Study of Charmonia at finite temperature in quenched lattice QCD

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Contents

We want to know
whether charmonium state can exist in QGP ?

- Introduction
  - motivation
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- Spectral function of charmonium
  - Our approach
  - Maximum entropy method
  - Standard fitting and more
  - Results
- Summary

Introduction

QGP searches in Heavy Ion Collision Experiments

Charmonium properties at T>0

for a signal of QGP

- Potential models analysis
  - Mass shift near Tc *Hashimoto et al. ('86)*
  - J/ψ suppression *Matsui&Satz ('86)*
- Correlation function in Lattice QCD
  - Spatial correlation of charmonium *Umeda et al. ('00)*
  - Reconstruction of the charmonium spectral function

Umeda et al.('02), Asakawa et al.('03), Datta et al('03)

Recent results indicate nonperturbative QGP !

First paper on the J/ψ suppression

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Mass Shift of Charmonium near Deconfining Temperature and Possible Detection in Lepton-Pair Production

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in Monte Carlo analyses.[7,8] A related question is whether charmoniumlike clusters may still exist in a quark-gluon plasma. We have made tentative calculations by screened Coulombic potential and found that possibility small. Thus, contribution to lepton pair in the J/ψ mass region from the deconfinement phase would be mainly thermal quark-antiquark annihilation.[18] In connection with this point, we make a com-

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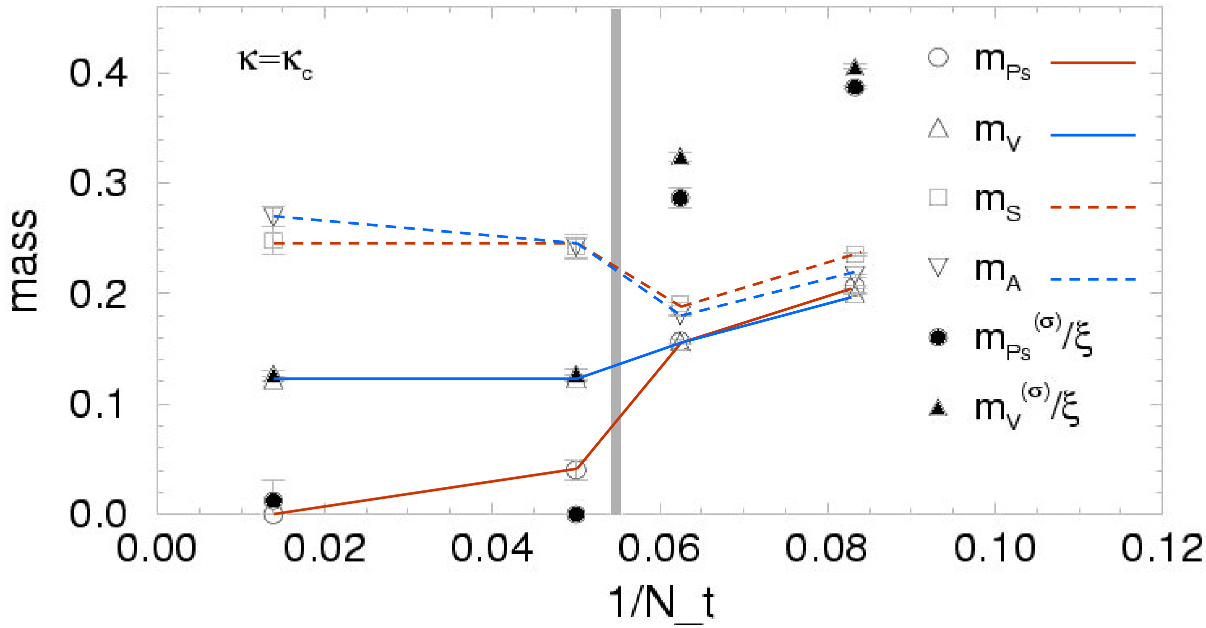
Previous studies 1

Light meson correlators with smeared operators.
using single exponential fit analysis



QCD-TARO('00)

- Consistent with NJL model analysis
- Chiral symmetry restoration above T_c

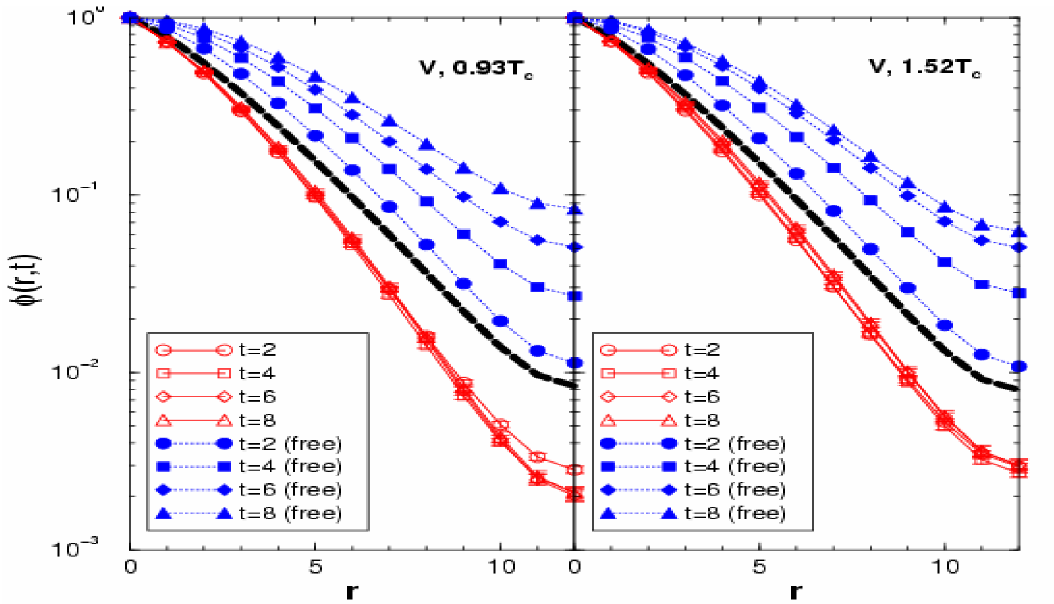
Previous studies 2

Spatial $c\bar{c}$ correlation

→ "Wave function" on the Coulomb gauge

$$\phi(r,t) \;=\; \omega_\Gamma(r,t)/\omega_\Gamma(0,t)$$

$$\omega_\Gamma(r,t) = \sum_{\vec{x}} \langle \bar{c}(\vec{x}+\vec{r},t)\Gamma c(\vec{x},t) O^\dagger(0)\rangle$$



$c\bar{c}$ strongly correlate even at T=1.5Tc

Umeda et al. ('01)

Spectral function of charmonium

Temporal correlators in lattice simulations

$$C(t) = \sum_{\vec{x}} \langle O(\vec{x}, t) O^\dagger(0) \rangle$$

$O(\vec{x}, t)$: meson operators

Spectral function $A(\omega)$ *Abrikosov et al. ('59)*

$$C(t) = \int d\omega K(t, \omega) A(\omega)$$

kernel: $K(t, \omega) = \dfrac{e^{-\omega t} + e^{-\omega(N_t - t)}}{1 - e^{-N_t \omega}}$

Our approach

- Anisotropic lattice
  fine resolution in temporal direction
- Our analysis procedure
  - Maximum entropy method
    fewer assumptions for the form of $A(w)$
  - Fit with ansatz for a spectral function
    need information on the form of $A(w)$
    with given form $A(w)$, more quantitative
    (Constrained curve fitting)

We use these two method in complementary manner

- Smeared operators
  enhancement of low frequency modes

Lattice setup

- Anisotropic quenched lattices: $20^3 \times N_t$
  $a_s/a_t=4$, $1/a_s=2.030(13)$ GeV
  Clover quark action with tadpole-imp.
  Matsufuru et al ('02)
- Temperatures:
  phase transition occurs at just Nt=28

Nt	160	32	30	29	27	26	24	20	16
T/Tc	≈0	0.88	0.93	0.97	1.04	1.08	1.17	1.40	1.75

statistics: 1000conf. x 16src. (500conf. for T=0)

Our lattices cover T/Tc=0~1.75
with the same lattice cutoff

Maximum entropy method (MEM)

Reconstruction of Spectral functions (SPFs)

Standard least square fit → ill-posed problem

MEM (based on Bayes' theorem)

Y.Nakahara et al. (99)

by Maximization of $Q = \alpha S - L$

L : Likelihood function(chi^2 term)

$$S = \int d\omega \left[A(\omega) - m(\omega) - A(\omega) \ln \frac{A(\omega)}{m(\omega)} \right]$$

m(w) : default model func.

using fit-form by Singular Value Decomposition

About MEM

- **Advantages**
  - Stabilized fitting
  - Suitable fit-form for SPFs (by SVD)
- **Disadvantages**
  - No intrinsically good default model function in lattice QCD
    → risk of bias from default model function
  - Rather complicated analysis
    → difficult to check the results

When data has good quality
(e.g. T=0, good statistics), MEM works well.

Nakahara et al. ('99), Yamazaki et al. ('01)

Application to T>0

On T>0 lattices, N_t is restricted to $1/Ta_t$

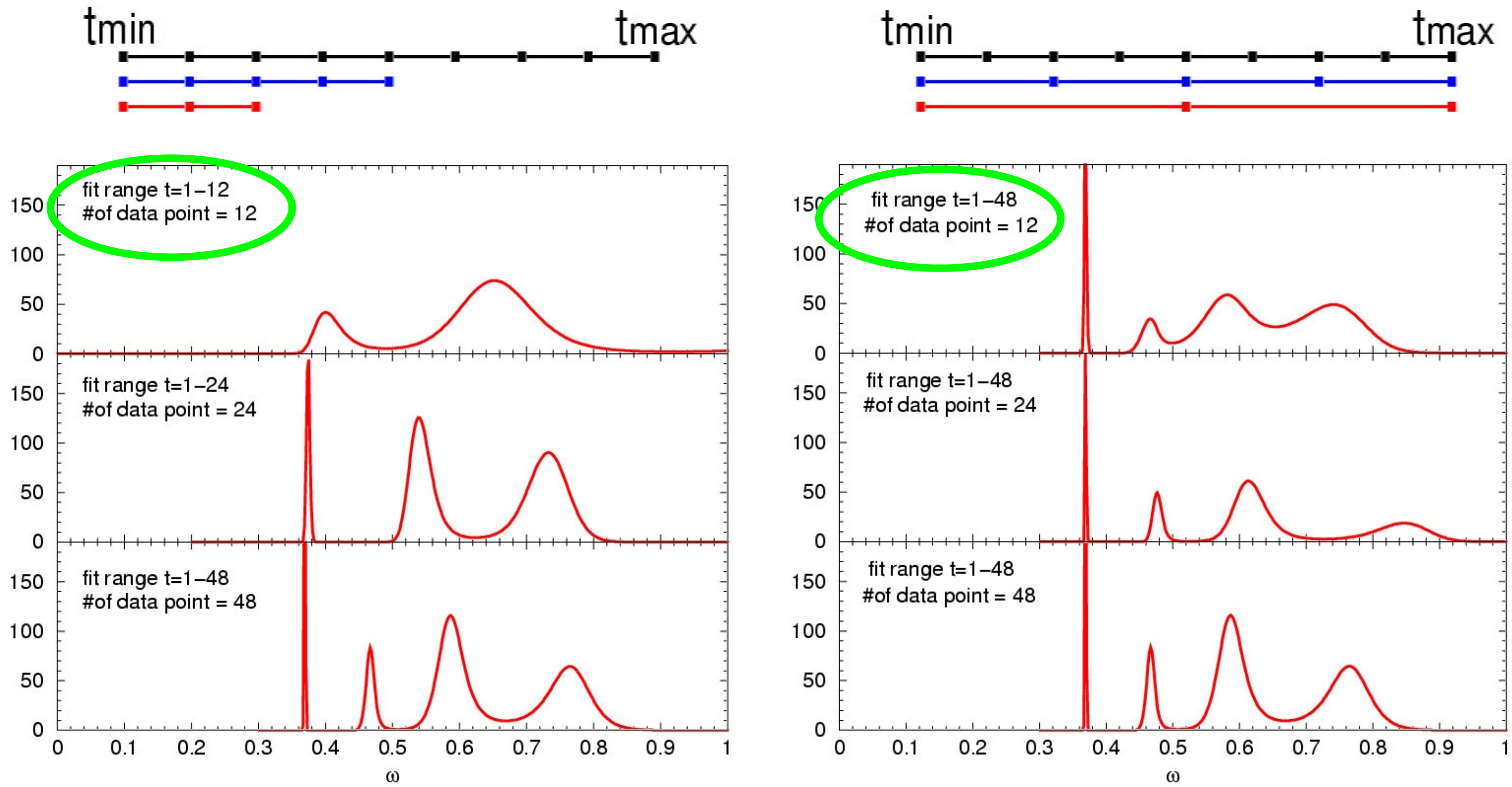
→We have to check
the reliability of the results

for example,

- using T=0 data
  in the same condition as T>0
- default model function dependence
- etc ...

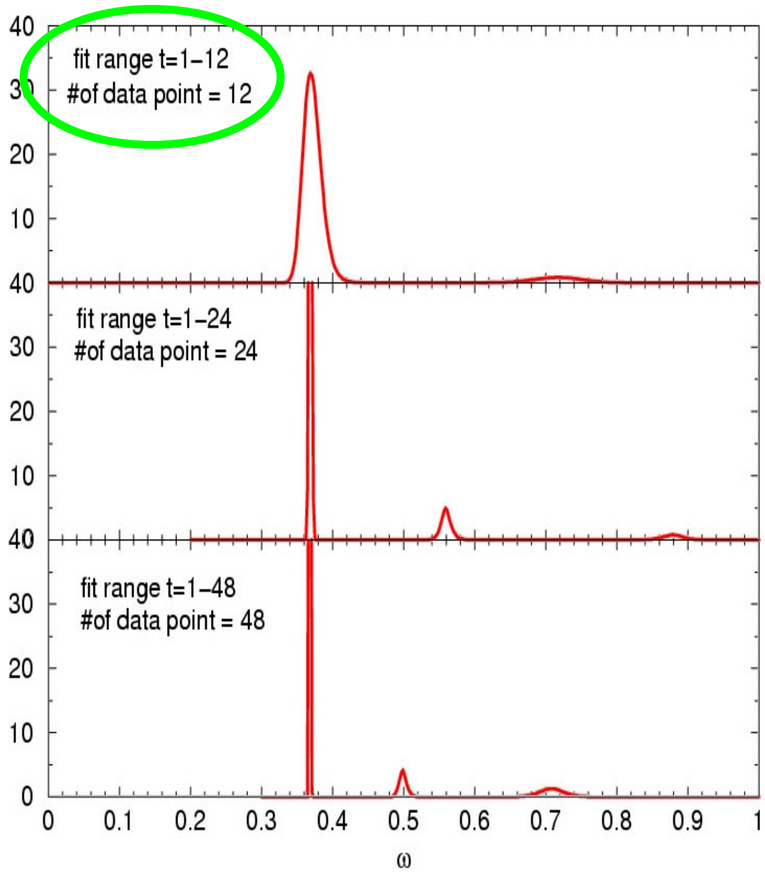
Before discussing our main results
we show these checks with our lattice data

Test with T=0 data



t_{max}=12 corresponds to data points at T≈1.2Tc

Test with T=0 data



- MEM does not work with small t_{max} (fit range $t = t_{min} - t_{max}$)
- Physical length is important rather than # of data points
- Smeared operators may improve this situation

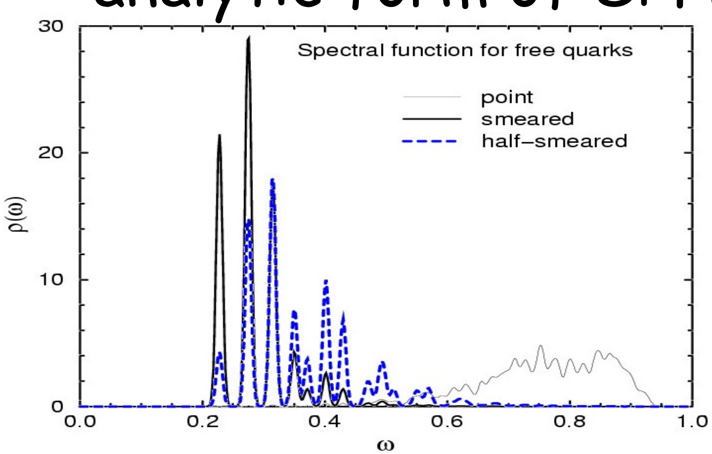
Problem of smeared operator

Smearing may cause mimic peak structure
→ we check it with several smeared operators

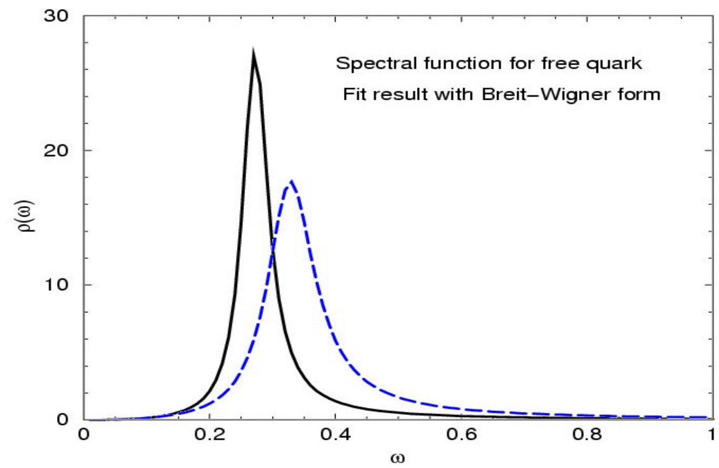
When the system has no bound state
the peak should be shifted against change of smearing

Case of free quarks

analytic form of SPFs

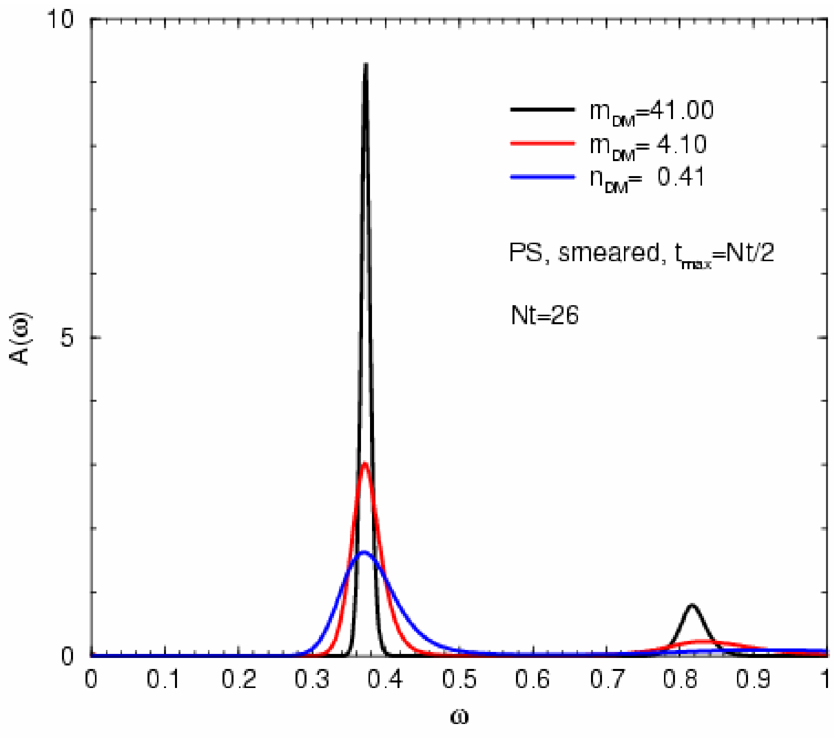


reconstructed SPFs



m(w) dependence

In this analysis we use the perturb. results for default model function $m(w)=m_{DM}w^2$



- peak position is stable
- width of SPFs is sensitive to m_{DM}

This result indicates that it is difficult to discuss width of SPFs

For quantitative study

We give up quantitative study using only MEM.

If we know a rough image of SPFs,
standard χ^2-fit (or constrained curve fit)
is appropriate for quantitative studies.

➔ We use MEM to find
a rough image (fit-form) of SPFs.

(Of course, it depends on lattice setup.)

Numerical results

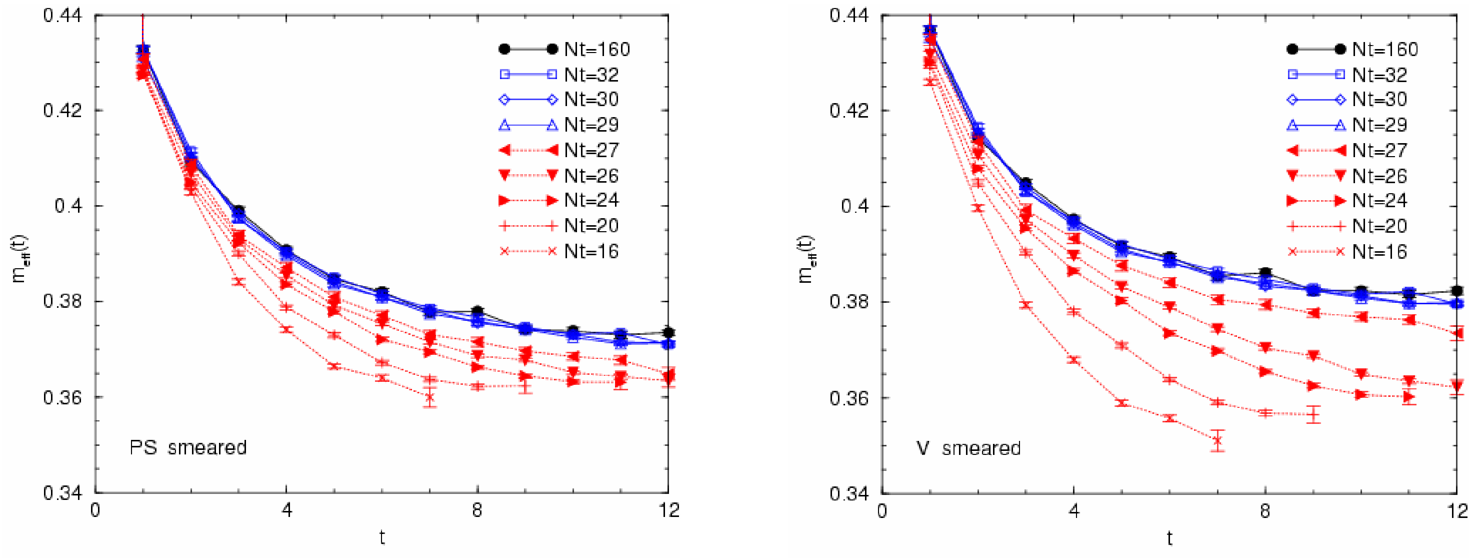
Nt	160	32	30	29	27	26	24	20	16
T/Tc	~0	0.88	0.93	0.97	1.04	1.08	1.17	1.40	1.75

statistics: 1000conf. x 16src. (500conf. for T=0)

Effective mass

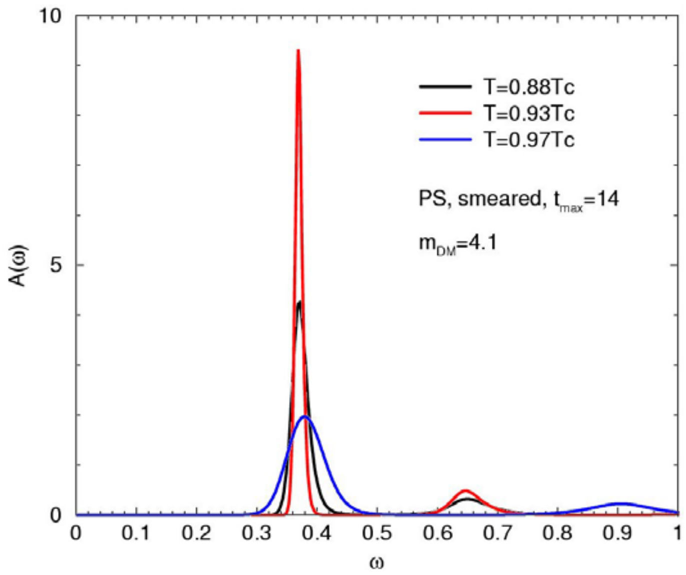
Temperature dependence of effective mass $m_{eff}(t)$

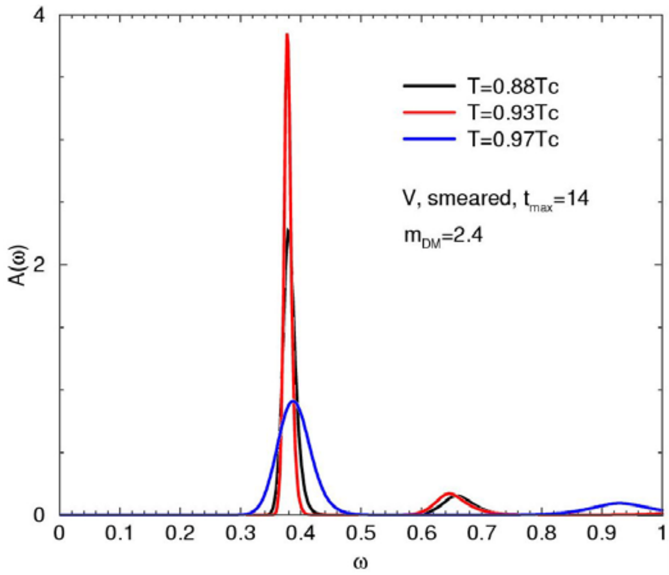
$$\frac{C(t)}{C(t+1)} = \frac{\cosh\left[m_{eff}(t)(N_t/2 - t)\right]}{\cosh\left[m_{eff}(t)(N_t/2 - t - 1)\right]}$$



- No change of correlators below T_c
- Significant change appears above T_c

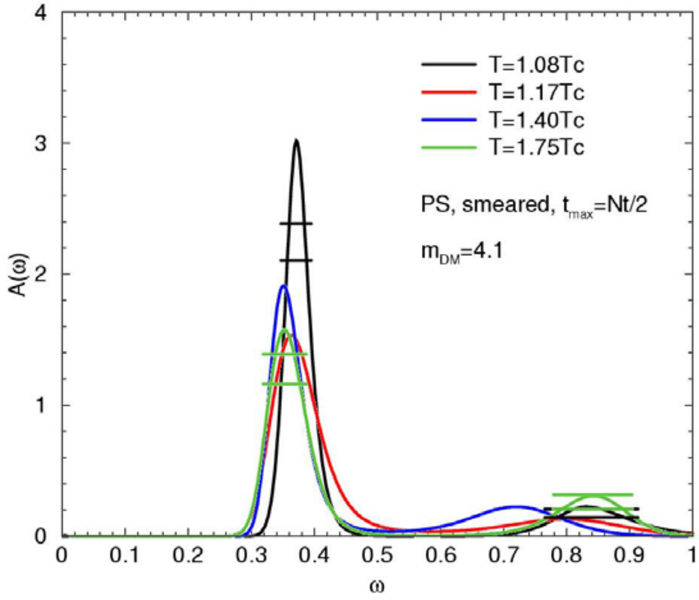
MEM result below Tc

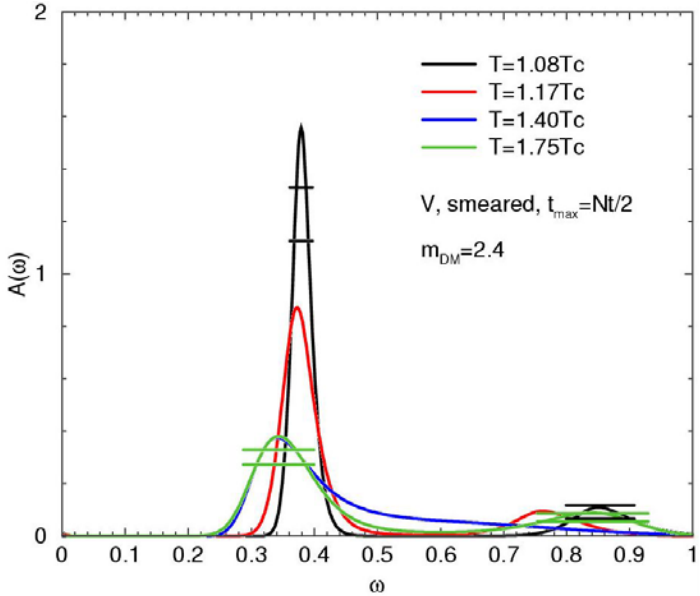




- correlators does not change below T_c
- pole-like structure → almost no width
- no mass shift from T=0

MEM result above Tc

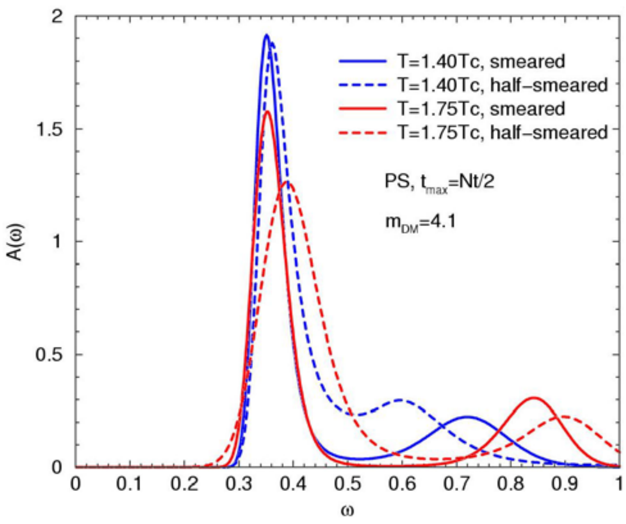


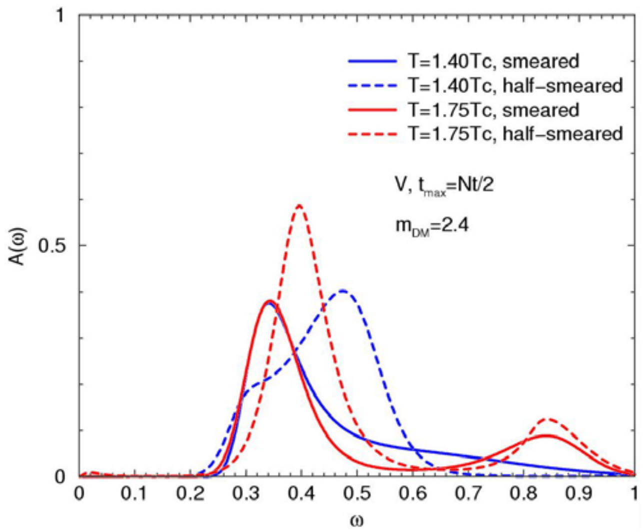


- correlators change gradually as T increases
- slightly wider structure than below T_c
- small or no mass shift from T=0
- vector channel (J/ψ) shows large change at high T

MEM result above Tc

comparison between smeared and half-smeared results at T/Tc=1.4 and 1.75 (Nt=20 and 16)





- in PS channel at Nt=20
  smeared & half-smeared results has similar peak
- in PS at Nt=16, V at Nt=20&16
  have similar behavior with free quark case

Constrained curve fitting (CCF)

A simple modification of standard least square fitting based on the Bayesian statistics

Lepage et al. ('02)

$$\chi^2 \to \chi^2_{arg} \equiv \chi^2 + \chi^2_{prior} \qquad \chi^2_{prior} \equiv \sum_i \frac{(c_i - \tilde{c}_i)^2}{\tilde{\sigma}^2_{c_i}}$$

c_i : fit parameters,

$\tilde{c}_i, \; \tilde{\sigma}_{c_i}$: input parameters as prior knowledge

- many-parameter fitting become stable
- bias from the input parameters (prior knowledge)

  → MEM results are suitable for prior knowledge

Function form ansatz

We suppose the shape of spectral function.
(MEM gives rough estimate of shape of spectral func.)

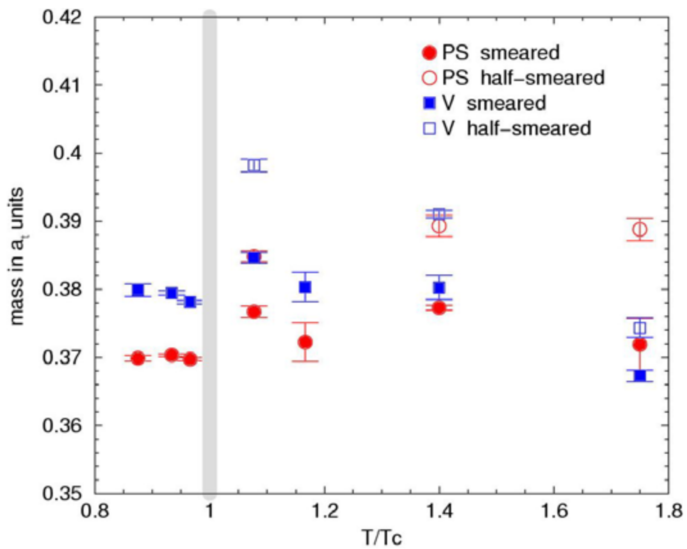
Breit-Wigner form:

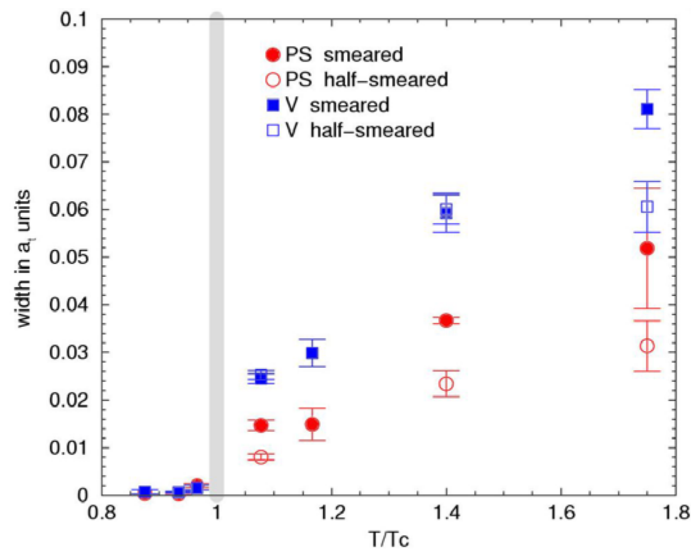
$$A(\omega) = \omega^2 \rho(\omega), \quad \rho(\omega) = \frac{C\Gamma m}{(\omega^2 - m^2)^2 - \Gamma^2 m^2}$$

C: overlap, m: mass, Γ: width

We apply multi-Breit-Wigner fit
using the Constrained curve fitting

Temperature dependence





CCF results has large systematic uncertainties from input parameters as prior knowledge

- small or no mass shift above and below T_c
- broad peak structure above T_c

Summary

We study spectral functions
from temporal charmonium correlators

We propose the analysis methods
MEM and fit analysis

- below Tc
  no mass shift and no width for PS and V channels
- above Tc
  peak structure at not so large T
  small or no mass shift
  finite width and grows as temperature increases

Notable points

- MEM does not solve ill-posed problem
  → MEM is a kind of
  constrained χ^2 fitting
  # of fit parameters < # of data points

- Prior knowledge of SPFs is needed
  as a default model function, m(w)
  The default model function
  plays crucial roles in MEM

Fit-form in MEM

using Singular Value Decomposition (SVD)

$$\begin{aligned} K(\tau,\omega) &= e^{-\omega\tau} + e^{-\omega(T-\tau)} \\ &= V(\tau,\tau')w(\tau',\tau'')U(\omega,\tau'')^t \end{aligned}$$

$w(\tau,\tau')$: diagonal matrix

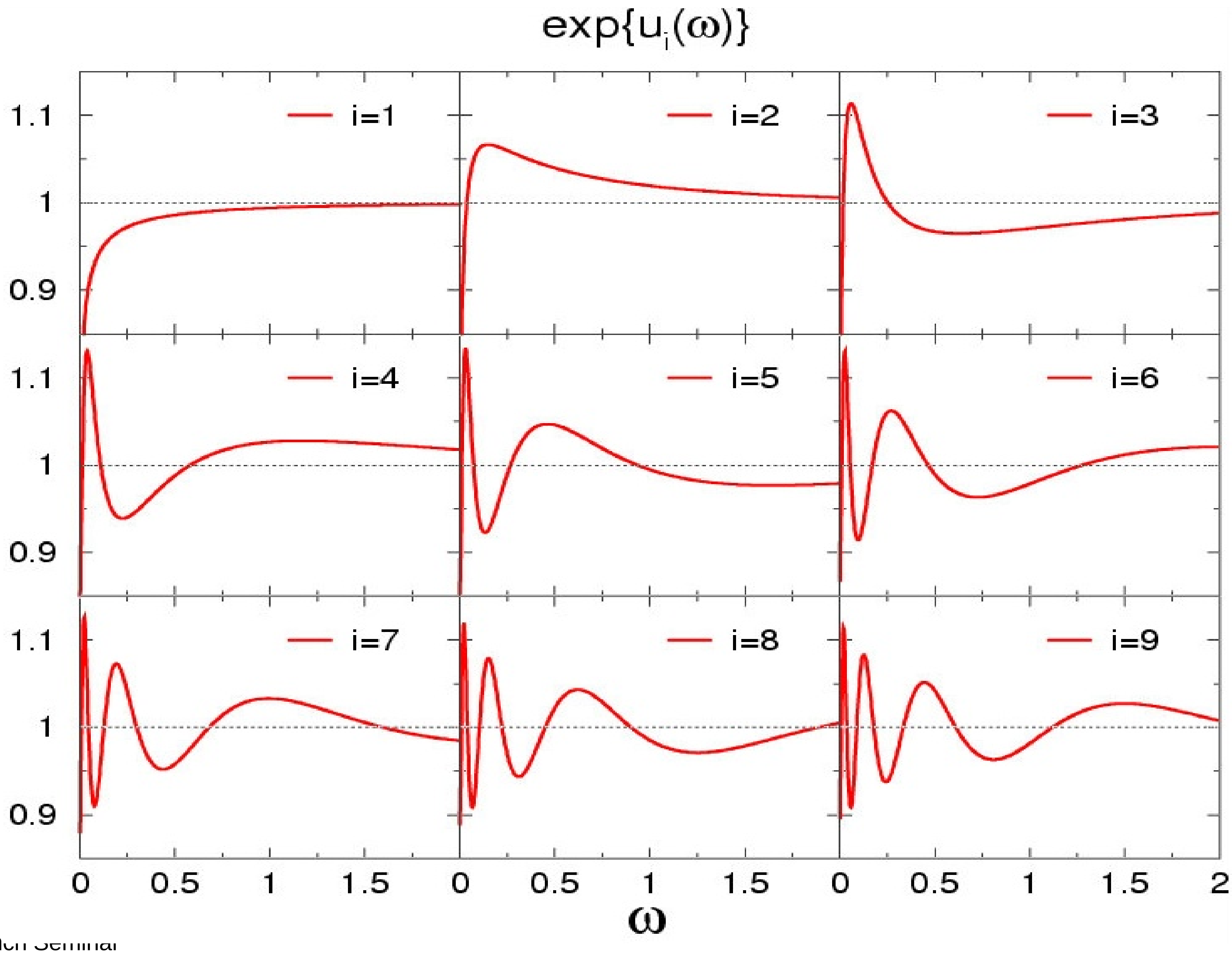
$u_i(\omega)(=U(\omega,\tau_i))$: basis in singular space

b_i : fit parameters

fit form for spactral function : A(w)

$$A(\omega) = m_0\omega^2 \prod_{i=1}^{N} \exp\{b_i, u_i(\omega)\}$$

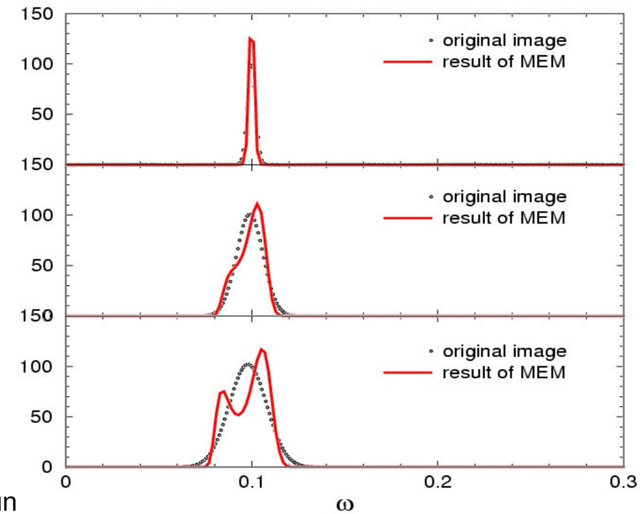
Singular Value Decomposition

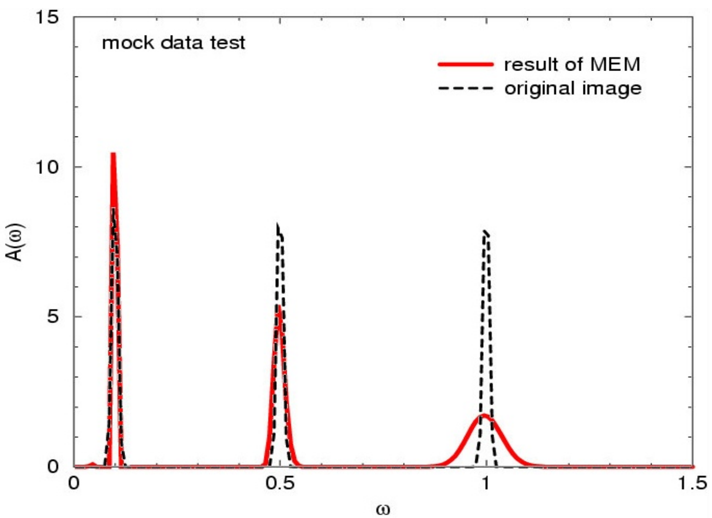


Singular Value Decomposition

- fit-form using SVD is suitable for SPFs but its resolution depends on energy ω.
- (sharp/broaden) peak at (low/high) energy region may be fake.

samples of mock data analysis





Quark action

$$S_q = \sum_{x,y} \bar{q}(x) K(x,y) q(y)$$

$$\begin{aligned} K(x,y) &= \delta_{xy} - \kappa_\tau \left[(1-\gamma_4) U_4(x) \delta_{x+\hat{4},y} + (1+\gamma_4) U_4(x-\hat{4}) \delta_{x-\hat{4},y} \right] \\ &\quad -\kappa_\sigma \sum_{i=1}^{3} \left[(r-\gamma_i) U_i(x) \delta_{x+\hat{i},y} + (r+\gamma_i) U_i(x-\hat{i}) \delta_{x-\hat{i},y} \right] \\ &\quad -\kappa_\sigma c_E \sum_{i=1}^{3} \sigma_{4i} F_{4i}(x) \delta_{x,y} - \kappa_\sigma c_B \sum_{i>j=1}^{3} \sigma_{ij} F_{ij}(x) \delta_{x,y}. \end{aligned}$$

- Constructed following the Fermilab approach
  El-Khadra et al. (97)
- $r = 1/\xi$ (action retains explicit Lorentz inv. form)
- tree-level Tadpole improved

Singular value decomposition

for any $M \times N$ matrix ($M>=N$), A, can be written as

$$A = U W V^T$$

U: $M \times N$ orthogonal matrix
W: $N \times N$ diagonal matrix
with positive or zero elements
V: $N \times N$ orthogonal matrix