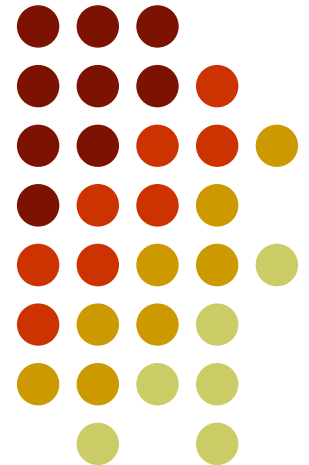


A study of charmonium dissociation temperatures using a finite volume technique in the lattice QCD

T. Umeda and H. Ohno
for the WHOT-QCD Collaboration



Komaba seminar, Tokyo Univ. , 14 Nov. 2007



Contents



- Introduction
 - J/ψ suppression scenarios
 - Spectral functions in Lattice QCD
 - Comments on MEM
- Charmonium in a finite box
 - Spectral function in a finite box
 - Finite volume technique
 - Variational analysis
 - Test with free quarks
- Numerical results
- Summary & outlook

Preliminary results !!

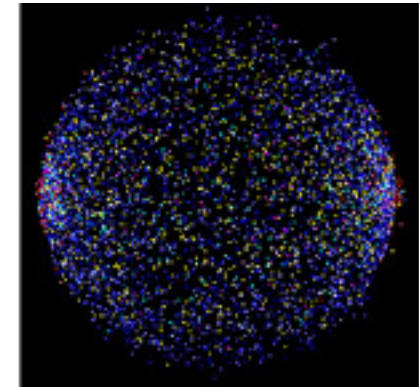
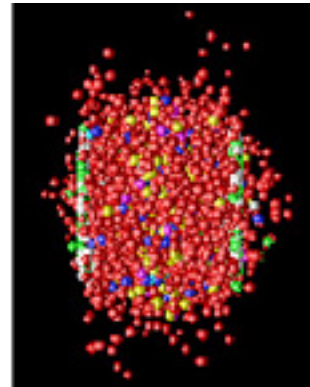
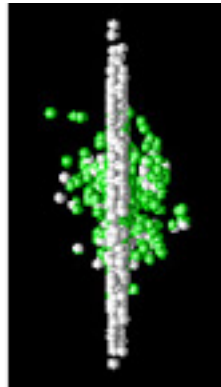
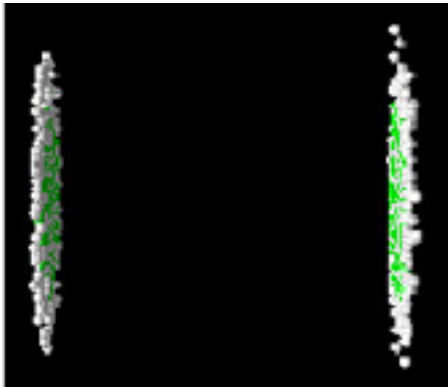


total 30 pages

Experiments



- **SPS** : CERN (– 2005)
Super Proton Synchrotron
- **RHIC**: BNL (2000 –)
Relativistic Heavy Ion Collider
- **LHC** : CERN (2009 –)
Large Hadron Collider





J/ψ suppression in Exp.

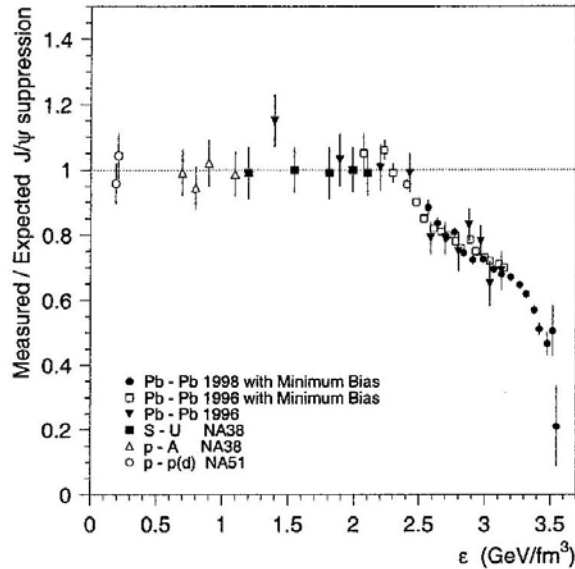


Fig. 7. Measured J/ψ production yields, normalised to the yields expected assuming that the only source of suppression is the ordinary absorption by the nuclear medium. The data is shown as a function of the energy density reached in the several collision systems.

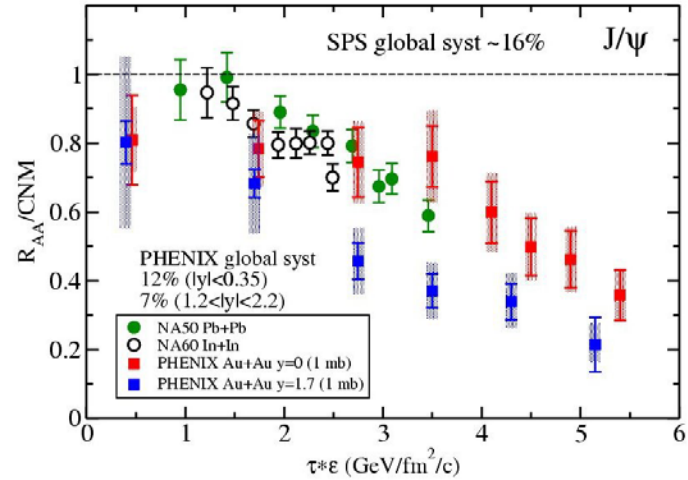
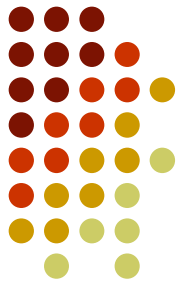


Figure 10: Survival fraction (R_{AA}/CNM) vs energy density comparison of PHENIX Au+Au suppression to that from NA38/50 at CERN.

Phy
NA5

Energy density $0.5-1.5 \text{ GeV}/\text{fm}^3 = 1.0 T_c$
 $10 \text{ GeV}/\text{fm}^3 = 1.5 T_c$
 $30 \text{ GeV}/\text{fm}^3 = 2.0 T_c$

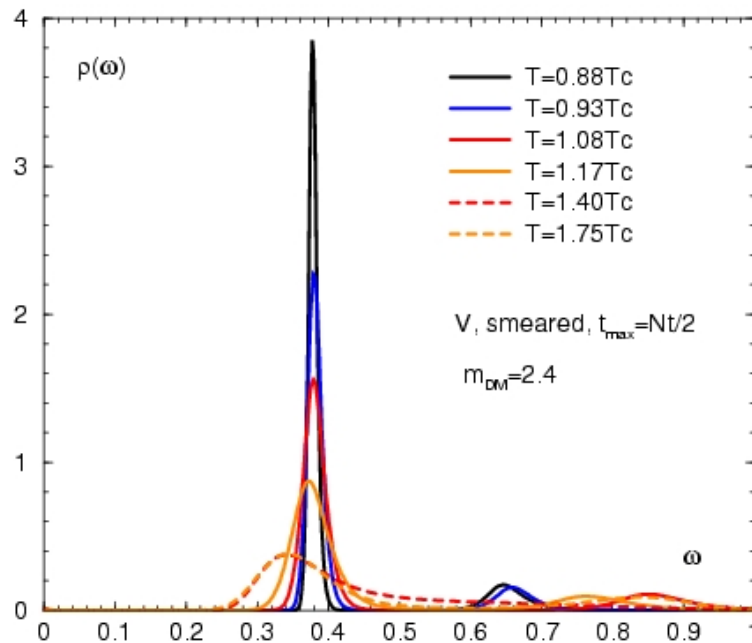
ation



Charmonium spectral function

$$\begin{aligned} C(t) &= \langle O(t)O^\dagger(0) \rangle \\ &= \int d\omega \rho(\omega) \frac{e^{-\omega t} + e^{-\omega(N_t-t)}}{1 - e^{-\omega N_t}} \end{aligned}$$

Maximum entropy method
 $C(t) \rightarrow \rho(\omega)$



Quenched QCD

- *T. Umeda et al., EPJC39S1, 9, (2005).*
- *S. Datta et al., PRD69, 094507, (2004).*
- *T. Hatsuda & M. Asakawa, PRL92, 012001, (2004).*
- *A. Jakovac et al., PRD75, 014506 (2007).*

Full QCD

- *G. Aarts et al., arXiv:0705.2198.*

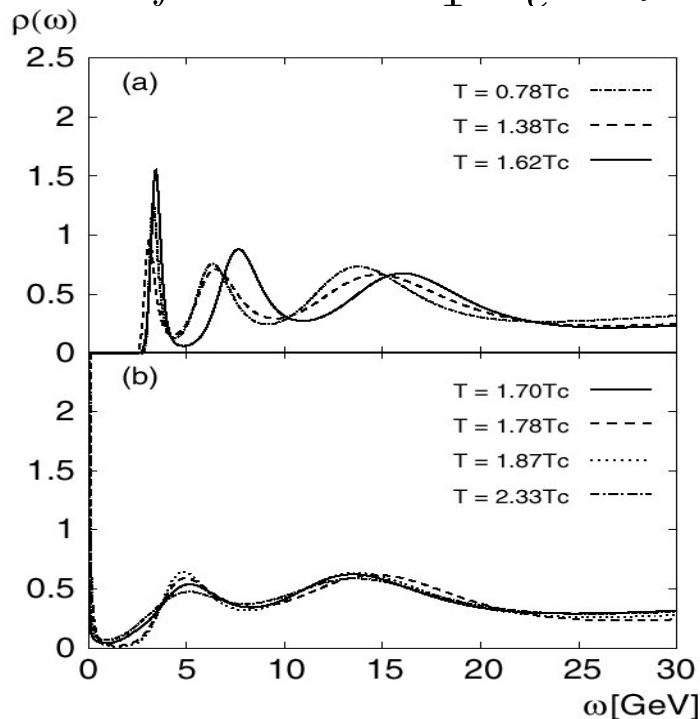


Charmonium spectral function

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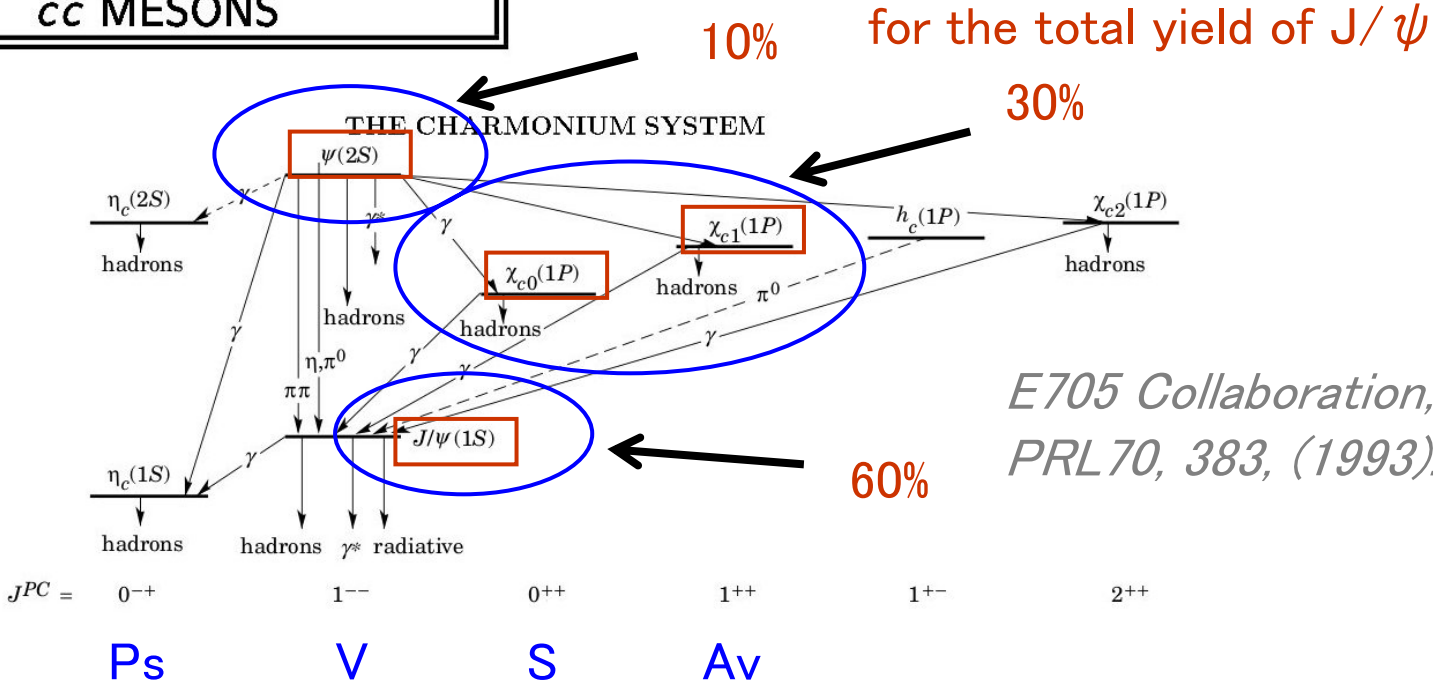
All studies indicate survival of J/ψ state above T_c ($1.5T_c$?)



Sequential J/ψ suppression

Particle Data Group (2006)

$c\bar{c}$ MESONS



E705 Collaboration, PRL 70, 383, (1993).

Dissociation temperatures of J/ψ and ψ' & χ_c are important for QGP phenomenology.

χ_c states dissolve ?

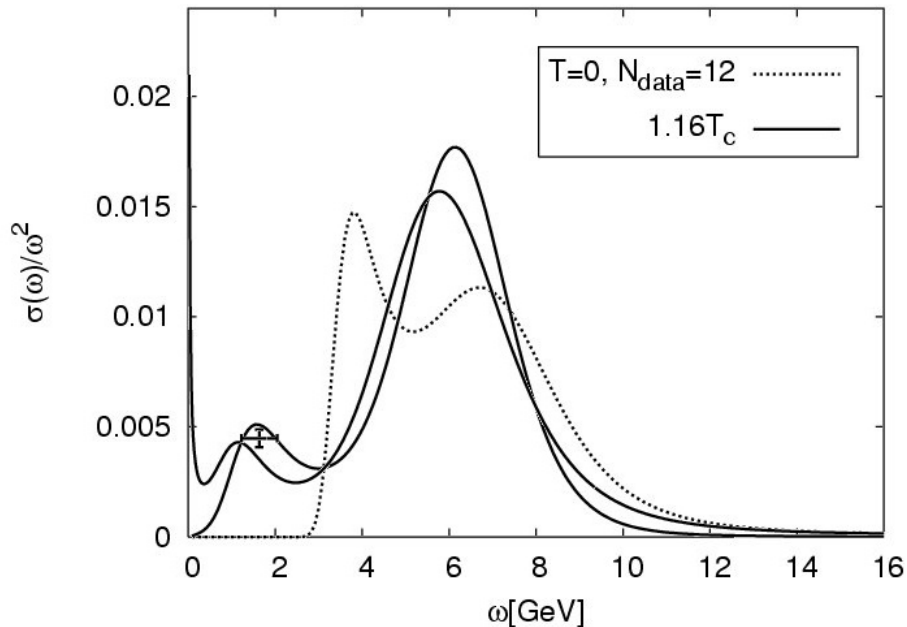


FIG. 19: The scalar spectral function for $\beta = 6.1$ at $T = 1.16T_c$ and at zero temperature reconstructed using $N_{data} = 12$. At finite temperature two default models $m(\omega) = 0.01$ and $m(\omega) = 0.038\omega^2$ have been used.

A.Jakovac et al., PRD75,014506)2007.

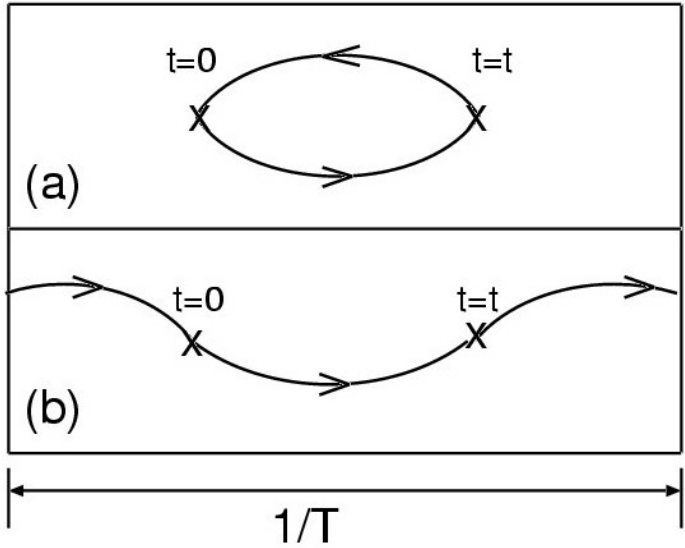
charmonium spectral func.
for P-wave states
 $\rightarrow \chi_c$ states

*S. Datta et al.,
PRD69, 094507 (2004).
A.Jakovac et al.,
PRD75, 014506 (2007).
G.Aarts et al.,
arXiv:0705.2198*

- the results of SPFs for P-states are not so reliable.
e.g. large default model dep.
- the drastic change just above T_c is reliable results.



Constant mode



$$\exp(-m_q t) \times \exp(-m_q t) = \exp(-2m_q t)$$

m_q is quark mass
or single quark energy

$$\exp(-m_q t) \times \exp(-m_q(L_t - t)) = \exp(-m_q L_t)$$

$L_t =$ temporal extent

Imaginary time formalism
 $L_t = 1/\text{Temp.}$
 gauge field : periodic b.c.
 quark field : anti-periodic b.c.

- confined phase ($T < T_c$)
 - $m_q = \infty$
 - the constant mode disappears
- deconfined phase ($T > T_c$)
 - $m_q =$ finite
 - the constant mode appears

Effective mass (local mass)



Definition of effective mass

$$\begin{aligned} C(t) &= A_0 e^{-m_0 t} + A_1 e^{-m_1 t} + \dots \\ &\equiv A e^{-m_{eff}(t)t} \end{aligned}$$

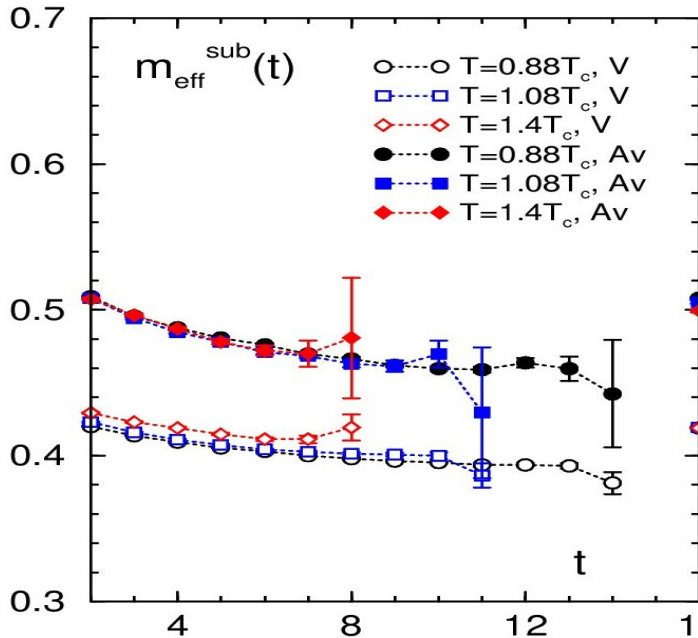
$$\frac{C(t)}{C(t+1)} = e^{-m_{eff}(t)}$$

$$m_{eff}(t) \rightarrow m_0 \quad \text{when} \quad (m_1 - m_0)t \gg 1$$

In the (anti) periodic b.c.

$$\frac{C(t)}{C(t+1)} = \frac{\cosh[m_{eff}(t)(N_t/2 - t)]}{\cosh[m_{eff}(t)(N_t/2 - t - 1)]}$$

χ_c states dissolve ?



Midpoint subtracted correlator

$$\bar{C}(t) = C(t) - C(N_t/2)$$

$$\frac{C(t)}{C(t+1)} = \frac{\cosh[m_{eff}(t)(N_t/2 - t)]}{\cosh[m_{eff}(t)(N_t/2 - t - 1)]}$$



$$\frac{\bar{C}(t)}{\bar{C}(t+1)} = \frac{\sinh^2 \left[\frac{1}{2} m_{eff}^{sub}(t) (N_t/2 - t) \right]}{\sinh^2 \left[\frac{1}{2} m_{eff}^{sub}(t) (N_t/2 - t - 1) \right]}$$

Small changes even in P-states (apart from the const. mode)



Comments on MEM

- MEM is a kind of constrained χ^2 fitting
of fit parameters \leq # of data points
- Prior knowledge of SPFs is needed
as a default model function, $m(\omega)$
The default model function
plays crucial roles in MEM

MEM maximizes $Q = \alpha S - L$

L : Likelihood function (χ^2 term)

α : parameter (to be integrated out)

$$S = \int d\omega \left[\rho(\omega) - m(\omega) - \rho(\omega) \ln \frac{\rho(\omega)}{m(\omega)} \right]$$

default model func.

Fit form in MEM



Singular Value Decomposition (SVD)

$$\begin{aligned} K(\tau, \omega) &= e^{-\omega\tau} + e^{-\omega(T-\tau)} \\ &= V(\tau, \tau') w(\tau', \tau'') U(\omega, \tau'')^t \end{aligned}$$

↖ diagonal matrix

fit form for spectral function : $\rho(\omega)$

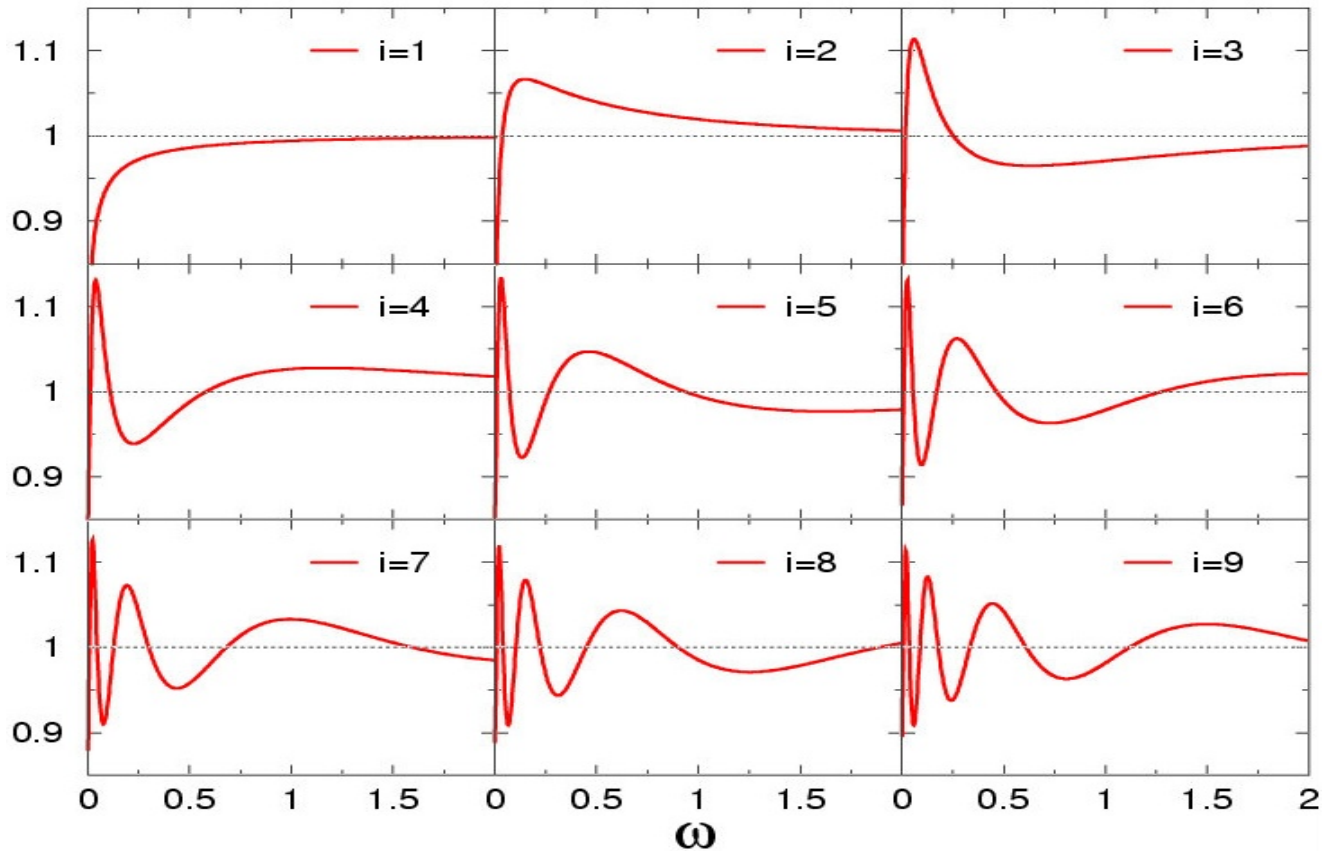
$$\rho(\omega) = m_0 \omega^2 \prod_{i=1}^N \exp \{ b_i u_i(\omega) \}$$

$u_i(\omega) (= U(\omega, \tau_i))$: basis in singular space

b_i : fit parameters

Fit form in MEM

$$\rho(\omega) = m_0 \omega^2 \prod_{i=1}^N \exp \{b_i u_i(\omega)\} \exp \{u_i(\omega)\}$$



Goal of our study

Dissociation temperatures for J/ψ , χ_c , $\psi(2S)$, ...
from Lattice QCD calculations

Comments on our Lattice QCD calculations

- Thermal equilibrium system
- Charmonium with zero total momentum
- Quenched approximation
 - $T_c \sim 260 \text{ MeV}$ ($\sim 190 \text{ MeV}$ in full QCD)
 - 1st order transition (cross over? in full QCD)
 - Static free energy vs T/T_c
 - $\sim 10\%$ deviation in hadron spectra
- Homogeneous in finite volume



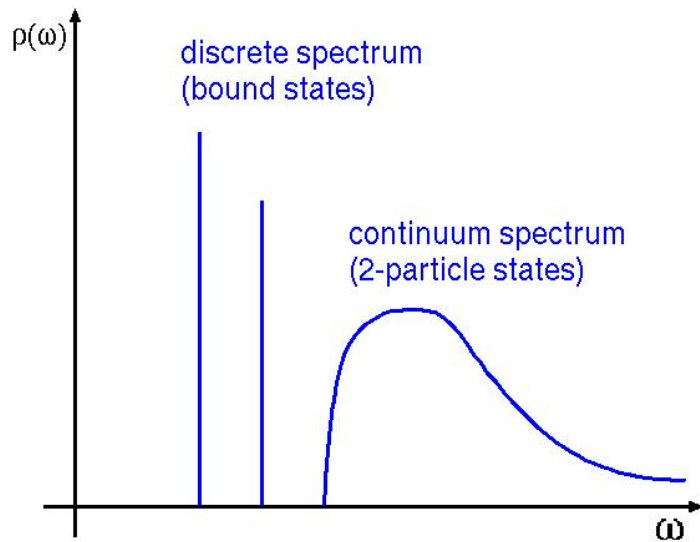


Spectral functions in finite volume

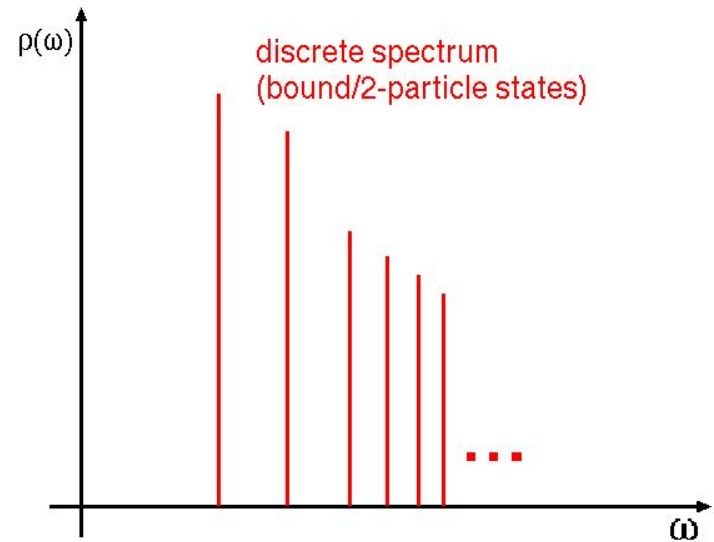
In a finite box of $(La)^3$, momenta are discretized

$$p/a = 2n\pi/L \quad (n = 0, 1, 2, \dots) \quad \text{for the periodic BC}$$

infinite volume case



finite volume case



bound state or scattering state ?



(1) volume dependence of the discrete spectra

bound states → small volume dependence

scattering state → volume dependence from discretized momenta

(2) wave function

bound states → localized shape

scattering states → broad shape

(3) boundary condition dependence

bound states → insensitive to B.C.

scattering states → sensitive to B.C.

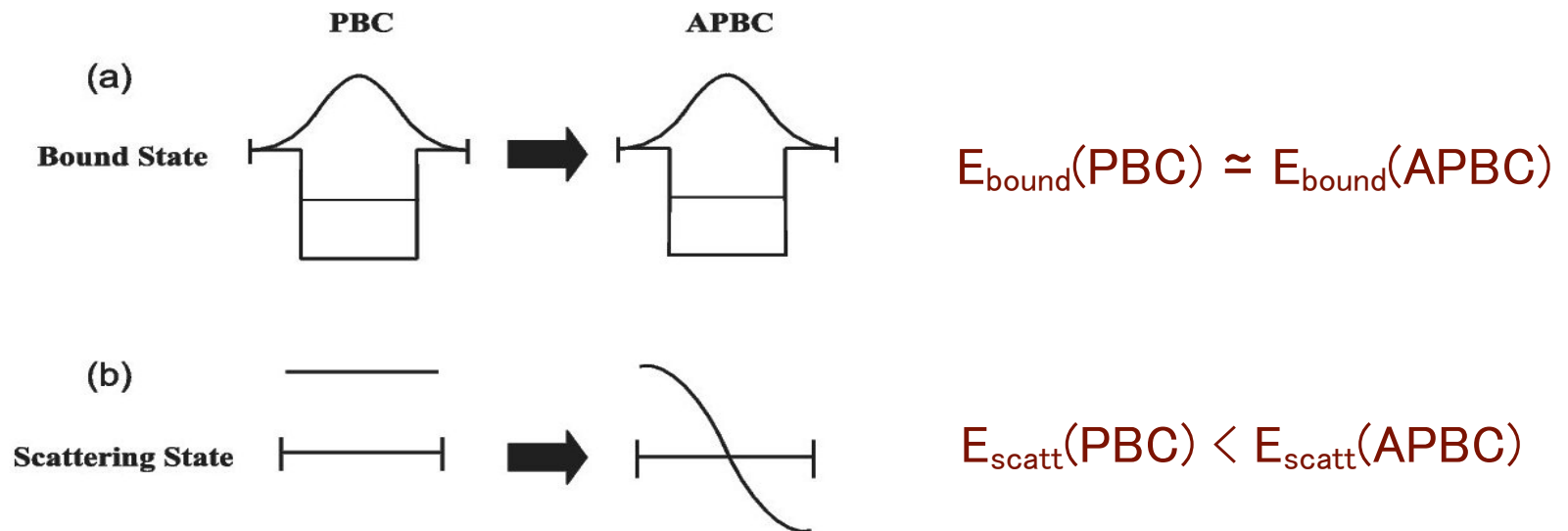
finite volume techniques



Spectrum on different (spatial) boundary conditions

Periodic B.C. $\rightarrow p/a = 2n\pi/L$ ($n = 0, 1, 2, \dots$)

Anti-periodic B.C. $\rightarrow p/a = (2n + 1)\pi/L$



H. Iida et al., PRD74, 074502 (2006).

Variational analysis

We prepare N operators for charmonia

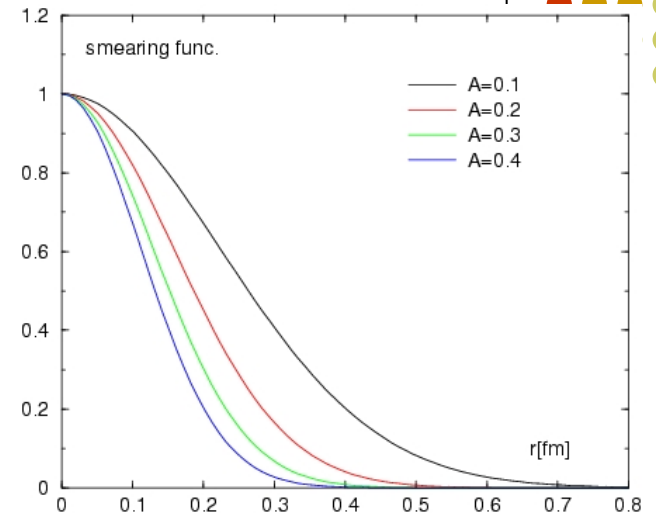
$$O_i^\Gamma(t) = \sum_{\vec{x}\vec{y}} \phi_i(\vec{y}) \bar{q}(\vec{x} - \vec{y}, t) \Gamma q(\vec{x}, t)$$
$$\phi_i(\vec{y}) \propto \exp(-A_i |\vec{x}|^2)$$

and calculate the correlator matrix

$$C_{ij}(t) = \langle O_i(t) O_j^\dagger(0) \rangle$$
$$= \sum_n V_{in} \lambda_n(t) V_{nj}^\dagger$$

$$\lambda_n(t) \propto \cosh\left(E_n\left(t - \frac{Nt}{2}\right)\right)$$

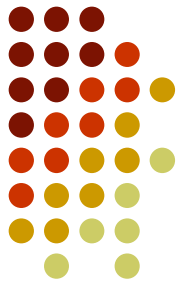
$$C(t)C(t_0)^{-1} = V\lambda(t - t_0)V^{-1}$$



Numerical results



- Test with free quarks
 - Wilson quarks in $20^3 \times 32$ lattice
 - bare quark mass is tuned as
$$2 * am_q \approx m_{\text{charmonium}}/8\text{GeV}$$
- Quenched QCD results

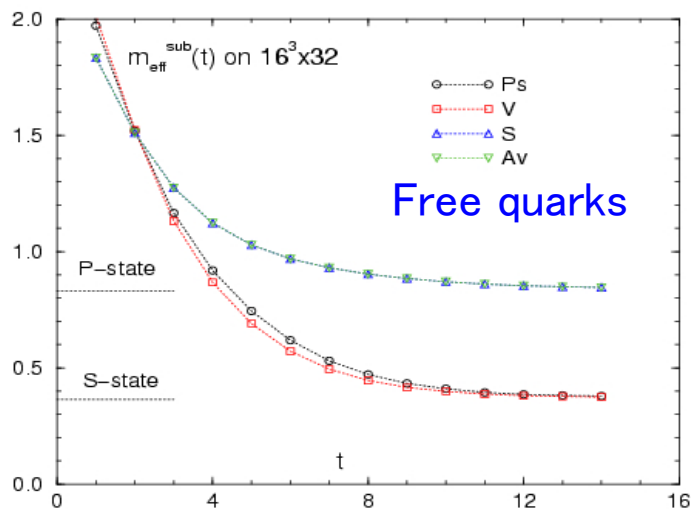
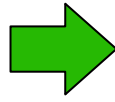
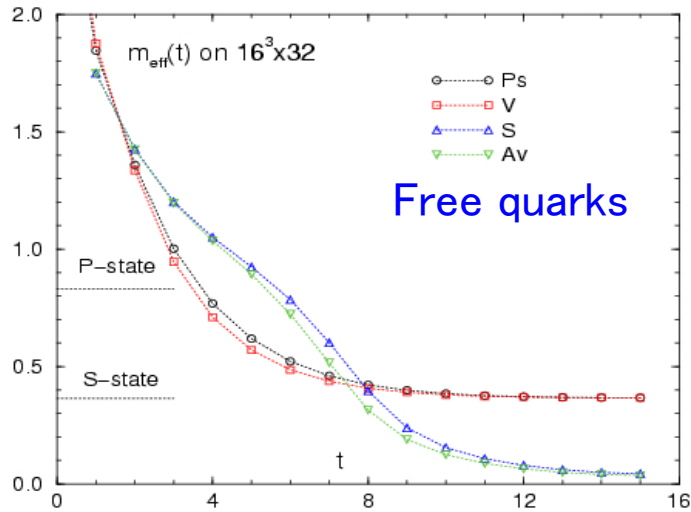


Midpoint subtraction

An analysis to avoid the constant mode

Midpoint subtracted correlator

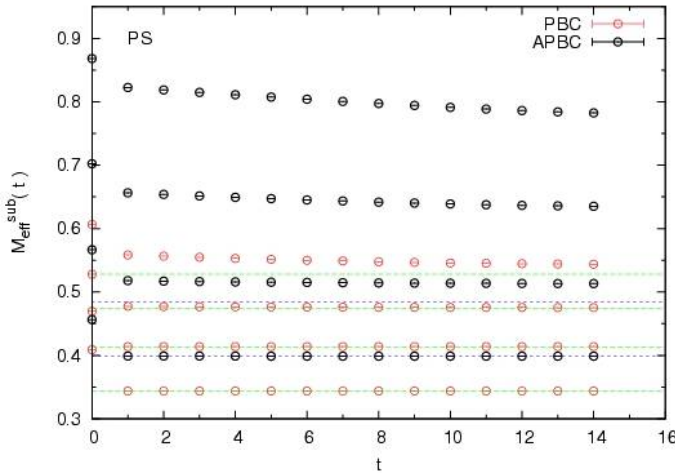
$$\bar{C}(t) = C(t) - C(N_t/2) \quad \rightarrow \quad \frac{\bar{C}(t)}{\bar{C}(t+1)} = \frac{\sinh^2 \left[\frac{1}{2} m_{eff}^{sub}(t) (N_t/2 - t) \right]}{\sinh^2 \left[\frac{1}{2} m_{eff}^{sub}(t) (N_t/2 - t - 1) \right]}$$



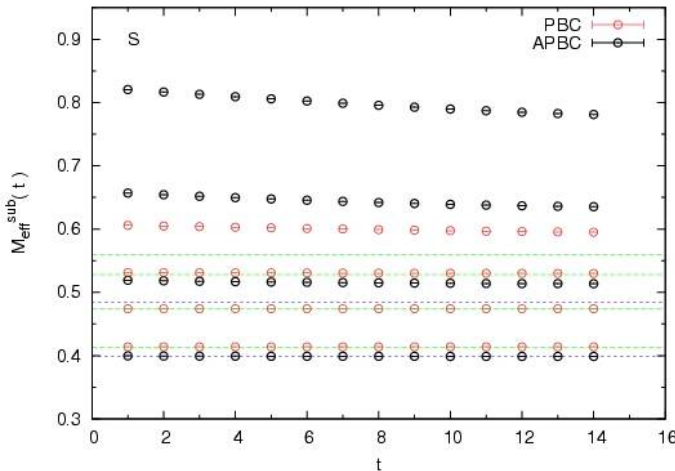


Test with free quarks

S-wave states



P-wave states



$$E(PBC) = 2\sqrt{m_q^2 + \left(\frac{2\vec{n}\pi}{L}\right)^2}$$

$$E(APBC) = 2\sqrt{m_q^2 + \left(\frac{(2\vec{n} + 1)\pi}{L}\right)^2}$$

S-wave :

PBC	1	APBC	2
$\vec{p}(1S) = \frac{\pi}{L}(0, 0, 0)$		$\frac{\pi}{L}(1, 1, 1)$	
$\vec{p}(2S) = \frac{\pi}{L}(2, 0, 0)$	3	$\frac{\pi}{L}(3, 1, 1)$	4

P-wave :

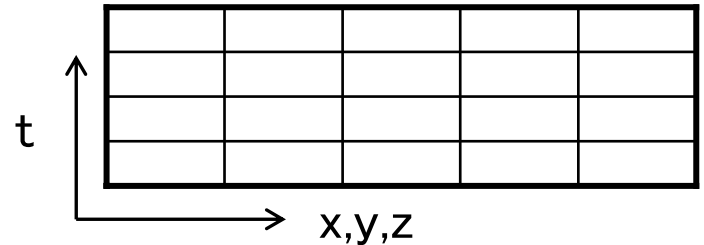
PBC	2	APBC	1
$\vec{p}(1P) = \frac{\pi}{L}(2, 0, 0)$		$\frac{\pi}{L}(-1, 1, 1)$	
$\vec{p}(2P) = \frac{\pi}{L}(2, 2, 0)$	3	$\frac{\pi}{L}(3, 1, 1)$	4

Lattice QCD results



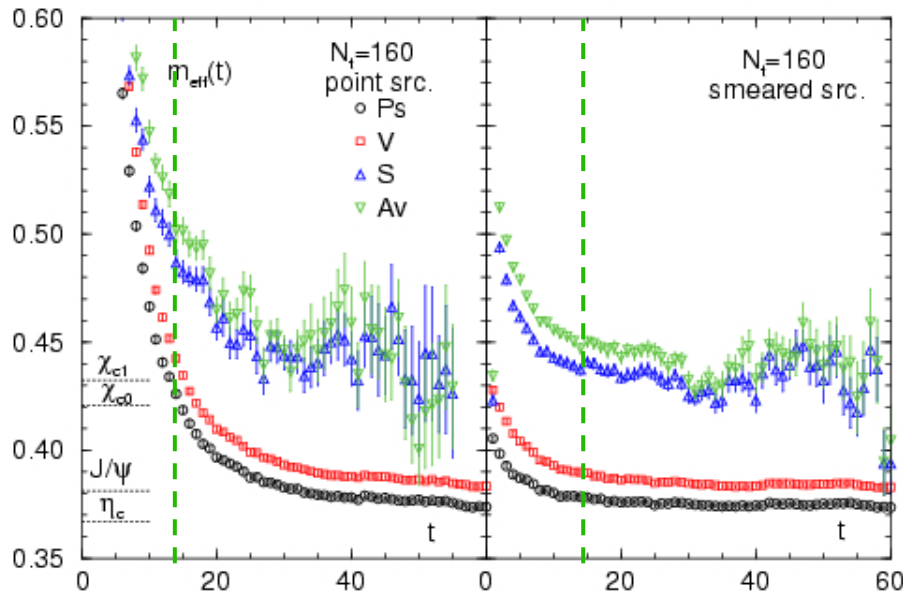
Lattice setup

- Quenched approximation (no dynamical quark effect)
- Anisotropic lattices
 - lattice size : $20^3 \times N_t$
 - lattice spacing : $1/a_s = 2.03(1) \text{ GeV}$,
 - anisotropy : $a_s/a_t = 4$
- Quark mass
 - charm quark (tuned with J/ψ mass)



N_τ	160	32	26	20	12
T/T_c	~ 0	0.88	1.08	1.40	2.33
# of conf.	60	100	100	100	100

At zero temperature



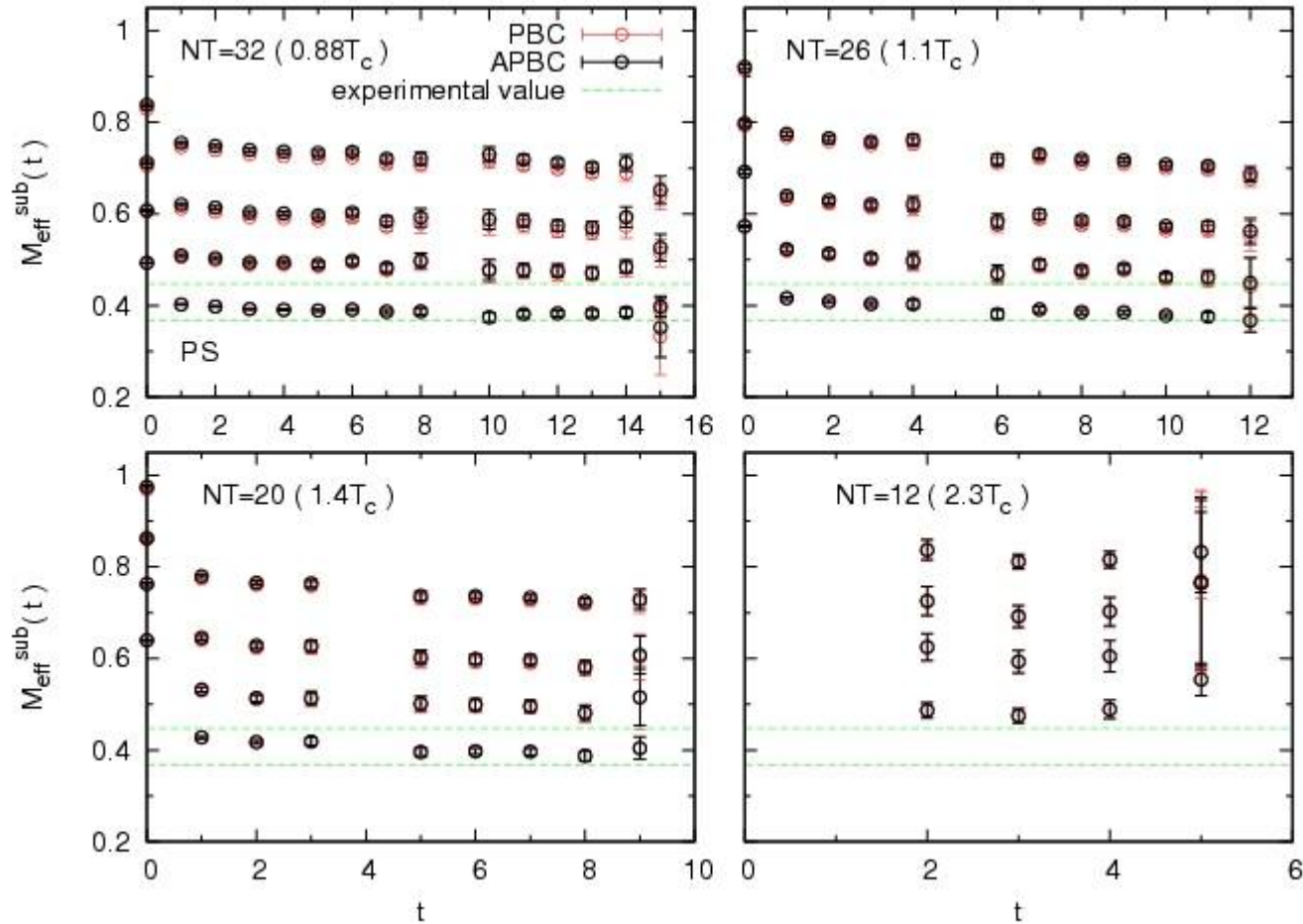
(our lattice results)
 $M_{PS} = 3033(19) \text{ MeV}$
 $M_V = 3107(19) \text{ MeV}$

(exp. results from PDG06)

$M_{\eta_c} = 2980 \text{ MeV}$
 $M_{J/\psi} = 3097 \text{ MeV}$
 $M_{\chi_{c0}} = 3415 \text{ MeV}$
 $M_{\chi_{c1}} = 3511 \text{ MeV}$

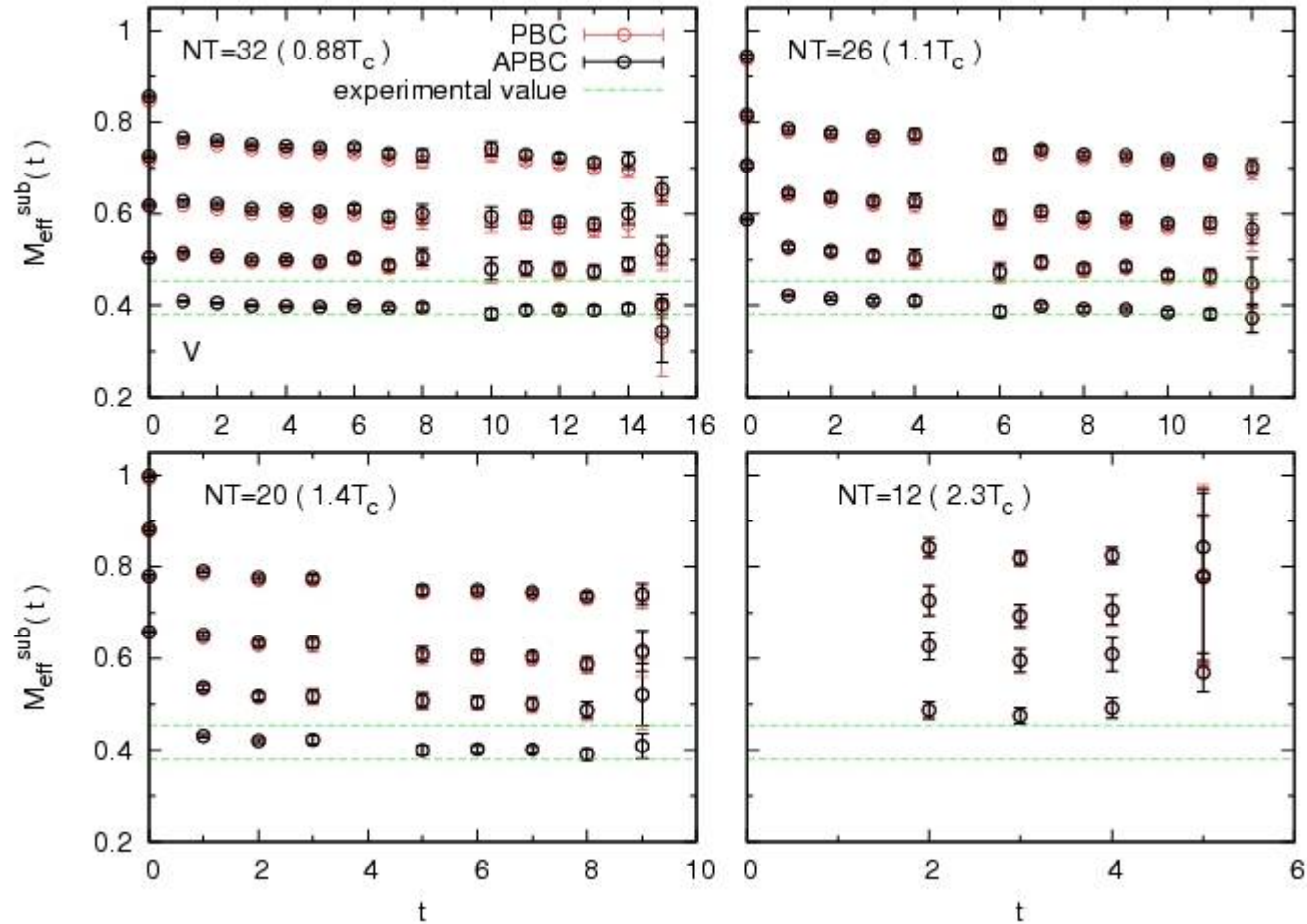
- In our lattice $N_t \simeq 28$ at T_c
 $t = 1 - 14$ is available near T_c
- Spatially extended (smeared) op. is discussed later

Results for PS channel



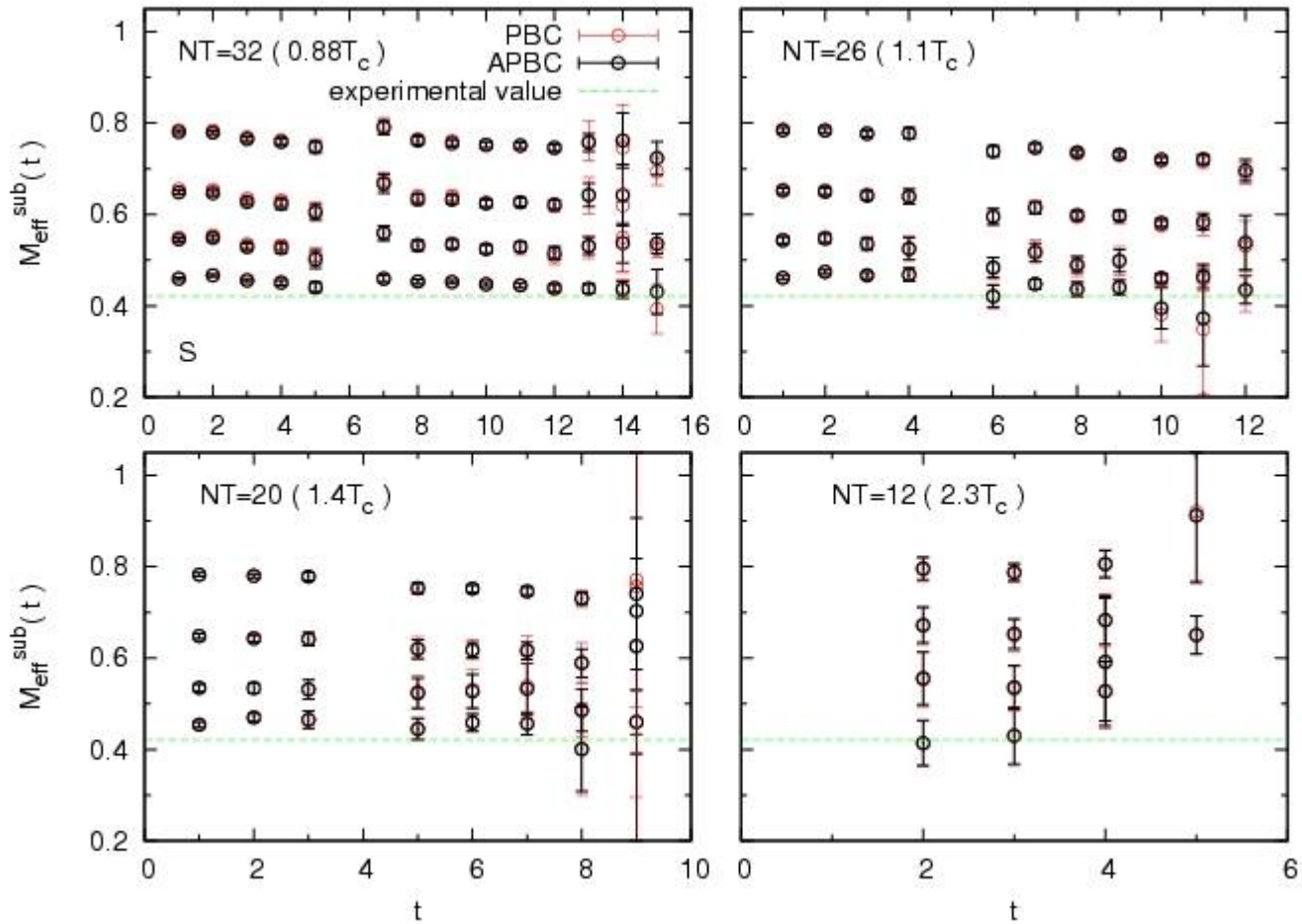


Results for V channel

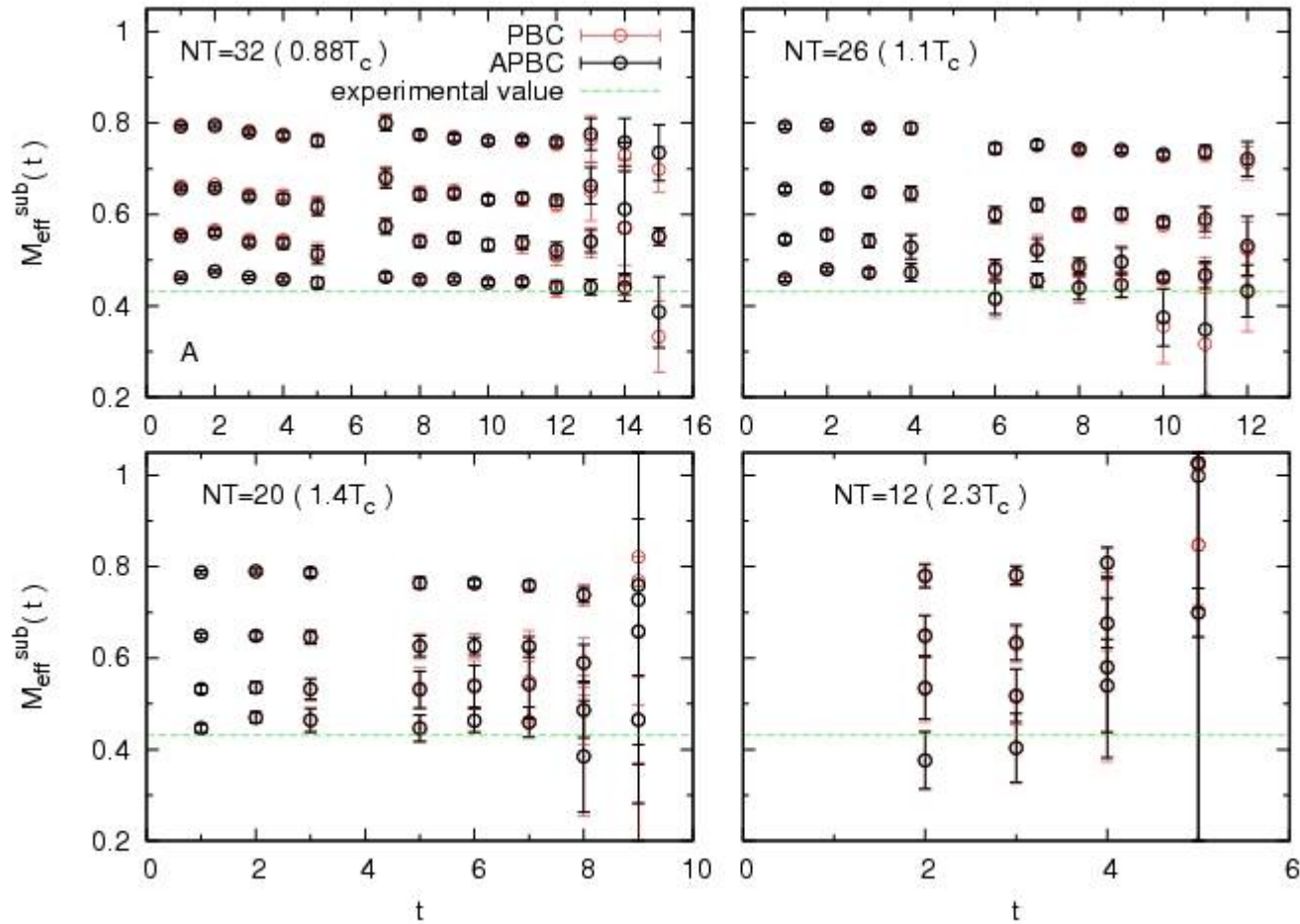




Results for S channel



Results for A_V channel



Summary



We investigated T_{dis} of charmonia from Lattice QCD using the finite volume technique

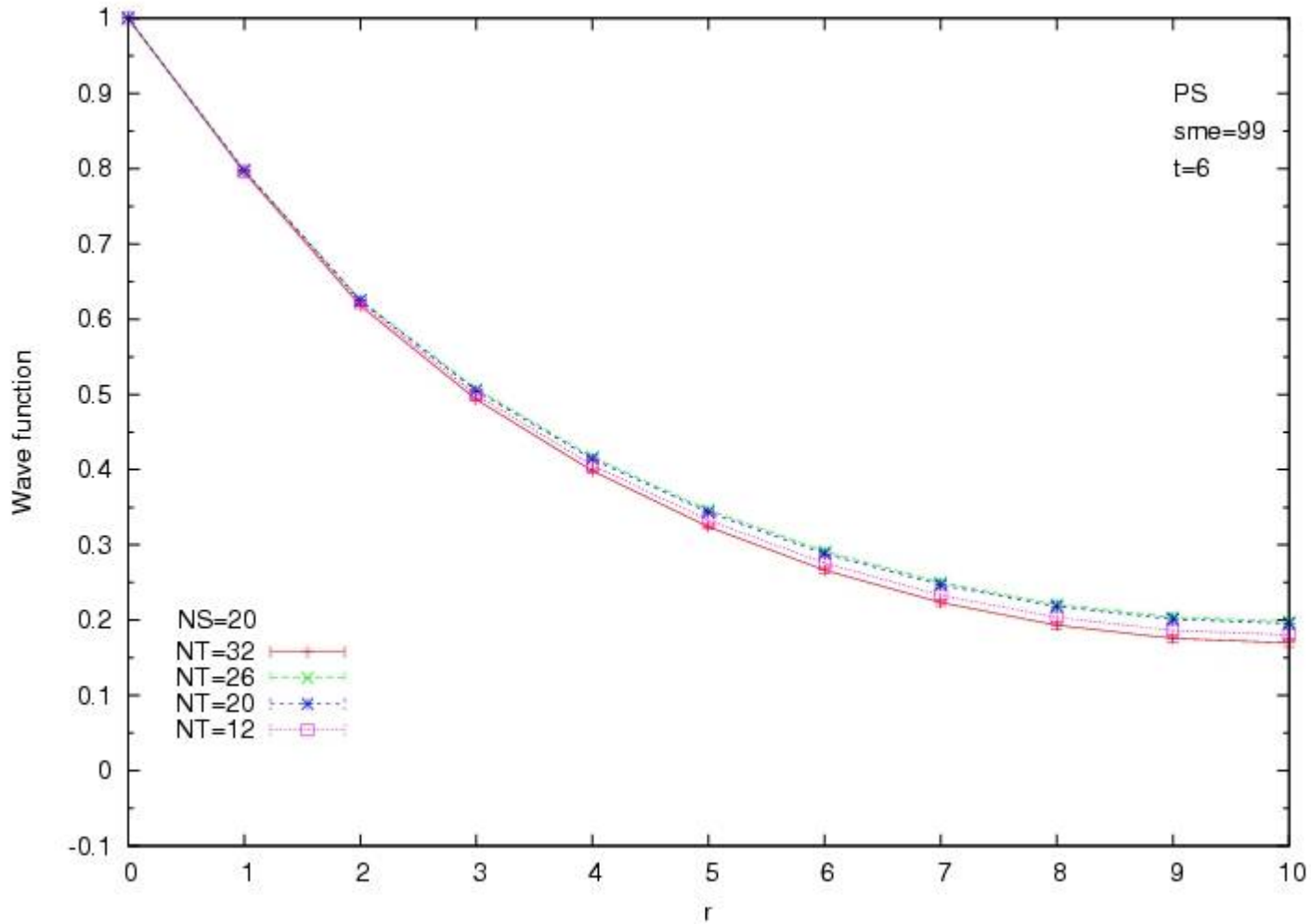
- boundary condition dependence
 - variational analysis
- Our approach can control the system with free quarks
 - In quenched QCD case,
 - we found no boundary condition dependence and no mass shift (except for $2.3T_c$)
 - Our results may suggest
 - both S & P states survive upto $2T_c$ (?)

Outlook

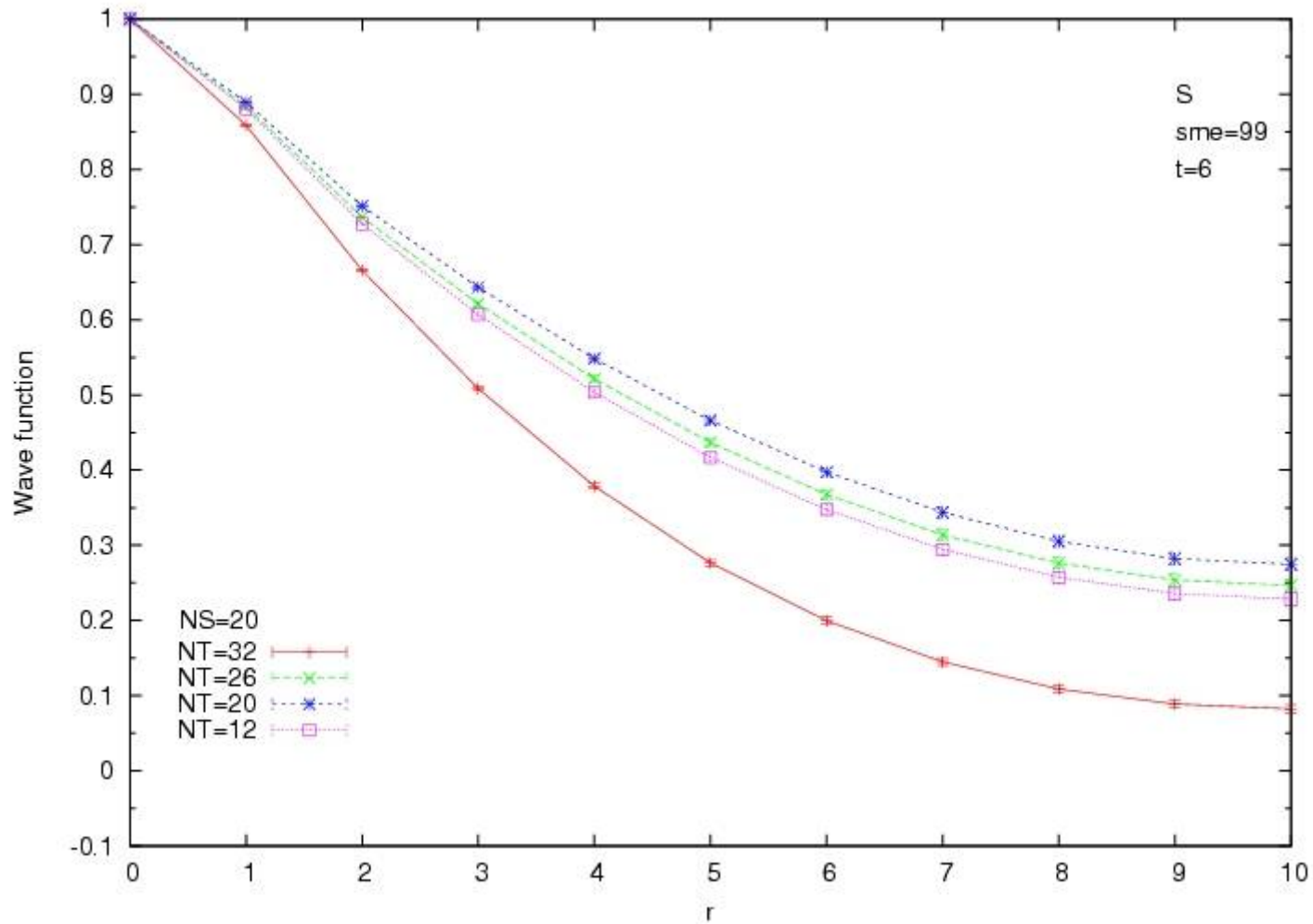
- more efficient smearing func.
- more configurations & temperatures
up to 1000confs. at each temp.
Nt=32,26,20,(16),12,(8)
- volume dependence
L=20, (16)
- wave function (Bethe–Salpeter amplitude)



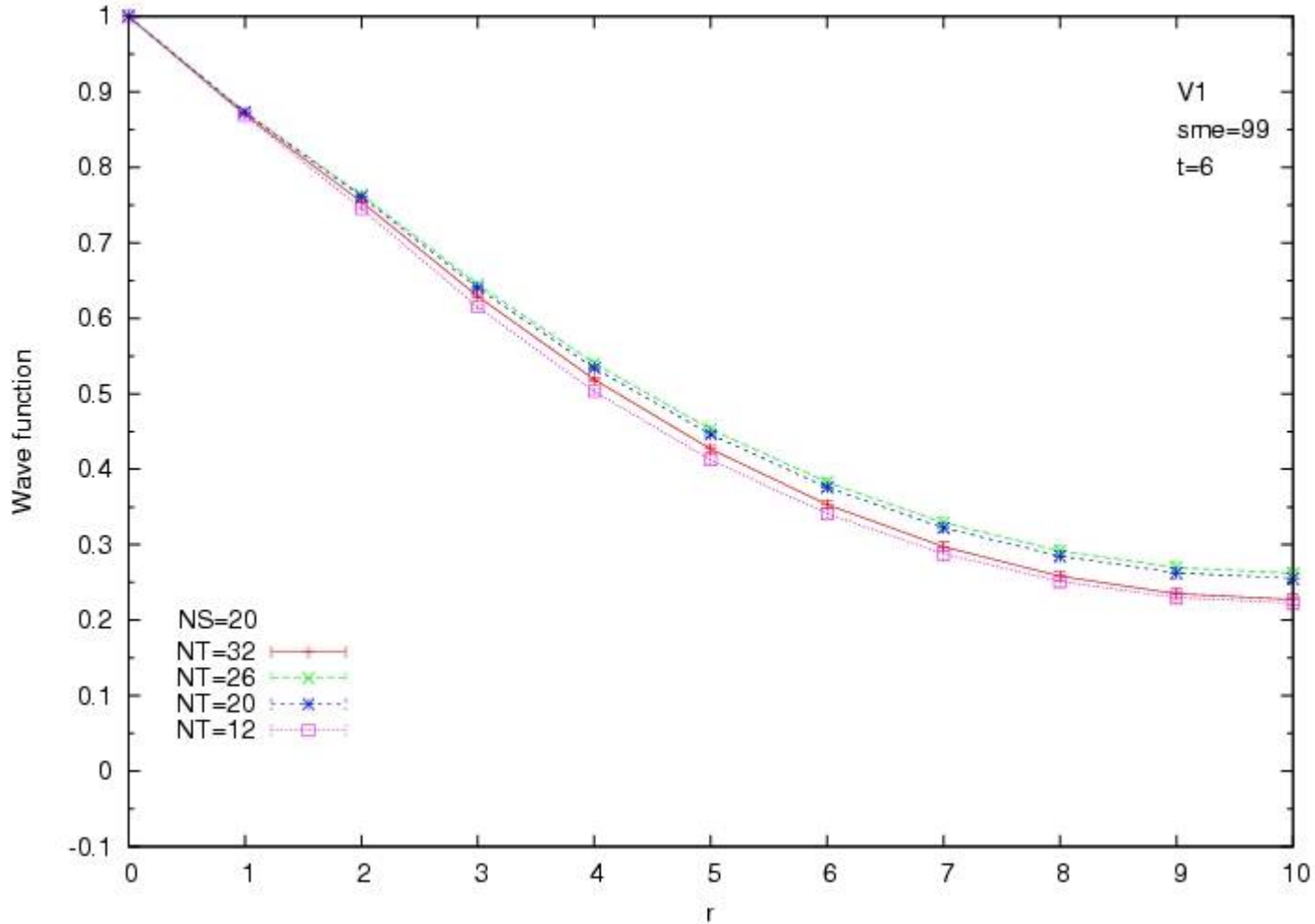
Wave functions at $T > 0$



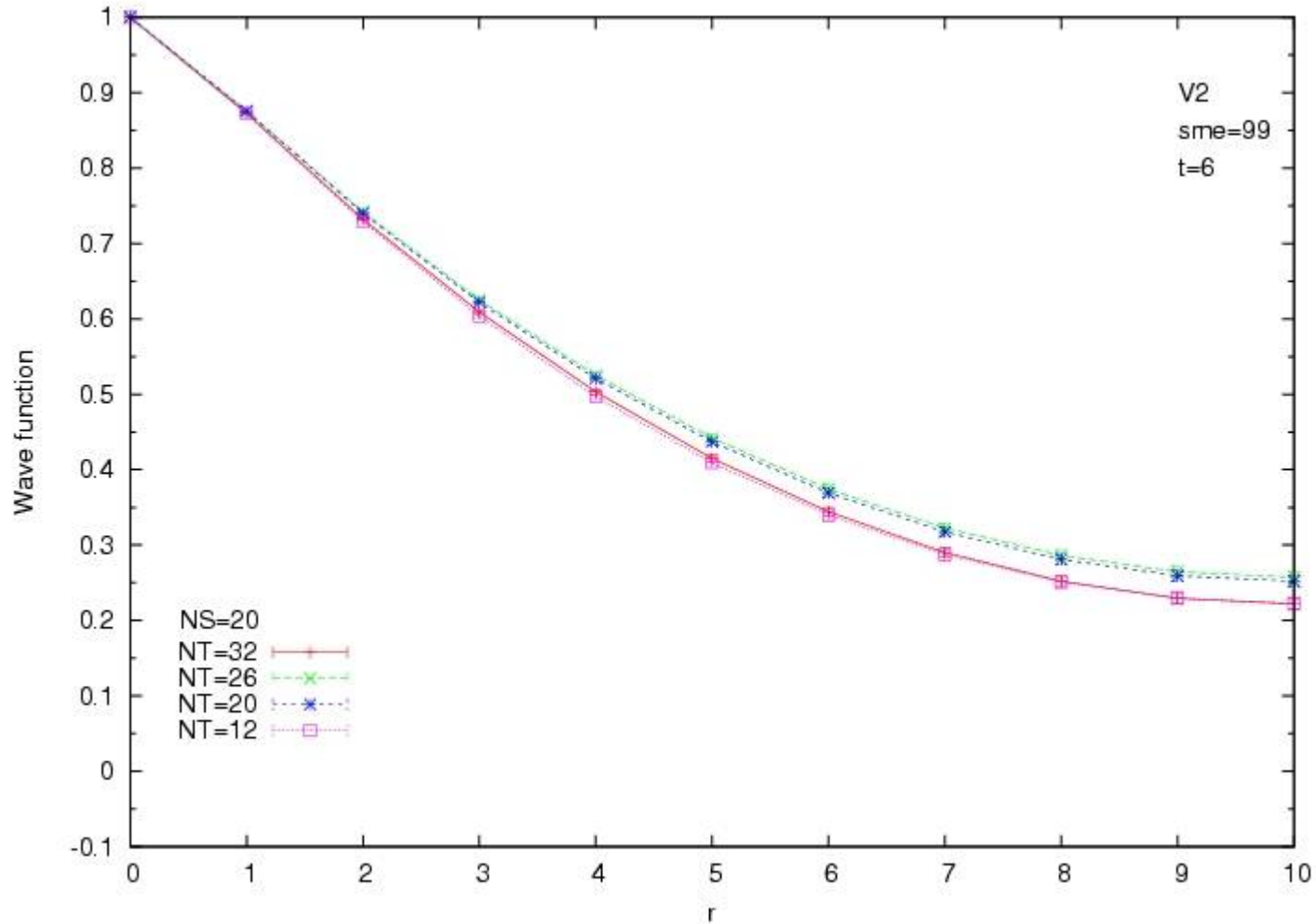
Wave functions at $T > 0$



Wave functions at $T > 0$

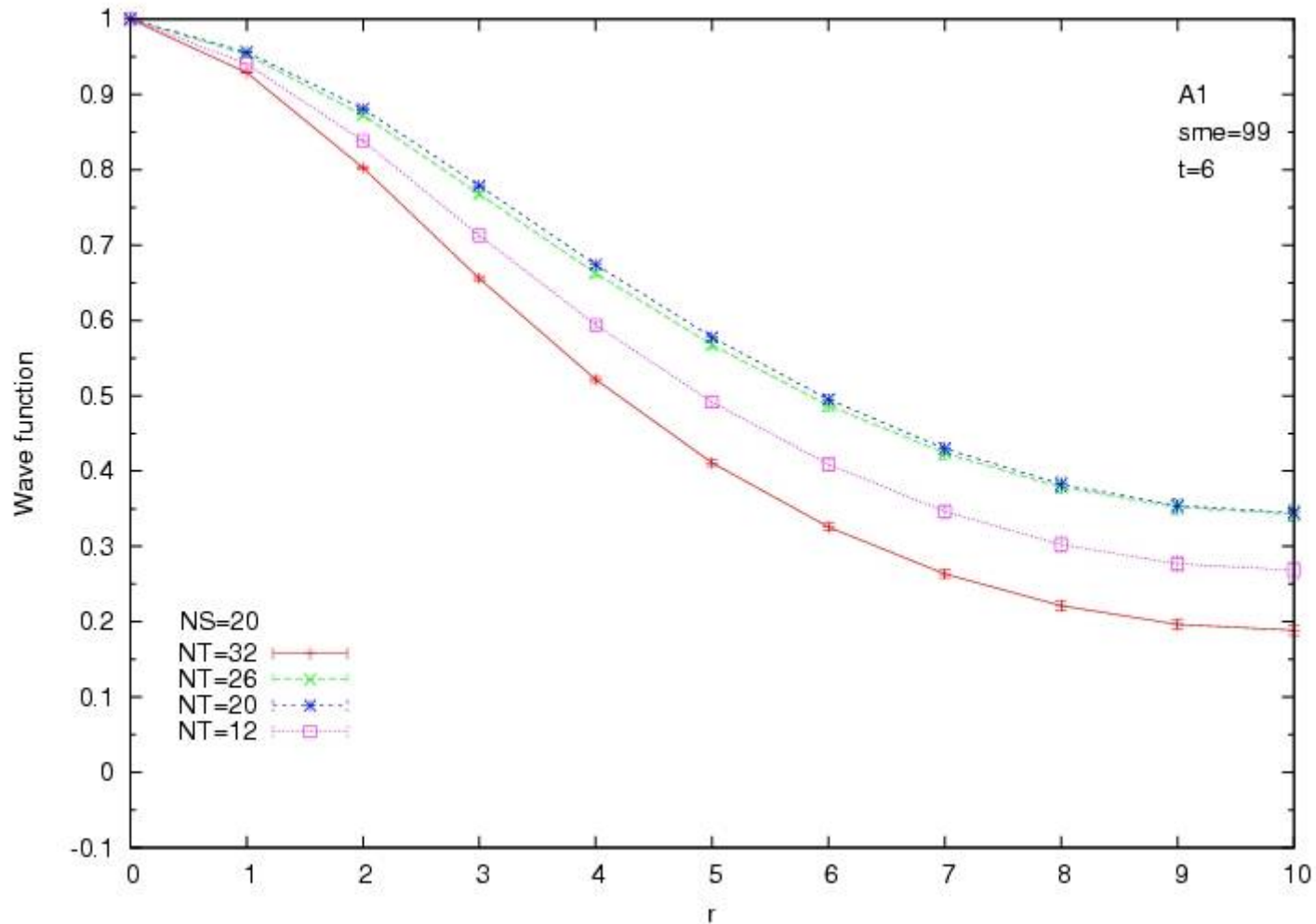


Wave functions at $T > 0$





Wave functions at $T > 0$





Wave functions at $T > 0$

