QCD thermodynamics with N_f=2+1 near the continuum limit at realistic quark masses

Takashi Umeda (BNL)

for the RBC – Bielefeld Collaboration

BROOKHAVEN NATIONAL LABORATORY

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Motivation & Approach

Quantitative study of QCD thermodynamics
from first principle calculation (Lattice QCD)

T_c, EoS, phase diagram, small μ, etc…

from recent studies, we know
these quantities strongly depend on m_q & N_f

Our aim is QCD thermodynamics with 2+1 flavor
at almost realistic quark masses

e.g. pion mass ≃ 150MeV, kaon mass ≃ 500MeV

- Choice of quark action
  → Improved Staggered quark action
- Continuum limit
  – N_t = 4, 6, (8) → a ≃ 0.24, 0.17, (0.12) fm

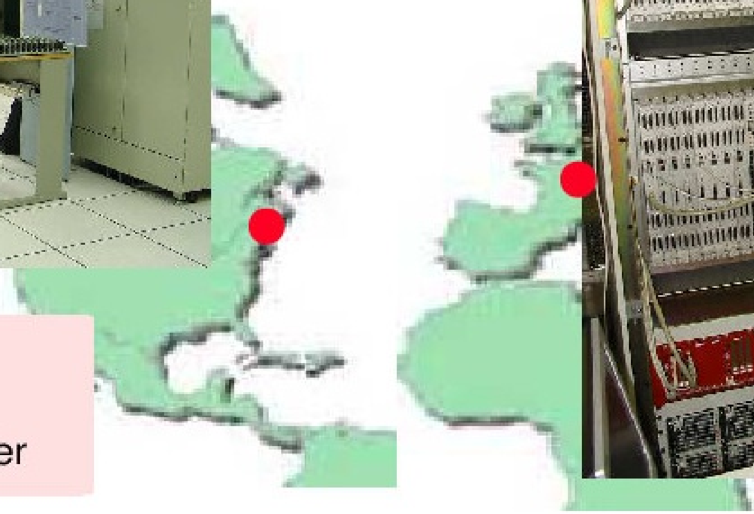
Computers

US/RBRC QCDOC

20.000.000.000.000 ops/sec

BI – apeNEXT

5.000.000.000.000 ops/sec



- critical temperature
- equation of state
- hadron properites in matter

today: [illegible] TFlops

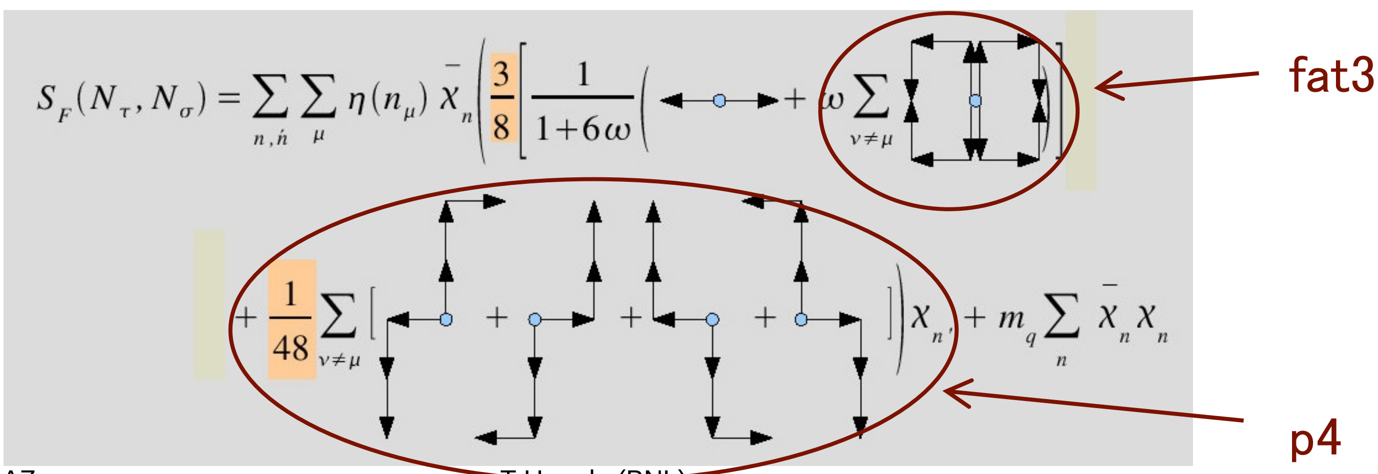
http://www.quark.phy.bnl.gov/~hotqcd

Choice of Lattice Action

Improved Staggered action : p4fat3 action

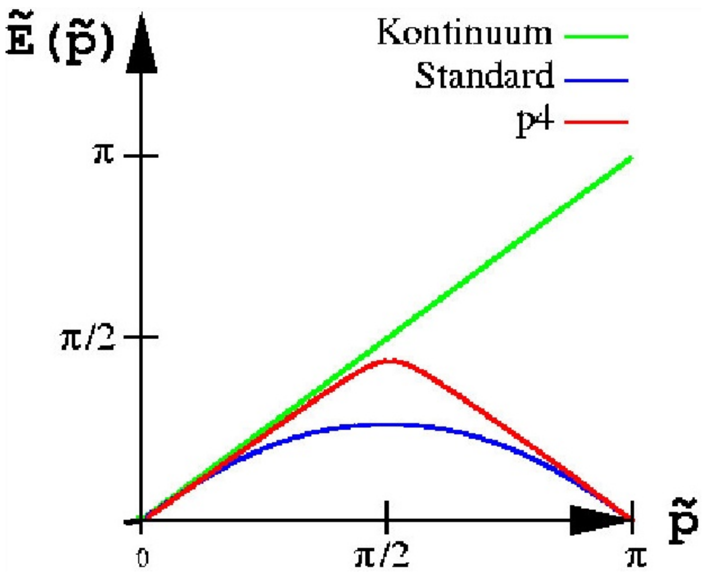
Karsch, Heller, Sturm (1999)

- gluonic part : Symanzik improvement scheme
  - remove cut-off effects of $O(a^2)$
  - tree level improvement $O(g^0)$
- fermion part : improved staggered fermion
  - remove cut-off effects & improve rotational sym.
  - improve flavor symmetry by smeared 1-link term



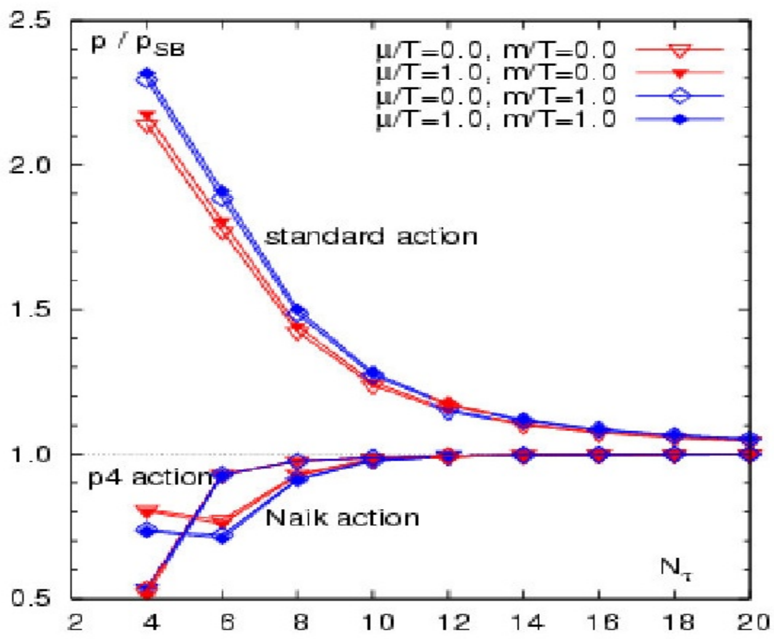
Properties of the p4-action

Dispersion relation



The free quark propagator is rotational invariant up to $O(p^4)$

pressure in high T limit



Bulk thermodynamic quantities show drastically reduced cut-off effects

flavor sym. is also improved by fat link

Contents of this talk

- Motivation and Approach
- Choice of lattice action
- Critical temperature
  - Simulation parameters
  - Critical β search
  - Scale setting by Static quark potential
  - Critical temperature
- Spatial string tension
- Conclusion

Simulation parameters

- Critical β search at T > 0

N_τ	$\hat{m}_s$	$\hat{m}_l$	V	#β values	max.# conf.
4	0.1	0.5 $\hat{m}_s$	8^3	10	40,000
		0.2 $\hat{m}_s$	8^3	6	12,000
4	0.065	0.4 $\hat{m}_s$	8^3, 16^3	10, 11	30,000, 60,000
		0.2 $\hat{m}_s$	8^3, 16^3	8, 7	30,000, 60,000
		0.1 $\hat{m}_s$	8^3, 16^3	9, 6	34,000, 50,000
		0.05 $\hat{m}_s$	8^3, 16^3	8, 5	30,000, 42,000
6	0.0040	0.4 $\hat{m}_s$	16^3	11	20,000
		0.2 $\hat{m}_s$	16^3	9	60,000
		0.1 $\hat{m}_s$	16^3	7	60,000

(∗) conf. = 0.5 MD traj.

to check
m_s dependence for Tc

- T=0 scale setting at $\beta_c(N_t)$

N_τ	$\hat{m}_s$	$\hat{m}_l$	β	# conf.	m_{ps}/m_v	a [fm]
4	0.1	0.5 $\hat{m}_s$	3.409	600	0.520(2)	0.2273(4)
		0.2 $\hat{m}_s$	3.371	238	0.372(5)	0.2336(7)
4	0.065	0.4 $\hat{m}_s$	3.362	500	0.410(2)	0.2312(7)
		0.2 $\hat{m}_s$	3.335	400	0.303(7)	0.2365(6)
		0.1 $\hat{m}_s$	3.310	750	0.212(7)	0.2458(5)
		0.05 $\hat{m}_s$	3.300	400	0.154(5)	0.2475(8)
6	0.0040	0.4 $\hat{m}_s$	3.500	294	0.461(4)	0.1558(7)
		0.2 $\hat{m}_s$	3.470	500	0.343(6)	0.1617(5)
		0.1 $\hat{m}_s$	3.455	410	0.248(4)	0.1670(5)

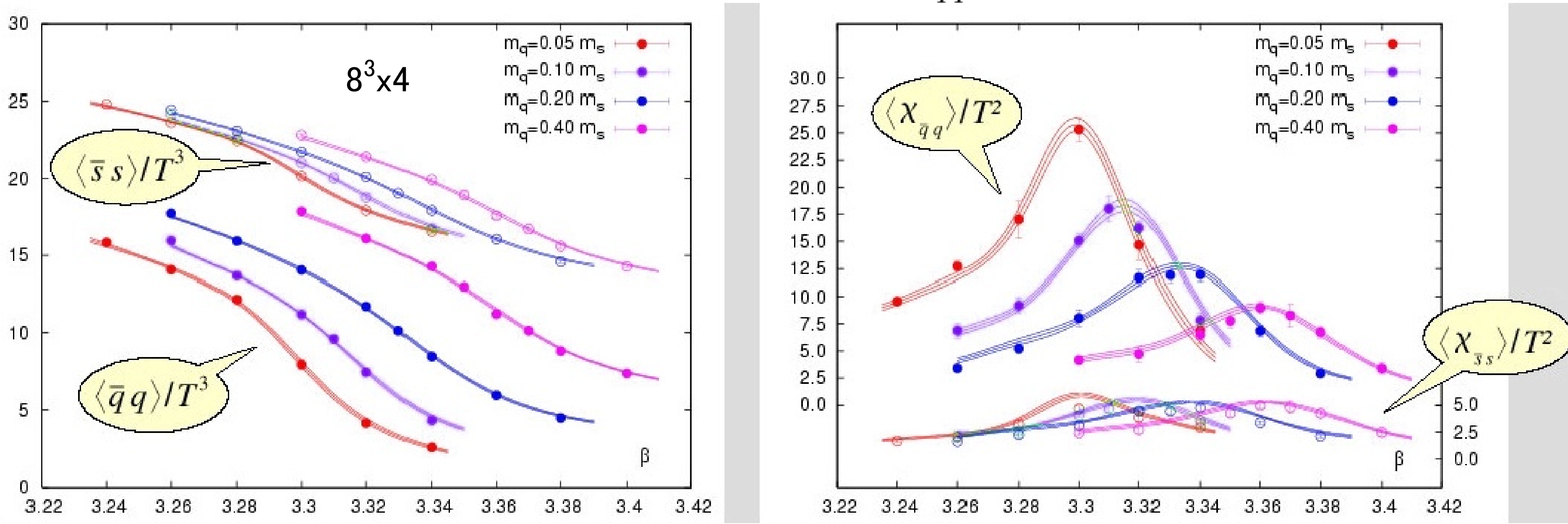
(∗) conf. = 5 MD traj.
after thermalization

Critical β search

chiral condensate:

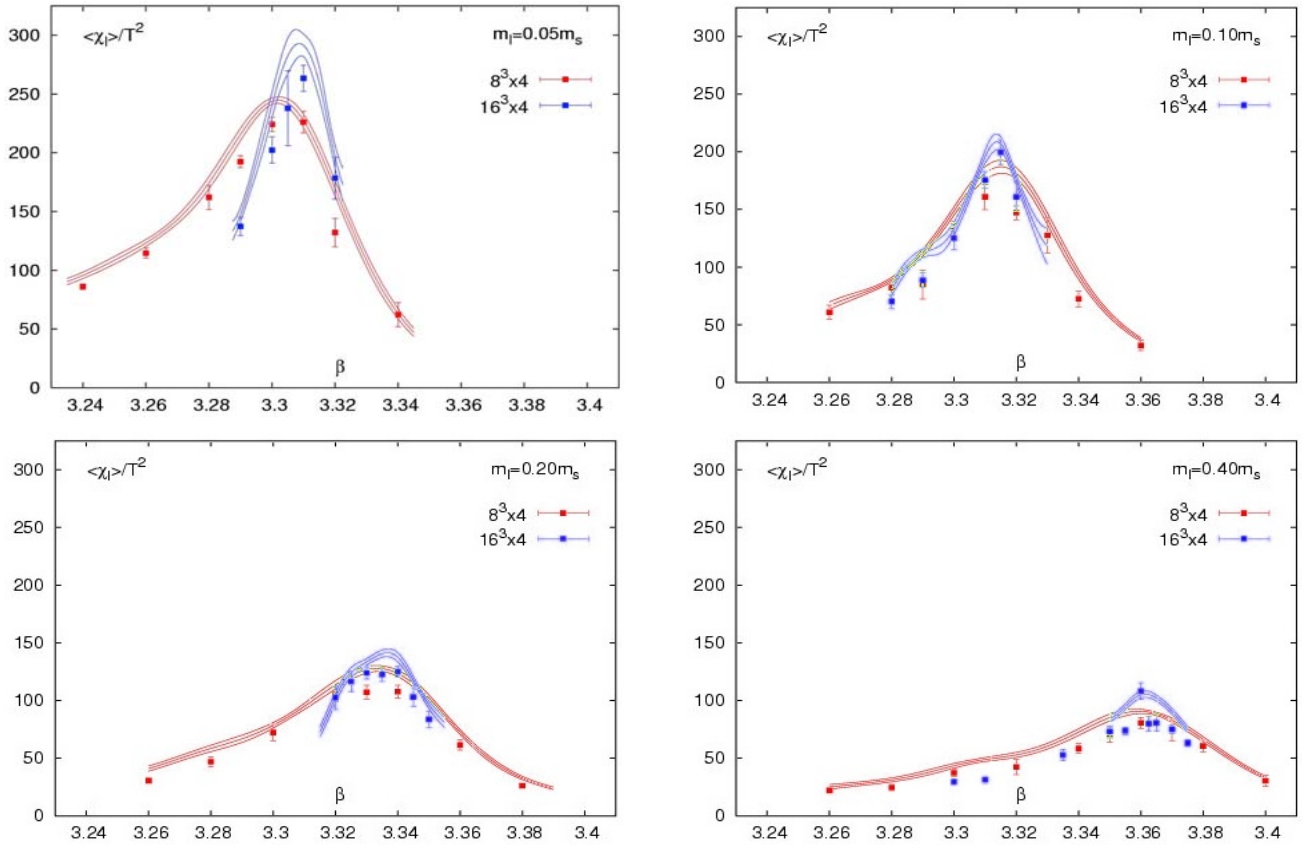
chiral susceptibility:

$$\langle \chi_{\bar{q}q} \rangle \equiv \langle (\bar{q}q)^2 \rangle - \langle \bar{q}q \rangle^2$$



- multi-histogram method (Ferrenberg-Swendson) is used
- β_c are determined by peak positions of the susceptibilities
- Transition becomes stronger for smaller light quark masses

Volume dependence of β_c



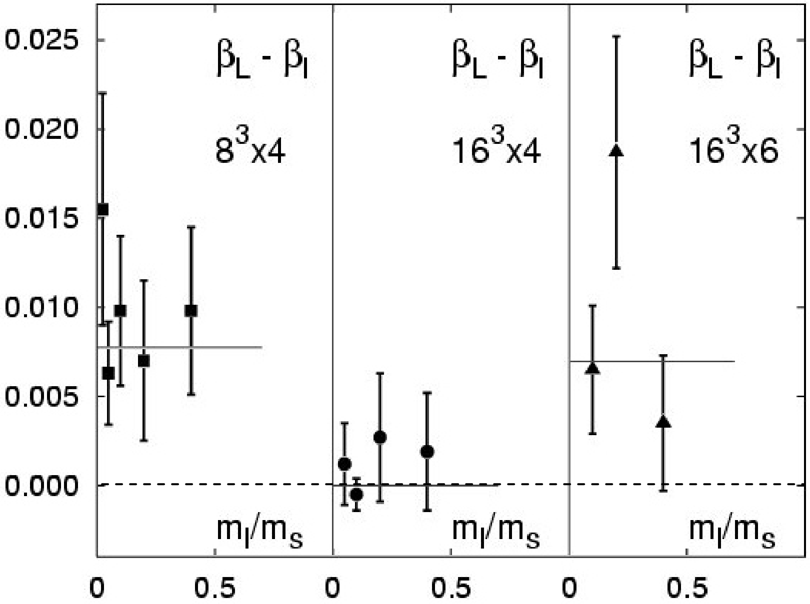
- No large change in peak height & position
  → consistent with crossover transition rather than true transition

Uncertainties in β_c

- Statistical error
  → jackknife analysis for peak-position of susceptibility

- We can find a difference between β_l and β_L
  → small difference but statistically significant
  β_l : peak position of χ_l
  β_L : peak position of χ_L

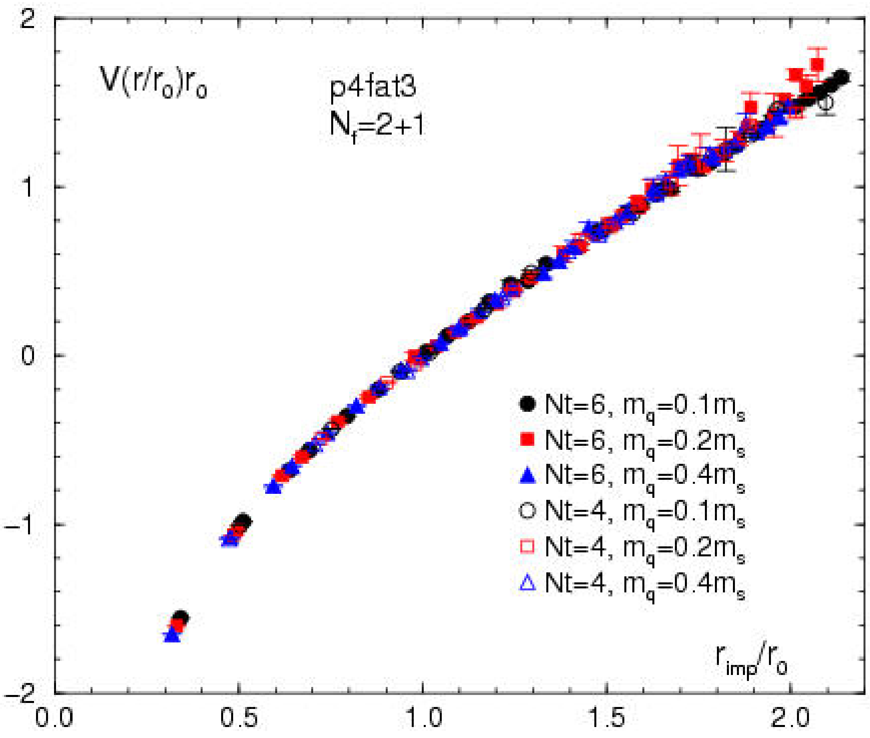
– the difference is negligible
  at 16^3x4 (N_s/N_t=4)
– no quark mass dependence
– the difference at 16^3x 6
  are taken into account as
  a systematic error in β_c



Scale setting at T=0

Lattice scale is determined by a static quark potential V(r)

$$r^2 \frac{dV_{\bar{q}q}(r)}{dr}|_{r=r_0} = 1.65$$



$$V_3(r) = -\frac{\alpha}{r_I} + \sigma r_I + C$$

$$V_4(r) = -\frac{\alpha}{r} + \sigma r - \alpha' \left(\frac{1}{r_I} - \frac{1}{r} \right) + C$$

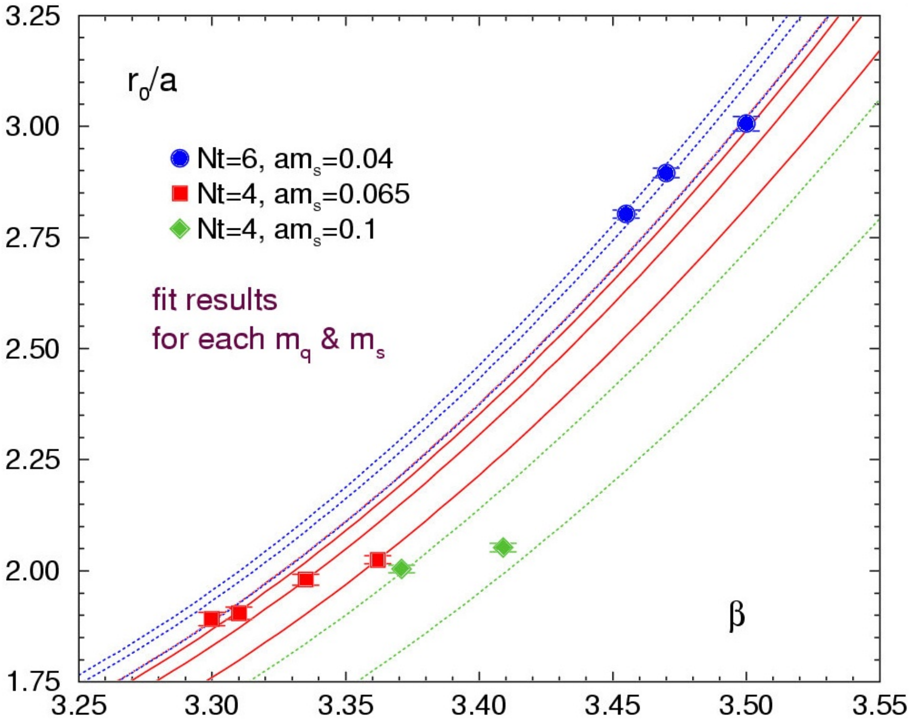
where, r_I is the improve dist.

- statistical error
  → jackknife error
- systematic errors
  → diff. between $V_3(r)$ & $V_4(r)$
  diff. in various fit range
  (r_{min}=0.15−0.3fm, r_{max}=0.7−0.9fm)

β & m_q dependence of r_0

RG inspired ansatz with 2−loop beta−function $R(\beta)$

$$\ln(r_0/a) = A(2m_l + m_s) - ln(R(\beta)) + B\left(\frac{R(\beta)}{R(3.4)}\right)^2 + C\left(\frac{R(\beta)}{R(3.4)}\right)^4 + D$$



$A = -1.53(11),\ \ B = -0.88(20)$
$C = \ 0.25(10),\ \ D = -2.45(10)$
$\chi^2/dof = 1.2$

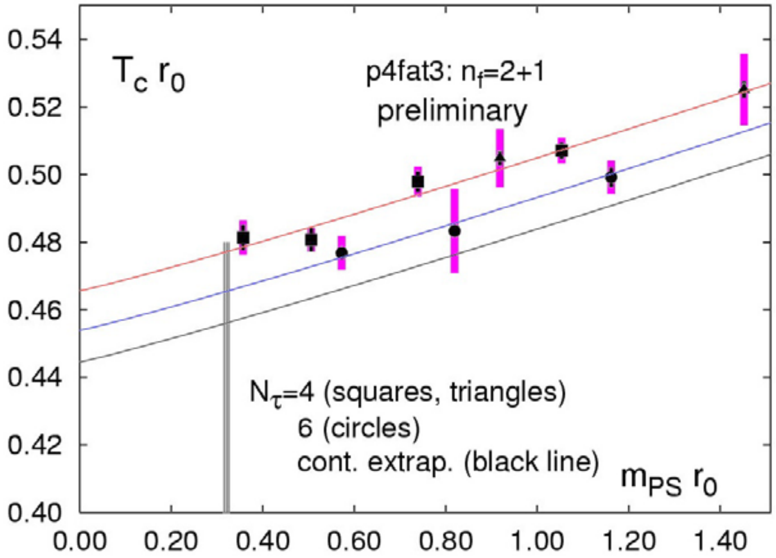
The fit result is used

(1) correction for the diff. between β_c & simulation β at T=0

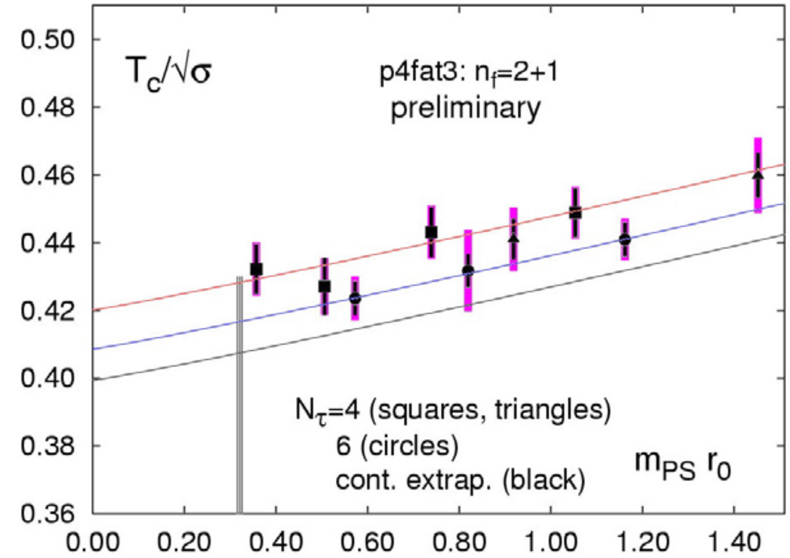
(2) convertion of sys. + stat. error in β_c to error of r_0/a

Critical temperature

ratio to Sommer scale:



ratio to string tension:



$$T_c r_0 = A(m_{ps} r_0)^d + B/N_\tau^2 + C$$ (d=1.08 from O(4) scaling)

$\Rightarrow$ $T_c r_0 = 0.456(7)^{+3}_{-1}, \quad T_c/\sqrt{\sigma} = 0.408(7)^{+3}_{-1}$ at phys. point,

(fit form dependence → d=1,2 : upper, lower errors)

Finally we obtain T_c=192(5)(4)MeV from r_0=0.469(7)fm

Preliminary result

Spatial string tension

Important to check theoretical concepts (dim. reduction) at high T

Static quark "potential" from Spatial Wilson loops

$$V_z(r) = \ln W(r,z)/W(r,z+1) \sim C - \frac{\alpha}{r_I} + \sigma_s r_I$$

$$\sqrt{\sigma_s(T)} = c g^2(T) T$$ (free parameters: c, Λ_σ)

$g^2(T)$ is given by the 2-loop RG equation

$$g^{-2}(T) = 2b_0 \ln \frac{T}{\Lambda_\sigma} + \frac{b_1}{b_0} \ln\left(2 \ln \frac{T}{\Lambda_\sigma}\right)$$

"c" should equal with 3-dim. string tension and
should be flavor independent, if dim. reduction works

- Numerical simulation

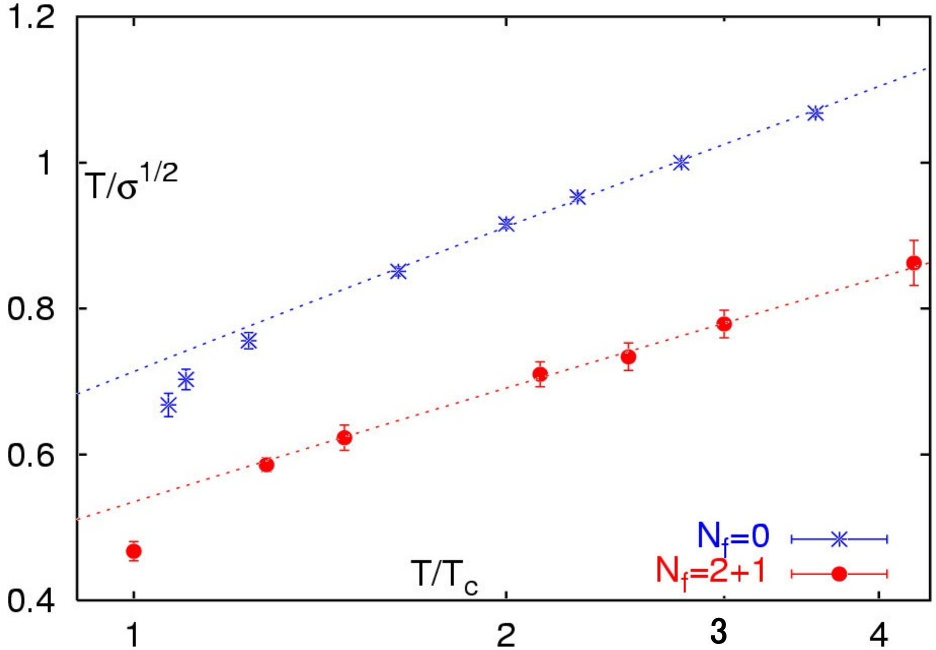
$16^3 \times 4, \quad m_q = 0.1 m_s$ fixed

β	3.31	3.41	3.46	3.61	3.68	3.76	3.94
m_q	0.0065	0.0052	0.0040	0.00325	0.0026	0.002	0.001625

Spatial string tension

$$\sqrt{\sigma_s(T)} = c g^2(T) T$$ (free parameters: c, Λ_σ)

$$g^{-2}(T) = 2b_0 \ln \frac{T}{\Lambda_\sigma} + \frac{b_1}{b_0} \ln\left(2 \ln \frac{T}{\Lambda_\sigma}\right)$$



Quenched case:

G.Boyd et al. Nucl.Phys.B469('96)419.

$$c = 0.566(13)$$
$$\frac{\Lambda_\sigma}{T_c} = 0.104(9)$$

2+1 flavor case:

$$c = 0.587(41)$$
$$\frac{\Lambda_\sigma}{T_c} = 0.114(27)$$

"c" is flavor independent within error

→ dim. reduction works well even for $T=2T_c$

Summary

N_f=2+1 simulation with realistic quark mass at N_t=4, 6

- critical temperature

  $T_c r_0$=0.456(7), (T_c=192(5)(4)MeV from r_0=0.469)

  – $T_c r_0$ is consistent with previous p4 result
  difference in T_c mainly comes from physical value of r_0

  – however, our value is about 10% larger than MILC result
  MILC collab., Phys. Rev. D71('05) 034504.

  – most systematic uncertainties are taken into account
  remaining uncertainty is in continuum extrapolation

- spatial string tension
  dimensional reduction works well even for T=2T_c