The charmonium wave functions at finite temperature from lattice QCD calculations

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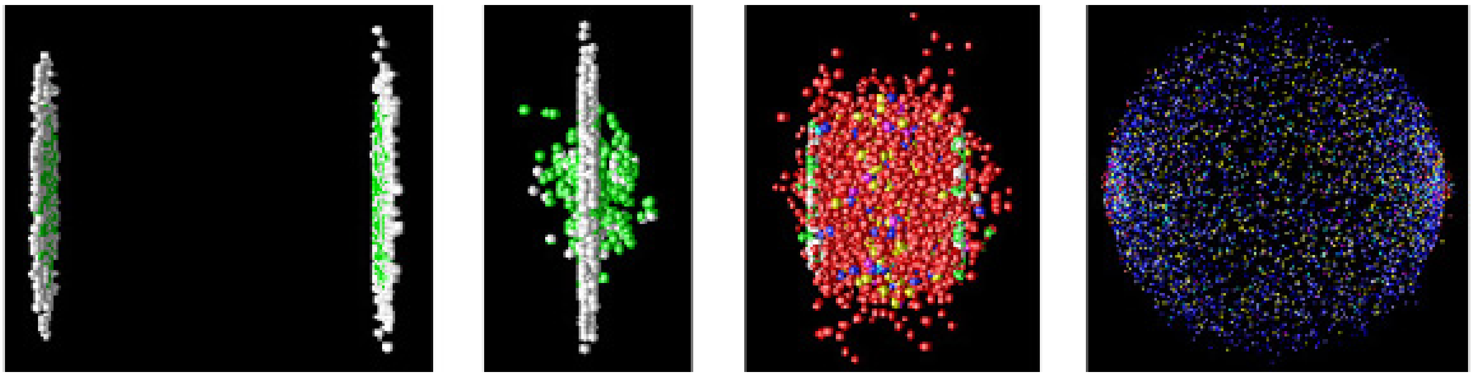
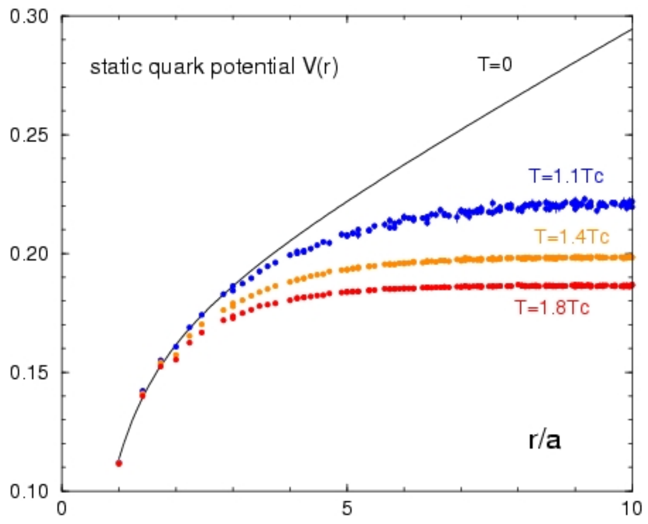
Contents of this talk

from the Phenix group web-site



- Introduction
  - -- Quark Gluon Plasma & J/ψ suppression
  - -- Lattice studies on J/ψ suppression
- Our approach to study charmonium dissociation
- Charmonium wave functions at T>0
- Summary & future plan

J/ψ suppression as a signal of QGP



Confined phase:
linear raising potential
→ bound state of c - $\bar{c}$

De-confined phase:
Debye screening
→ scattering state of c - $\bar{c}$

T.Hashimoto et al.('86), Matsui&Satz('86)

<u>Lattice QCD calculations:</u>

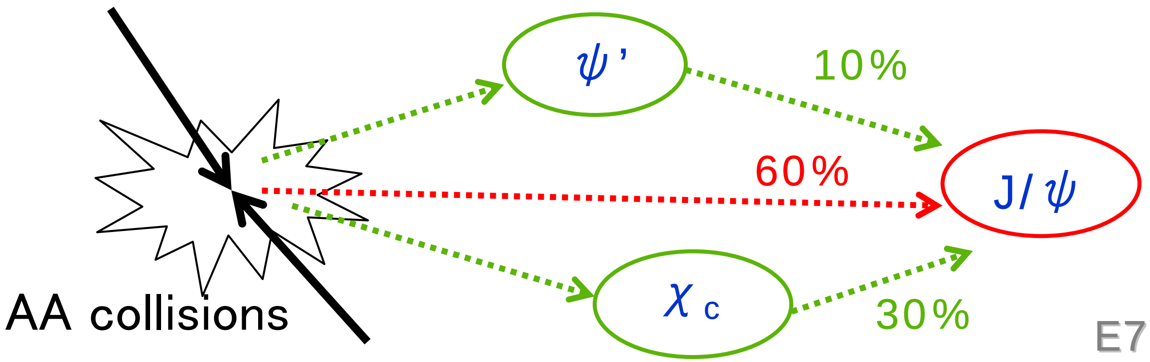
Spectral function by MEM: T.Umeda et al.('02), S.Datta et al.('04), Asakawa&Hatsuda('04), A.Jakovac et al.('07), G.Aatz et al.('06)

Wave func.: T.Umeda et al.('00)

B. C. dep.: H.Iida et al. ('06)

→ all calculations conclude that J/ψ survives till $1.5T_c$ or higher

Sequential J/ψ suppression scenario



E705 Collab.('93)

J/ψ (1S)	:	$J^{PC} = 1^{--}$	M=3097MeV	(Vector)
ψ (2S)	:	$J^{PC} = 1^{--}$	M=3686MeV	(Vector)
χ_{c0} (1P)	:	$J^{PC} = 0^{++}$	M=3415MeV	(Scalar)
χ_{c1} (1P)	:	$J^{PC} = 1^{++}$	M=3511MeV	(AxialVector)

PDG('06)

It is important to study dissociation temperatures for not only J/ψ but also ψ(2S), χ_c's

Current status on charmonium T_{dis}

- Lattice QCD studies (by MEM analysis) indicate
  - J/ψ may survive up to $T=1.5T_c$ or higher
  - χ_c may dissolve just above T_c
    e.g. A.Jakovac et al. (2007)
  - no results on excited states, ψ'
- The 2nd statement may be misleading (!)
  small change even in P-wave state
  up to $1.4T_c$ w/o the constant mode
- On the other hand,
  the potential model studies suggest
  charmonium dissociation may also
  provide small change in the correlators
  e.g. A.Mocsy et al. (2007)

Therefore we would like to investigate T_{dis}
using new approaches with Lattice QCD
without Bayesian analyses

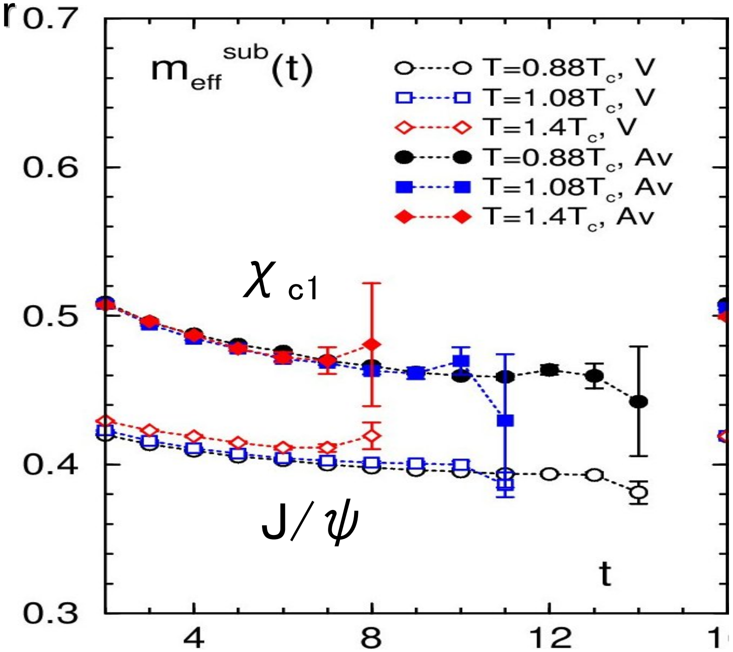
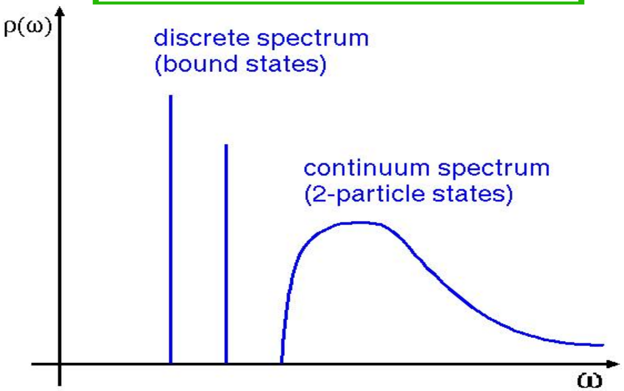


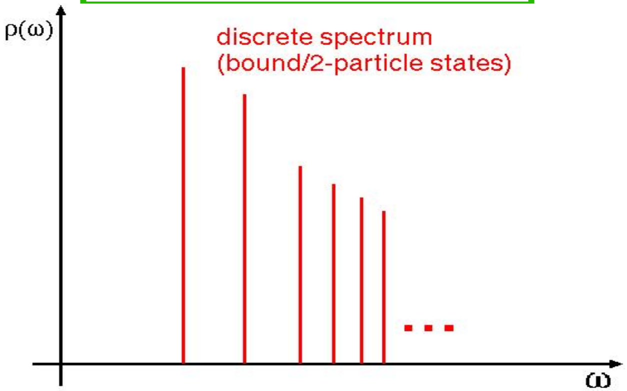
Fig: Temp. dependence of $M_{eff}(t)$ for J/ψ, χ_{c1} w/o constant mode.
T.Umeda (2007)

Another approach to study charmonium at T>0

infinite volume case



finite volume case



In a finite volume,
discrete spectra does not always indicate bound states !

In order to study a few lowest states,
the variational analysis is one of the most reliable approaches !

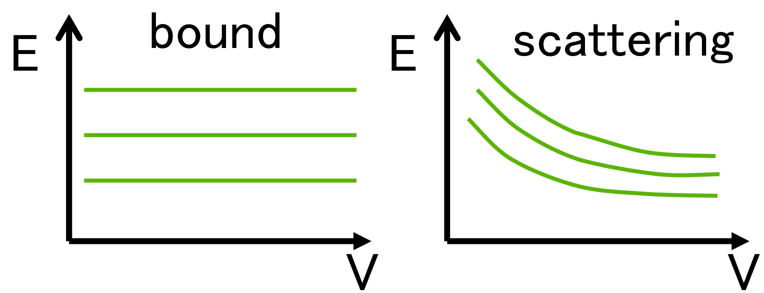
N x N correlation matrix : C(t)

$$C(t)\psi = \lambda(t,t_0)C(t_0)\psi \qquad \lambda_i(t,t_0) = e^{-E_i(t-t_0)}$$

Bound state or scattering state ?

We know three ways to identify the state in a finite volume

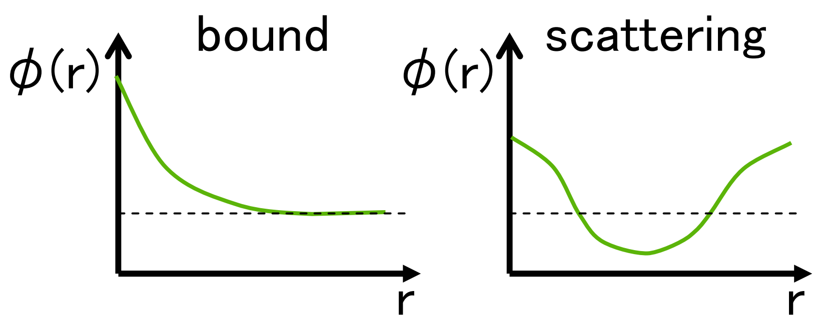
■ Volume dependence



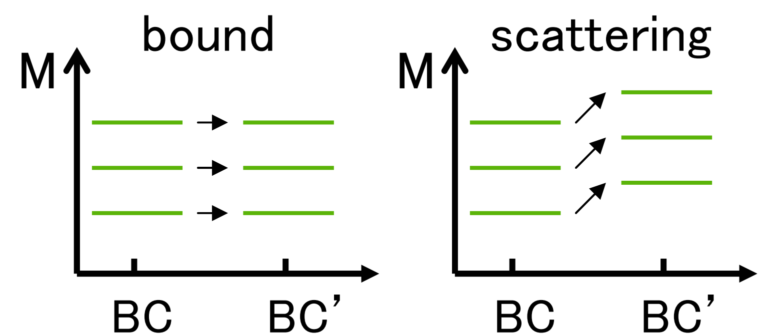
E : energy
V : volume

Φ(r): wave function
r : c - c̄ distance

■ Wave function



■ Boundary Condition (B.C.) dep.



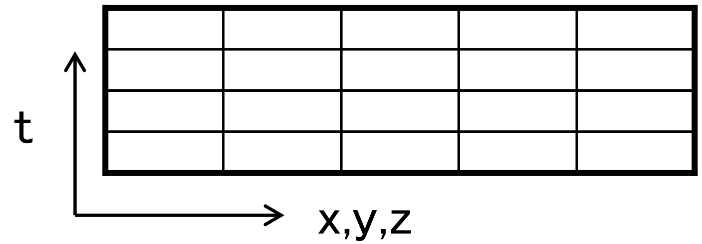
H.Iida et al.('06), N.Ishii et al.('05)

Lattice setup

- Quenched approximation (no dynamical quark effect)
- Anisotropic lattices

  lattice spacing : $a_s = 0.0970(5)$ fm

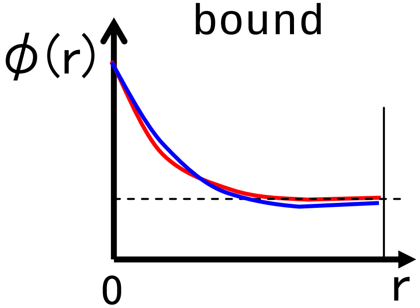
  anisotropy : $a_s/a_t = 4$



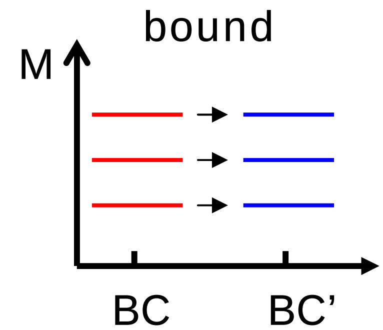
- r_s=1 to suppress doubler effects
- Variational analysis with 4 x 4 correlation matrix

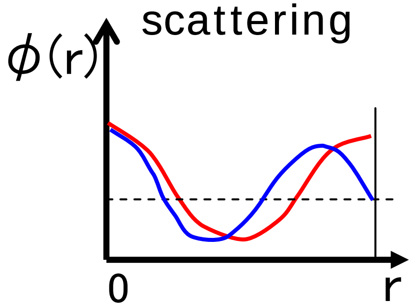
N_t	32	26	20	16	12
T/T_c	0.88	1.08	1.40	1.75	2.33
# of conf.					
$V=16^3$	300	300	300	300	300
$V=20^3$	300	300	300	300	300
$V=32^3$	—	—	—	—	100

Boundary condition dependence

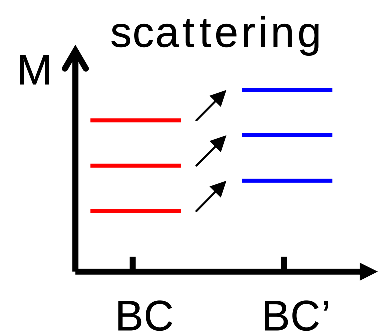


The wave functions are localized,
their energies are insensitive to B.C.



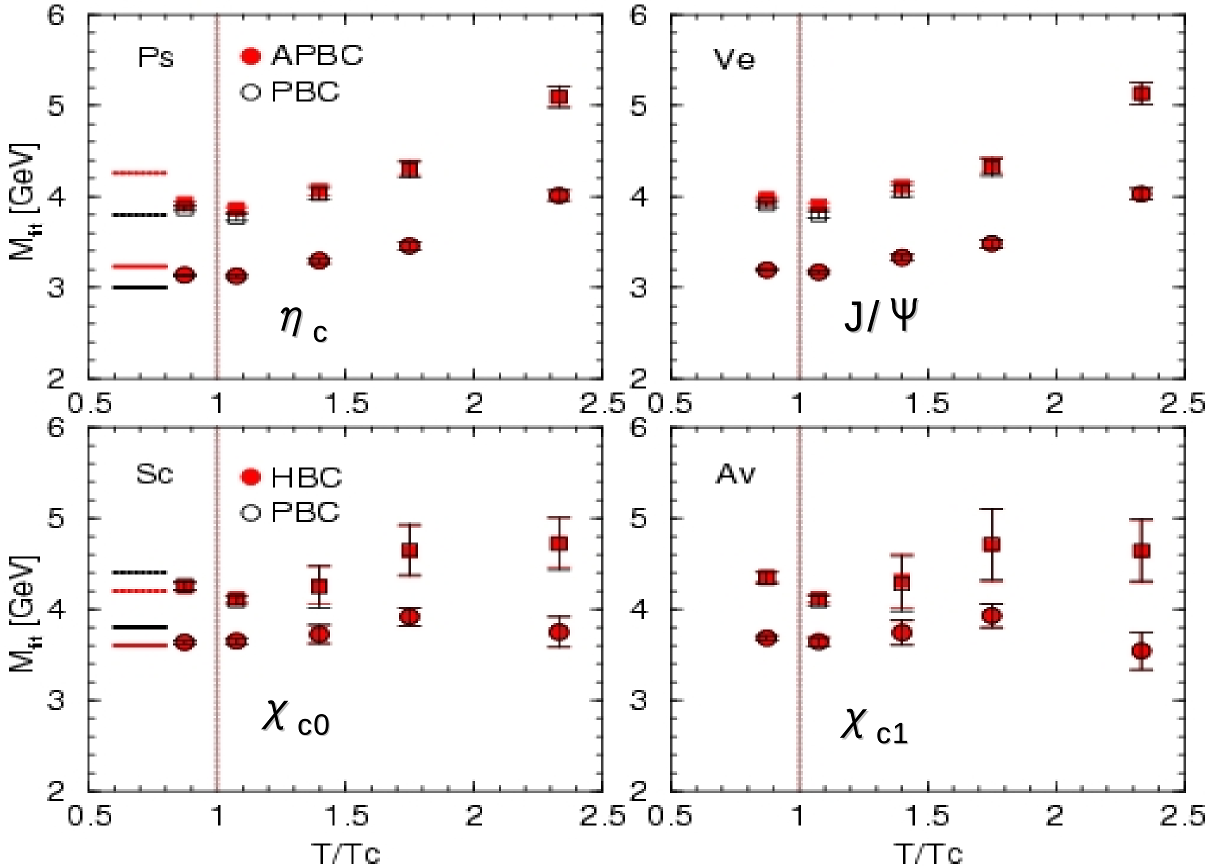


The momenta depends on BC,
the scattering state energies
are sensitive to B.C.



The idea has been originally applied for the charmonium study in H. Iida et al., PRD74, 074502 (2006).

Temperature dependence of charmonium spectra



$$q(x_i + L_i) = b_i q(x_i)$$

$b_i = 1$: periodic

$b_i = -1$: anti-periodic

PBC : b=(1, 1, 1)

APBC : b=(-1,-1,-1)

HBC : b=(-1, 1, 1)

an expected gap in $V=(2\text{fm})^3$ (free quark case) ~ 200MeV

- No significant differences in the different B.C.
- Analysis is difficult at higher temperature ($2T_c \sim$)

Wave functions at finite temperature

Temp. dependence of (Bethe-Salpeter) “Wave function”

$$
\begin{aligned}
BS(\vec{r},t) &= \sum_{\vec{x}} \langle \bar{q}(\vec{x}+\vec{r},t)\Gamma q(\vec{x},t)\bar{q}(\vec{0},0)\Gamma q(\vec{0},0)\rangle \\
\Psi(|\vec{r}|,t) &= BS(\vec{r},t)/BS(\vec{r}_0,t)
\end{aligned}
$$

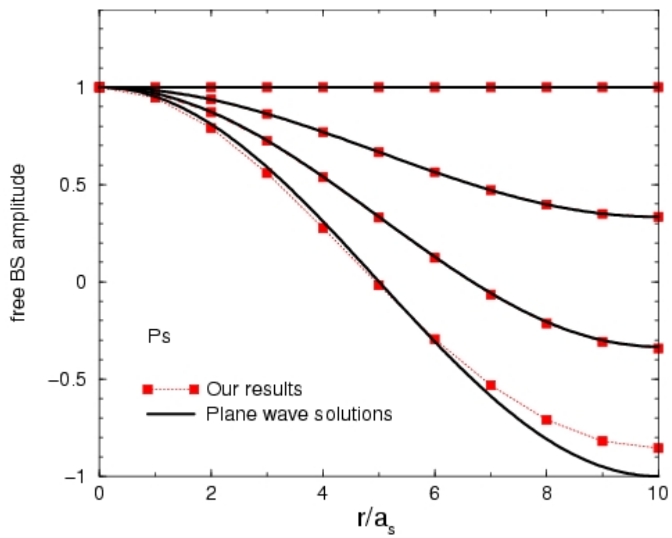
$$
\Gamma = \begin{cases}
\gamma_5 & \text{(Ps)} \\
\gamma_i & \text{(Ve)} \quad (i=1,2,3) \\
\sum_j \left(\overrightarrow{\partial}_j \gamma_j - \overleftarrow{\partial}_j \gamma_j \right) & \text{(Sc)} \\
\sum_{j,k} \epsilon_{ijk} \left(\overrightarrow{\partial}_j \gamma_k - \overleftarrow{\partial}_j \gamma_k \right) & \text{(Av)} \quad (i=1,2,3)
\end{cases}
$$

using the eigen functions of the variational method

→ we can extract the wave functions of each states

Wave functions in free quark case

Test with free quarks ($L_s/a=20$, $ma=0.17$)
in case of S-wave channels



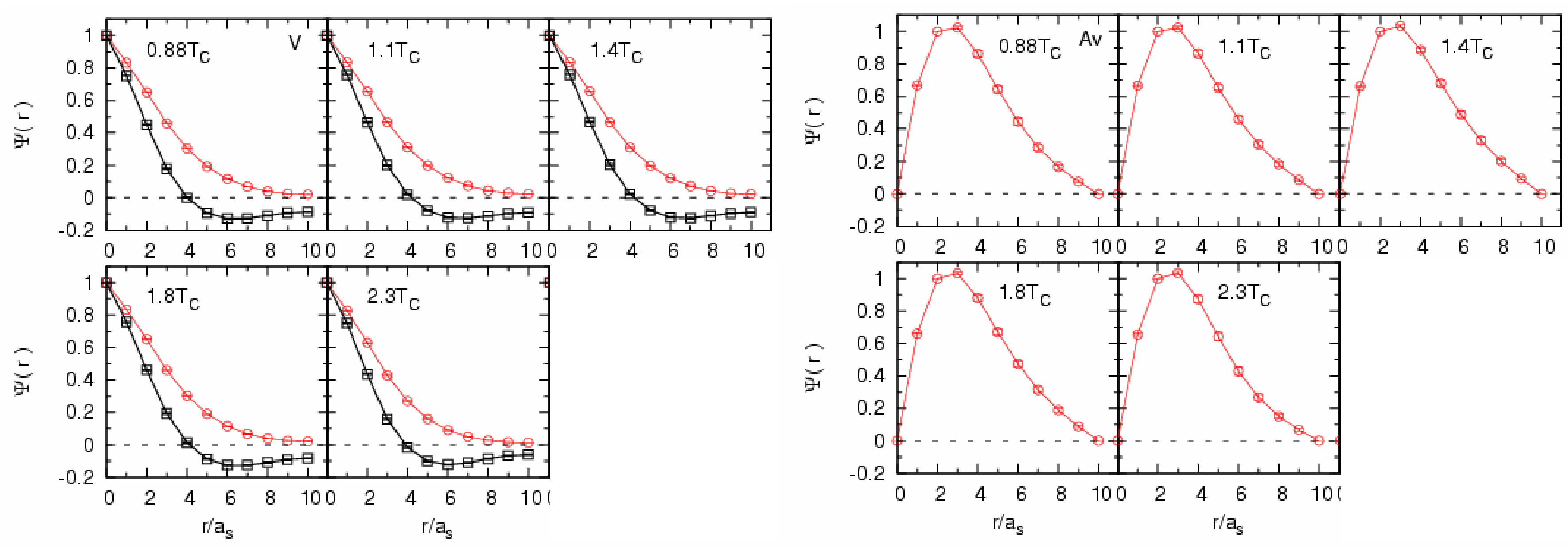
- Free quarks make trivial waves with an allowed momentum in a box

$$\Psi_k(|\vec{r}|,t) = \frac{\sum_{\vec{p}=\vec{k}} \cos(p_1 r_1)\cos(p_2 r_2)\cos(p_3 r_3)}{\sum_{\vec{p}=\vec{k}} 1}$$

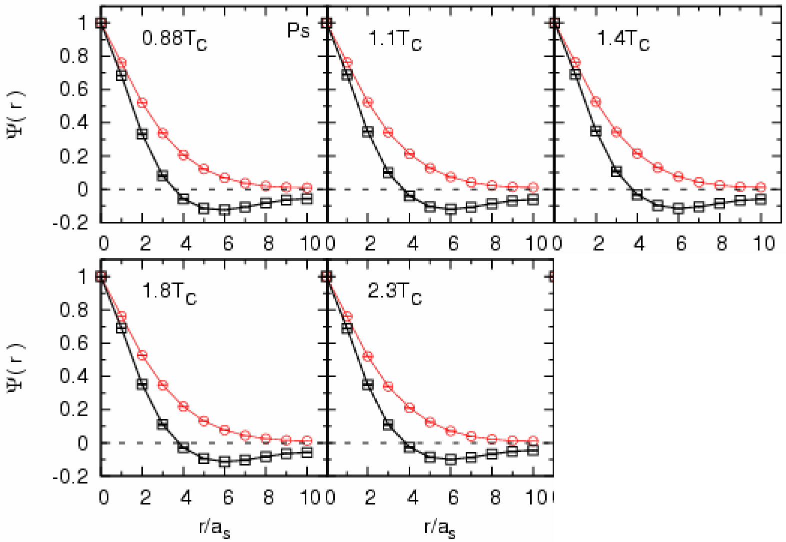
- The wave function is constructed with eigen functions of 6 x 6 correlators

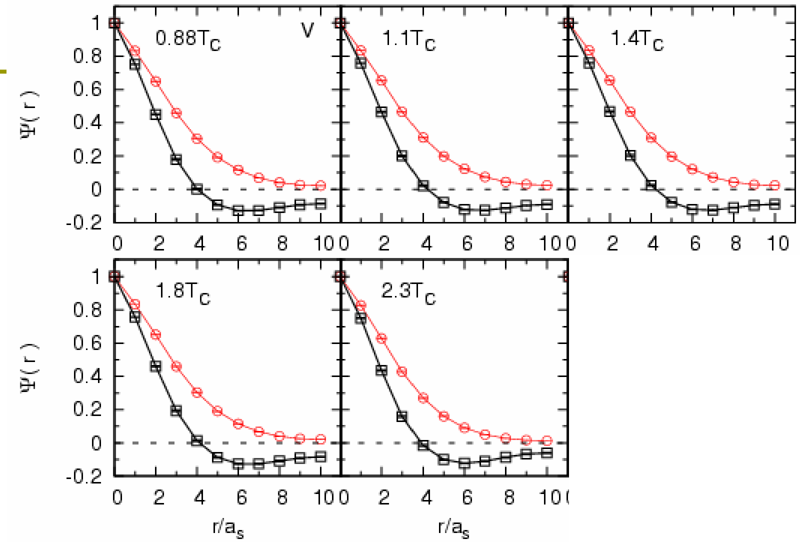
- Our method well reproduces the analytic solutions (!)

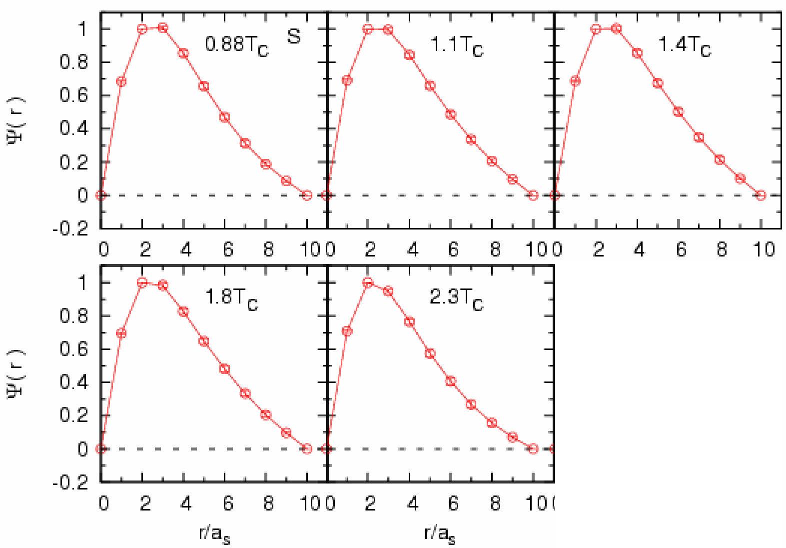
Charmonium wave functions at finite temperatures

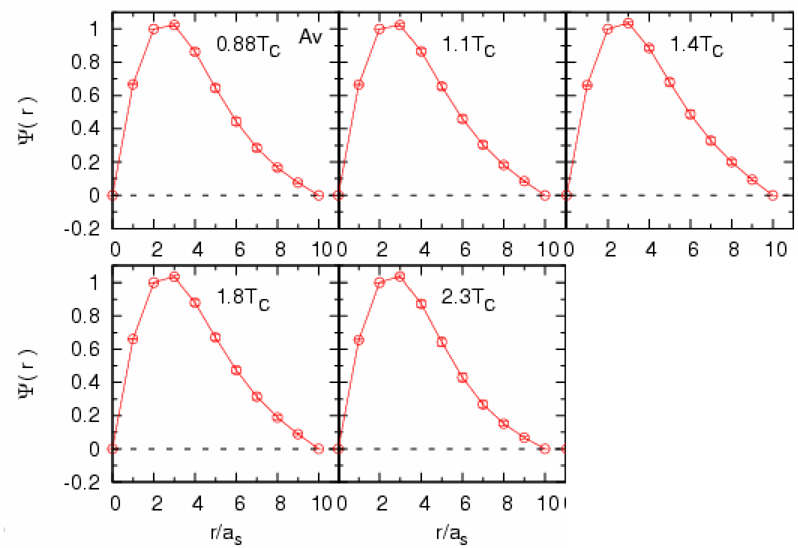


- Small temperature dependence in each channels
- Clear signals of bound states even at T=2.3Tc (!)

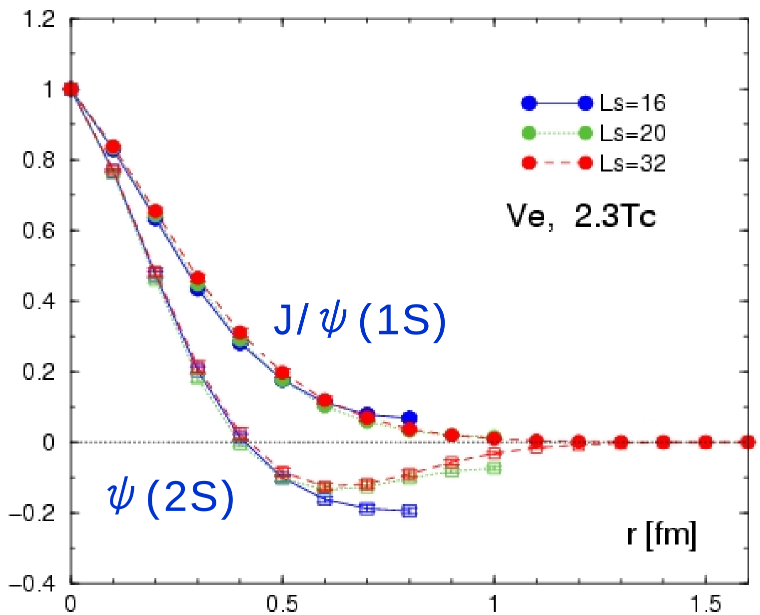

Ps
0.88T_C
1.1T_C
1.4T_C
1.8T_C
2.3T_C
Ψ(r)
r/a_s

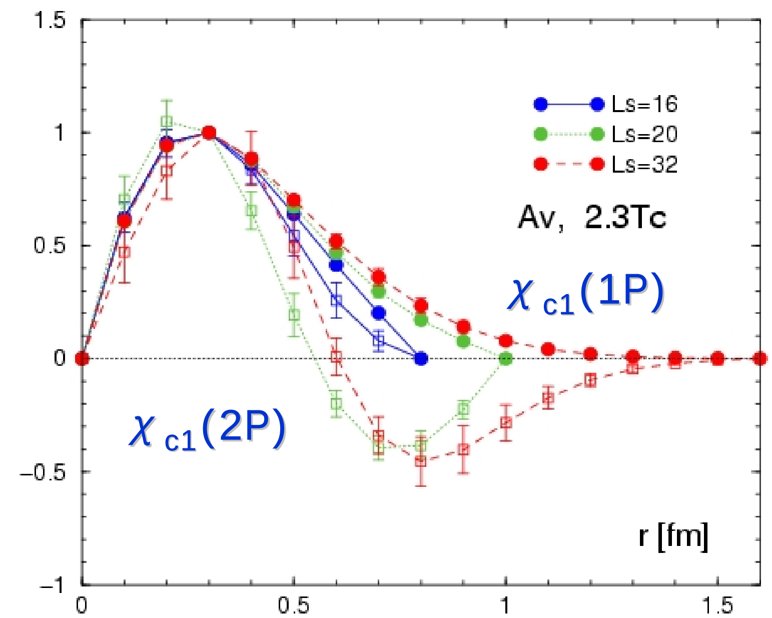

V
0.88T_C
1.1T_C
1.4T_C
1.8T_C
2.3T_C
Ψ(r)
r/a_s


S
0.88T_C
1.1T_C
1.4T_C
1.8T_C
2.3T_C
Ψ(r)
r/a_s


Av
0.88T_C
1.1T_C
1.4T_C
1.8T_C
2.3T_C
Ψ(r)
r/a_s

Volume dependence at T=2.3Tc





- Clear signals of bound states even at T=2.3Tc (!)
- Large volume is necessary for P-wave states.

Summary and future plan

We investigated T_{dis} of charmonia from Lattice QCD
using another approach to study charmonium at T>0
without Bayesian analysis

- boundary condition dependence
- Wave function (Volume dependence)

No evidence for unbound $c\text{-}\bar{c}$ quarks up to T = 2.3 Tc

→ The result may affect the scenario of J/ψ suppression.

Future plan

- higher temperature calculations ($T=3T_c$ or higher)
- Full QCD calculations (Nf=2+1 Wilson is now in progress)