非等方格子上での　クォーク作用の非摂動繰り込み

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２００５年3月２７日
日本物理学会（東京理科大学野田キャンパス）

Motivation

Hadronic matrix elements with high precision

- Relativistic quark
- Fermilab approach, AKT
- NRQCD
- Nonperturbtive HQET (Alpha collab.)

Our motivation:
study the heavy quark action
on anisotropic lattice

Why anisotropic lattice ?

Anisotropic lattice: $a_\tau < a_\sigma$

(anisotropy: $\xi = a_\sigma / a_\tau$)

Our expectation:

for $m_Q a_\tau \ll 1$,(not necessarily $m_Q a_\sigma \ll 1$)
mass dep. of parameters in the action
are so small.

Nonperturbative renormalization techniqe
(based on the PCAC relation) is possible.

Parameters we have to tune

Quark action on the anisotropic lattice:

$$S_F = \sum_{x,y} \bar{\psi}(x) K(x,y) \psi(y)$$

$$K(x,y) = \delta_{x,y} - \kappa_\tau \left[(1-\gamma_4)U_4(x)\delta_{x+\hat{4},y} + (1+\gamma_4)U_4^\dagger(x-\hat{4})\delta_{x-\hat{4},y}\right]$$
$$-\kappa_\sigma \sum_i \left[(r-\gamma_i)U_i(x)\delta_{x+\hat{i},y} + (r+\gamma_i)U_i^\dagger(x-\hat{i})\delta_{x-\hat{i},y}\right]$$
$$-\kappa_\sigma c_E \sum_i \sigma_{4i} F_{4i}(x)\delta_{x,y} + r\kappa_\sigma c_B \sum_{i>j} \sigma_{ij} F_{ij}(x)\delta_{x,y}$$

($\gamma_F \equiv \kappa_\sigma / \kappa_\tau$, c_E , c_B) should be determined.

$r = 1/\xi$ is to control the O(ma) error.

Is mass dependence really small ?

- One-loop calc. for Z_A , Z_V

  Harada et al. Phys. Rev. D64 (2001) 074501

- Nonperturbative calc. for γ_F

  Matsufuru et al., Phys. Rev. D64(2001)114503

  Harada et al., Phys. Rev. D66(2002)014509

- Application to Heavy-light decay constant

  Matsufuru et al., hep-lat/0209090

Encouraging result for fully O(a) improvement !

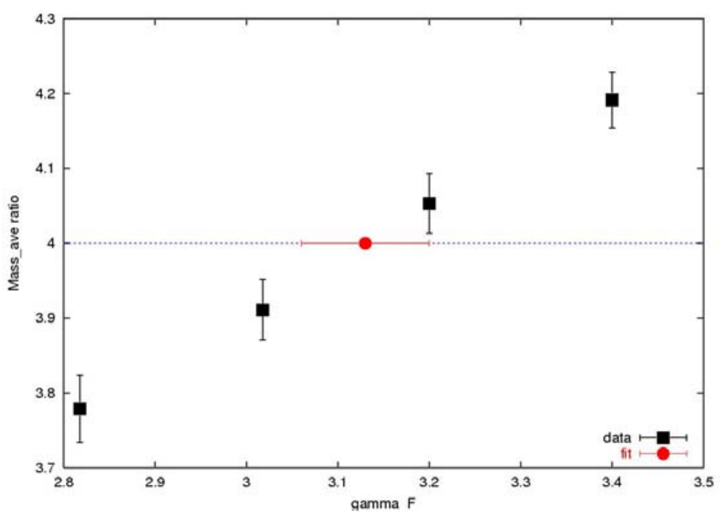
Fully nonperturb. O(a) improvement

We have to tune (γ_F, c_E, c_B) simultaneously.

tuning → γ_F; fixed → c_E, c_B

Using Physical isotropic condition

(spatial meson mass) = (temporal meson mass)



γ_F vs $M_\sigma a_\sigma / M_\tau a_\tau$ (=4)

with tree-level c_E & c_B

Well determined !

Fully nonperturb. O(a) improvement

tuning

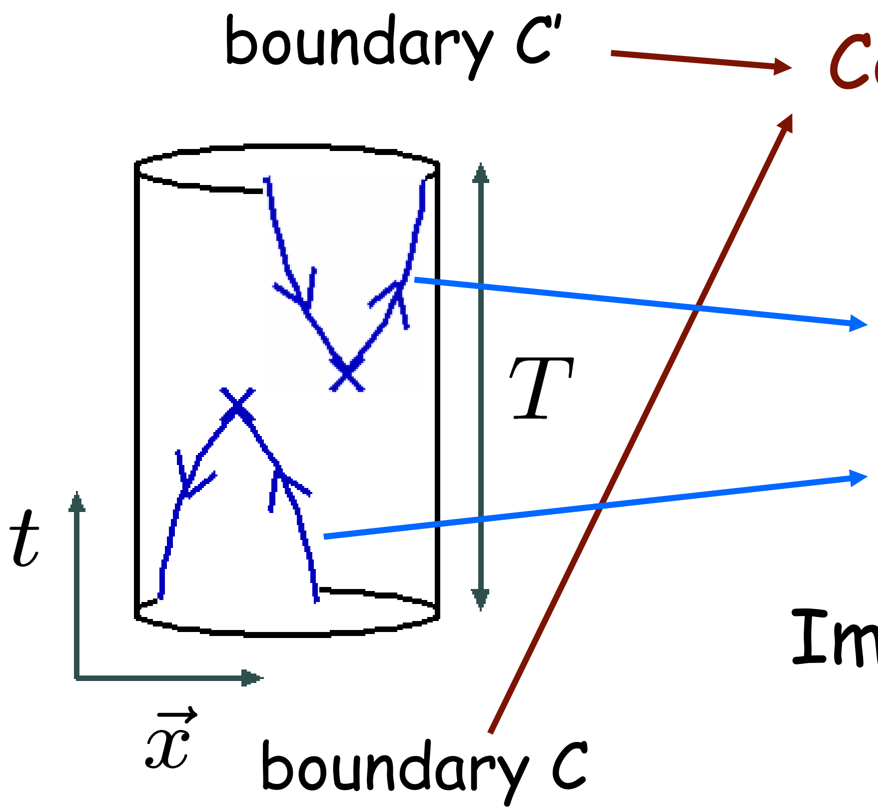
We have to tune (γ_F, c_E, c_B) simultaneously.

fixed

Schrödinger Functional method (PCAC relation)

Fully nonperturb. O(a) improvement

Schrödinger Functional method (PCAC relation)



Constant Chromo-Electric background field

$$\partial_\mu A'^a_\mu = 2m' P'^a$$

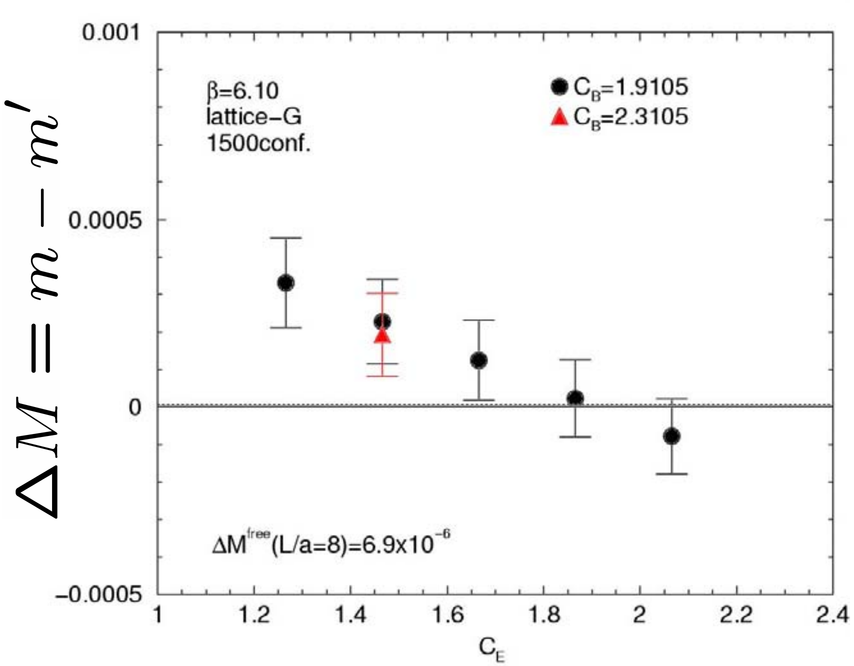
$$\partial_\mu A^a_\mu = 2m P^a$$

Improvement cond. for c_E

$$m = m' + O(a^2)$$

Fully nonperturb. O(a) improvement

Schrödinger Functional method (PCAC relation)



- at high beta consistent with (mean-field) tree-level

  Well determined !

- c_B dependence is weak (only chromo-elect. field is induced)

Fully nonperturb. O(a) improvement

We have to tune (γ_F, c_E, c_B) simultaneously.

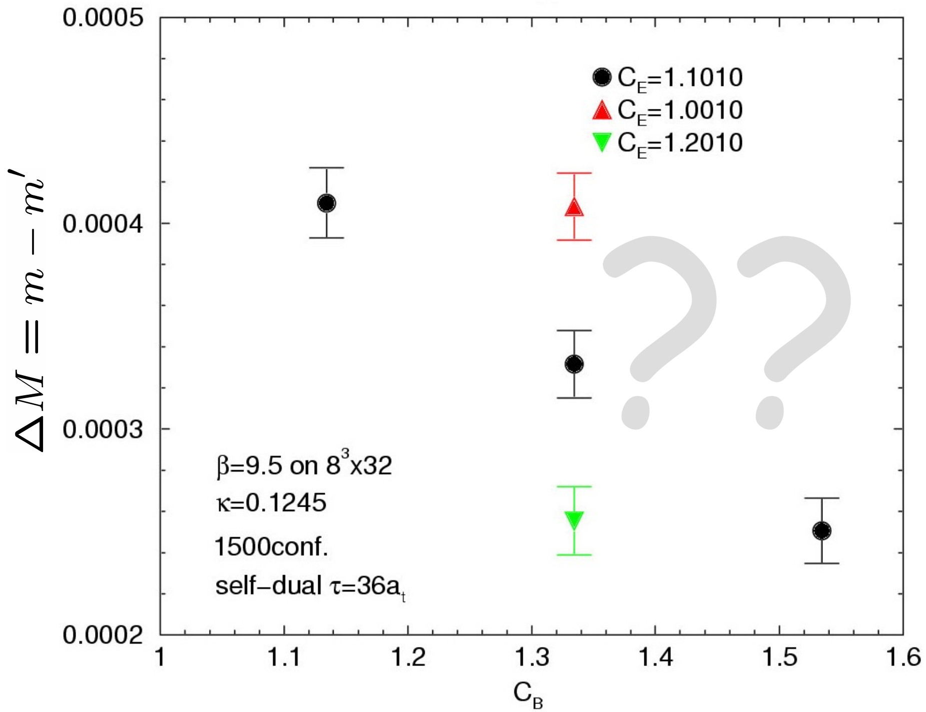
fixed (γ_F, c_E) — tuning (c_B)

(1) Self-dual background field

$$A_k(t,\vec{x}) = b(t) I_k$$

$$[I_j, I_k] = \epsilon_{jkl} I_l$$

$$b(t) = 1/(\tau - t)$$



Fully nonperturb. O(a) improvement

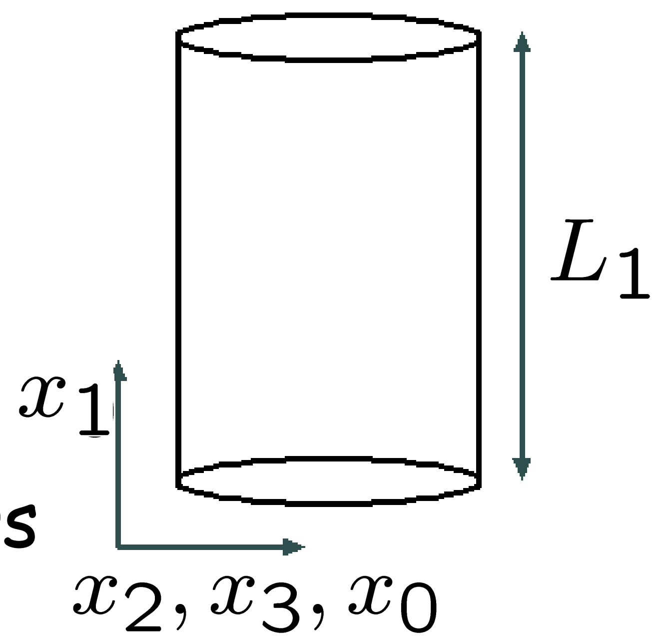
We have to tune (γ_F, c_E, c_B) simultaneously.

fixed (γ_F, c_E) — tuning (c_B)

(2) Constant Chromo-Magnetic background field

$$-\kappa_\sigma(r-\gamma_i)U_i(x)\delta_{x+\hat{i},y}$$

$r \neq 1$ causes oscillating modes from doublers



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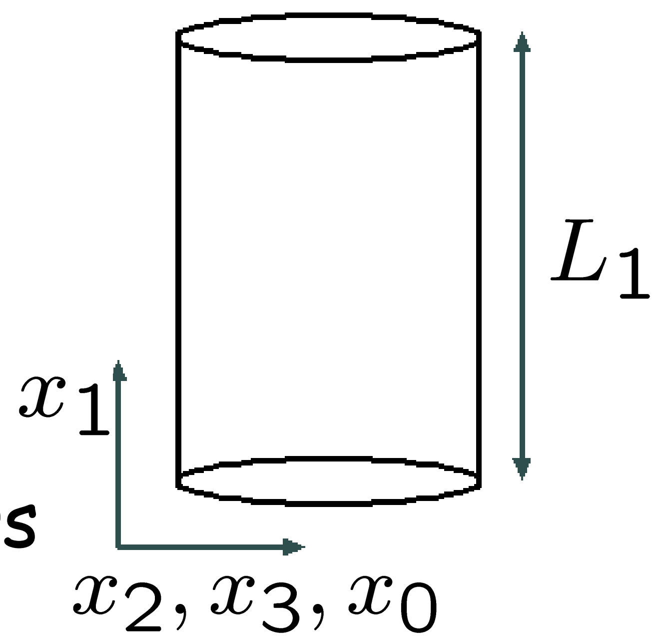


Fully nonperturb. O(a) improvement

(2) Constant Chromo-Magnetic background field

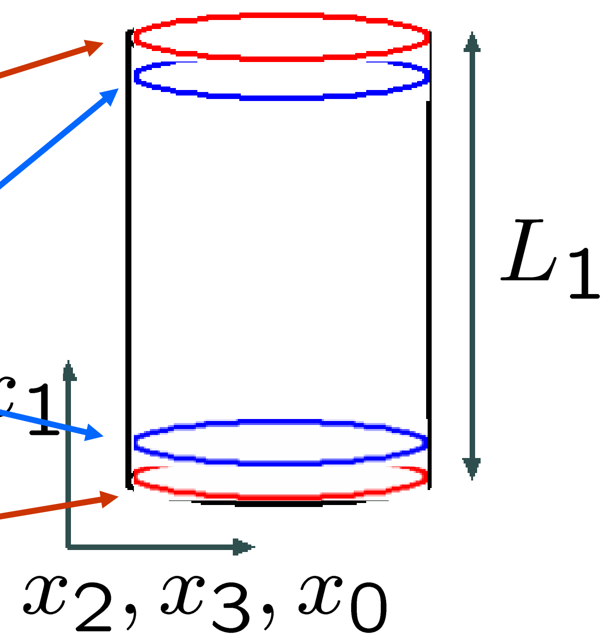
$$P_{\pm} = \frac{1}{2}(1 \pm \gamma_1)$$

(Dirichlet) $P_{-}\psi(x)|_{x_1=L_1} = \rho'$

(Neumann) $a_\sigma D_1 P_{-}\psi(x)|_{x_1=L_1} = 0$

(Neumann) $a_\sigma D_1 P_{+}\psi(x)|_{x_1=0} = 0$

(Dirichlet) $P_{+}\psi(x)|_{x_1=0} = \rho$



Summary

- Anisotropic lattice is efficient
  for heavy quark physics in practical.
- γ_F, c_E can be determined.
- c_B could be determined.

Outlook

- Application to heavy-light matrix elements.
  ➔ High precision comp. actually possible ?
- Extension to dynamical QCD.

Contents

Nonperturbative improvement
for anisotropic lattice quark action

Is mass dependence really small ?

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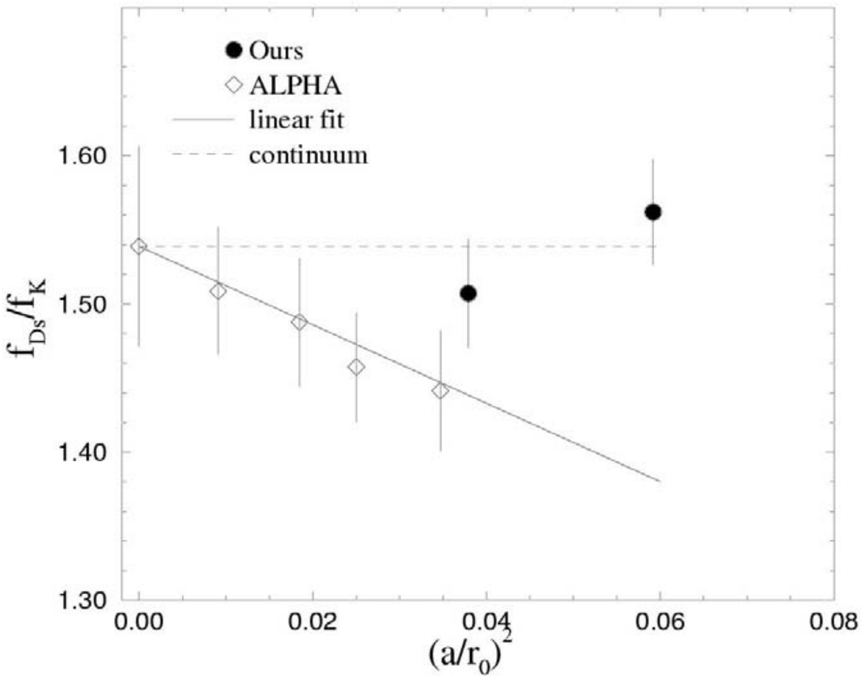
  Matsufuru et al., hep-lat/0209090



Our result
vs
ALPHA's result

Scaling violation is small
↓
$O(m_q a)$ is under control

Encouraging result for fully O(a) improvement !