

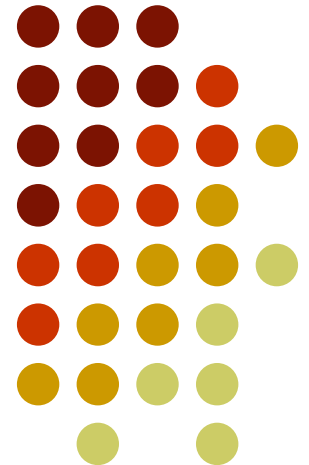
Constant mode and charmonium dissociation temperatures in finite temperature lattice QCD

Takashi Umeda



This talk is based on the Phys. Rev. D75 094502 (2007)
[hep-lat/0701005]

E,H,Qken joint seminar, Nagoya Univ., 24 July 2007



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- Overview
- J/ψ suppression
- Charmonium in Lattice QCD
- Sequential J/ψ suppression

■ Quenched QCD calculations

- Lattice setup
- T dependence of charmonium correlators
- Constant mode in meson correlators

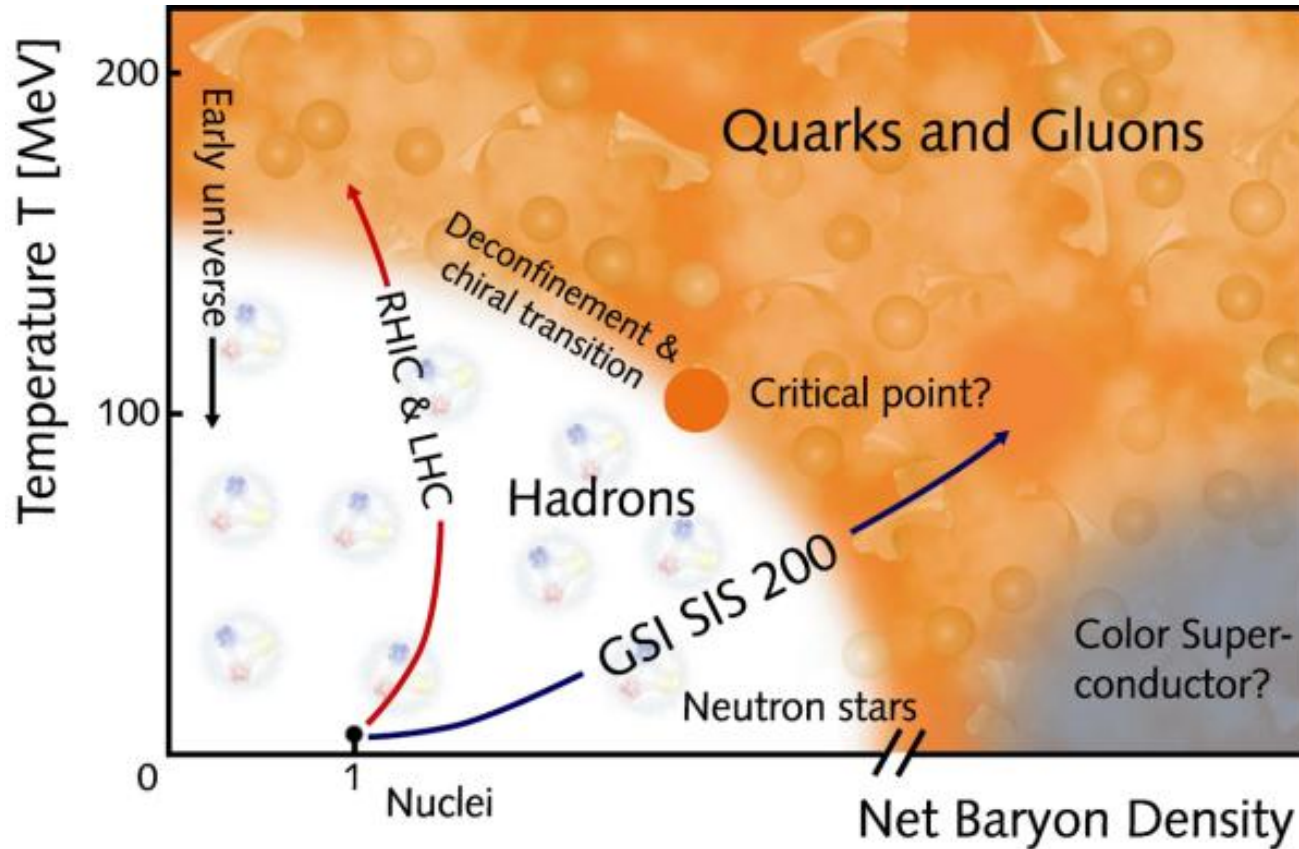
■ Discussion & Conclusion

total 36 pages



Quark Gluon Plasma (QGP)

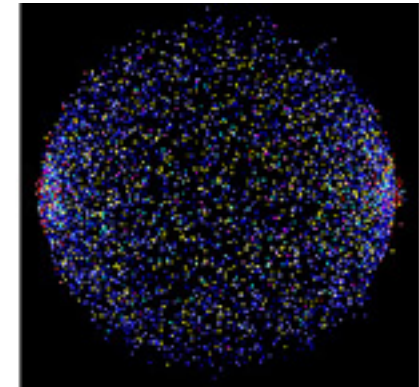
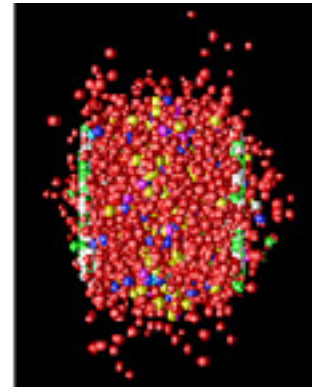
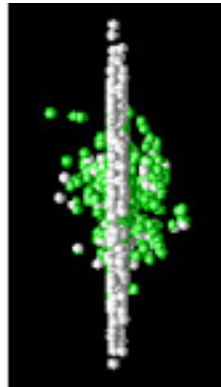
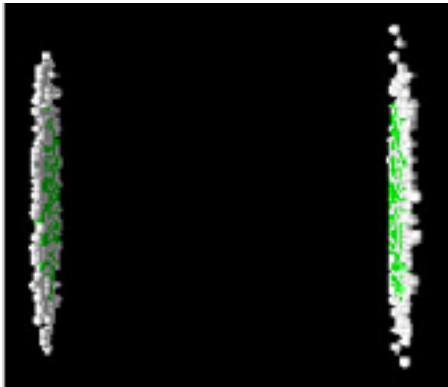
from Munster Univ. web-site



Experiments



- **SPS** : CERN (– 2005)
Super Proton Synchrotron
- **RHIC**: BNL (2000 –)
Relativistic Heavy Ion Collider
- **LHC** : CERN (2009 –)
Large Hadron Collider

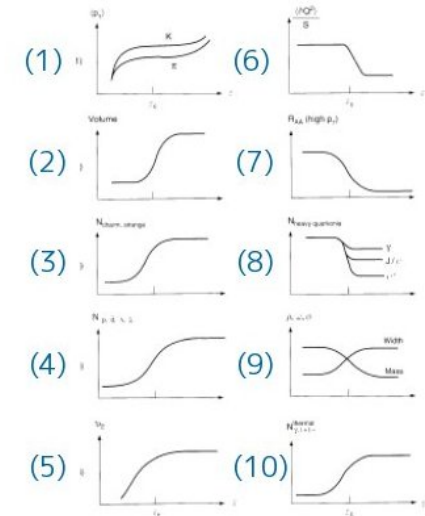


Signatures of QGP



Signatures of QGP

- (1) Average transverse momentum
- (2) Volume
- (3) Enhance of strangeness and charm
- (4) Enhance of anti-particles
- (5) Elliptic flow (v_2)
- (6) Fluctuations conserved charges
- (7) Suppression of high- p_T hadrons
- (8) Heavy quarkonium
- (9) Modification of light vector mesons
- (10) Thermal photons and dileptons



K. Yagi, T. Hatsuda, and Y. Miake, "Quark-Gluon Plasma"

2006/12/9

Heavy Ion Cafe

2

J/ψ suppression



*T.Hashimoto et al.,
PRL57 (1986) 2123.
T.Matsui & H.Satz,
PLB178 (1986) 416.*

Talk by Ozawa @ Heavy Ion Cafe

J/ψ suppression in Exp.

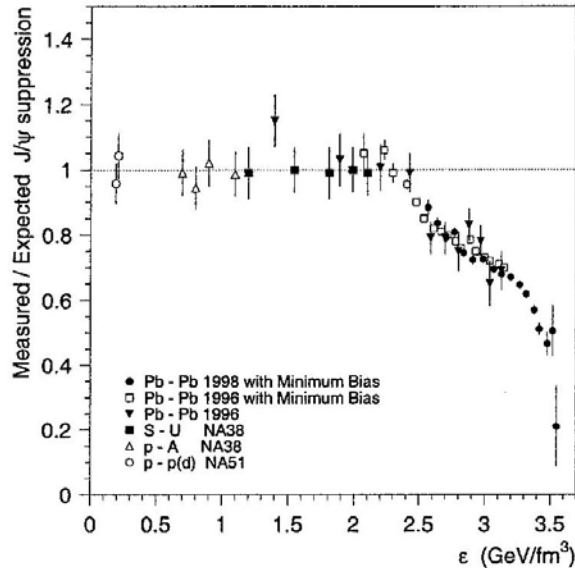


Fig. 7. Measured J/ψ production yields, normalised to the yields expected assuming that the only source of suppression is the ordinary absorption by the nuclear medium. The data is shown as a function of the energy density reached in the several collision systems.

Phys.Lett.B477(2000)28
NA50 Collaboration

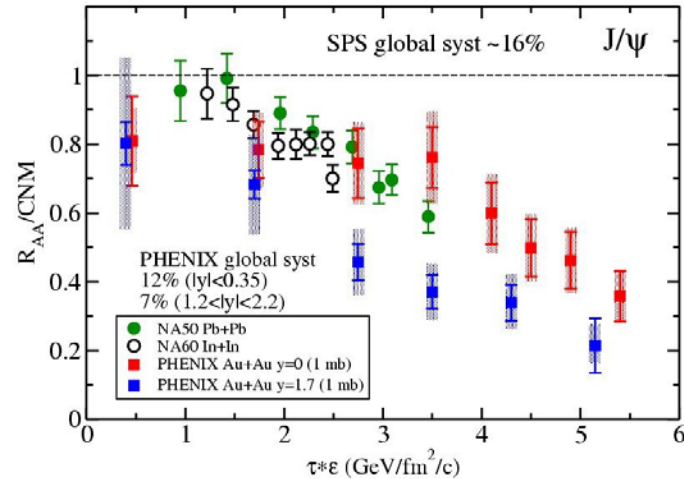


Figure 10: Survival fraction (R_{AA}/CNM) vs energy density comparison of PHENIX Au+Au suppression to that from NA38/50 at CERN.

QM2006
PHENIX Collaboration

“Charmonium states in QGP exist or not ?” from Lattice QCD



Charmonium in Lattice QCD

Lattice QCD enables us to perform
nonperturbative calculations of QCD

$$\langle X \rangle = \frac{1}{Z_{QCD}} \int Dq(x) D\bar{q}(x) DA_\mu(x) X(q, \bar{q}, A_\mu) e^{-S_{QCD}}$$

Path integral by MC integration

QCD action on a lattice

Wilson quark,
Staggered (KS) quark,
Domain Wall quark, etc

Input parameters (lattice setup) :

- (1) gauge coupling \rightarrow lattice spacing (a) \rightarrow continuum limit
- (2) quark masses
- (3) (Imaginary) time extent \rightarrow Temperature ($T=1/Nta$)

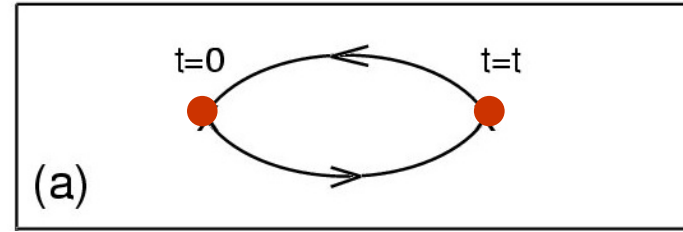


Charmonium correlation function

$$\langle X \rangle = \frac{1}{Z_{QCD}} \int Dq(x) D\bar{q}(x) DA_\mu(x) X(q, \bar{q}, A_\mu) e^{-S_{QCD}}$$

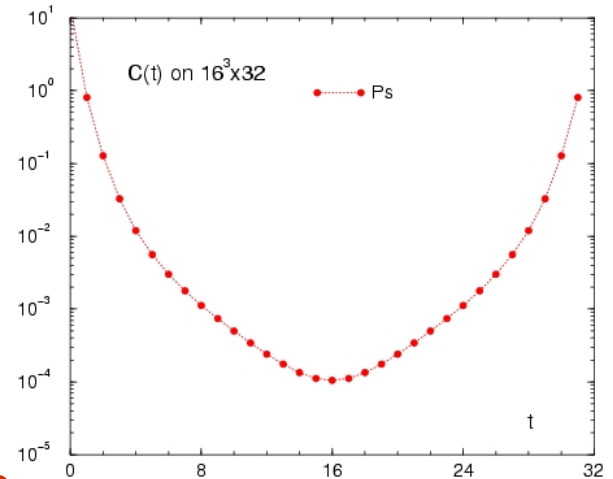
Correlation function

$$C(t) = \langle O(t) O^\dagger(0) \rangle$$



Charmonium operators

Pseudoscalar (Ps)	$J^{PC} = 0^{-+}$	η_c, \dots
Vector (V)	$J^{PC} = 1^{--}$	$J/\psi, \psi(2S), \dots$
Scalar (S)	$J^{PC} = 0^{++}$	χ_{c0}, \dots
Axialvector (Av)	$J^{PC} = 1^{++}$	χ_{c1}, \dots

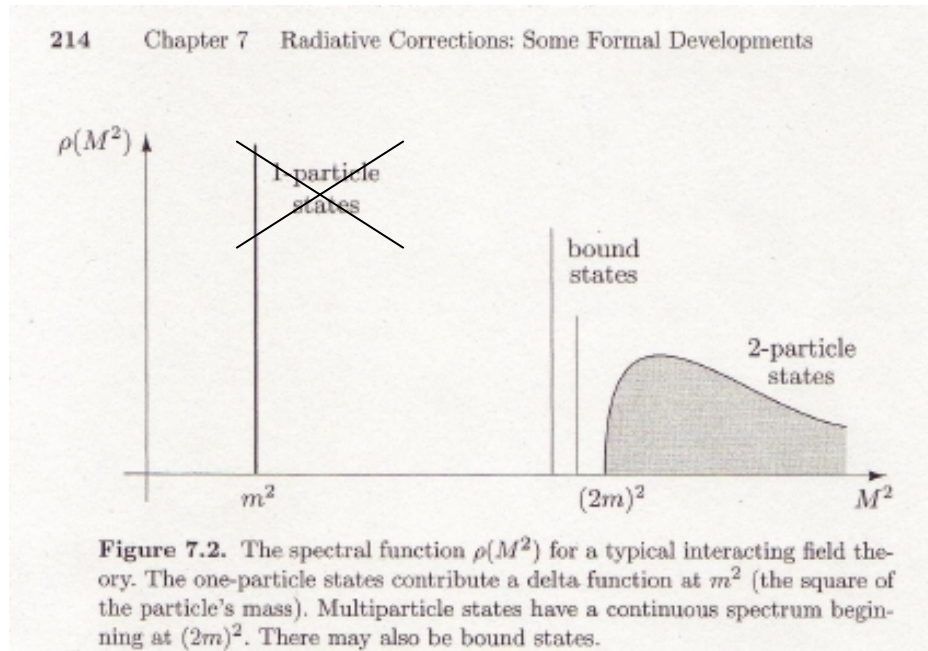


LQCD provides $C(t)$ of charmonium at $T > 0$

Charmonium spectral function



$$\begin{aligned} C(t) &= \langle O(t)O^\dagger(0) \rangle \\ &= \int d\omega \rho(\omega) \frac{e^{-\omega t} + e^{-\omega(N_t-t)}}{1 - e^{-\omega N_t}} \end{aligned}$$

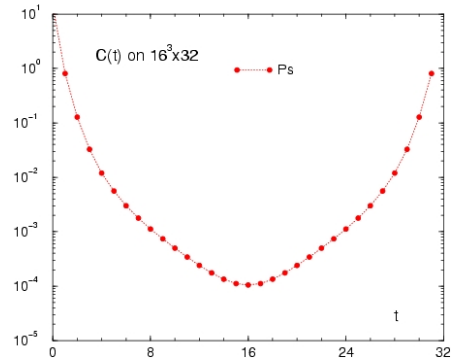
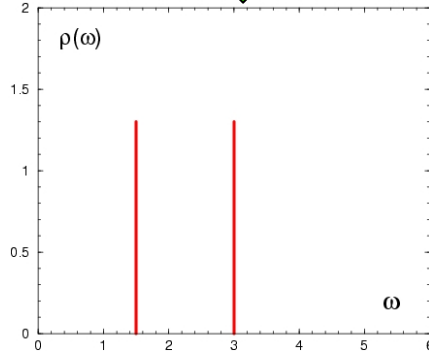
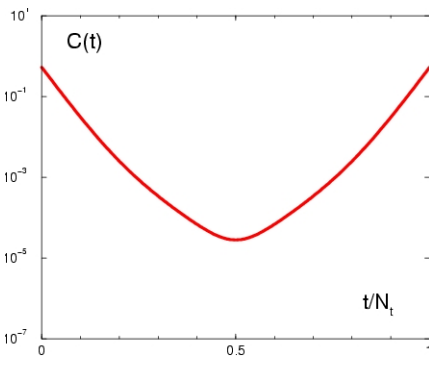
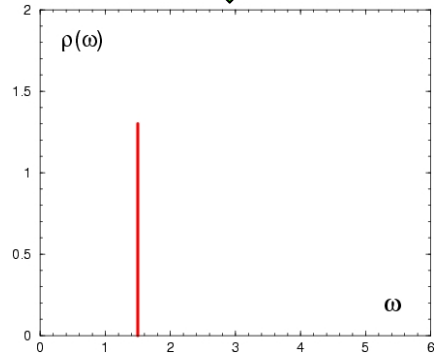
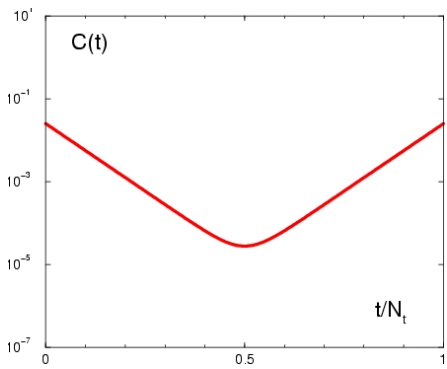
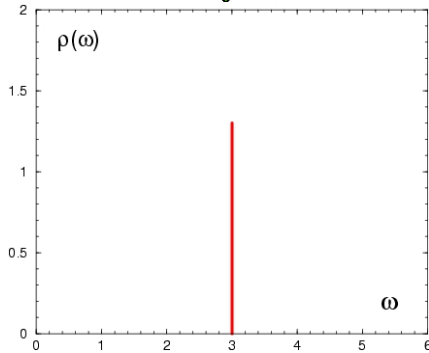
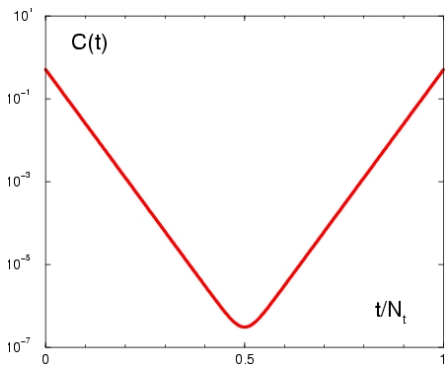


from “An Introduction to Quantum Field Theory”
Michael E. Peskin, Perseus books (1995)



Example of lattice results

$$\begin{aligned}
 C(t) &= \langle O(t)O^\dagger(0) \rangle \\
 &= \int d\omega \rho(\omega) \frac{e^{-\omega t} + e^{-\omega(N_t-t)}}{1 - e^{-\omega N_t}}
 \end{aligned}$$

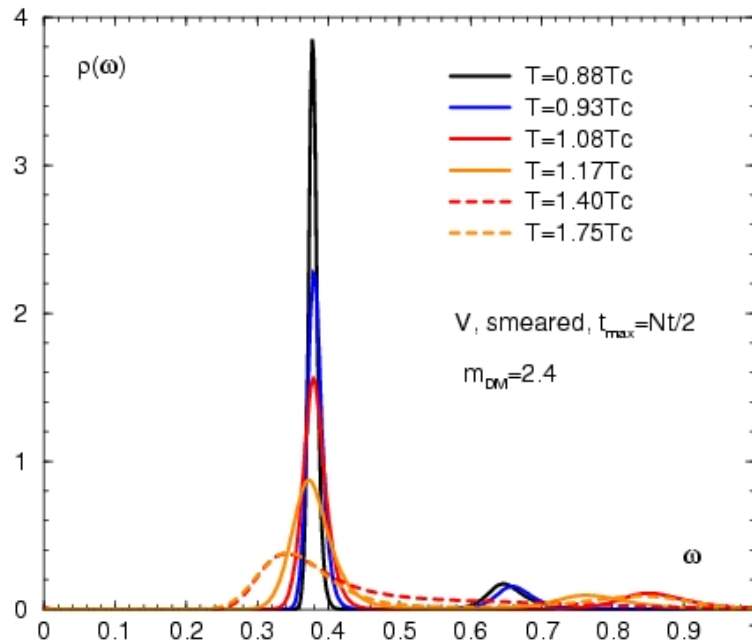




Charmonium spectral function

$$\begin{aligned} C(t) &= \langle O(t)O^\dagger(0) \rangle \\ &= \int d\omega \rho(\omega) \frac{e^{-\omega t} + e^{-\omega(N_t-t)}}{1 - e^{-\omega N_t}} \end{aligned}$$

Maximum entropy method
 $C(t) \rightarrow \rho(\omega)$



Quenched QCD

- *T. Umeda et al., EPJC39S1, 9, (2005).*
- *S. Datta et al., PRD69, 094507, (2004).*
- *T. Hatsuda & M. Asakawa, PRL92, 012001, (2004).*
- *A. Jakovac et al., PRD75, 014506 (2007).*

Full QCD

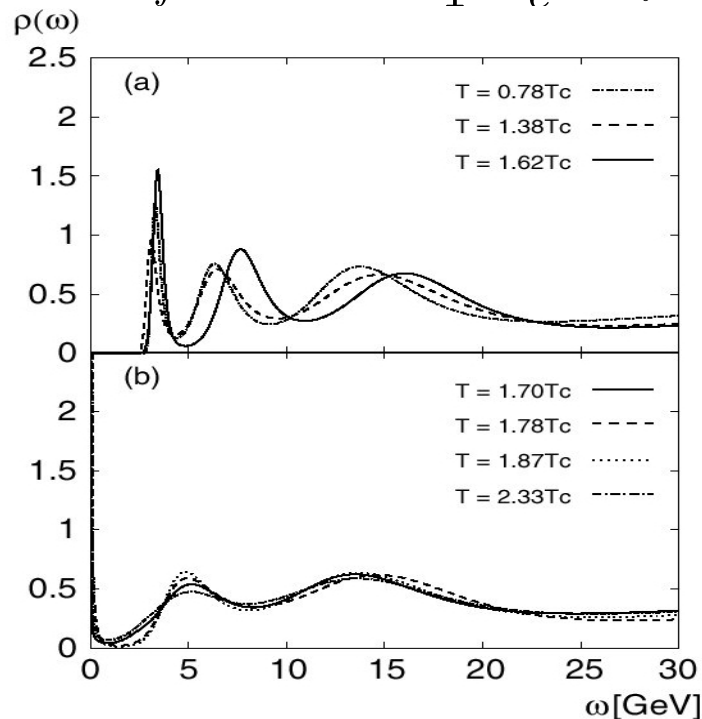
- *G. Aarts et al., hep-lat/0610065.*



Charmonium spectral function

$$C(t) = \langle O(t)O^\dagger(0) \rangle$$
$$= \int d\omega \rho(\omega) \frac{e^{-\omega t} + e^{-\omega(N_t-t)}}{1 - e^{-\omega N_t}}$$

Maximum entropy method
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Quenched QCD

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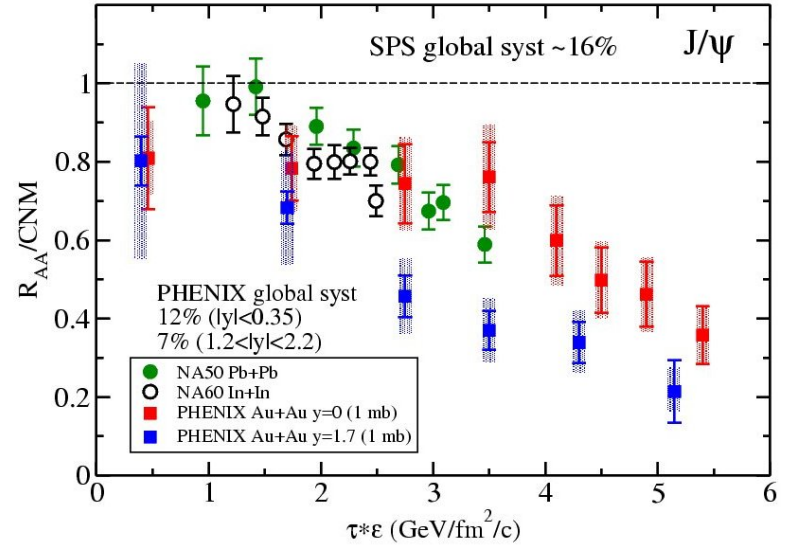
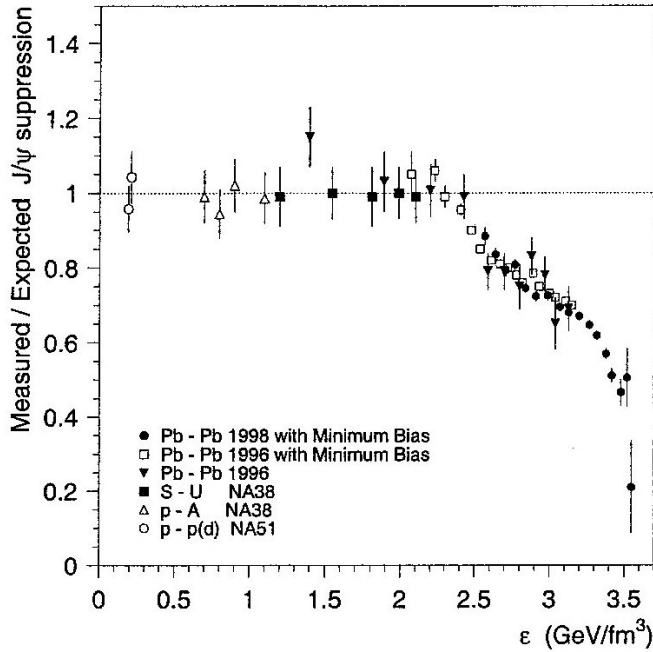
Full QCD

- G. Aarts et al.,
hep-lat/0610065.

All studies indicate survival of J/ψ state above T_c ($1.5T_c$?)



J/ψ suppression in Exp. (?)



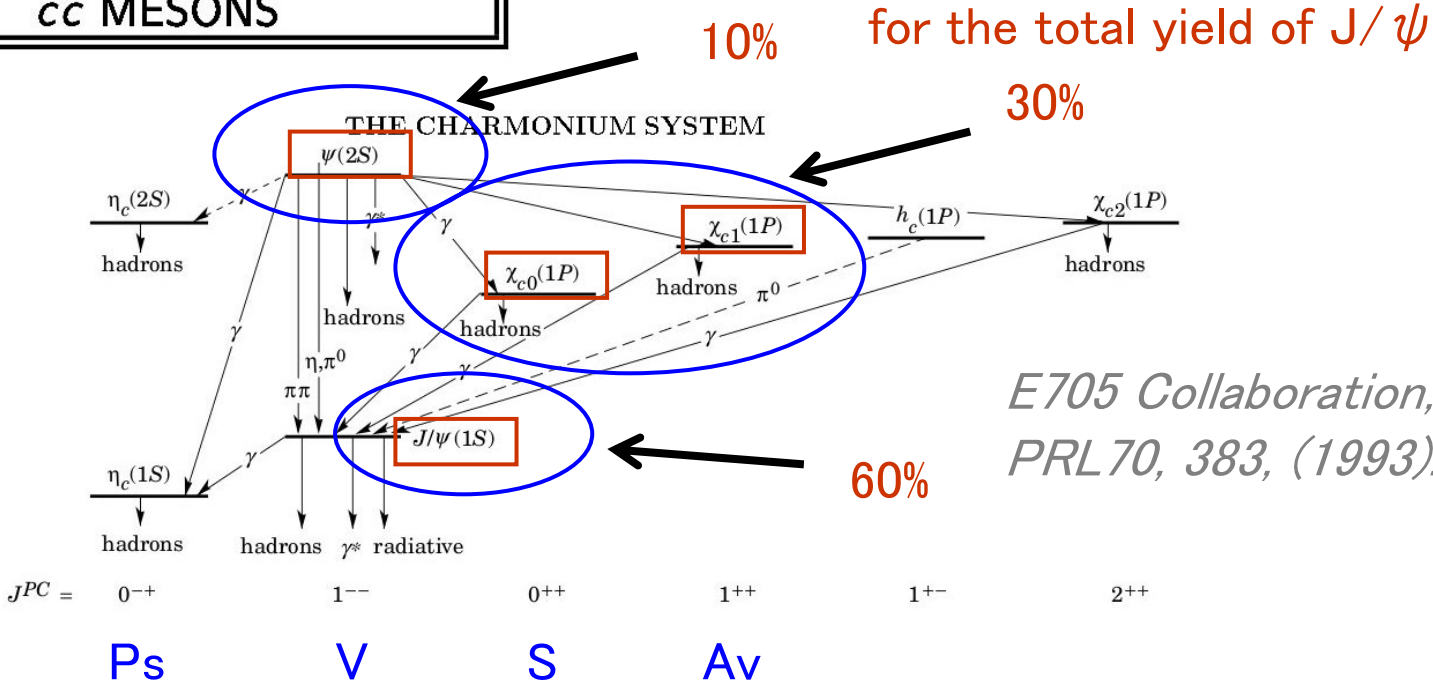
Energy density $0.5-1.5 \text{ GeV}/\text{fm}^3 = 1.0 T_c$
 $10 \text{ GeV}/\text{fm}^3 = 1.5 T_c$
 $30 \text{ GeV}/\text{fm}^3 = 2.0 T_c$



Sequential J/ψ suppression

Particle Data Group (2006)

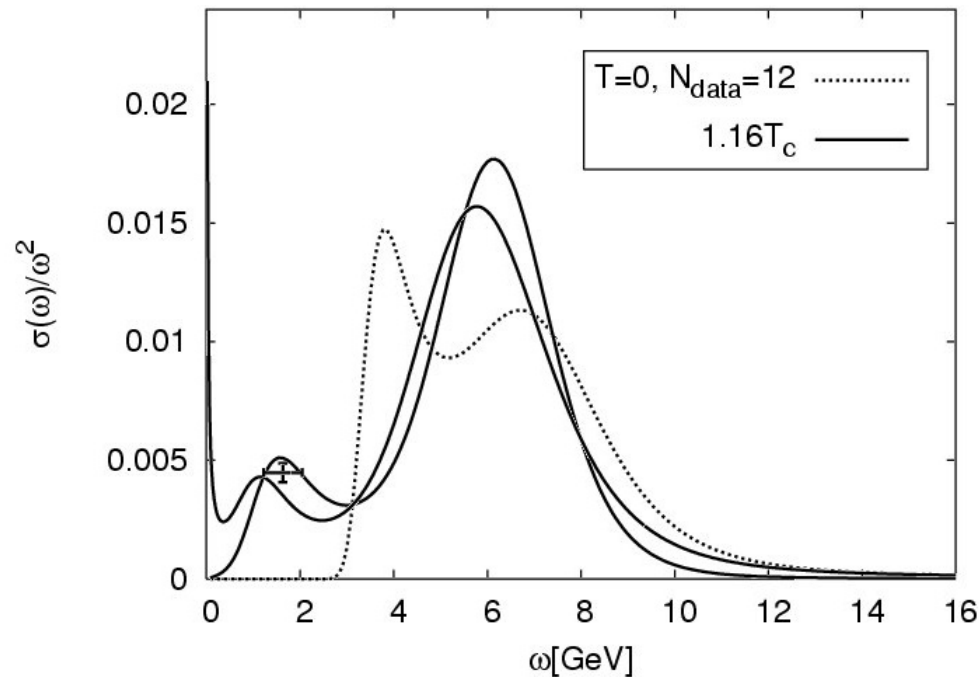
$c\bar{c}$ MESONS



E705 Collaboration, PRL 70, 383, (1993).

Dissociation temperatures of J/ψ and ψ' & χ_c are important for QGP phenomenology.

χ_c states on the lattice



*S. Datta et al.,
PRD69, 094507 (2004).
A. Jakovac et al.,
PRD75, 014506 (2007).*



FIG. 19: The scalar spectral function for $\beta = 6.1$ at $T = 1.16T_c$ and at zero temperature reconstructed using $N_{data} = 12$. At finite temperature two default models $m(\omega) = 0.01$ and $m(\omega) = 0.038\omega^2$ have been used.

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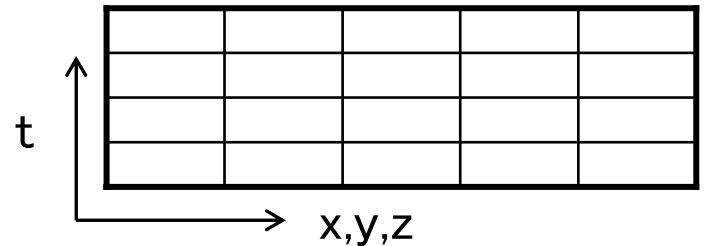
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Lattice QCD results



Lattice setup

- Quenched approximation (no dynamical quark effect)
- Anisotropic lattices
 - lattice size : $20^3 \times N_t$
 - lattice spacing : $1/a_s = 2.03(1) \text{ GeV}$,
 - anisotropy : $a_s/a_t = 4$
- Quark mass
 - charm quark (tuned with J/ψ mass)



N_τ	160	32	26	20
T/T_c	~ 0	0.88	1.08	1.4
# of conf.	60	300	300	300

Effective mass (local mass)



Definition of effective mass

$$\begin{aligned} C(t) &= A_0 e^{-m_0 t} + A_1 e^{-m_1 t} + \dots \\ &\equiv A e^{-m_{eff}(t)t} \end{aligned}$$

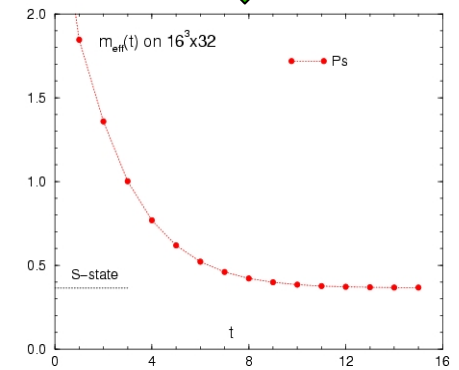
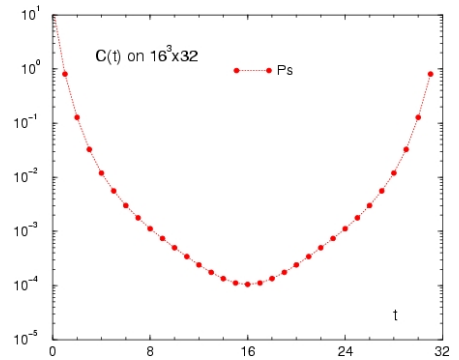
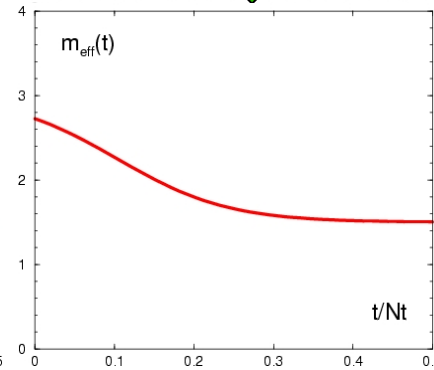
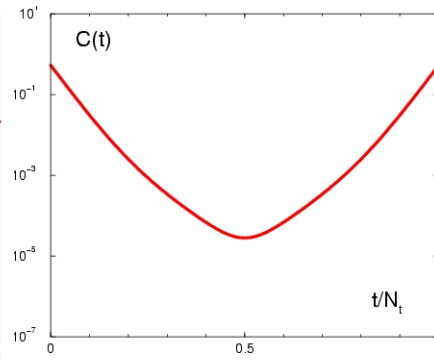
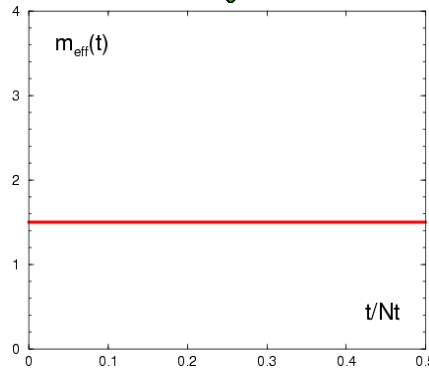
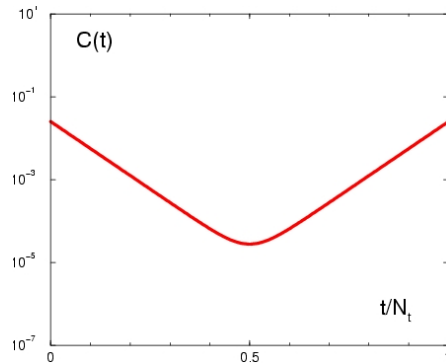
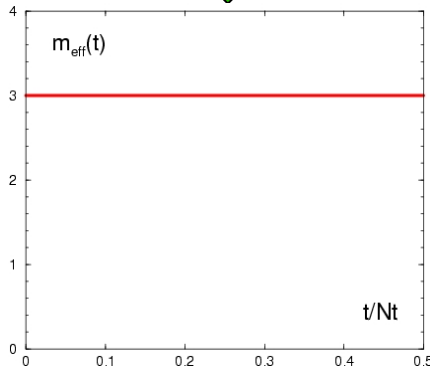
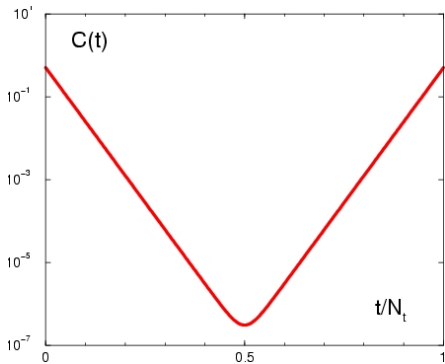
$$\frac{C(t)}{C(t+1)} = e^{-m_{eff}(t)}$$

$$m_{eff}(t) \rightarrow m_0 \quad \text{when} \quad (m_1 - m_0)t \gg 1$$

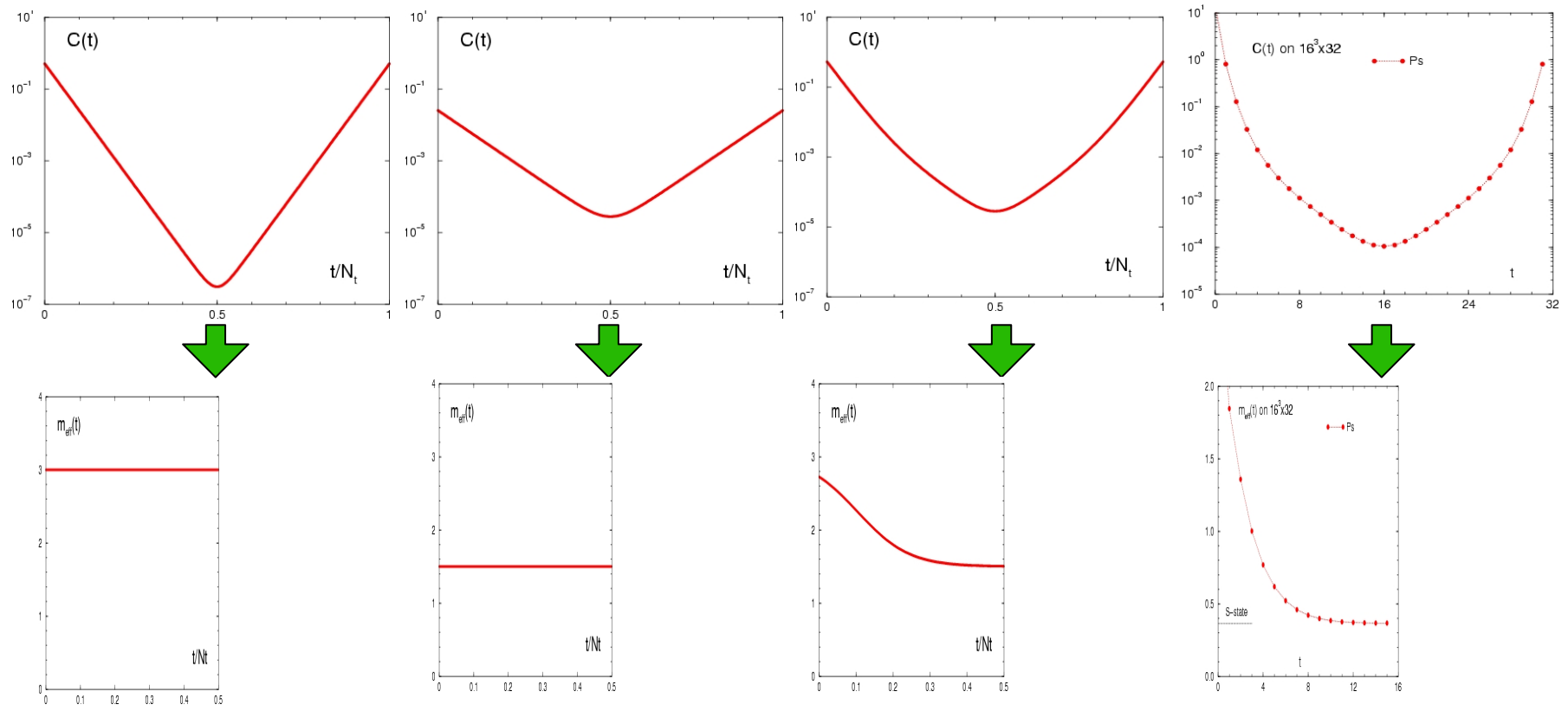
In the (anti) periodic b.c.

$$\frac{C(t)}{C(t+1)} = \frac{\cosh[m_{eff}(t)(N_t/2 - t)]}{\cosh[m_{eff}(t)(N_t/2 - t - 1)]}$$

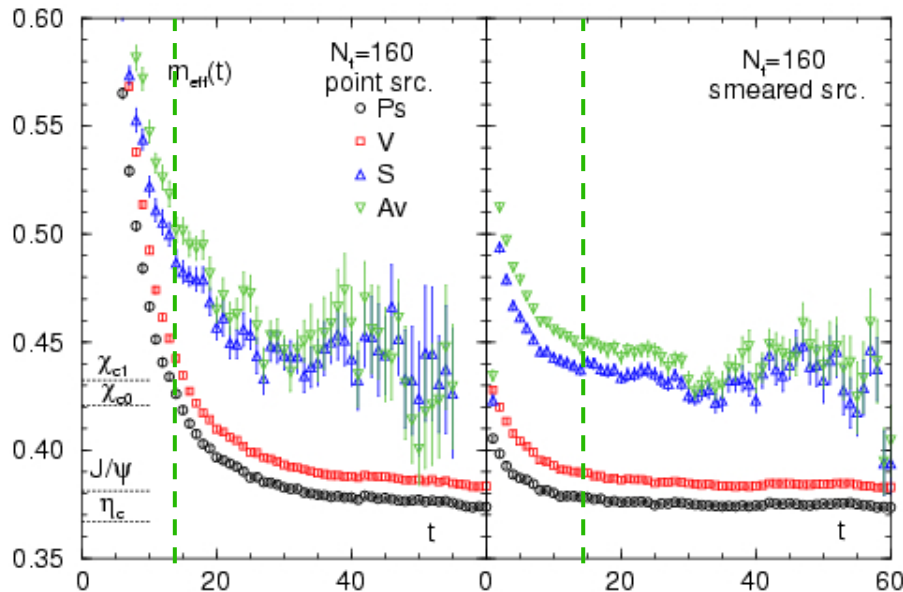
Effective mass (local mass)



Effective mass (local mass)



At zero temperature

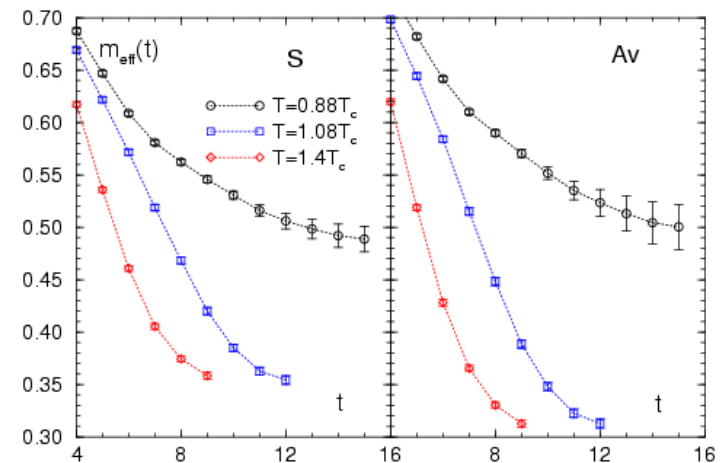
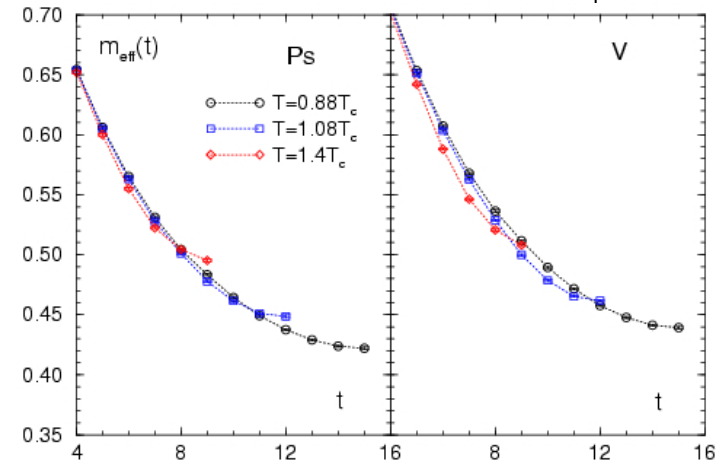
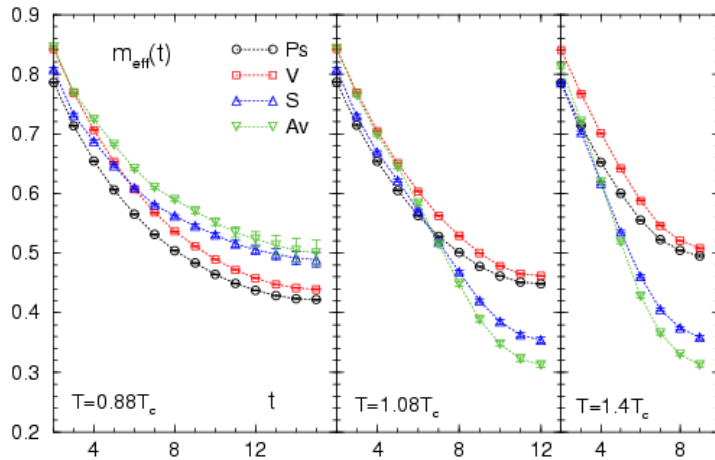


(our lattice results)
 $M_{PS} = 3033(19) \text{ MeV}$
 $M_V = 3107(19) \text{ MeV}$

(exp. results from PDG06)
 $M_{\eta_c} = 2980 \text{ MeV}$
 $M_{J/\psi} = 3097 \text{ MeV}$
 $M_{\chi_{c0}} = 3415 \text{ MeV}$
 $M_{\chi_{c1}} = 3511 \text{ MeV}$

- In our lattice $N_t \simeq 28$ at T_c
 $t = 1 - 14$ is available near T_c
- Spatially extended (smeared) op. is discussed later

Quenched QCD at $T > 0$

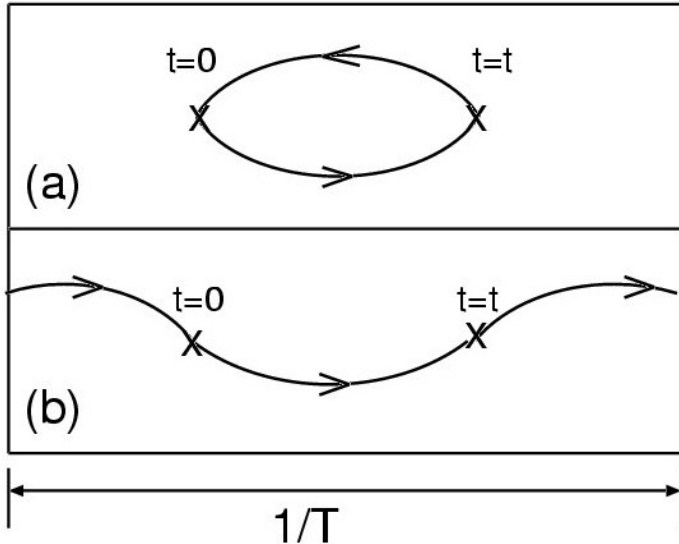


- small change in S-wave states
→ survival of J/ψ & η_c at $T > T_c$
- drastic change in P-wave states
→ dissociation of χ_c just above T_c (?)

*S. Datta et al.,
PRD69, 094507 (2004). etc...*



Constant mode



Pentaquark (KN state):
 two pion state:
 → Dirichlet b.c.
*c.f. T.T.Takahashi et al.,
 PRD71, 114509 (2005).*

$$\exp(-m_q t) \times \exp(-m_q t) = \exp(-2m_q t)$$

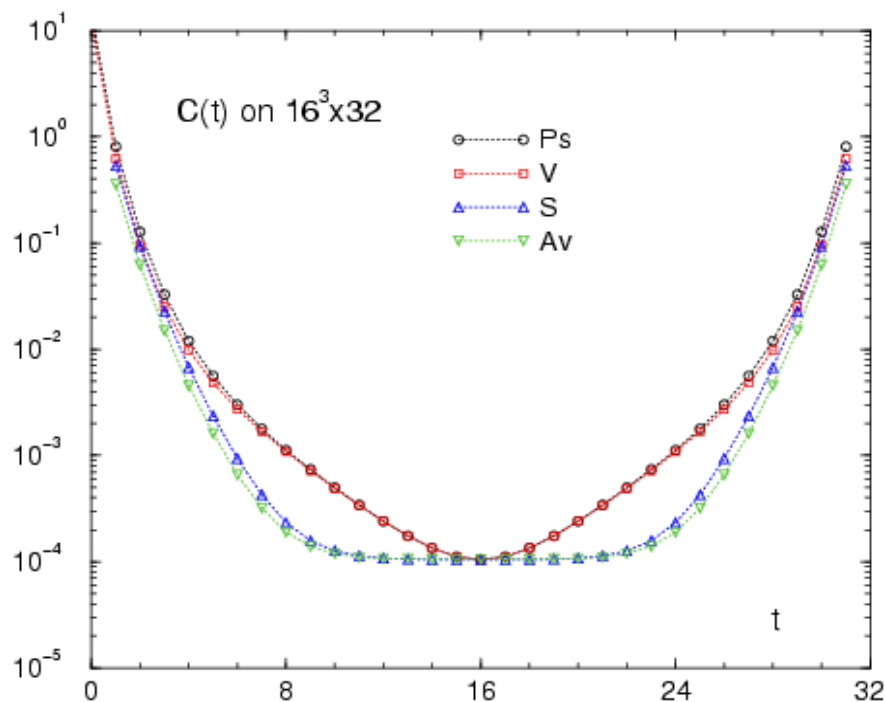
m_q is quark mass
 or single quark energy

$$\exp(-m_q t) \times \exp(-m_q(L_t - t)) = \exp(-m_q L_t)$$

$L_t =$ temporal extent

- in imaginary time formalism
 $L_t = 1/Temp.$
 gauge field : periodic b.c.
 quark field : anti-periodic b.c.
- in confined phase: m_q is infinite
 → the effect appears
 only in deconfined phase

Free quark calculations

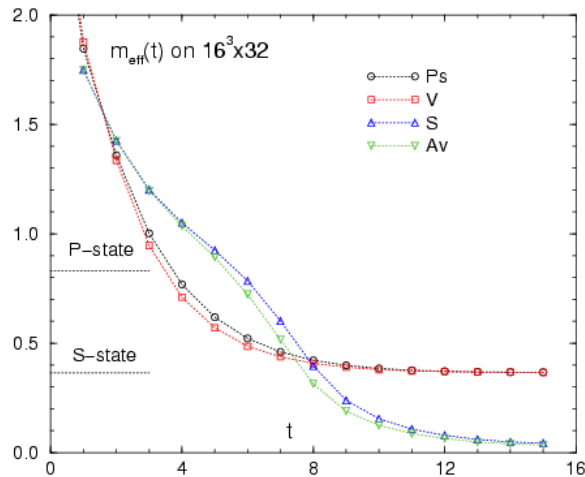
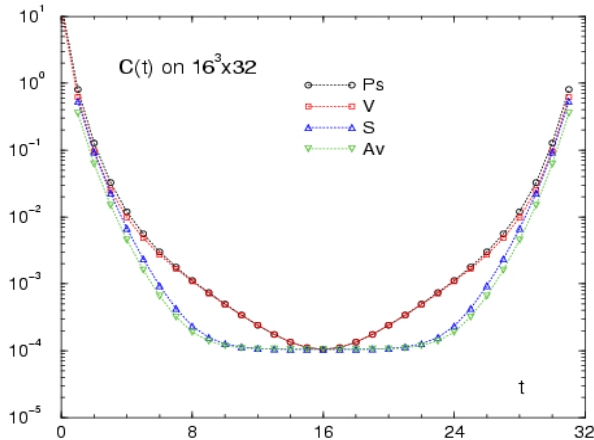


- $16^3 \times 32$ isotropic lattice
- Wilson quark action
with $m_q a = 0.2$

Obvious constant contribution
in P-wave states



Free quark calculations



Continuum form of the correlators
calculated by S. Sasaki

$$C(t) = \sum_{\vec{p}} \frac{4}{\cosh(E_p N_t / 2)} \times \left\{ \begin{array}{ll} (E_p^2 \cosh[2E_p(t - N_t/2)]) & \text{for } \Gamma = \gamma_5 \\ ((E_p^2 - p_i^2) \cosh[2E_p(t - N_t/2)] + p_i^2) & \text{for } \Gamma = \gamma_i \\ -(p^2 \cosh[2E_p(t - N_t/2)] + (E_p^2 - p^2)) & \text{for } \Gamma = 1 \\ -((p^2 - p_i^2) \cosh[2E_p(t - N_t/2)] + (E_p^2 - p^2 + p_i^2)) & \text{for } \Gamma = \gamma_i \gamma_5 \end{array} \right.$$

where

E_p : single quark energy with relative mom. p

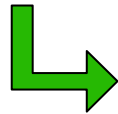
$$p^2 = \sum_i p_i^2$$

Physical interpretation



$$\begin{aligned} \rho_{\Gamma}(\omega) = & \Theta(\omega^2 - 4m_q^2) \frac{N_c}{8\pi\omega} \sqrt{\omega^2 - 4m_q^2} [1 - 2n_F(\omega/2)] \\ & \times [\omega^2 (a_H^{(1)} - a_H^{(2)}) + 4m^2 (a_H^{(2)} - a_H^{(3)})] \\ & + 2\pi\omega\delta(\omega) N_c [(a_H^{(1)} + a_H^{(2)}) I_1 + (a_H^{(2)} - a_H^{(3)}) I_2] \end{aligned}$$

*F. Karsch et al.,
PRD68, 014504 (2003).
G. Aarts et al.,
NPB726, 93 (2005).*



constant contribution remains
in the continuum form & infinite volume

The constant term is related to some transport coefficients.

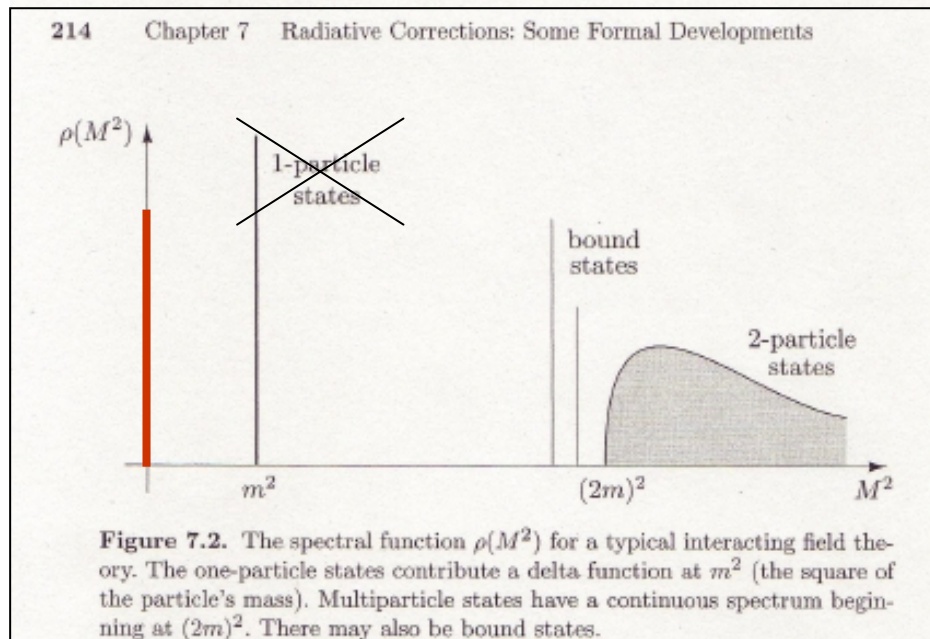
From Kubo-formula, for example, a derivative of the SPF
in the V channel is related to the electrical conductivity σ .

$$\sigma = \frac{1}{6} \frac{\partial}{\partial \omega} \rho_V(\omega) \Big|_{\omega=0}$$



Without constant mode

How much does SPF change at the region $\omega \gg T$



*from "An Introduction to Quantum Field Theory"
Michael E. Peskin, Perseus books (1995)*

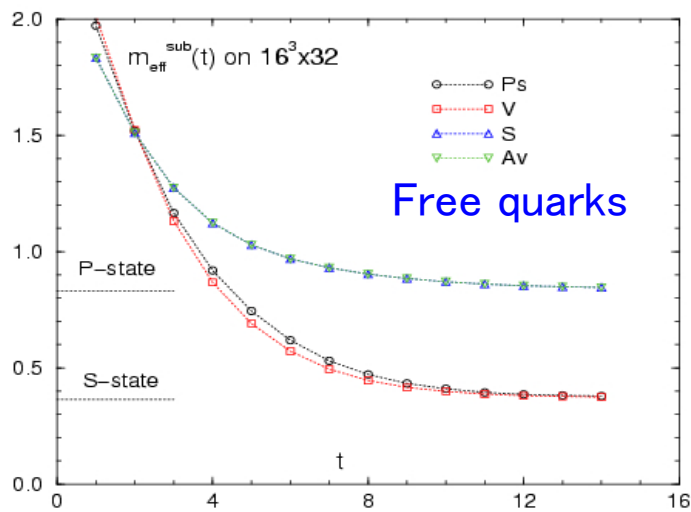
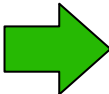
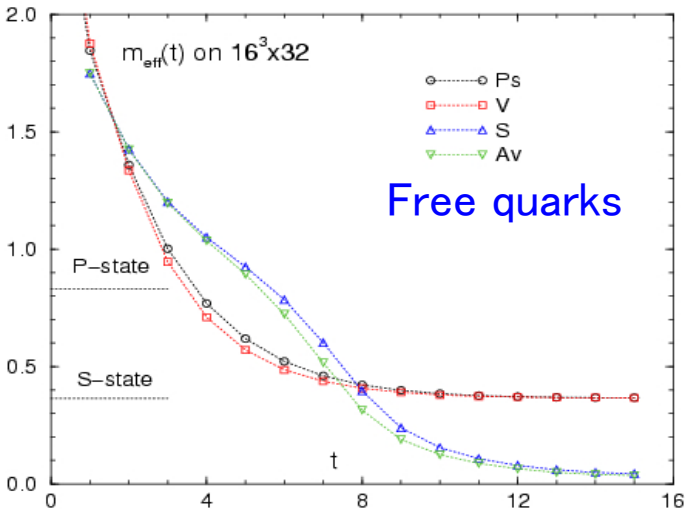


Midpoint subtraction

An analysis to avoid the constant mode

Midpoint subtracted correlator

$$\bar{C}(t) = C(t) - C(N_t/2) \quad \rightarrow \quad \frac{\bar{C}(t)}{\bar{C}(t+1)} = \frac{\sinh^2 \left[\frac{1}{2} m_{eff}^{sub}(t) (N_t/2 - t) \right]}{\sinh^2 \left[\frac{1}{2} m_{eff}^{sub}(t) (N_t/2 - t - 1) \right]}$$



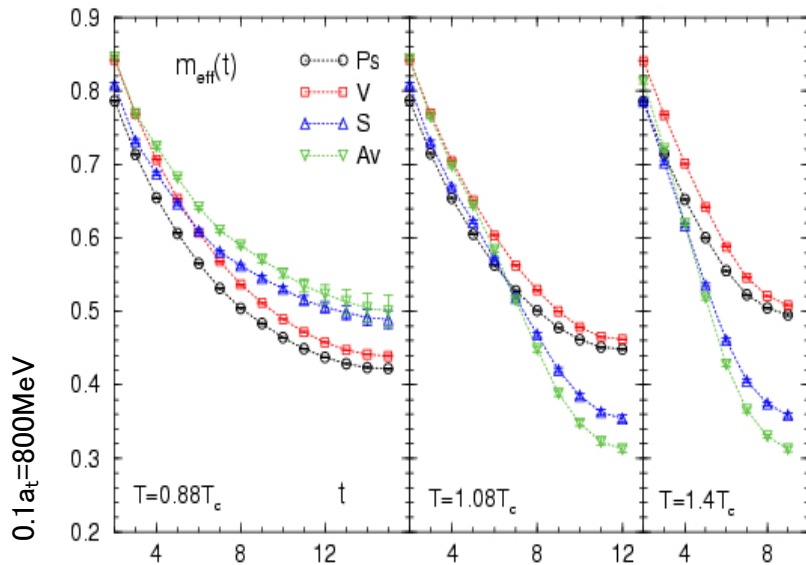


Midpoint subtraction analysis

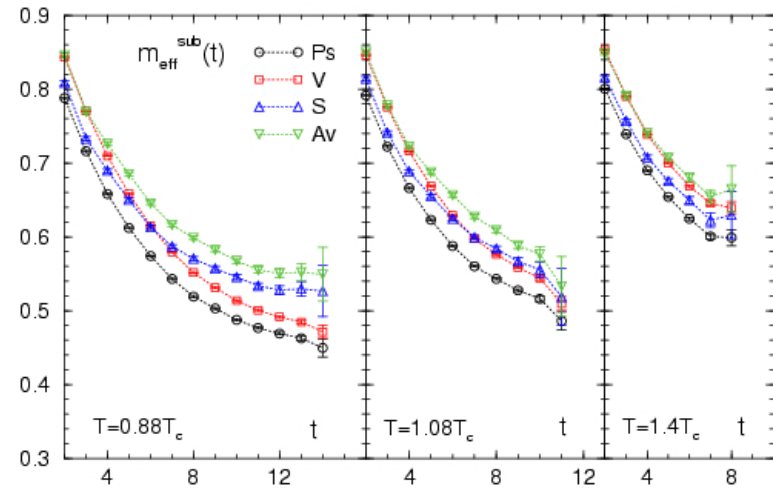
$$\bar{C}(t) = C(t) - C(N_t/2) \quad \frac{\bar{C}(t)}{\bar{C}(t+1)} = \frac{\sinh^2 \left[\frac{1}{2} m_{eff}^{sub}(t) (N_t/2 - t) \right]}{\sinh^2 \left[\frac{1}{2} m_{eff}^{sub}(t) (N_t/2 - t - 1) \right]}$$



usual effective masses at $T > 0$



subtracted effective mass

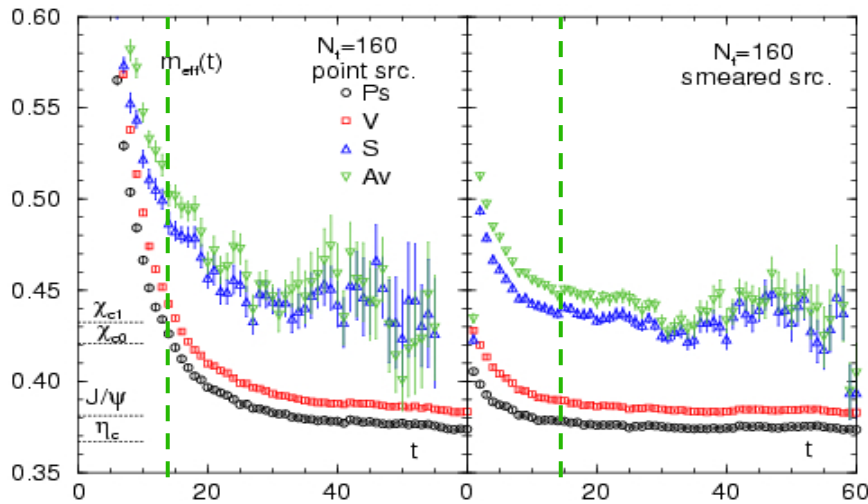


the drastic change in P-wave states disappears in $m_{eff}^{sub}(t)$

→ the change is due to the constant mode



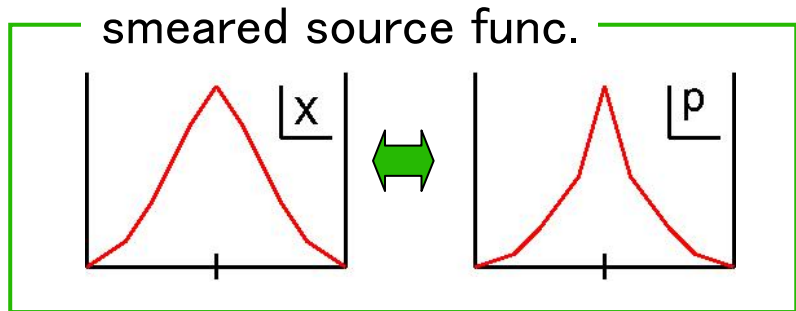
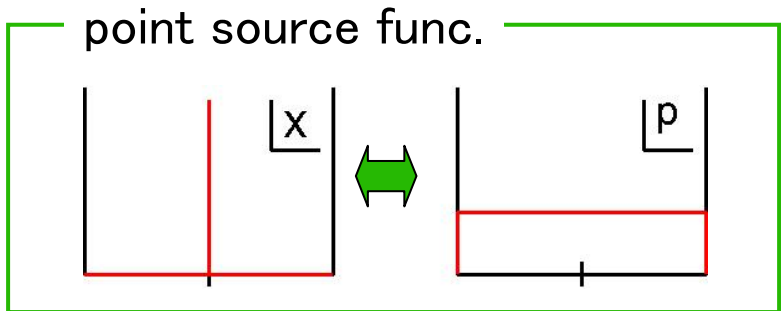
Results with extended op.



Spatially extended operators:

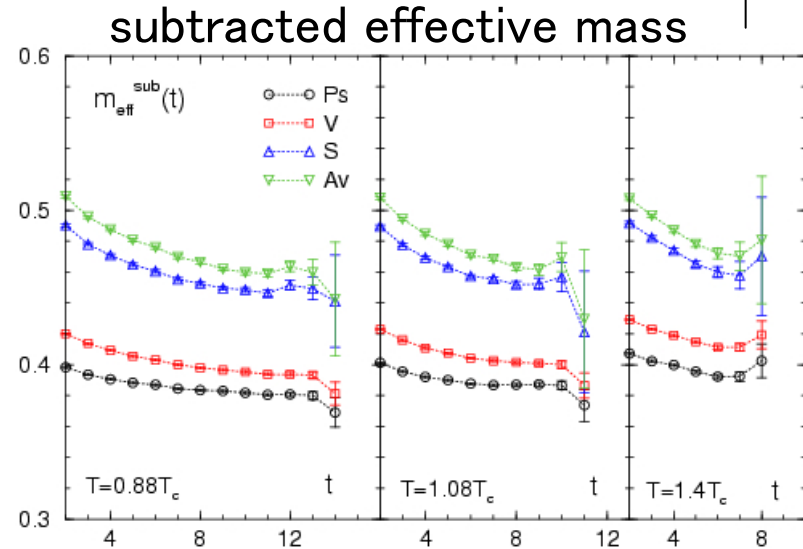
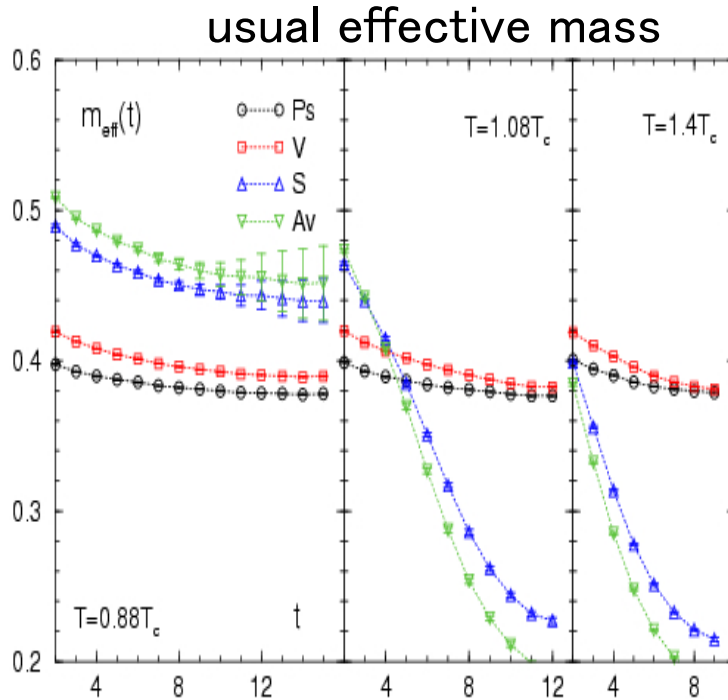
$$O_{\Gamma}(\vec{x}, t) = \sum_{\vec{y}} \phi(\vec{y}) \bar{q}(\vec{x} - \vec{y}, t) \Gamma q(\vec{x}, t)$$

with a smearing func. $\phi(x)$
in Coulomb gauge



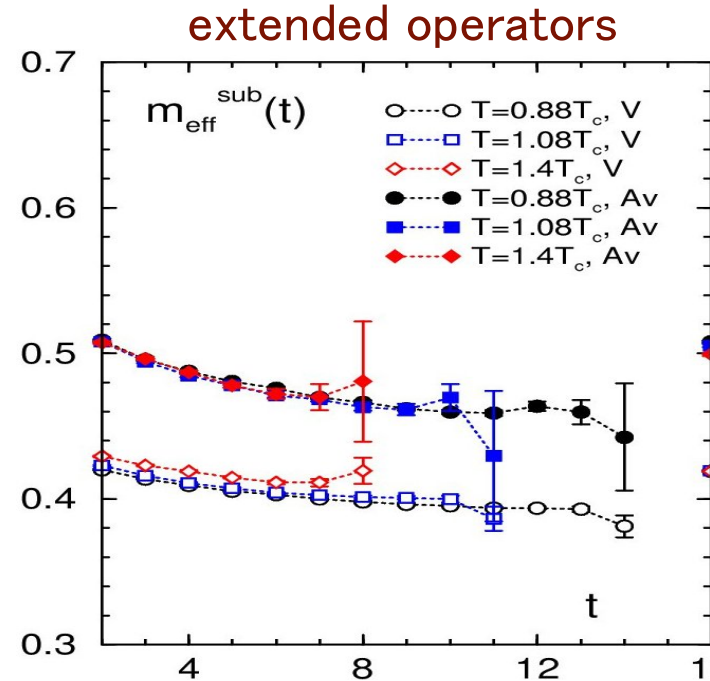
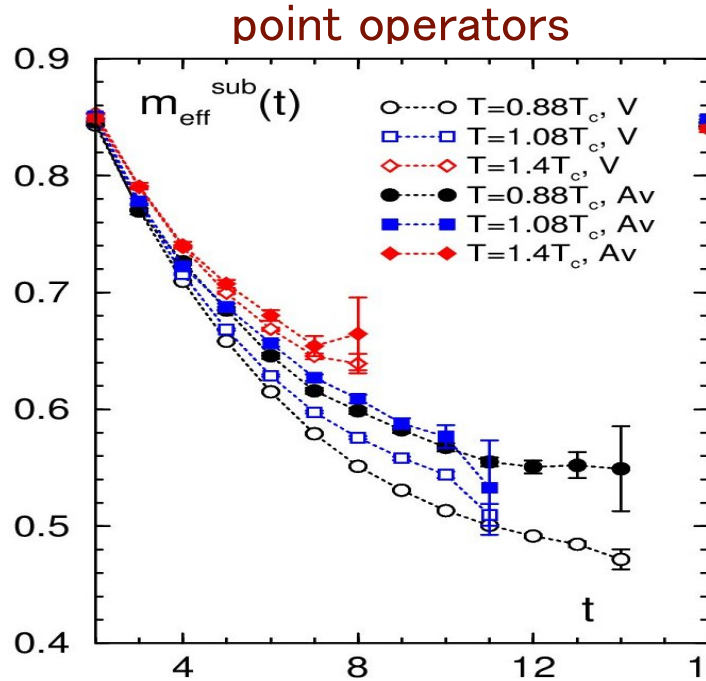
The extended op. yields large overlap with lowest states

Results with extended op.



- extended op. enhances overlap with const. mode
- small constant effect is visible in V channel
- no large change above T_c in $m_{\text{eff}}^{\text{sub}}(t)$

Discussion



The drastic change of P-wave states is due to the const. contribution.

→ There are small changes in SPFs (except for $\omega=0$ peak).

Why several MEM studies show the dissociation of χ_c states ?

Discussion

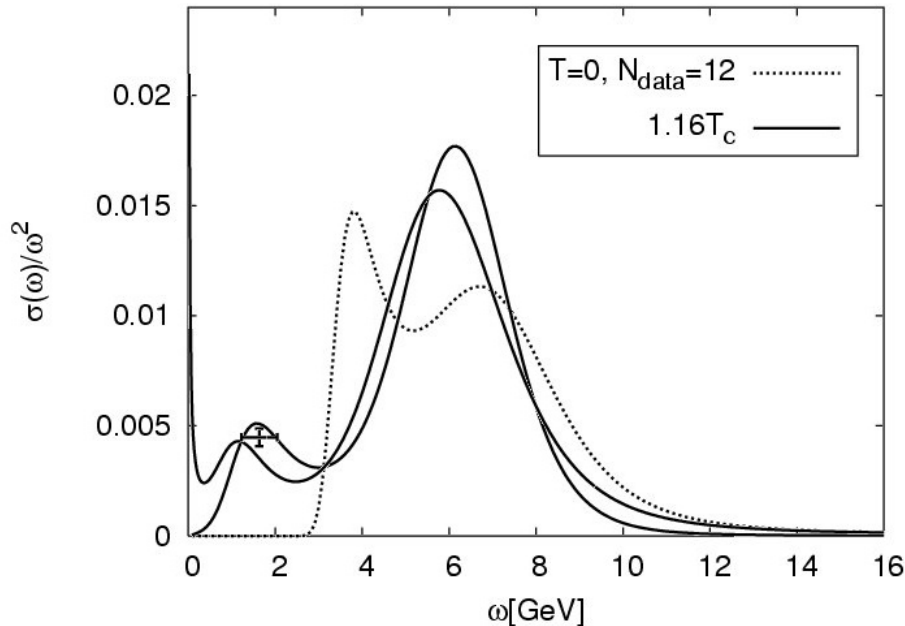


FIG. 19: The scalar spectral function for $\beta = 6.1$ at $T = 1.16T_c$ and at zero temperature reconstructed using $N_{data} = 12$. At finite temperature two default models $m(\omega) = 0.01$ and $m(\omega) = 0.038\omega^2$ have been used.

A. Jakovac et al., hep-lat/0611017.

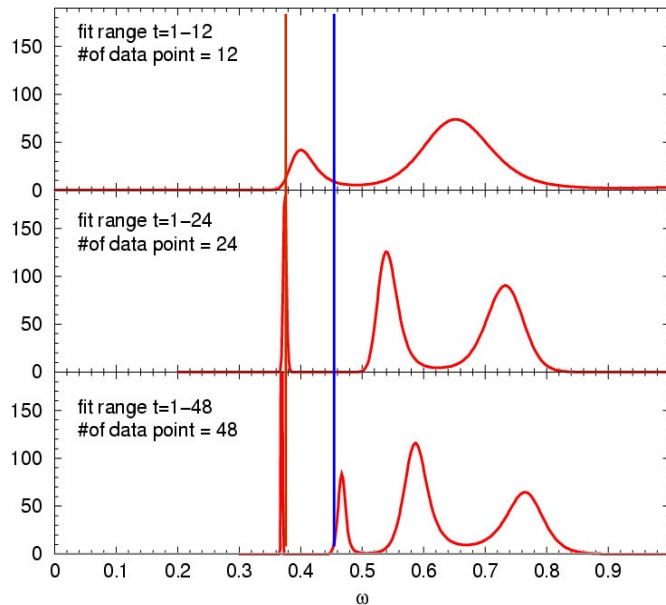
They concluded that

- the results of SPFs for P-states are not so reliable.
e.g. large default model dep.
- the drastic change just above T_c is reliable results.

Difficulties in MEM analysis



MEM test using $T=0$ data



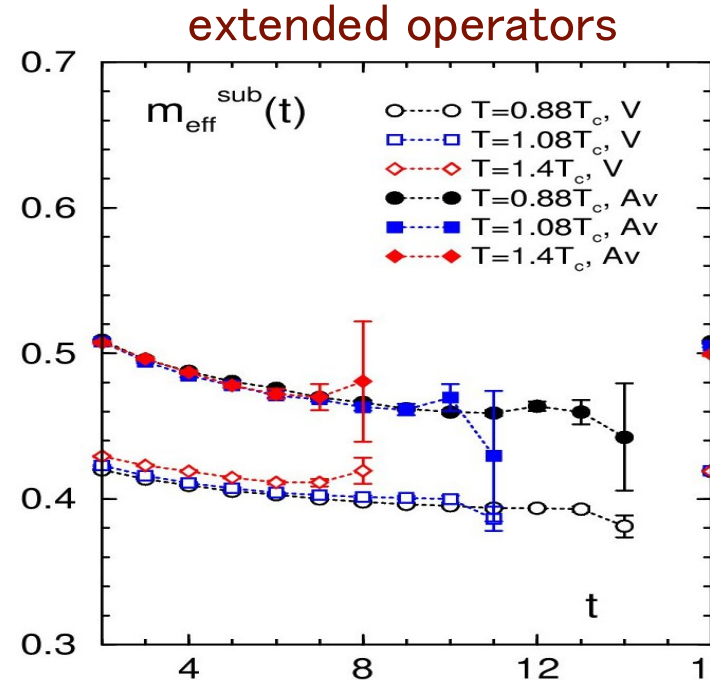
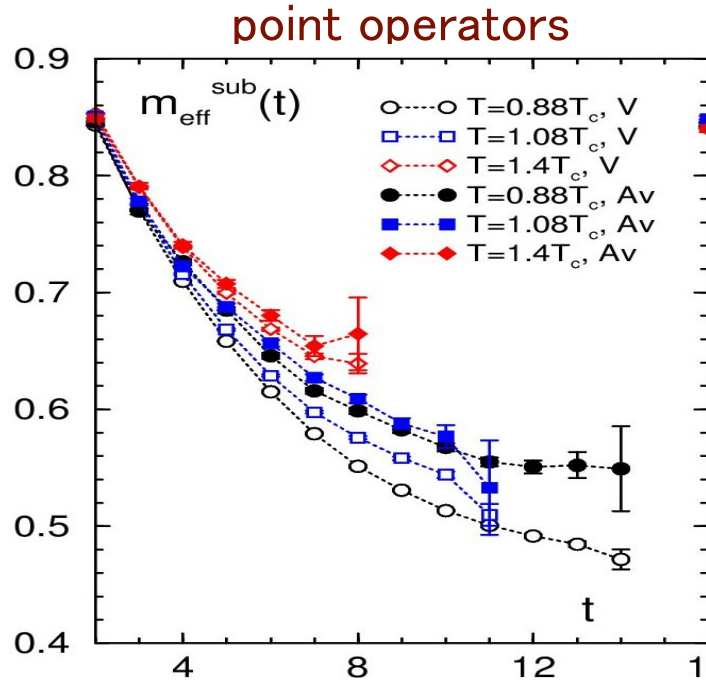
data
for $T/T_c=1.2$

data
for $T/T_c=0.6$

data
for $T/T_c=0$

MEM analysis sometimes fails if data quality is not sufficient
Furthermore P-wave states have larger noise than that of S-wave states

Discussion



The drastic change of P-wave states is due to the constant mode.

→ There are small changes in SPFs (except for $\omega=0$).

Why several MEM studies show the dissociation of χ_c states ?



Conclusion

- There is the constant mode in charmonium correlators above T_c
- The drastic change in χ_c states is due to the constant mode
→ the survival of χ_c states above T_c , at least $T=1.4T_c$.

The result may affect the scenario of J/ψ suppression.

In the MEM analysis,

one has to check consistency of the results at $\omega \gg T$ using, e.g., midpoint subtracted correlators.

$$\bar{C}(t) = C(t) - C(N_t/2)$$
$$\bar{C}(t) = \int_0^\infty d\omega \rho_\Gamma(\omega) K^{sub}(\omega, t),$$
$$K^{sub}(\omega, t) = \frac{\sinh^2(\frac{\omega}{2}(N_t/2 - t))}{\sinh(\omega N_t/2)}$$

First paper on the J/ψ suppression



photo : Prof. Osamu Miyamura

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PHYSICAL REVIEW LETTERS

27 OCTOBER 1986

Mass Shift of Charmonium near Deconfining Temperature and Possible Detection in Lepton-Pair Production

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(Received 27 May 1986)

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PHYSICAL REVIEW LETTERS

27 OCTOBER 1986

in Monte Carlo analyses.^{7,8} A related question is whether charmoniumlike clusters may still exist in a quark-gluon plasma. We have made tentative calculations by screened Coulombic potential and found that possibility small. Thus, contribution to lepton pair in the J/ψ mass region from the deconfinement phase would be mainly thermal quark-antiquark annihilation.¹⁸ In connection with this point, we make a com-

lin, 1985), p. 1.

⁴R. D. Pisarski, Phys. Lett. **110B**, 155 (1982).

⁵R. D. Pisarski and F. Wilczek, Phys. Rev. D **29**, 338 (1984).

⁶L. McLerran and B. Svetitsky, Phys. Rev. D **24**, 450 (1981).

⁷M. Fukugita, T. Kaneko, and A. Ukawa, Phys. Lett. **154B**, 185 (1985).

⁸C. Dorninger, H. Leeb, and H. Markum, Z. Phys. C **70**

Discussion

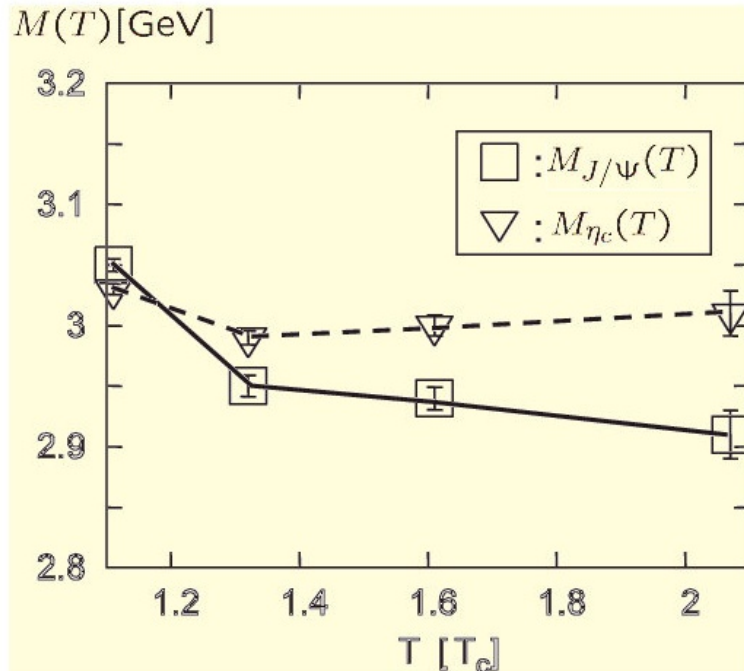


FIG. 8. Temperature dependence of the pole mass (on PBC) of J/Ψ and η_c for $(1.11-2.07)T_c$. The squares denote $M_{J/\Psi}(T)$ and the inverse triangles denote $M_{\eta_c}(T)$. There occurs the level inversion of J/Ψ and η_c above $1.3T_c$.

H. Iida et al., PRD74, 074502 (2006).

Several groups have presented

- almost no change in Ps channel
- small but visible change in V channel

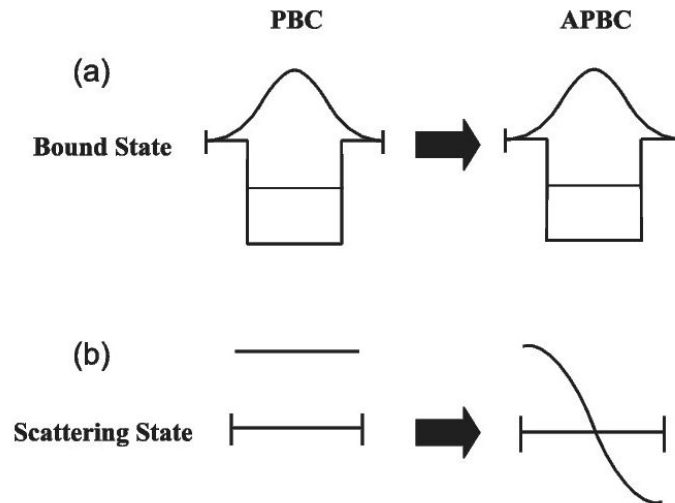


These results can be explained by the constant contribution.

- no constant in Ps channel
- small constant in V channel (proportional to p_i^2) in free quark case

Discussion

H. Iida et al., PRD74, 074502 (2006).



S-wave states : PBC $p=(0, 0, 0)$ < xAPBC $p=(\pi/L, 0, 0)$
 P-wave states : PBC $p=(2\pi/L, 0, 0)$ > xAPBC $p=(\pi/L, 0, 0)$
 P-const. : PBC $p=(0, 0, 0)$ < xAPBC $p=(\pi/L, 0, 0)$

Discussion



Polyakov loop sector dependence

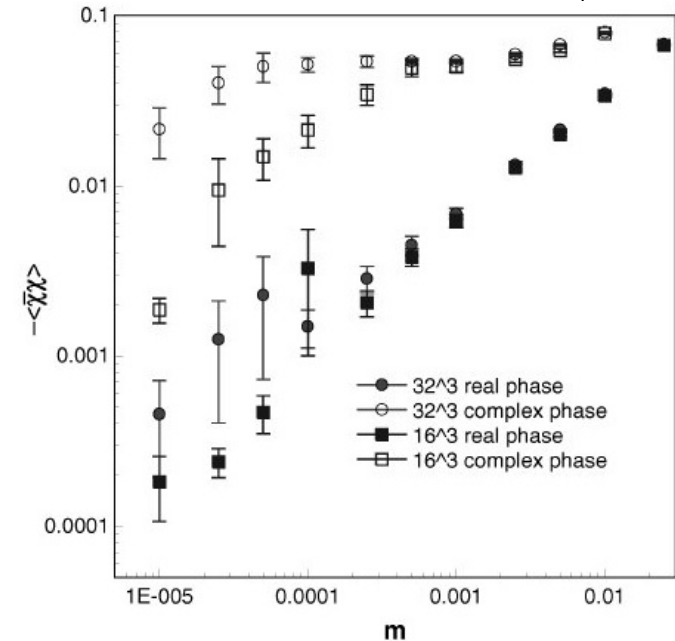
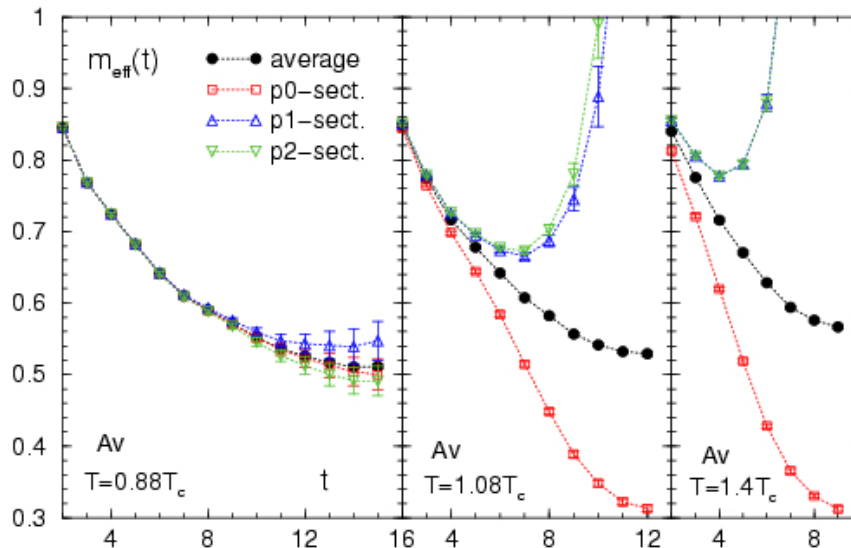


FIG. 1. The chiral condensate $\langle \bar{\chi}\chi \rangle$ plotted as a function of quark mass for a pure gauge calculation on $16^3 \times 4$ and $32^3 \times 4$ lattices. The real phase (closed points) is the most physical [$\det(D - m)$ is largest for this phase]. No evidence is seen for the expected anomalous behavior, $\langle \bar{\chi}\chi \rangle \sim m^{-1}$ as $m \rightarrow 0$.

*S. Chandrasekharan et al.,
PRL82, 2463, (1999).*



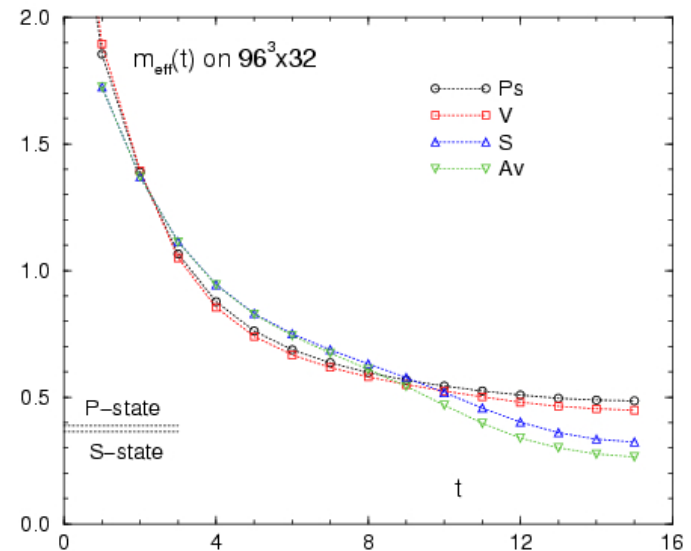
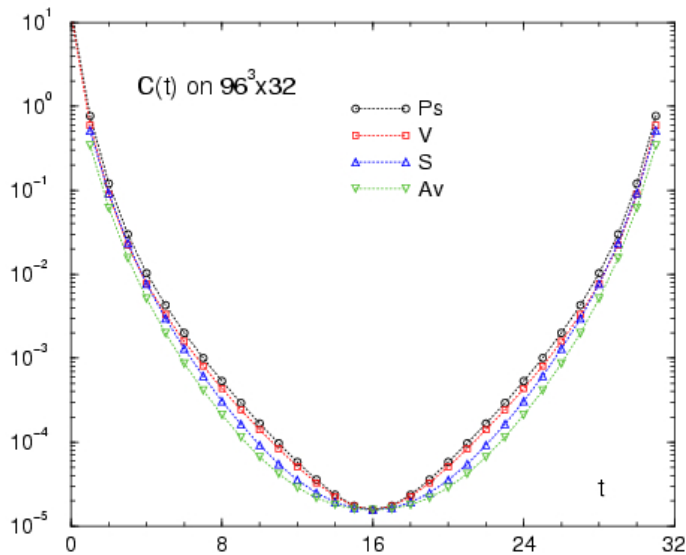
Volume dependence

Size of the constant contribution depends on the volume N_s^3

The dependence is negligible at $N_s/N_t \gtrsim 2$

■ Results on $96^3 \times 32$

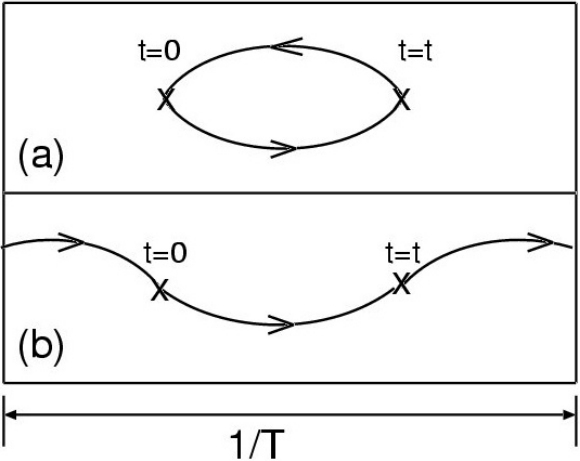
($N_s/N_t=3 \leftarrow$ similar to $T>0$ quench QCD calculation)





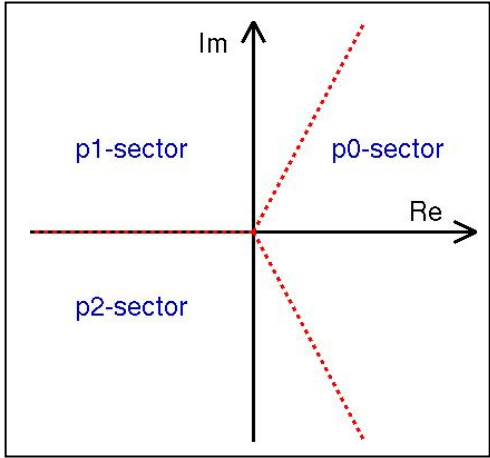
Z_3 symmetrization

Here we consider the Z_3 transformation



Z_3 symmetric

Z_3 asymmetric



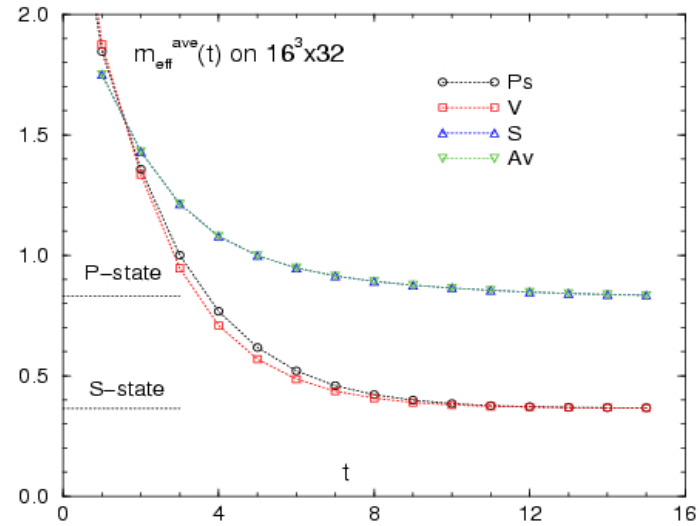
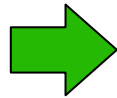
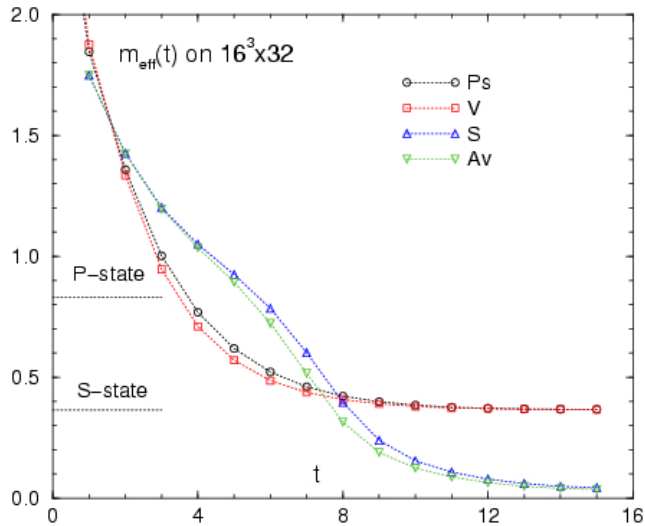
The asymmetry of diag-(b) is coming from a factor of $\text{Re}[\exp(-i2\pi n/3)]$

$$C^{ave}(t) = \frac{1}{3} (C^{p0}(t) + C^{p1}(t) + C^{p2}(t))$$

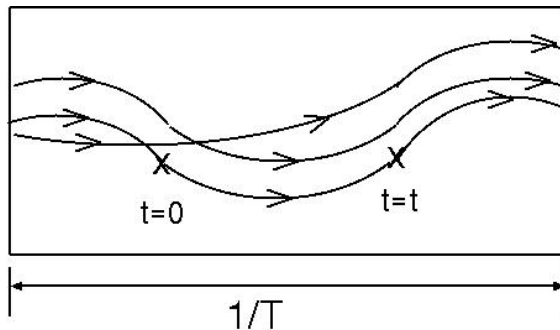
Z_3 asym. terms are removed because $\sum_{n=0}^2 \text{Re}(e^{-i2\pi n/3}) = 0$



Averaged correlators



However, this is not an exact method to avoid the constant contribution.

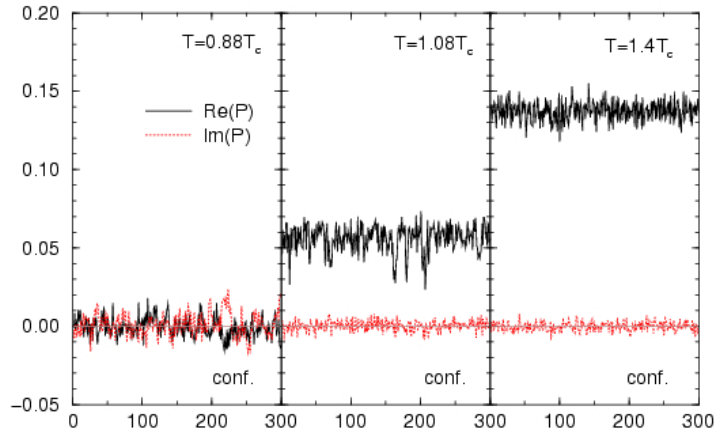


The 3 times wrapping diagram is also Z_3 symmetric.

→ the contribution is not canceled.
 but, $O(\exp(-m_q N_t)) \gg O(\exp(-3m_q N_t))$

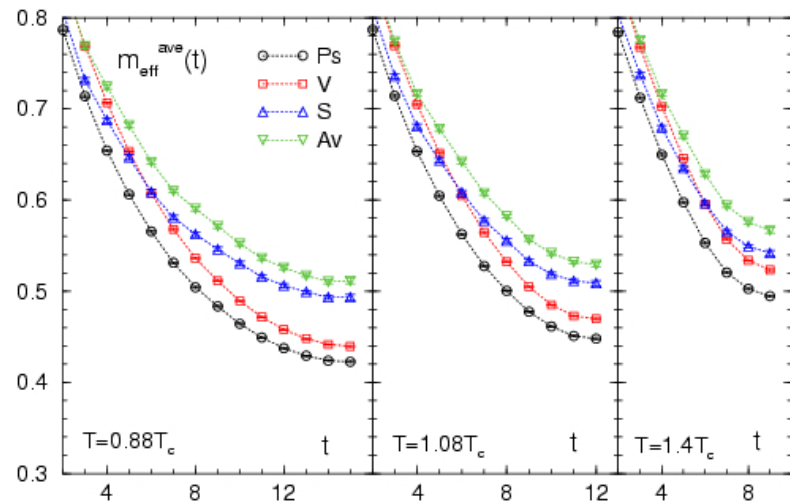
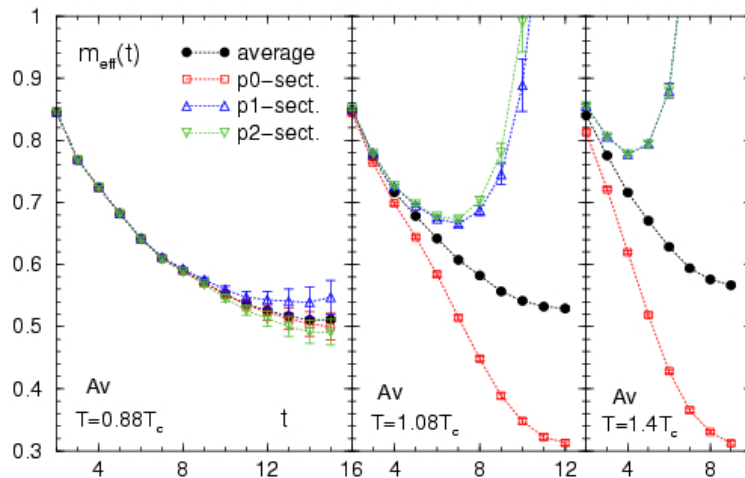


Polyakov loop sector dependence



- after Z_3 transformation
const. $\rightarrow \text{Re}(\exp(-i2\pi n/3)) * \text{const.}$
- even below T_c , small const. effect enhances the stat. fluctuation.
- drastic change in P-states disappears.

Results for Av channel



Discussion



point operators

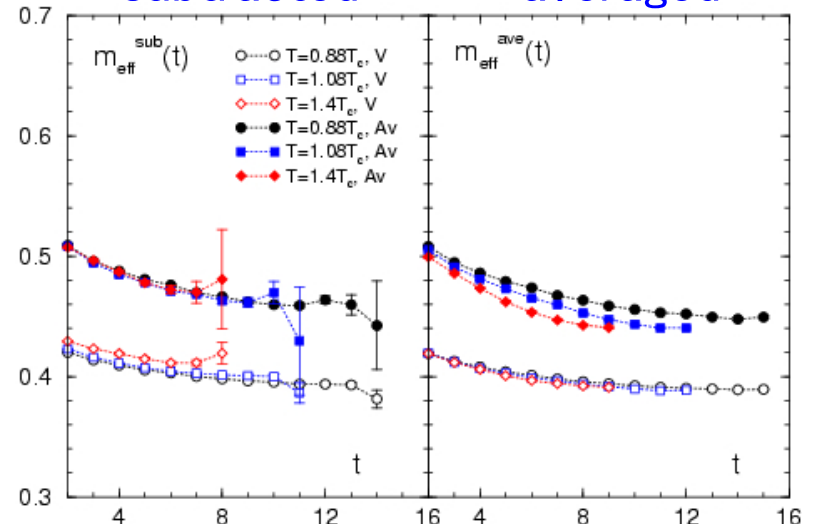
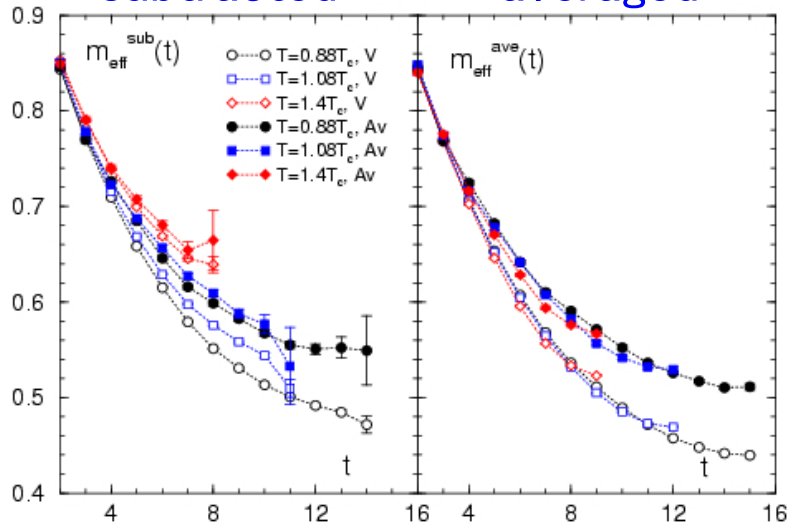
extended operators

subtracted

averaged

subtracted

averaged



The drastic change of P-wave states is due to the const. contribution.
 → There are small changes in SPFs (except for $\omega=0$).

Why several MEM studies show the dissociation of χ_c states ?



Spectral representation

Spectral function of the correlator

$$C(t) = \int_0^\infty d\omega \rho_\Gamma(\omega) K(\omega, t),$$

$$K(\omega, t) = \frac{\cosh(\omega(N_t/2 - t))}{\sinh(\omega N_t/2)}$$

*F. Karsch et al.,
PRD68, 014504 (2003).
G. Aarts et al.,
NPB726, 93 (2005).*

$$\rho_\Gamma(\omega) = \Theta(\omega^2 - 4m_q^2) \frac{N_c}{8\pi\omega} \sqrt{\omega^2 - 4m_q^2} [1 - 2n_F(\omega/2)]$$

$$\times [\omega^2 (a_H^{(1)} - a_H^{(2)}) + 4m^2 (a_H^{(2)} - a_H^{(3)})]$$

$$+ 2\pi\omega\delta(\omega) N_c [(a_H^{(1)} + a_H^{(2)}) I_1 + (a_H^{(2)} - a_H^{(3)}) I_2]$$

$$I_1 = -2 \int_{\vec{k}} n'_F(\omega_{\vec{k}})$$

with

$$I_2 = -2 \int_{\vec{k}} \frac{k^2}{\omega_{\vec{k}}^2} n'_F(\omega_{\vec{k}})$$

	Γ	$a_H^{(1)}$	$a_H^{(2)}$	$a_H^{(3)}$	$a_H^{(1)} - a_H^{(2)}$	$a_H^{(2)} - a_H^{(3)}$	$a_H^{(1)} + a_H^{(2)}$	$a_H^{(2)} - a_H^{(3)}$
Ps	γ_5	1	-1	-1	2	0	0	0
V	γ_i	3	-1	-3	4	2	2	2
S	1	1	-1	1	2	-2	0	-2
Av	$\gamma_i \gamma_5$	3	-1	3	4	-4	2	-4

最後のテーブルいらんかも

chiral symmetry in massless limit