Lattice calculations on Heavy flavor
~ Open and Hidden charm states above Tc ~

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Heavy Flavor Productions
& Hot/Dense Quark Matter

Brookhaven Natl. Lab. at 12 Dec. 2005

First of all ...

"What is a role of Lattice QCD
for the study of QGP in Heavy Ion Experiments ?"

Heavy Ion Collisions include complicated interactions
among large number of particles

→ We have to investigate the experimental results
based on theoretical model predictions.

- "First principle calculations" is possible by Lattice QCD

however it is restricted in some ideal systems
such as thermal equilibrium, finite volume, etc.

⇓

- Lattice QCD can provide some basic parameters & parts
for phenomenological models

e.g. EOS, T_c , Order of phase trans. etc.

Subject of this talk

- In the QGP phase, heavy quarkonium states exist or not ?
- If it exists, what temperature does it dissolve at ?

where we assume the following system

- thermal equilibrium
- homogeneous in finite volume with periodic b.c.
- zero baryon chemical potential

Of course the situation is rather different from
that of actual experiments

One of the most important parts
of "J/ψ suppression" as a phenomenological model

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- I. Spectral function (SPF) of charmonium at T>0
  - How to calculate SPFs on lattice
  - Maximum Entropy Method
  - Numerical results
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  - Numerical results
- Summary

I : SPFs of charmonium at T>0

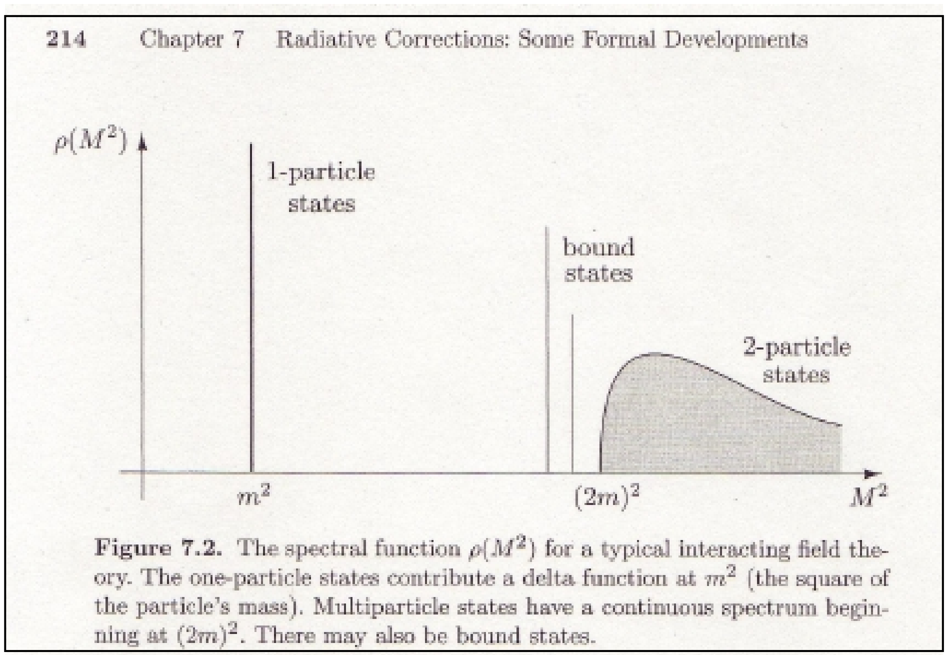
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Figure 7.2. The spectral function $\rho(M^2)$ for a typical interacting field theory. The one-particle states contribute a delta function at m^2 (the square of the particle's mass). Multiparticle states have a continuous spectrum beginning at $(2m)^2$. There may also be bound states.

from "An Introduction to Quantum Field Theory"
Michael E. Peskin, Perseus books (1995)

How to calculate the SPFs on lattice

Lattice QCD can calculate correlation functions
of operators with specific quantum numbers

Thermal hadron correlation function $C_H(\tau, T) = \sum_{\vec{r}} \langle J_H(\tau, \vec{r}) J_H^{\dagger}(\tau, \vec{0}) \rangle$

$$C_H(\tau, T) = \int_0^{\infty} d\omega \; \sigma_H(\omega, T) \; \frac{\cosh(\omega(\tau - \frac{1}{2T}))}{\sinh(\frac{\omega}{2T})}$$

on the lattice

$$\tau = n_\tau a, \qquad \omega = n_\omega \Delta\omega$$

We can get the SPFs σ(w,T) from the correlation function $C_H(\tau, T)$
This is so-called “Inverse problem”
usually, brute force χ^2 analysis fails (ill-posed problem)

Furthermore the situation becomes worse as Temp. increases
because finite temperature lattice → $0 \leq \tau \leq 1/T$

Maximum Entropy Method

MEM (based on Bayes' theorem)

M.Asakawa et al. Prog.Part.Nucl.Phys.46(2001) 459

searches for the most probable shape of SPFs

by maximization of $Q = \alpha S - L$

L : Likelihood function (χ^2 term)

$$S = \int d\omega \left[\sigma(\omega) - m(\omega) - \sigma(\omega) \ln \frac{\sigma(\omega)}{m(\omega)} \right]$$

m(w) : default model func.

α : parameter → integrated out with a prior prob.

- result depend on the default model function
- at high temperature

  it is difficult to check the reliability even if MEM is used.

  melting of bound state? simple failure of MEM?

Papers for charmonium SPFs at T>0

- Umeda, Matsufuru and Nomura (quenched QCD)

  T.Umeda et al, Eur.Phys.J.C37S1 (2004) 9.(hep-lat/0211003)

- Bielefeld group (quenched QCD)

  S.Datta et al., Phys.Rev.D69(2004)094507.

- Asakawa and Hatsuda (quenched QCD)

  M.Asakawa and T.Hatsuda, Phys. Rev. Lett. 92 (2004) 012001

- Trin-lat group (2-flavors QCD)

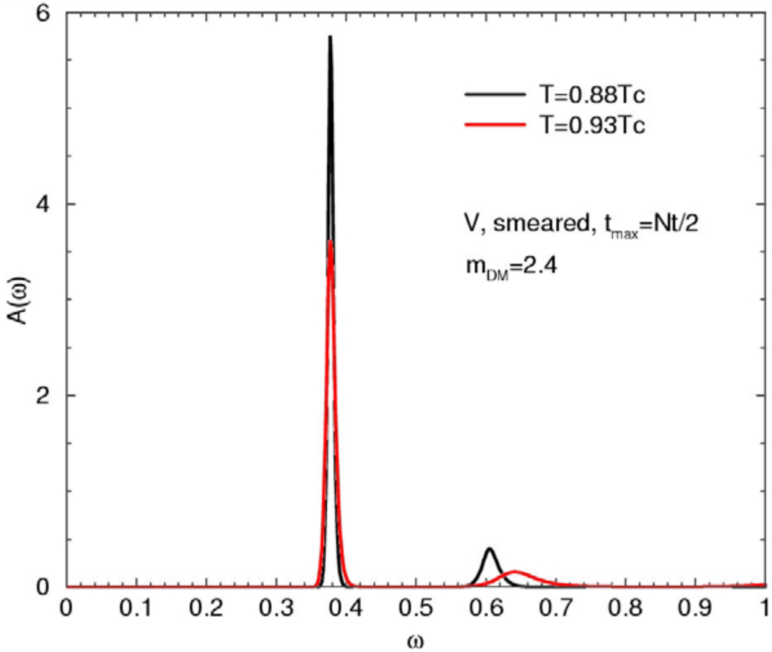
  R.Morrin et al., hep-lat/0509115 (Lattice'05)

- others

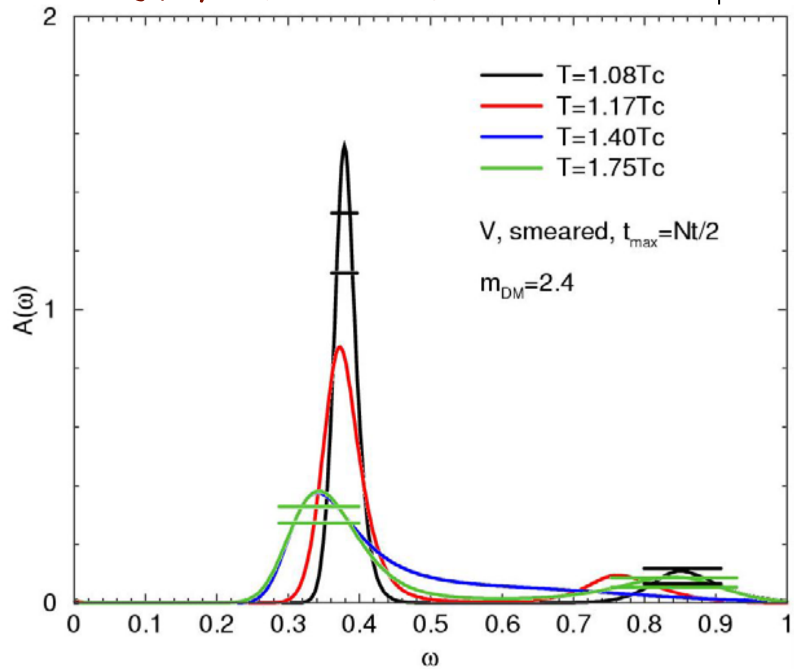
All study supports an existence of hadronic mode just above T_c

Results by T.Umeda et al.

J/ψ below Tc



J/ψ above Tc



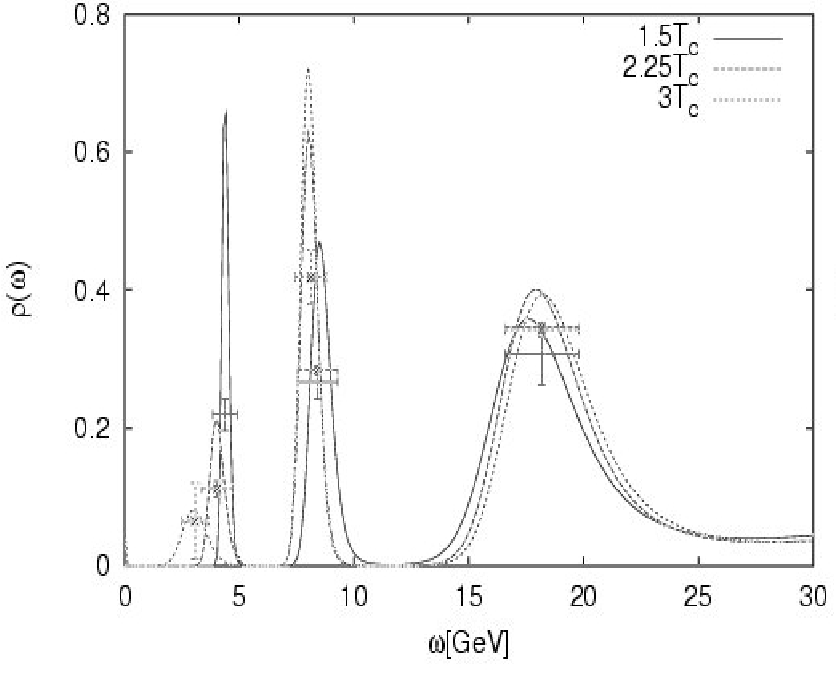
T<Tc : no mass shift, almost same SPF as T=0

T>Tc : peak structure survives up to 1.2Tc (?)

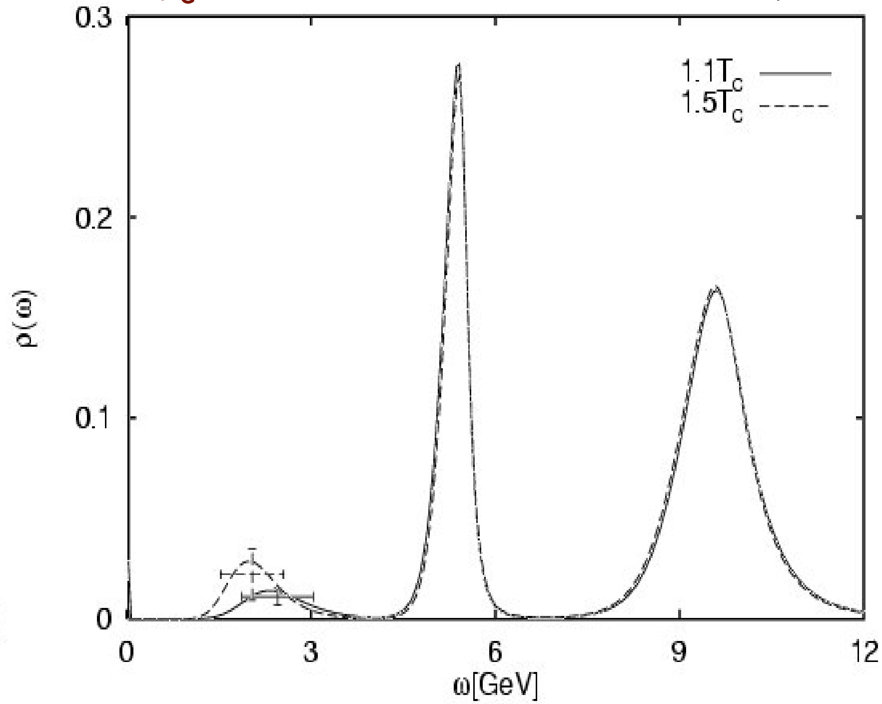
η_c : up to 1.4Tc, J/ψ : up to 1.2Tc (at least)

Bielefeld group results

J/ψ above Tc



χ_c above Tc

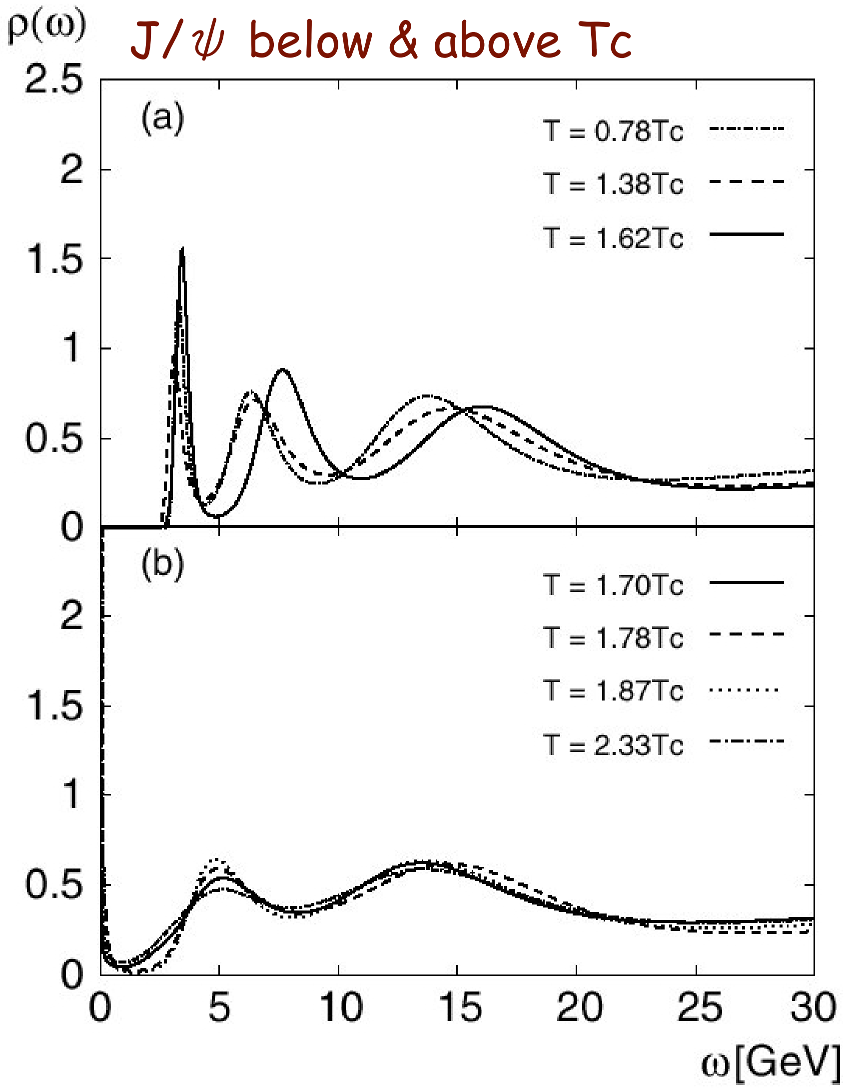


- J/ψ , η_c states survive up to 2.25T_c
- χ_c state may dissolve just above T_c

(in pp collisions, about 40% of J/ψ production comes from decay of the excited states χ_c and ψ')

Hatsuda & Asakawa result

J/ψ below & above Tc



SPF has peak at
the same place as T=0
upto 1.6Tc
for J/ψ & η_c channels

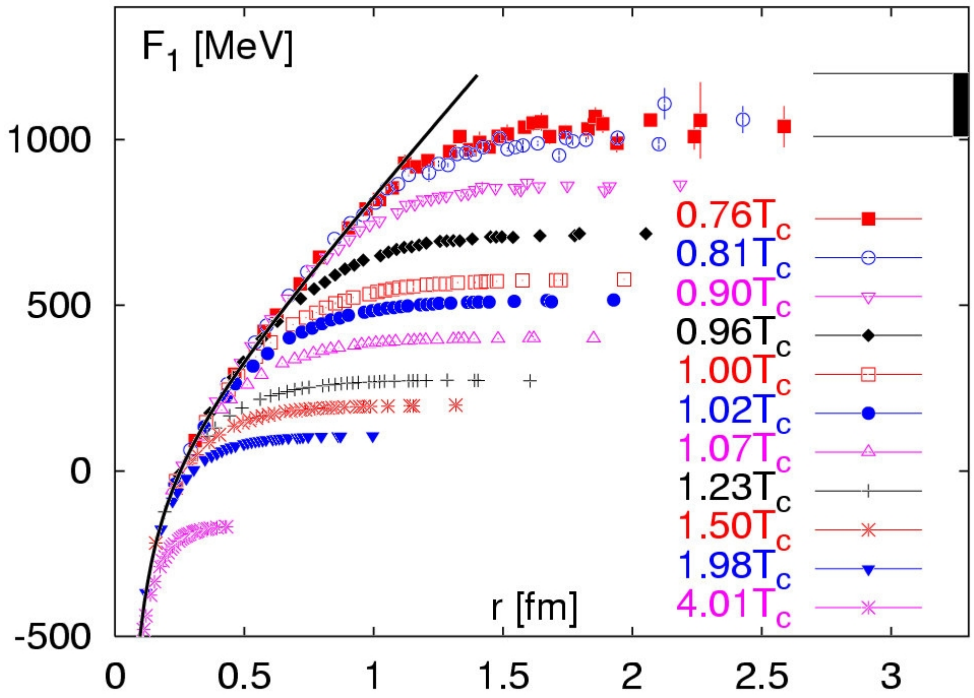
Comment for these results

- From these MEM results,
  - J/ψ , η_c states could survive even in QGP phase
  - dissociation temp. may be 1.4~2 T_c (large uncertainty)
  - χ_c state may disappear just above T_c

- many problems remain...
  - most studies does not include dynamical quark effects
  - reliability of the results at high temperature

At present we have only qualitative results !!

II : Potential model analysis



Color singlet free energy in 2-flavors QCD
from Phys.Rev.D71(2005) ,O.Kaczmarek and F.Zantow.

Digression:

First paper on the J/ψ suppression

VOLUME 57, NUMBER 17 PHYSICAL REVIEW LETTERS 27 OCTOBER 1986

Mass Shift of Charmonium near Deconfining Temperature and Possible Detection in Lepton-Pair Production

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photo: Prof. Osamu Miyamura

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in Monte Carlo analyses.[7,8] A related question is whether charmoniumlike clusters may still exist in a quark-gluon plasma. We have made tentative calculations by screened Coulombic potential and found that possibility small. Thus, contribution to lepton pair in the J/ψ mass region from the deconfinement phase would be mainly thermal quark-antiquark annihilation.[18] In connection with this point, we make a com-

lin, 1985), p. 1.
[4]R. D. Pisarski, Phys. Lett. **110B**, 155 (1982).
[5]R. D. Pisarski and F. Wilczek, Phys. Rev. D **29**, 338 (1984).
[6]L. McLerran and B. Svetitsky, Phys. Rev. D **24**, 450 (1981).
[7]M. Fukugita, T. Kaneko, and A. Ukawa, Phys. Lett. **154B**, 185 (1985).
[8]C. Dorninger, H. Leeb, and H. Markum, Z. Phys. C 29

to return to the subject...

Potential model analysis

Original idea for "J/ψ suppression" is based on
Schrödinger eq. with
the free energy of static $Q\bar{Q}$ system on the lattice.

At present,
we have results of static quark free energy in full QCD.
e.g. O.Kaczmarek and F.Zantow, Phys.Rev.D71(2005).
These are quantitative results !!
potential model analysis is still useful for this problem.

"Early potential model predictions" contract recent MEM studies
→ for charmonium state as a (color singlet) hadronic mode
color averaged free energy is used as an eff. potential
(weighted average of
color singlet & octet contributions)

$V_{eff}(r)$ for heavy quarkonium at T>0

Free energy F ? Internal energy U ?

thermodynamical relation : F = U - TS

Which definition should be used as the potential ?
→ still an open problem

In the paper(*) by Shuryak and Zahed :

"the answer should depend on
the relation between the time scales involved."

t_{bound} : time associated with the bound state, e.g. $\langle \dot{r}/r \rangle^{-1}$

t_{heat} : time needed to transfer heat to matter

$t_{bound} \gg t_{heat}$ (isothermal) → Free energy

$t_{bound} \ll t_{heat}$ (adiabatic) → Internal energy

(*) *Phys.Rev.D70(2004)054507.*

Results: model dependence

- free energy : S.Digal et al. Phys.Rev.D64(2001)094015
- linear comb. of both: W.M.Alberico et al. hep-ph/0507084
- internal energy: C.Y.Wong hep-ph/0509088

state	J/ψ	χ_c	ψ'	Υ	χ_b	Υ'	χ_b'	Υ''
$E_s^i[GeV]$	0.64	0.20	0.005	1.10	0.67	0.54	0.31	0.20
T_d/T_c	1.1	0.74	0.1-0.2	2.31	1.13	1.1	0.83	0.75
T_d/T_c	~ 1.42	~ 1.05	unbound	~ 3.3	~ 1.22	~ 1.18	-	-
T_d/T_c	1.78-1.92	1.14-1.15	1.11-1.12	$\gtrsim 4.4$	1.60-1.65	1.4-1.5	~ 1.2	~ 1.2

Table 1: Estimated dissociation temperatures T_d in units of T_c obtained from potential models using free energies [8] (green), a linear combination of F_1 and U_1 [10] (blue) and internal energies [9] (red) as effective T-dependent potentials.

J/ψ : T_d is about $1.1T_c \sim 2\ T_c$
χ_c : T_d is less than $1.2T_c$

Summary of this talk

- From recent MEM results,
  J/ψ , η_c states could survive even in QGP phase
  dissociation temp. may be 1.4~2 T_c (large uncertainty)
  χ_c state may disappear just above T_c
- Recent potential model studies
  give similar result with MEM (even large uncertainty !!)

Next step

- Improvement of MEM analysis
  & dynamical quark effects
- Suitable effective potential for this problem