ud}}{\partial a}\left\langle \frac{\partial S}{\partial \kappa_{ud}}\right\rangle_{sub} + a\frac{\partial \kappa_s}{\partial a}\left\langle \frac{\partial S}{\partial \kappa_s}\right\rangle_{sub}\right)$$

Inverse matrix method

Phys. Rev. D64 (2001) 074510

(1) Collect T=0 lattice results of #param. observables

(2) Fit them as functions of coupling param.

(3) Determine LCP's

(4) Invert the coupling param. dependence of observables along a LCP.

Nf=2+1 QCD
→ β , κ_{ud}, κ_s
e.g. m_ρ ,
m_π / m_ρ ,
$m_{\eta ss} / m_\phi$

$$\begin{pmatrix} \frac{\partial \beta}{\partial (m_V a)} & \frac{\partial K}{\partial (m_V a)} \\ \frac{\partial \beta}{\partial (m_{PS}/m_V)} & \frac{\partial K}{\partial (m_{PS}/m_V)} \end{pmatrix} = \begin{pmatrix} \frac{\partial (m_V a)}{\partial \beta} & \frac{\partial (m_{PS}/m_V)}{\partial \beta} \\ \frac{\partial (m_V a)}{\partial K} & \frac{\partial (m_{PS}/m_V)}{\partial K} \end{pmatrix}^{-1}$$

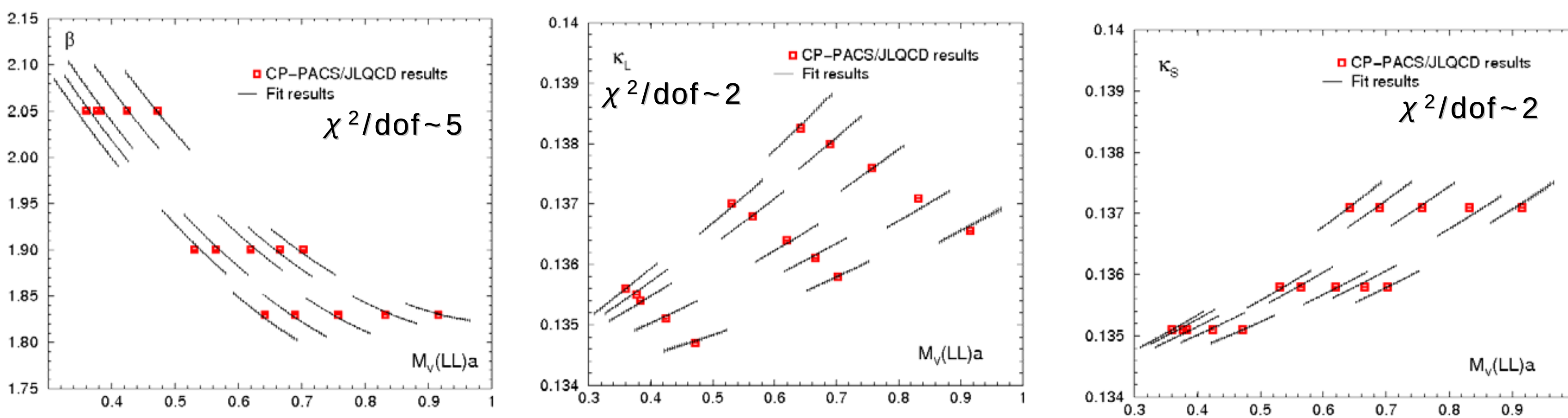
in case of Nf=2

Beta-functions from CP-PACS/JLQCD results

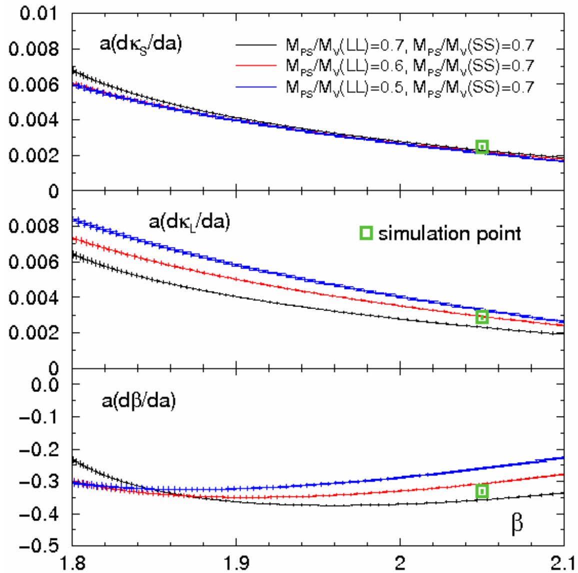
Direct fit method Phys. Rev. D64 (2001) 074510

fit β , κ_{ud}, κ_s as functions of $(am_\rho), \left(\frac{m_\pi}{m_\rho}\right), \left(\frac{m_{\eta ss}}{m_\phi}\right)$

$$\begin{pmatrix}\beta\\ \kappa_L\\ \kappa_S\end{pmatrix} = \vec{c}_1 + \vec{c}_2(am_\rho) + \vec{c}_3(am_\rho)^2 + \vec{c}_4\left(\frac{m_\pi}{m_\rho}\right) + \vec{c}_5\left(\frac{m_\pi}{m_\rho}\right)^2 + \vec{c}_6(am_\rho)\left(\frac{m_\pi}{m_\rho}\right)$$
$$+\vec{c}_7\left(\frac{m_{\eta ss}}{m_\phi}\right) + \vec{c}_8\left(\frac{m_{\eta ss}}{m_\phi}\right)^2 + \vec{c}_9(am_\rho)\left(\frac{m_{\eta ss}}{m_\phi}\right) + \vec{c}_{10}\left(\frac{m_\pi}{m_\rho}\right)\left(\frac{m_{\eta ss}}{m_\phi}\right)$$



First trial calculations on these configurations



$$\left(a\frac{\partial\beta}{\partial a},\ a\frac{\partial\kappa_{ud}}{\partial a},\ a\frac{\partial\kappa_s}{\partial a} \right)_{\text{simulation point}}$$

$$= (-0.330(3), 0.00288(5), 0.00247(5)) \quad m_\rho$$

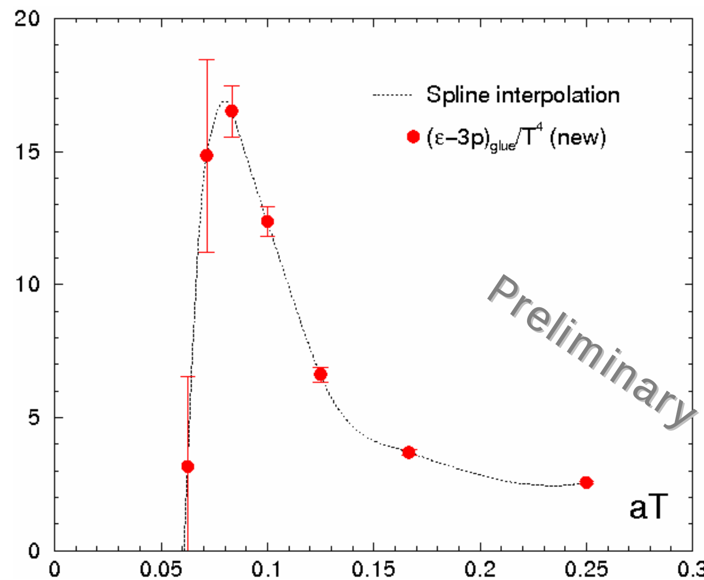
$$= (-0.340(3), 0.00286(5), 0.00242(5)) \quad m_{K^*}$$

$$= (-0.345(3), 0.00285(5), 0.00242(5)) \quad m_\phi$$

Preliminary !

gluon contribution to the trace anomaly

$$\left.\frac{\epsilon - 3p}{T^4}\right|_{\text{gluon}} = a\frac{\partial\beta}{\partial a}\frac{N_t^3}{N_s^3}\left\langle \frac{\partial S}{\partial \beta} \right\rangle_{\text{sub}}$$



(cf.1) peak height ~ 7 (KS N_f=2+1 N_t=8)
(cf.2) peak height ~ 13 (Wilson N_f=2 N_t=4)
gluon ~ 45, quark ~ -32

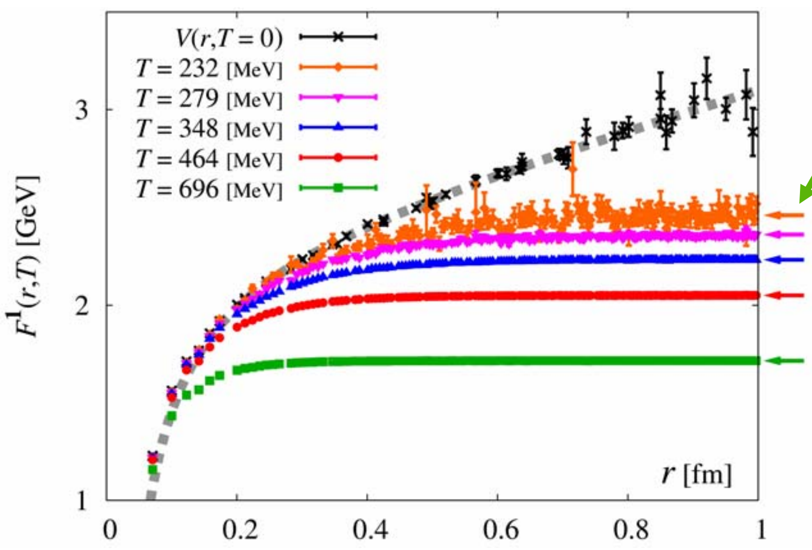
Heavy quark free energy at $T>T_c$

- HQ free energy in the color singlet channel

$$F^1(|\vec{x}-\vec{y}|,T) = -T\ln\langle \mathrm{Tr}\Omega^\dagger(\vec{x})\Omega(\vec{y})\rangle$$

Fixed scale approach : equal renormalization for all T

→ no T-dependent adjustments needed for the constant term in $F_1(r,T)$



T=0 data by CP-PACS/JLQCD

$$2F_Q = -2T\ln\langle \mathrm{Tr}\Omega\rangle$$

(2 x single quark free energy)

- Temp. insensitivity of $F^1(r,T)$ at short distances
- Q's are screened at T>Tc at long distances

Perspectives

- Beta functions

  More work needed

  Reweighting method to directly calculated beta functions

  at the simulation point ?

- Equation of state

  Fermion part measurement

- Nf=2+1 QCD just at the physical point

  the physical point (pion mass ~ 140MeV)

  with N_f=2+1 Wilson quarks (PACS-CS)

- Finite density

  We can combine our approach with the Taylor expansion method,

  to explore EOS at $\mu \neq 0$

Thank you for your attention !!!