The charmonium wave functions at finite temperature from lattice QCD calculations

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- Summary & future plan
**J/ψ suppression as a signal of QGP**

**Confined phase:**
- Linear raising potential
- \( \rightarrow \) bound state of \( c - \bar{c} \)

**De-confined phase:**
- Debye screening
- \( \rightarrow \) scattering state of \( c - \bar{c} \)

**T. Hashimoto et al. (‘86), Matsui & Satz (‘86)**

**Lattice QCD calculations:**
- **Spectral function by MEM:** T. Umeda et al. (‘02), S. Datta et al. (‘04), Asakawa & Hatsuda (‘04), A. Jakovac et al. (‘07), G. Aatz et al. (‘06)
- **Wave func.:** T. Umeda et al. (‘00)
- **B. C. dep.:** H. Iida et al. (‘06)

\( \rightarrow \) all calculations conclude that J/ψ survives till \( 1.5T_c \) or higher
Sequential $J/\psi$ suppression scenario

It is important to study dissociation temperatures for not only $J/\psi$ but also $\psi(2S)$, $\chi_c$’s.
Lattice QCD studies (by MEM analysis) indicate:
- $J/\psi$ may survive up to $T=1.5T_c$ or higher
- $\chi_c$ may dissolve just above $T_c$
  \[ \text{e.g. A.Jakovac et al. (2007)} \]
- No results on excited states, $\psi'$

The 2nd statement may be misleading (!)
- Small change even in P-wave state up to $1.4T_c$ w/o the constant mode

On the other hand,
- The potential model studies suggest charmonium dissociation may also provide small change in the correlators
  \[ \text{e.g. A.Mocsy et al. (2007)} \]

Therefore we would like to investigate $T_{\text{dis}}$
- Using new approaches with Lattice QCD without Bayesian analyses

**Fig:** Temp. dependence of $M_{\text{eff}}(t)$ for $J/\psi$, $\chi_{c1}$ w/o constant mode.

\[ T.\text{Umeda (2007)} \]
Another approach to study charmonium at $T>0$

**infinite volume case**

\[ \rho(\omega) \]
- discrete spectrum (bound states)
- continuum spectrum (2-particle states)

**finite volume case**

\[ \rho(\omega) \]
- discrete spectrum (bound/2-particle states)

In a finite volume, discrete spectra does not always indicate bound states!

In order to study a few lowest states, the variational analysis is one of the most reliable approaches!

\[
N \times N \text{ correlation matrix: } C(t) \\
C'(t)\psi = \lambda(t, t_0)C(t_0)\psi \quad \lambda_i(t, t_0) = e^{-E_i(t-t_0)}
\]
Bound state or scattering state?

We know three ways to identify the state in a finite volume:

- Volume dependence
  - Bound state
  - Scattering state

- Wave function
  - Bound state
  - Scattering state

- Boundary Condition (B.C.) dependence

\[ E : \text{energy} \]
\[ V : \text{volume} \]
\[ \Phi(r) : \text{wave function} \]
\[ r : \text{c - \bar{c} distance} \]

H.Iida et al. ('06), N.Ishii et al. ('05)
Lattice setup

- Quenched approximation (no dynamical quark effect)
- Anisotropic lattices
  - lattice spacing: \( a_s = 0.0970(5) \) fm
  - anisotropy: \( a_s/a_t = 4 \)
- \( r_s = 1 \) to suppress doubler effects
- Variational analysis with 4 x 4 correlation matrix

<table>
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<tr>
<th>( N_t )</th>
<th>32</th>
<th>26</th>
<th>20</th>
<th>16</th>
<th>12</th>
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<td>( T/T_c )</td>
<td>0.88</td>
<td>1.08</td>
<td>1.40</td>
<td>1.75</td>
<td>2.33</td>
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<td># of conf.</td>
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<tr>
<td>( V=20^3 )</td>
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<tr>
<td>( V=32^3 )</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>100</td>
</tr>
</tbody>
</table>
The idea has been originally applied for the charmonium study in H. Iida et al., PRD74, 074502 (2006).

Boundary condition dependence

The wave functions are localized, their energies are insensitive to B.C.

The momenta depends on BC, the scattering state energies are sensitive to B.C.

The idea has been originally applied for the charmonium study in H. Iida et al., PRD74, 074502 (2006).
Temperature dependence of charmonium spectra

No significant differences in the different B.C.

Analysis is difficult at higher temperature (2T_c~)

\[ q(x_i + L_i) = b_i q(x_i) \]

- \( b_i = 1 \) : periodic
- \( b_i = -1 \) : anti-periodic

PBC : \( b=(1, 1, 1) \)
APBC : \( b=(-1,-1,-1) \)
HBC : \( b=(-1, 1, 1) \)

an expected gap in \( V=(2\text{fm})^3 \)
(free quark case)
\[ \sim 200\text{MeV} \]
Wave functions at finite temperature

Temp. dependence of (Bethe-Salpeter) "Wave function"

\[
BS(\vec{r}, t) = \sum_{\vec{x}} \langle \vec{q}(\vec{x} + \vec{r}, t) | \Gamma \rangle q(\vec{x}, t) q(\vec{0}, 0) | \Gamma \rangle q(\vec{0}, 0)
\]

\[
\Psi(|\vec{r}|, t) = \frac{BS(\vec{r}, t)}{BS(\vec{r}_0, t)}
\]

\[
\Gamma = \begin{cases} 
\gamma_5 & (Ps) \\
\gamma_i & (Ve) \quad (i = 1, 2, 3) \\
\sum_j (\overrightarrow{\partial}_j \gamma_j - \overleftarrow{\partial}_j \gamma_j) & (Sc) \\
\sum_{j,k} \epsilon_{ijk} (\overrightarrow{\partial}_j \gamma_k - \overleftarrow{\partial}_j \gamma_k) & (Av) \quad (i = 1, 2, 3)
\end{cases}
\]

using the eigen functions of the variational method

→ we can extract the wave functions of each states
Wave functions in free quark case

Test with free quarks ($L_s/a=20$, $m_a=0.17$) in case of S-wave channels

- Free quarks make trivial waves with an allowed momentum in a box
  \[ \Psi_k(|r|, t) = \frac{\sum_{\vec{p}=\vec{k}} \cos(p_1 r_1) \cos(p_2 r_2) \cos(p_3 r_3)}{\sum_{\vec{p}=\vec{k}} 1} \]

- The wave function is constructed with eigen functions of 6 x 6 correlators

- Our method well reproduces the analytic solutions ( ! )
Charmonium wave functions at finite temperatures

- Small temperature dependence in each channels
- Clear signals of bound states even at $T=2.3T_c$ (!)
Volume dependence at $T=2.3T_c$

- Clear signals of bound states even at $T=2.3T_c$ ( ! )
- Large volume is necessary for P-wave states.
Summary and future plan

We investigated $T_{\text{dis}}$ of charmonia from Lattice QCD using another approach to study charmonium at $T>0$ without Bayesian analysis

- boundary condition dependence
- Wave function (Volume dependence)

No evidence for unbound $c\bar{c}$ quarks up to $T = 2.3 \ T_c$

→ The result may affect the scenario of $J/\psi$ suppression.

Future plan

- higher temperature calculations ($T=3T_c$ or higher)
- Full QCD calculations (Nf=2+1 Wilson is now in progress)