Remarks on the Maximum Entropy Method applied to finite temperature lattice QCD

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Introduction

Spectral function (SPF) includes
much information of hadronic correlators

- E.g. SPF of charmonium at T>0
  is important for Physics of QGP.
  Maximum Entropy Method (MEM) is
  adopted by some groups for this study.
- Recently MEM is also applied to many
  research area in lattices QCD.

But MEM is not a Magic Tool to get the SPFs !

Notable points

- MEM is a kind of
  constrained χ^2 fitting
  # of fit parameters < # of data points

- Prior knowledge of SPFs is needed
  as a default model function, m(w)
  The default model function
  plays crucial roles in MEM

Outline of MEM

Reconstruction of Spectral functions (SPFs)

$$C(\tau) = \int d\omega K(\tau,\omega) A(\omega)$$

hadron correlator → $C(\tau)$

kernel $\left(= e^{-\omega\tau} + e^{-\omega(T-\tau)} \right)$ → $K(\tau,\omega)$

by Maximization of $Q = \alpha S - L$

const. (to be integ. out) → α

L : Likelihood function(chi^2 term)

$$S = \int d\omega \left[A(\omega) - m(\omega) - A(\omega) \ln \frac{A(\omega)}{m(\omega)} \right]$$

default model func. → $m(\omega)$

using fit-form by Singular Value Decomposition

Fit-form in MEM

using Singular Value Decomposition (SVD)

$$
\begin{aligned}
K(\tau,\omega) &= e^{-\omega\tau} + e^{-\omega(T-\tau)} \\
&= V(\tau,\tau')w(\tau',\tau'')U(\omega,\tau'')^t
\end{aligned}
$$

$w(\tau,\tau')$: diagonal matrix

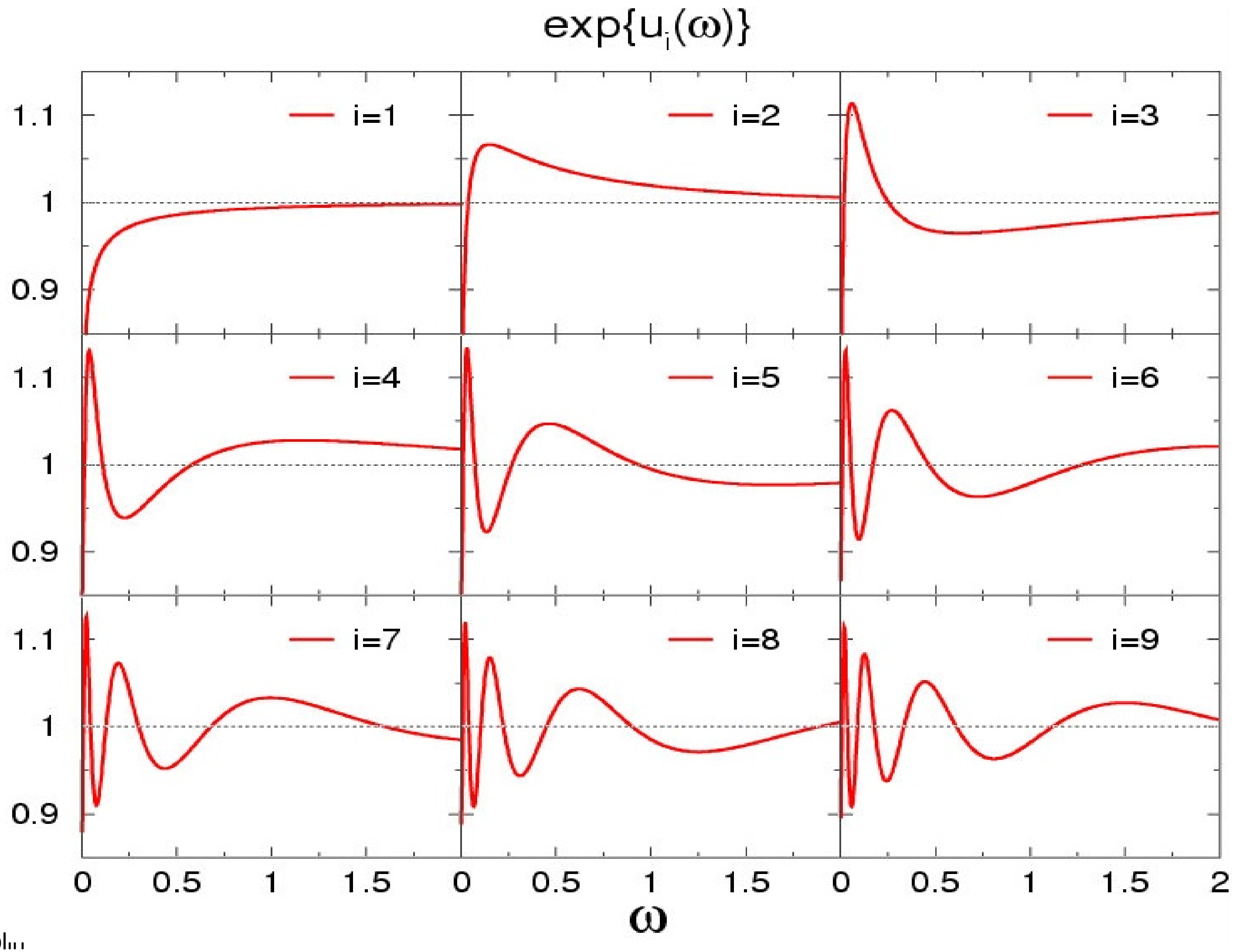
$u_i(\omega)(=U(\omega,\tau_i))$: basis in singular space

b_i : fit parameters

fit form for spactral function : A(w)

$$
A(\omega) = m_0\omega^2 \prod_{i=1}^{N} \exp\{b_i, u_i(\omega)\}
$$

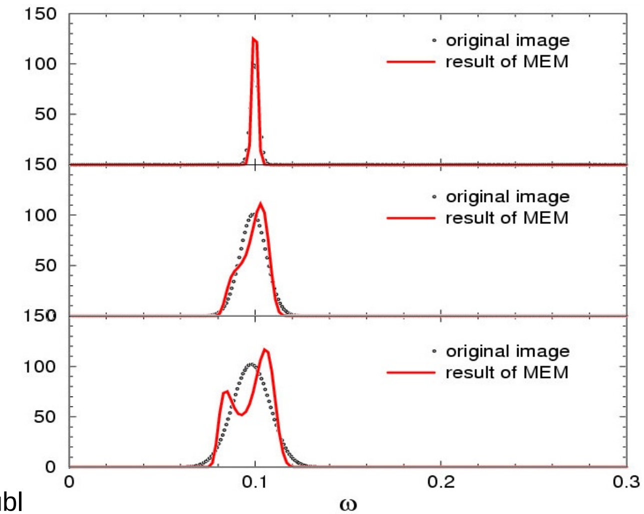
Singular Value Decomposition

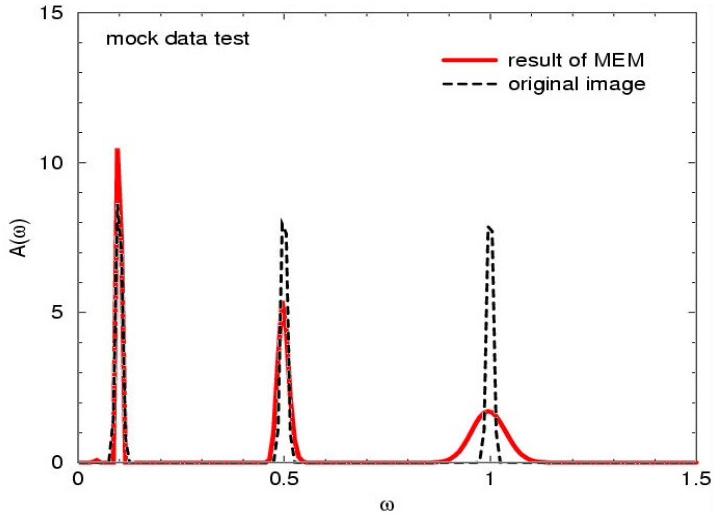


Singular Value Decomposition

- fit-form using SVD is suitable for SPFs but its resolution depends on energy ω.
- (sharp/broaden) peak at (low/high) energy region may be fake.

samples of mock data analysis





Default model function

MEM needs prior knowledge for m(w),
which is relevant to Entropy term in Q.

In principle, m(w) should include
only fact we know.

In MEM, roughly speaking,
when both results have same χ^2,
closer result to m(w) is preferred
through a maximization of Q

※ As a practical importance, the entropy term stabilizes many-parameter fit.

Choice of default model function

In QCD, prior knowledge for SPFs is,
for example,

(1) positive definite
(2) perturbative results at high energy

but the latter is not appropriate
on the lattice !

In many studies, perturbative result are used in default model functions.

....

Summary of MEM

- Advantages
  - Stabilized fitting
  - Suitable fit-form for SPFs (by SVD)
- Disadvantages
  - No intrinsically good default model funcion.
  - Rather complicated analysis
    → difficult to check the results

When data has good quality
(e.g. T=0, good statistics), MEM works well.

Application to T>0

On T>0 lattices, Nt is restricted to $1/Ta_t$

→We have to check
the reliability of the results

for example,

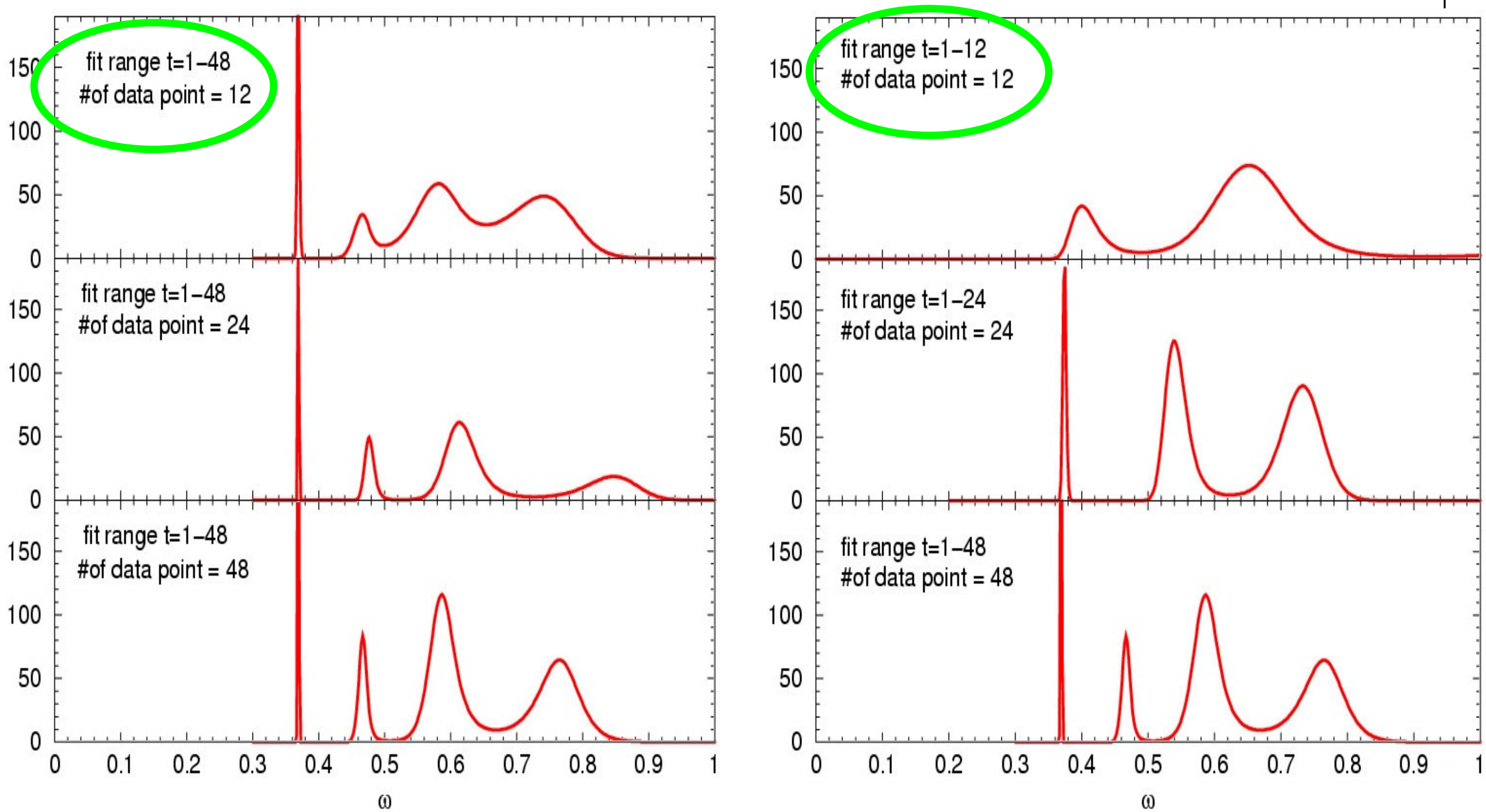
- using T=0 data
  in the same condition as T>0
- default model function dependence
- fit range dependence

We show these checks with our lattice data

quenched, clover + plaq. actions

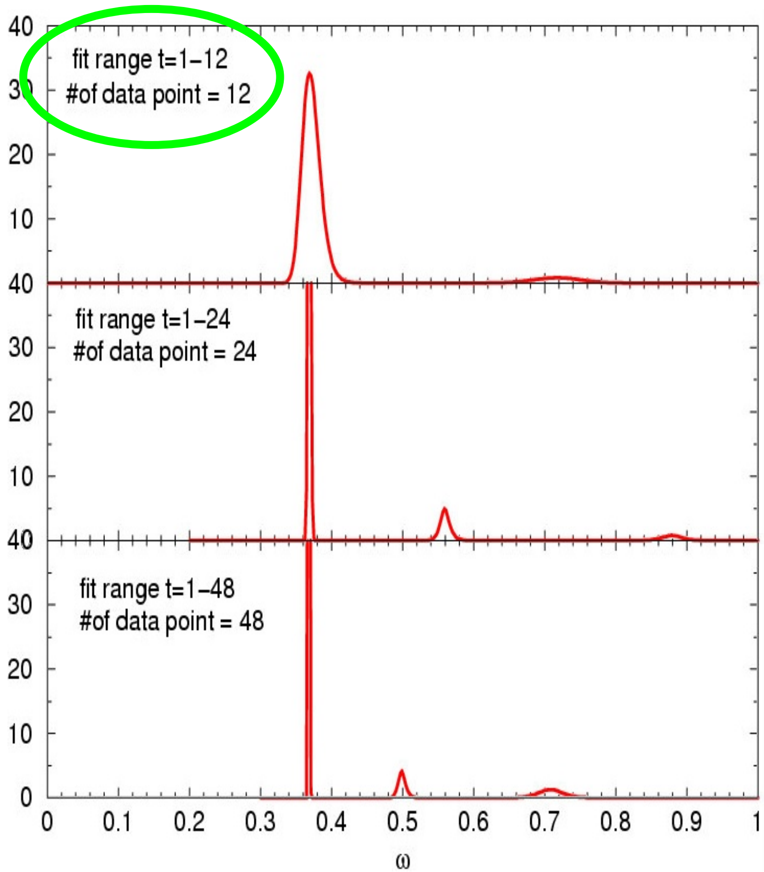
$$1/a_\tau = 8GeV, 1/a_\sigma = 2GeV$$

Test with T=0 data



t_{max}=12 corresponds to data points at T≈1.2Tc

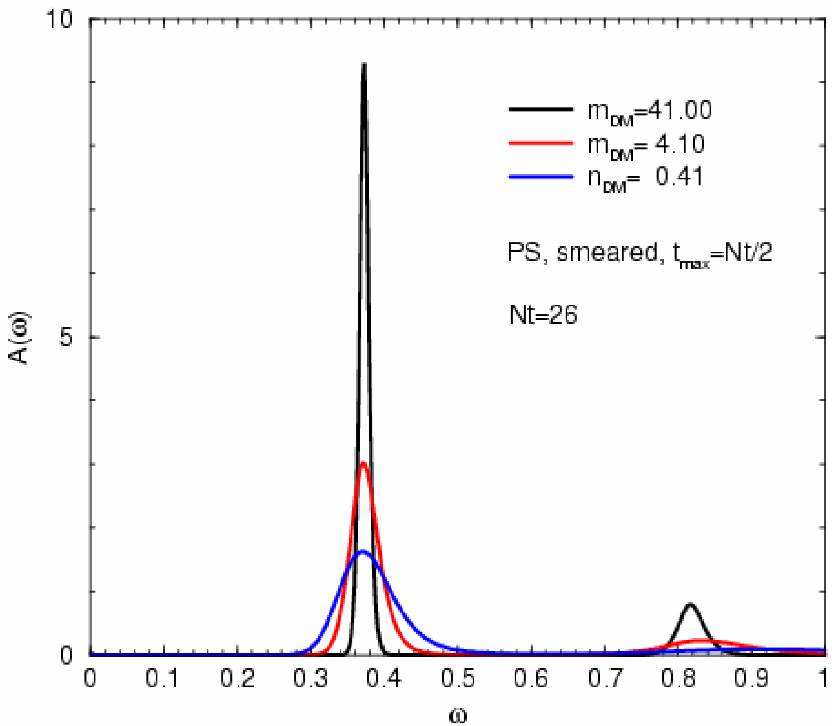
Test with T=0 data



- MEM does not work with small t_{max} (fit range $t = t_{min} - t_{max}$)
- Physical length is important rather than # of data points
- Smeared operators may improve this situation

m(w) dependence

In this analysis we use the perturb. results for default model function $m(w)=m_{DM}w^2$



- peak position is stable
- width of SPFs is sensitive to m_{DM}

This result indicates that it is difficult to discuss width of SPFs

For quantitative study

In our paper, Eur. Phys. J. C39s1 (2004) 9-26.
hep-lat/0211003

we gave up quantitative study using only MEM.

If we know a rough image of SPFs,
standard χ^2-fit (or constrained curve fit)
is appropriate for quantitative studies.

➔ We used MEM to find
a rough image (fit-form) of SPFs.

(Of course, it depends on lattice setup.)

Finally we want to say ...

MEM is powerful tool

to extract the SPFs from correlators,

however we have to use it carefully

taking its properties into account.

Abstract

We make some remarks on the Maximum Entropy Method (MEM) for studies of the spectral function of hadronic correlators in finite temperature lattice QCD.

We discuss the virtues and subtlety of MEM for the cases that one does not have enough number of data points such as at finite temperature.

Taking these points into account, we suggest several tests which one should examine to keep the reliability for the results, and also apply them using mock and lattice QCD data.

Contents

Notable points

- MEM does not solve ill-posed problem
  → MEM is a kind of
  constrained χ^2 fitting
  # of fit parameters < # of data points

- Prior knowledge of SPFs is needed
  as a default model function, m(w)
  The default model function
  plays crucial roles in MEM