A constant contribution in meson correlators at finite temperature

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This talk is based on the hep-lat/0701005.

Riken Lunch Seminar, BNL, Jan. 25, 2007

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  - J/ψ suppression and charmonium states
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Introduction

Charmonium properties at T>0 play the key role
for understanding the QGP formation in Heavy Ion Collis. exp.

Potential model analysis

Mass shift near Tc *T. Hashimoto et al., PRL57, 2123 (1986).*

J/ψ suppression *T. Matsui & H. Satz, PLB178, 416 (1986).*

Spectral function in lattice QCD

T. Umeda et al., EPJC39S1, 9, (2005).

S. Datta et al., PRD69, 094507, (2004).

T. Hatsuda & M. Asakawa, PRL92, 012001, (2004).

A. Jakovac et al., hep-lat/0611017.

G. Aarts et al., hep-lat/0610065.

In the studies of SPFs on the lattice,
all studies indicate survival of J/ψ state above T_c ($1.5T_c$?)

Introduction

Recently "indirect J/ψ suppression" is discussed.

total yield of J/ψ =

direct production of J/ψ (60%)

+ decay from higher states, ψ' & χ_c (40%)

L. Antoniazzi et al. (E705 Collab.), PRL70, 383, (1993).

→ If the higher states dissociate at lower temp.

a part of the J/ψ suppression may be observed

in exp. at a lower temp. than that of J/ψ

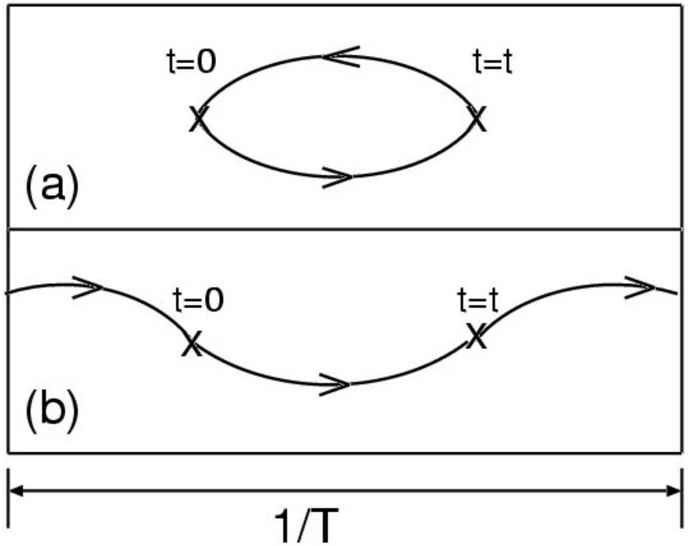
Dissociation temperatures of these states

(not only J/ψ but also ψ'& χ_c)

are important for QGP phenomenology.

Constant contribution

Now we consider the meson correlator with p=0 & $m_{q1}=m_{q2}$



$\exp(-m_q t) \times \exp(-m_q t)$
$= \exp(-2m_q t)$

m_q is quark mass
or single quark energy

$\exp(-m_q t) \times \exp(-m_q(L_t-t))$
$= \exp(-m_q L_t)$

L_t is 1/T

Pentaquark (KN state):
two pion state:
→ Dirichlet b.c.
c.f. T.T.Takahashi et al., PRD71, 114509 (2005).

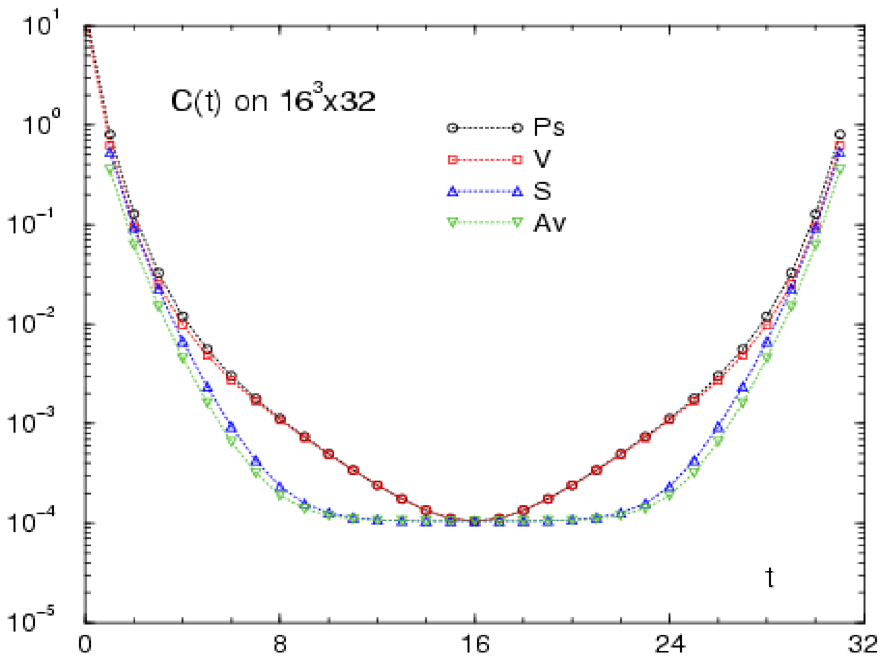
- in imaginary time formalism
  - gauge field : periodic b.c.
  - quark field : anti-periodic b.c.
- in confined phase: m_q is infinite

→ the effect appears
only in deconfined phase

Free quark calculations

$$C(t) = \sum_{\vec{x}} \langle O_\Gamma(\vec{x},t) O_\Gamma^\dagger(\vec{0},0)\rangle,$$
$$O_\Gamma(\vec{x},t) = \bar{q}(\vec{x},t)\Gamma q(\vec{x},t),$$
$$\Gamma = \gamma_5, \gamma_i, 1 \text{ and } \gamma_i\gamma_5$$

for Ps, V, S and Av channels



- 16^3 x 32 isotropic lattice
- Wilson quark action with $m_q a = 0.2$

Obvious constant contribution in P–wave states

Effective mass (local mass)

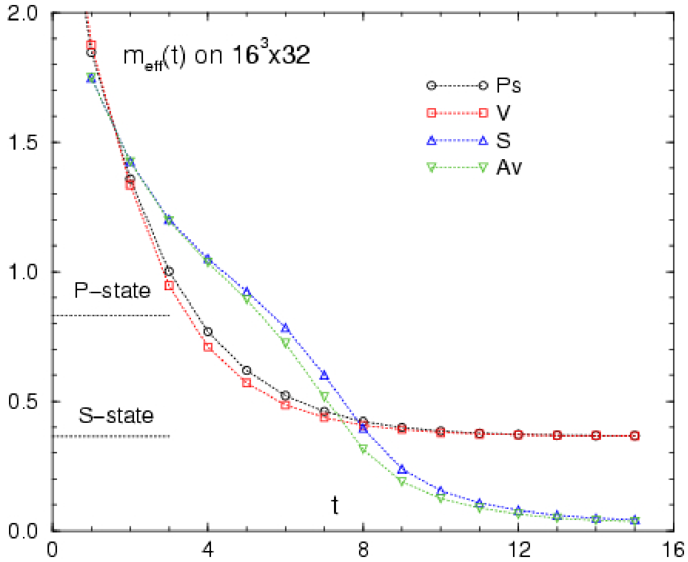
Definition of effective mass

$$C(t) = Ae^{-m_{eff}t}$$

$$\frac{C(t)}{C(t+1)} = e^{-m_{eff}}$$

In the (anti) periodic b.c.

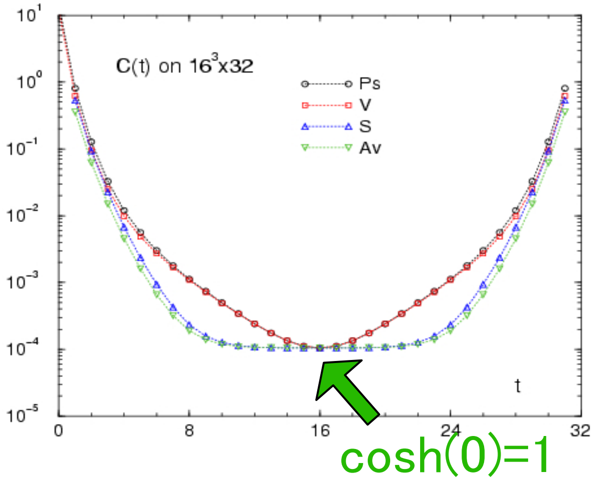
$$\frac{C(t)}{C(t+1)} = \frac{\cosh[m_{eff}(t)(N_t/2-t)]}{\cosh[m_{eff}(t)(N_t/2-t-1)]}$$

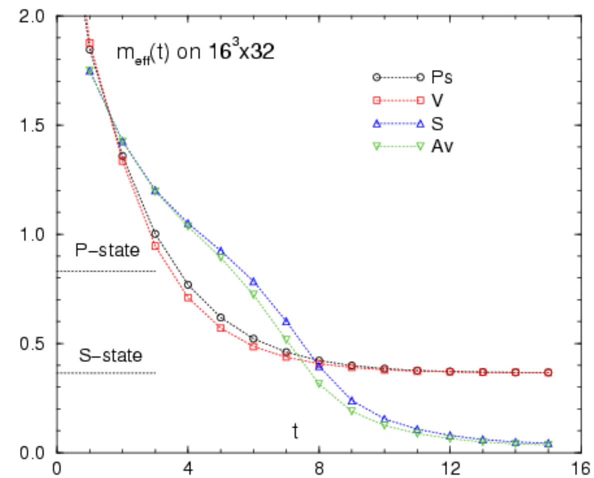


- for S–wave states:
  lowest state energy is $2E_q(p=0)$
  where $E_q(p=0) = \ln(m_q+1)$
- for P–wave states:
  lowest state energy is $2E_q(p=p_{min})$
  where $p_{min}=2\pi/L_s$

free Wilson quark dispersion relation

Free quark calculations





Continuum form of the correlators

calculated by S. Sasaki

$$C(t) = \sum_{\vec{p}} \frac{16}{\cosh^2 E_p N_t} \times$$

$$\begin{cases} \left(E_p^2 \cosh\left[2E_p(t - N_t/2)\right]\right) & \text{for } \Gamma = \gamma_5 \\ \left((E_p^2 - p_i^2)\cosh\left[2E_p(t - N_t/2)\right] + p_i^2\right) & \text{for } \Gamma = \gamma_i \\ -\left(p^2 \cosh\left[2E_p(t - N_t/2)\right] + (E_p^2 - p^2)\right) & \text{for } \Gamma = 1 \\ -\left((p^2 - p_i^2)\cosh\left[2E_p(t - N_t/2)\right] + (E_p^2 - p^2 + p_i^2)\right) & \text{for } \Gamma = \gamma_i\gamma_5 \end{cases}$$

where

E_p : single quark energy with relative mom. p

$$p^2 = \sum_i p_i^2$$

Spectral representation

Spectral function of the correlator

F. Karsch et al., PRD68, 014504 (2003).
G. Aarts et al., NPB726, 93 (2005).

$$C(t) = \int_0^\infty d\omega \rho_\Gamma(\omega) K(\omega, t),$$

$$K(\omega, t) = \frac{\cosh(\omega(N_t/2 - t))}{\sinh(\omega N_t/2)}$$

$$\rho_\Gamma(\omega) = \Theta(\omega^2 - 4m_q^2)\frac{N_c}{8\pi\omega}\sqrt{\omega^2 - 4m_q^2}[1 - 2n_F(\omega/2)] \times[\omega^2(a_H^{(1)} - a_H^{(2)}) + 4m^2(a_H^{(2)} - a_H^{(3)})] + 2\pi\omega\delta(\omega)N_c[(a_H^{(1)} + a_H^{(2)})I_1 + (a_H^{(2)} - a_H^{(3)})I_2]$$

with

$$I_1 = -2\int_{\vec{k}} n'_F(\omega_{\vec{k}})$$

$$I_2 = -2\int_{\vec{k}} \frac{k^2}{\omega_{\vec{k}}^2} n'_F(\omega_{\vec{k}})$$

	Γ	$a_H^{(1)}$	$a_H^{(2)}$	$a_H^{(3)}$	$a_H^{(1)} - a_H^{(2)}$	$a_H^{(2)} - a_H^{(3)}$	$a_H^{(1)} + a_H^{(2)}$	$a_H^{(2)} - a_H^{(3)}$
Ps	γ_5	1	-1	-1	2	0	0	0
V	γ_i	3	-1	-3	4	2	2	2
S	1	1	-1	1	2	-2	0	-2
Av	$\gamma_i\gamma_5$	3	-1	3	4	-4	2	-4

chiral symmetry in massless limit

Physical interpretation

$$\rho_\Gamma(\omega) = \Theta(\omega^2 - 4m_q^2)\frac{N_c}{8\pi\omega}\sqrt{\omega^2 - 4m_q^2}[1 - 2n_F(\omega/2)]$$
$$\times[\omega^2(a_H^{(1)} - a_H^{(2)}) + 4m^2(a_H^{(2)} - a_H^{(3)})]$$
$$+2\pi\omega\delta(\omega)N_c[(a_H^{(1)} + a_H^{(2)})I_1 + (a_H^{(2)} - a_H^{(3)})I_2]$$

constant contribution remains
in the continuum form & infinite volume

The constant term is related to some transport coefficients.

From Kubo-formula, for example, a derivative of the SPF
in the V channel is related to the electrical conductivity σ.

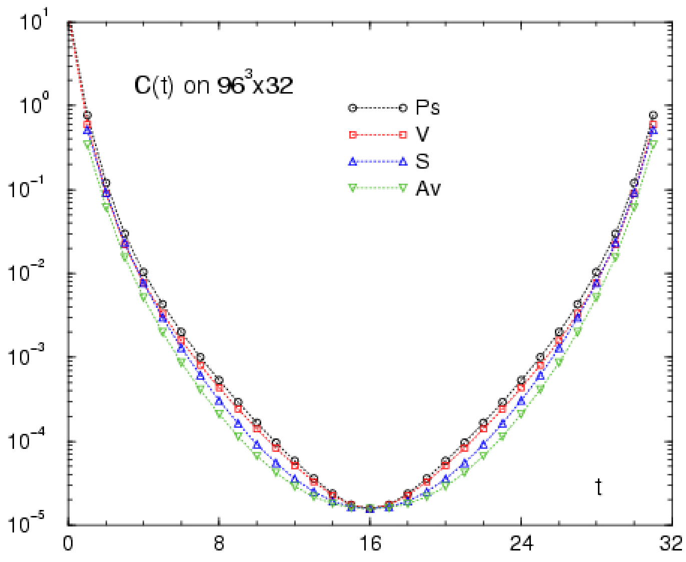
$$\sigma = \frac{1}{6}\frac{\partial}{\partial\omega}\rho_V(\omega)\Big|_{\omega=0}$$

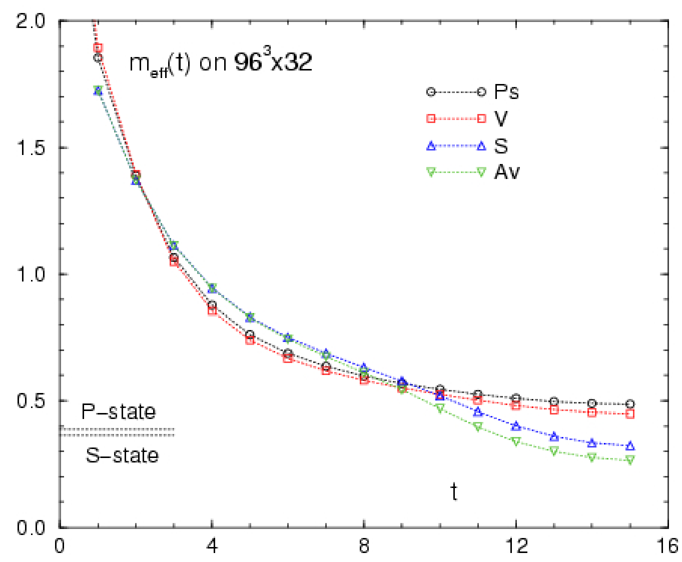
Volume dependence

Size of the constant contribution depends on the volume N_s^3

The dependence is negligible at $N_s/N_t \gtrsim 2$

- Results on 96^3 x 32
  (N_s/N_t=3 ← similar to T>0 quench QCD calculation)





Removing constant contrib. (1)

An analysis to avoid the constant contribution

Midpoint subtracted correlator

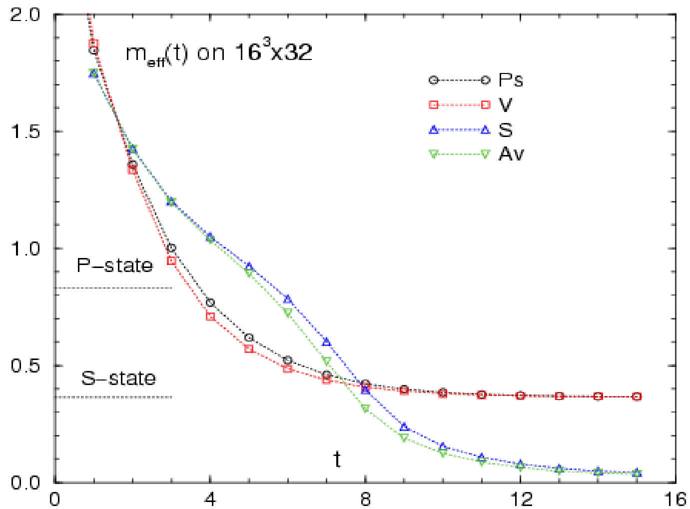
$$\bar{C}(t) = C(t) - C(N_t/2)$$

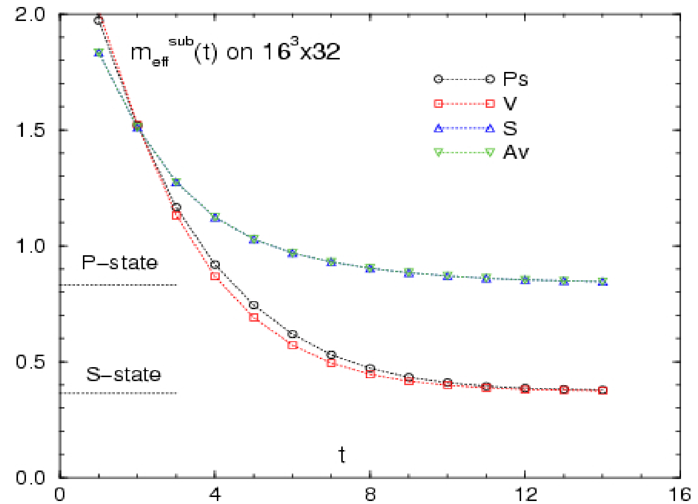
$$\frac{\bar{C}(t)}{\bar{C}(t+1)} = \frac{\sinh^2\left[\frac{1}{2}m_{eff}^{sub}(t)(N_t/2-t)\right]}{\sinh^2\left[\frac{1}{2}m_{eff}^{sub}(t)(N_t/2-t-1)\right]}$$

SPFs are unchanged

$$\bar{C}(t) = \int_0^\infty d\omega \rho_\Gamma(\omega) K^{sub}(\omega,t),$$

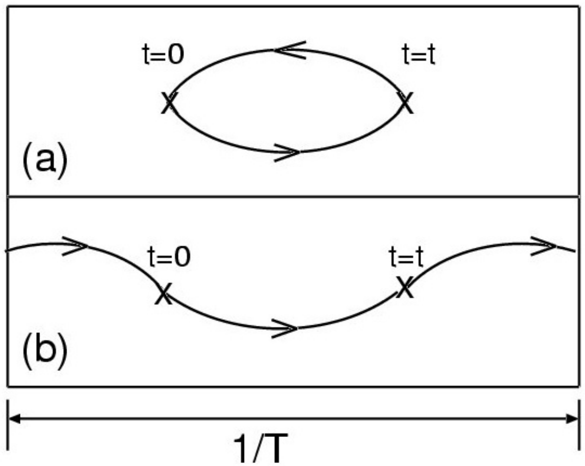
$$K^{sub}(\omega,t) = \frac{\sinh^2(\frac{\omega}{2}(N_t/2-t))}{\sinh(\omega N_t/2)}$$





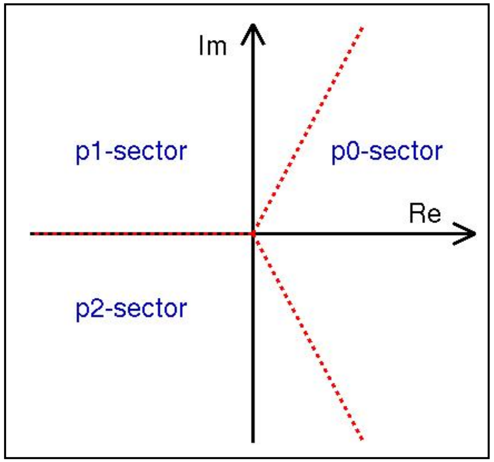
Removing constant contrib. (2)

Here we consider the Z_3 transformation



Z_3 symmetric

Z_3 asymmetric

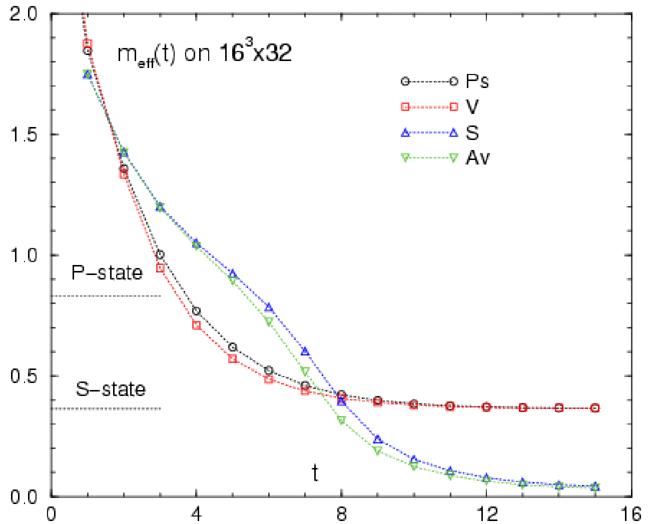


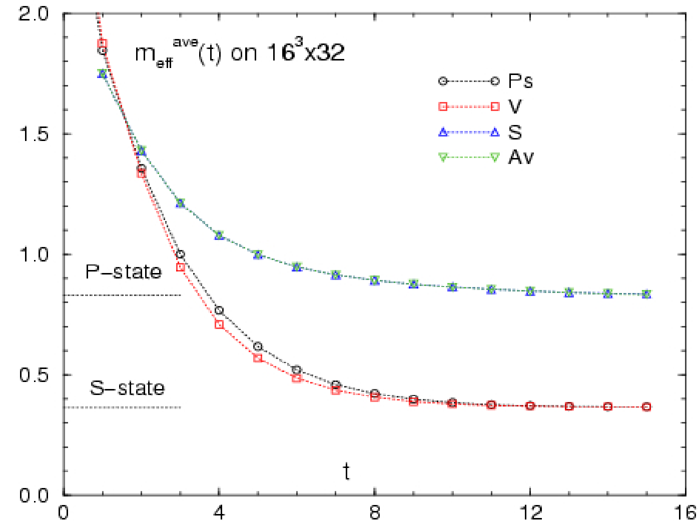
The asymmetry of diag−(b)
is coming from a factor of Re[exp(−i2 π n/3)]

$$C^{ave}(t) = \frac{1}{3}\left(C^{p0}(t) + C^{p1}(t) + C^{p2}(t)\right)$$

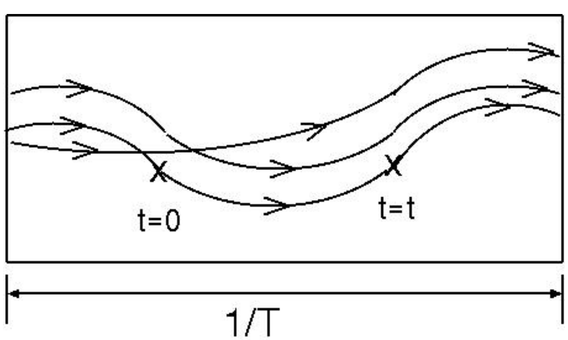
Z_3 asym. terms are removed because $\sum_{n=0}^{2} Re(e^{-i2\pi n/3}) = 0$

Averaged correlators





However, this is not an exact method to avoid the constant contribution.



The 3 times wrapping diagram is also Z_3 symmetric.

→ the contribution is not canceled.

but, $O(\exp(-m_q N_t)) \gg O(\exp(-3 m_q N_t))$

Quenched QCD at T>0

We demonstrate the effects of the constant contribution for charmonium correlators in quenched QCD at T>0

Lattice setup

anisotropic lattices : $20^3 \times N_t$

$1/a_s = 2.03(1)$ GeV, $a_s/a_t = 4$

Clover quark action with tadpole imp. on anisotropic lattice

H. Matsufuru et al., PRD64, 114503 (2001).

$r_s=1$ to reduce cutoff effects in higher energy states

F. Karsch et al., PRD68, 014504 (2003).

N_T	160	32	26	20
T/T_c	~ 0	0.88	1.04	1.40
# of conf.	60	300	300	300

equilib. is 20K sweeps
each config. is separated by 500 sweeps

(※ in a_t units, $80^3 \times N_t$ → N_s/N_t=3–4 at $T>T_c$)

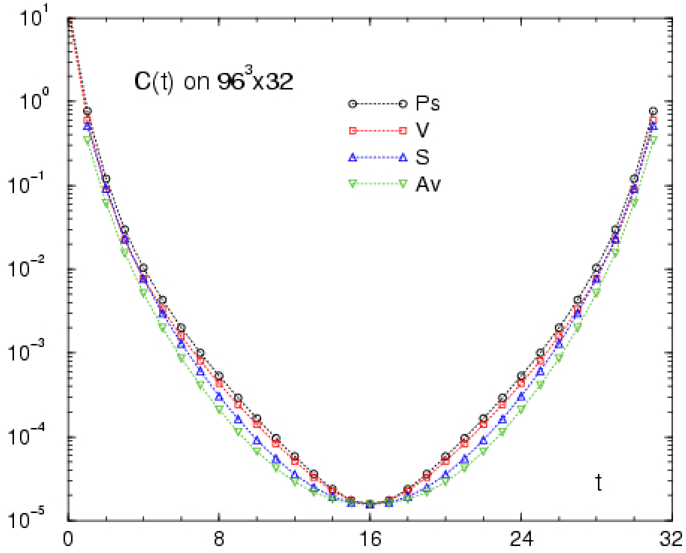
Volume dependence

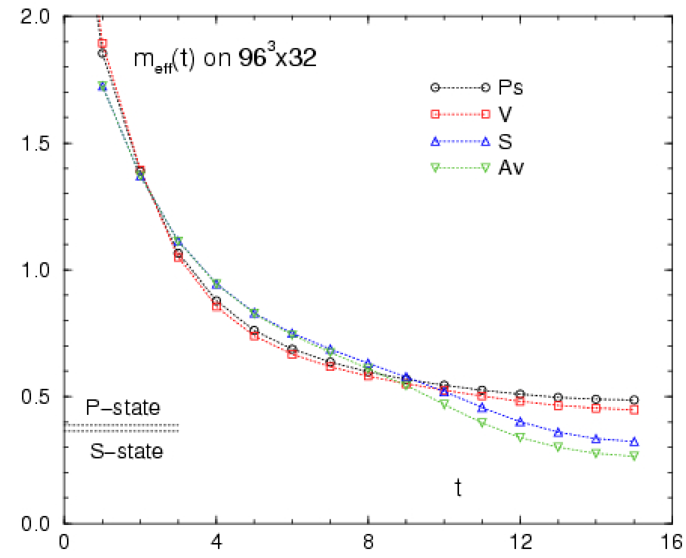
Size of the constant contribution depends on the volume N_s^3

The dependence is negligible at $N_s/N_t \gtrsim 2$

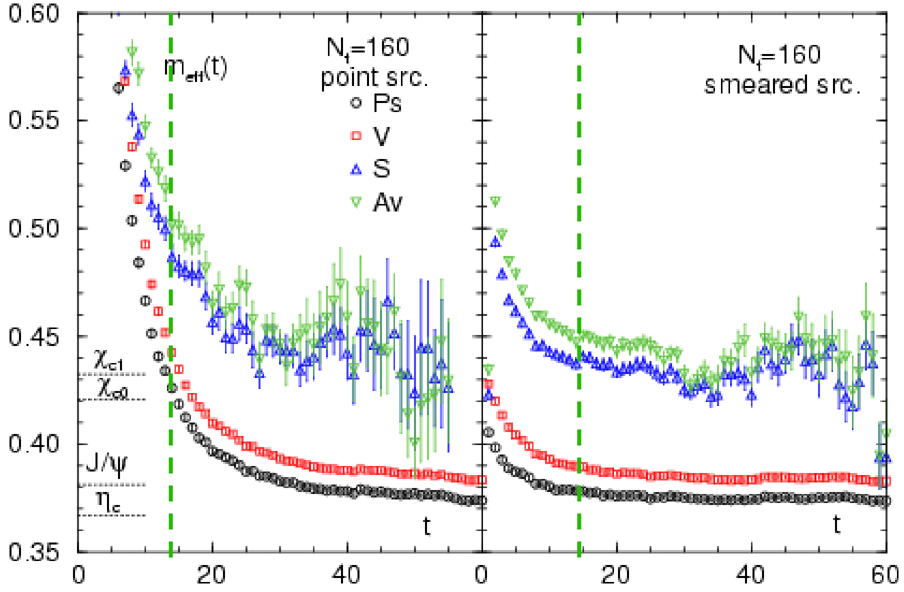
- Results on 96^3 x 32

  ($N_s/N_t=3$ ← similar to T>0 quench QCD calculation)





Zero Temp. results



(our lattice results)

M_{PS} = 3033(19) MeV

M_V = 3107(19) MeV

(exp. results from PDG06)

$M_{\eta c}$ = 2980 MeV

$M_{J/\psi}$ = 3097 MeV

$M_{\chi c0}$ = 3415 MeV

$M_{\chi c0}$ = 3511 MeV

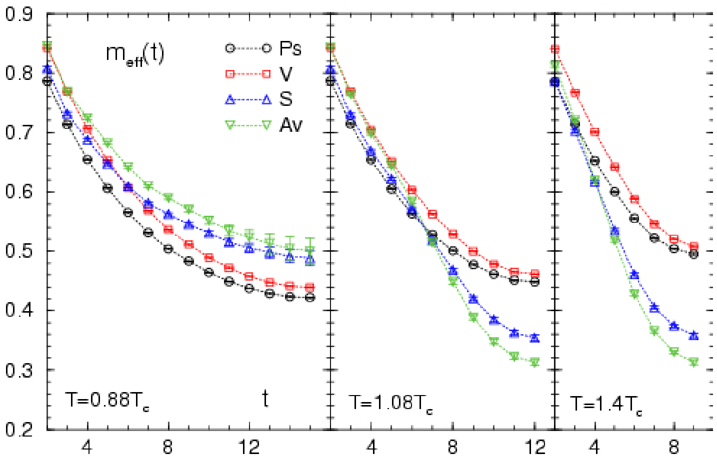
- One can expect there is no plateau T>0 with the point op. when the SPF does not change at T>0

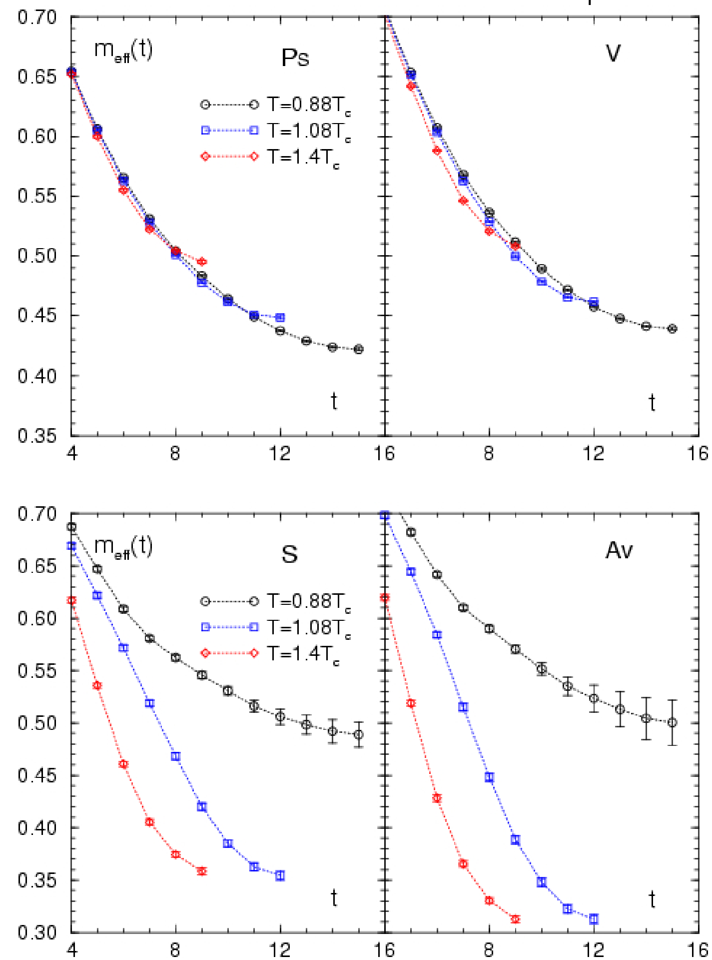
  $N_t \simeq 28$ at T_c

  t = 1 – 14 is available near T_c

- Spatially extended (smeared) op. is discussed later

Quenched QCD at T>0





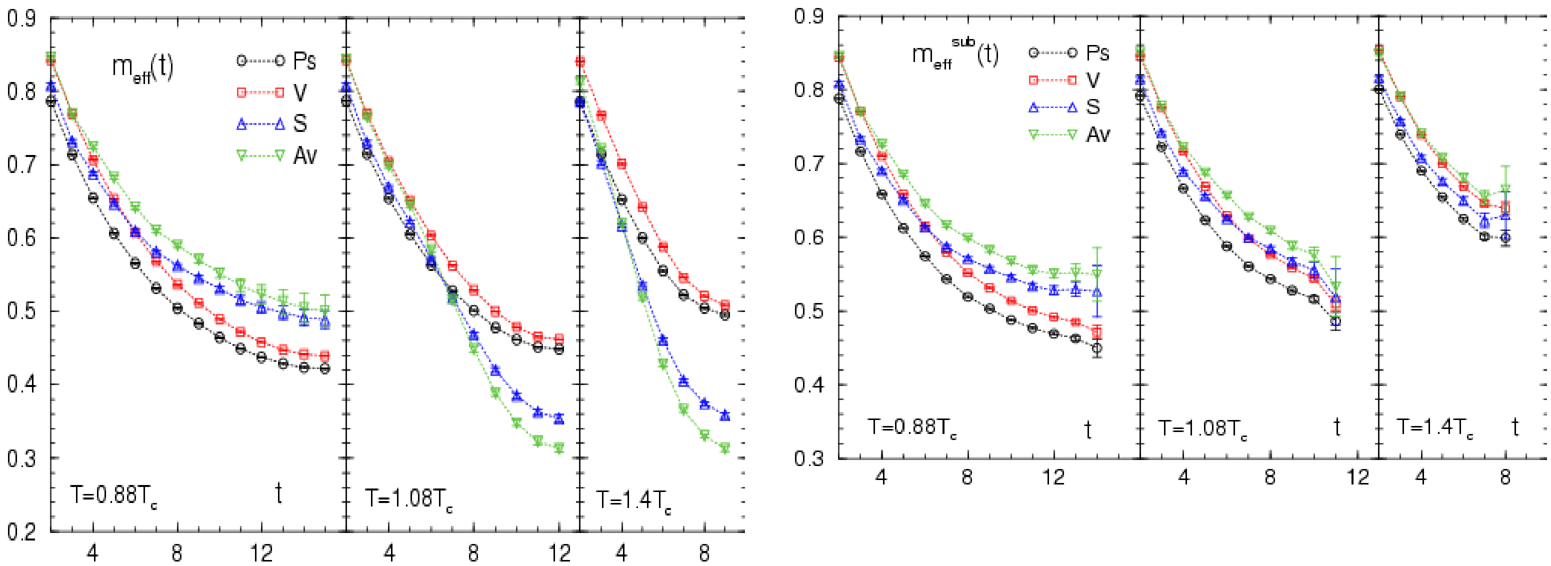
- small change in S−wave states
  → survival of J/ψ & η_c at $T>T_c$
- drastic change in P−wave states
  → dissociation of χ_c just above Tc (?)

S. Datta et al.,
PRD69, 094507 (2004). etc...

Midpoint subtraction analysis

$$\bar{C}(t) = C(t) - C(N_t/2) \qquad \frac{\bar{C}(t)}{\bar{C}(t+1)} = \frac{\sinh^2\left[\frac{1}{2}m_{eff}^{sub}(t)(N_t/2-t)\right]}{\sinh^2\left[\frac{1}{2}m_{eff}^{sub}(t)(N_t/2-t-1)\right]}$$

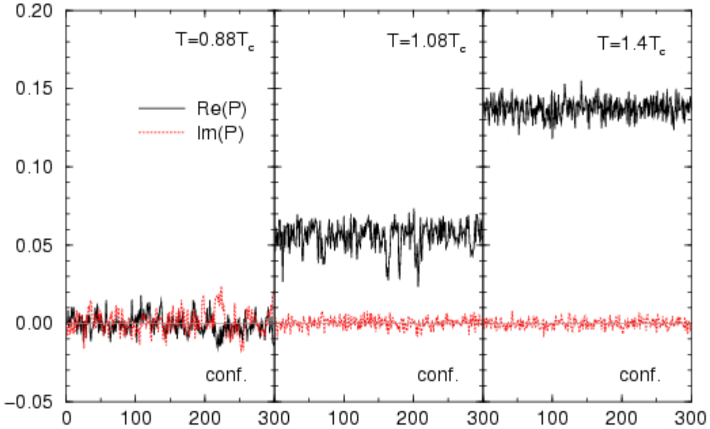
usual effective masses at T>0



the drastic change in P-wave states disappears in $m_{eff}^{sub}(t)$

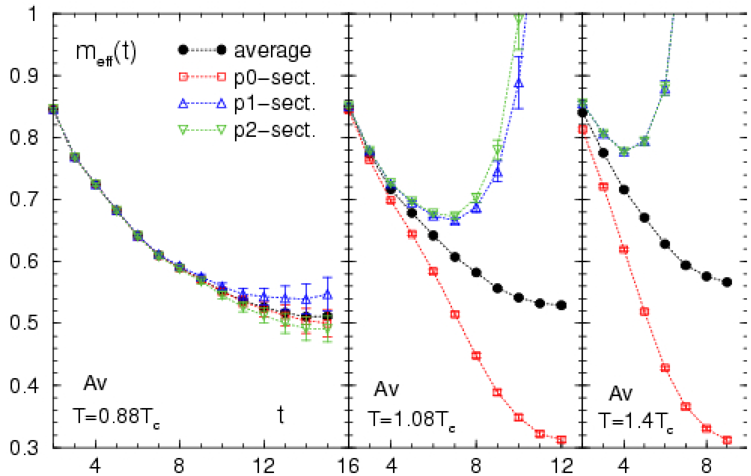
→ the change is due to the constant contribution

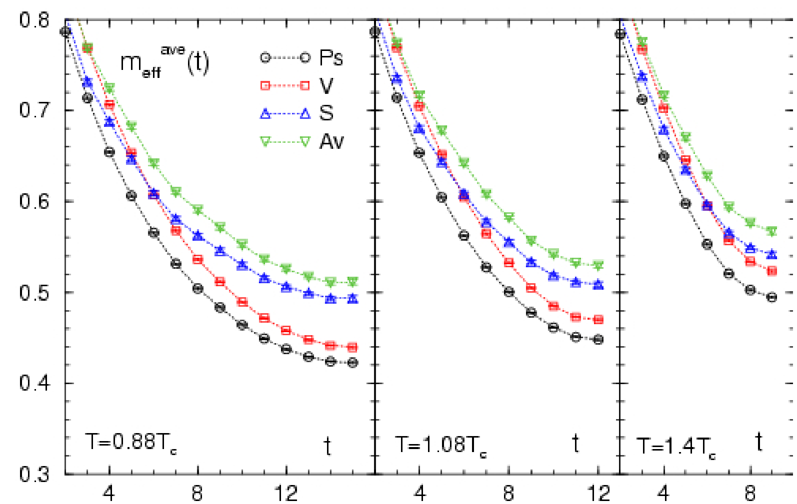
Polyakov loop sector dependence



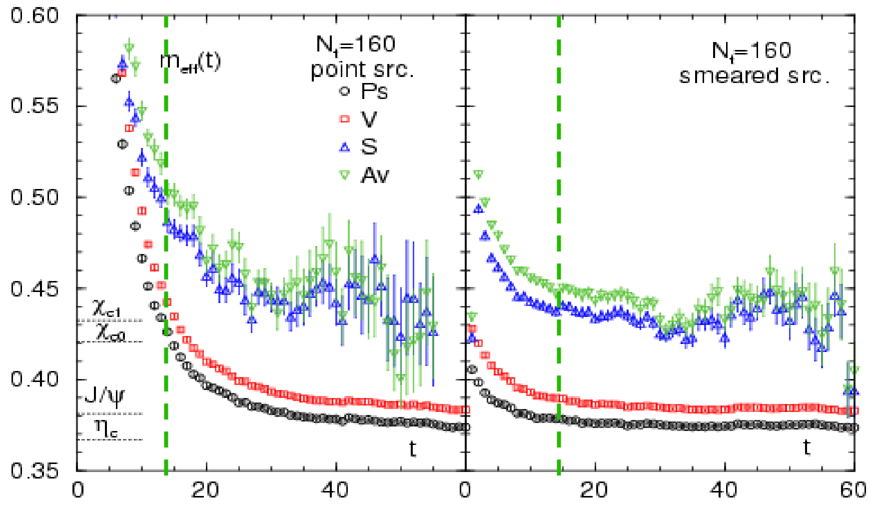
- after Z_3 transformation
  const. → Re(exp(−i2 π n/3))*const.
- even below T_c, small const. effect enhances the stat. fluctuation.
- drastic change in P-states disappears.

Results for Av channel





Results with extended op.

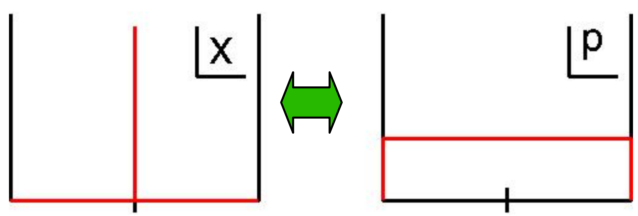


Spatially extended operators:

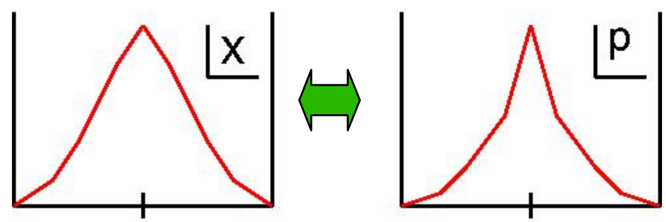
$$O_\Gamma(\vec{x},t) = \sum_{\vec{y}} \phi(\vec{y})\bar{q}(\vec{x}-\vec{y},t)\Gamma q(\vec{x},t)$$

with a smearing func. $\phi(x)$
in Coulomb gauge

point source func.



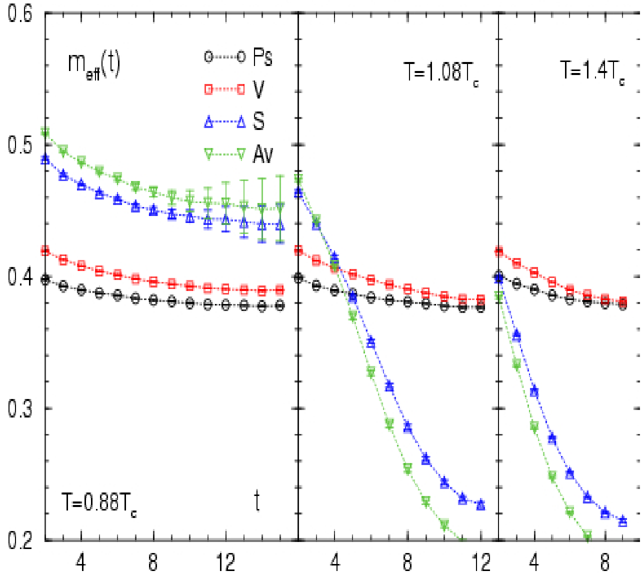
smeared source func.



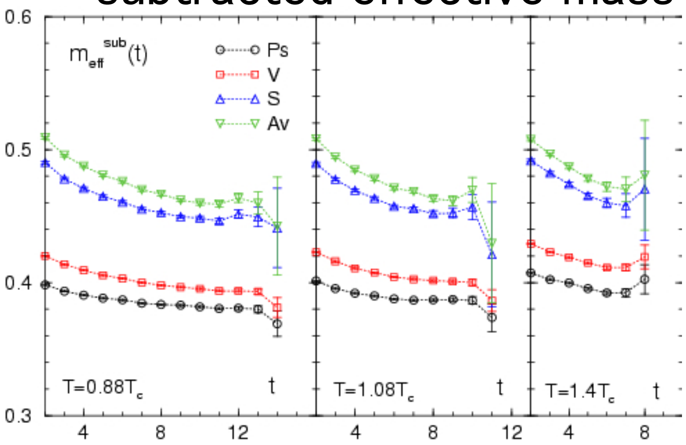
The extended op. yields large overlap with lowest states

Results with extended op.

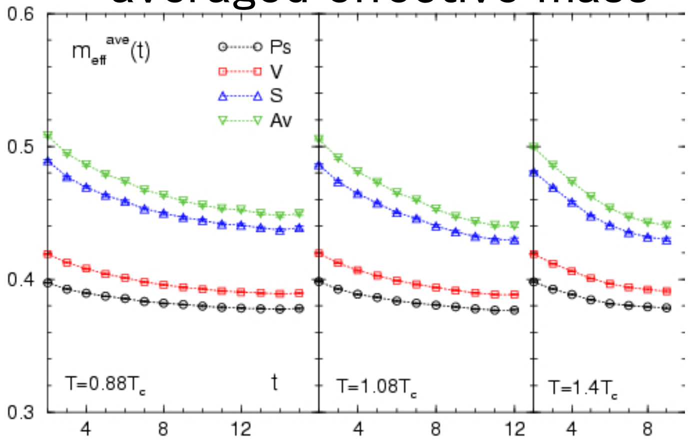
usual effective mass



subtracted effective mass

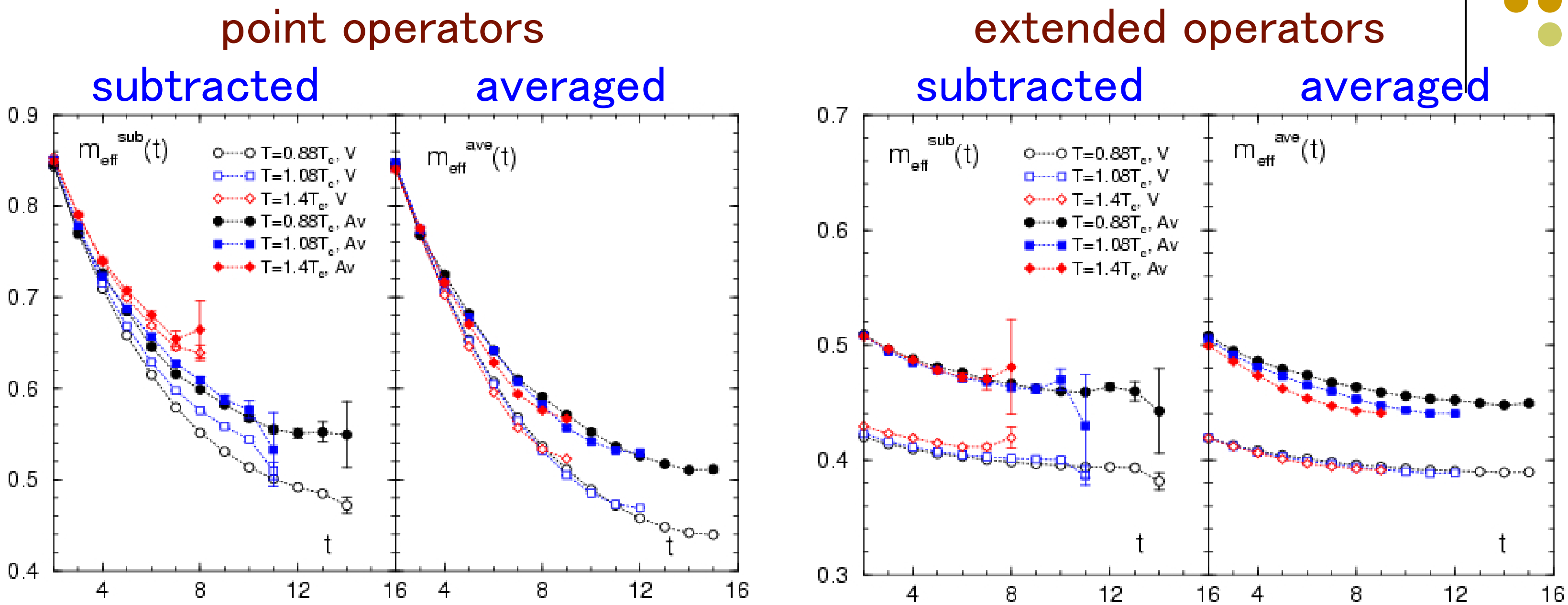


averaged effective mass



- extended op. enhances overlap with const. mode
- small constant effect is visible in V channel
- no large change above T_c in $m_{eff}^{sub}(t)$ and $m_{eff}^{ave}(t)$

Discussion



The drastic change of P-wave states is due to the const. contribution.

→ There are small changes in SPFs (except for $\omega=0$).

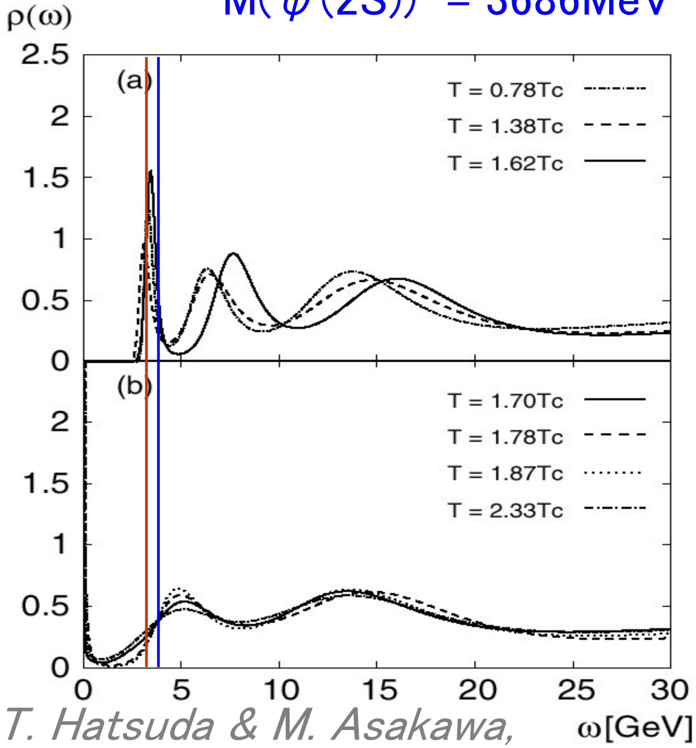
Why several MEM studies show the dissociation of χ_c states ?

→ above T_c, χ_c state is not the lowest state

Analysis of non-lowest state is difficult even if MEM is applied.

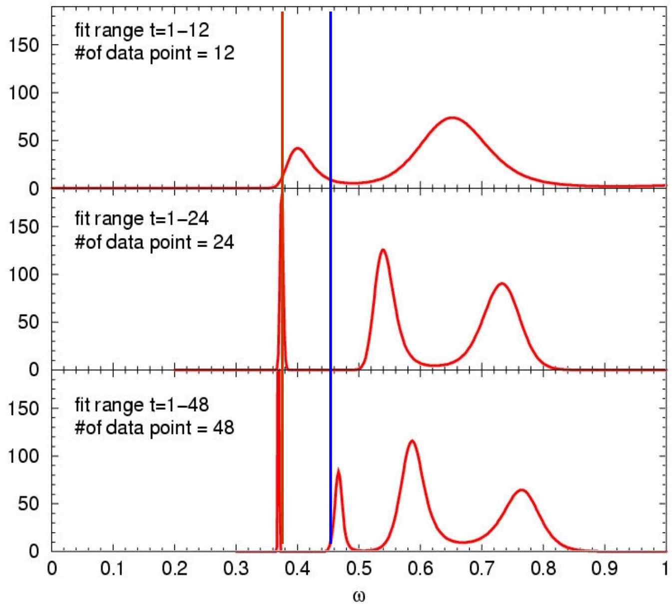
Difficulties on non-lowest states

M(J/ψ(1S)) = 3097MeV
M(ψ(2S)) = 3686MeV



T. Hatsuda & M. Asakawa, PRL92, 012001 (2004).

MEM test using T=0 data



data for T/T_c=1.2

data for T/T_c=0.6

data for T/T_c=0

It is difficult to reproduce the non-lowest states peak at T>0

Furthermore P-wave states have larger noise than S-wave states

Discussion

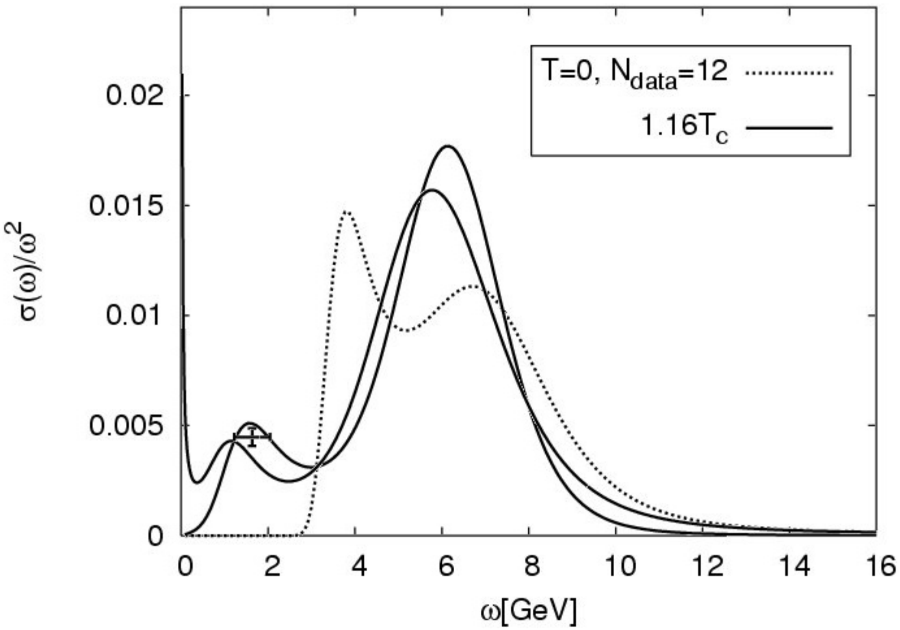


FIG. 19: The scalar spectral function for $\beta = 6.1$ at $T = 1.16T_c$ and at zero temperature reconstructed using $N_{data} = 12$. At finite temperature two default models $m(\omega) = 0.01$ and $m(\omega) = 0.038\omega^2$ have been used.

A.Jakovac et al., hep-lat/0611017

Most systematic & reliable calc. using MEM for charmonium SPFs

They concluded that

- the results of SPFs for P-states are not so reliable.
  e.g. large default model dep.
- the drastic change just above Tc is reliable results.

In MEM results, we sometimes find a kind of divergence at ω=0
→ it may & should be caused by the constant contribution

Discussion

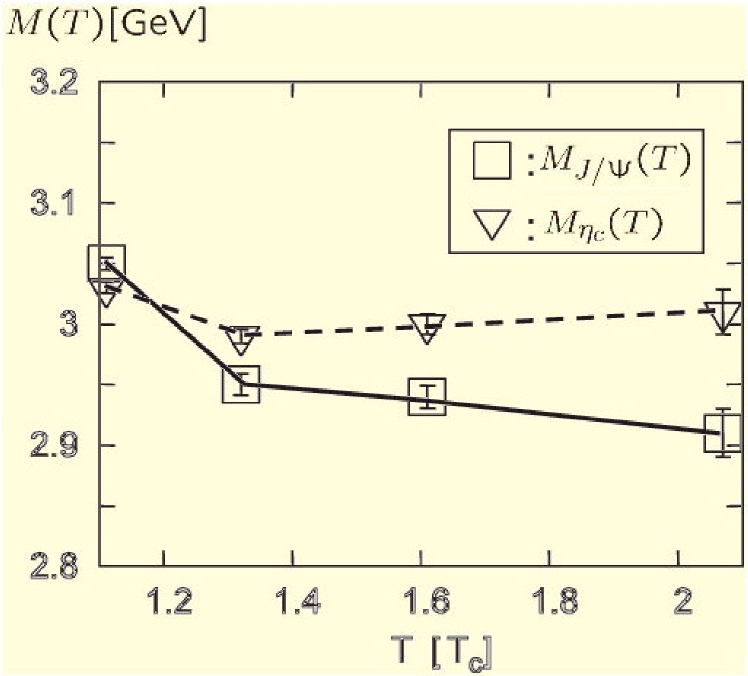


FIG. 8. Temperature dependence of the pole mass (on PBC) of J/Ψ and η_c for $(1.11\text{–}2.07)T_c$. The squares denote $M_{J/\Psi}(T)$ and the inverse triangles denote $M_{\eta_c}(T)$. There occurs the level inversion of J/Ψ and η_c above $1.3T_c$.

H. Iida et al., PRD74, 074502 (2006)

Several groups show

- almost no change in Ps channel
- small but visible change in V channel

These results can be explained by the constant contribution.

- no constant in Ps channel
- small constant in V channel (proportional to p_i^2)

in free quark case

Summary

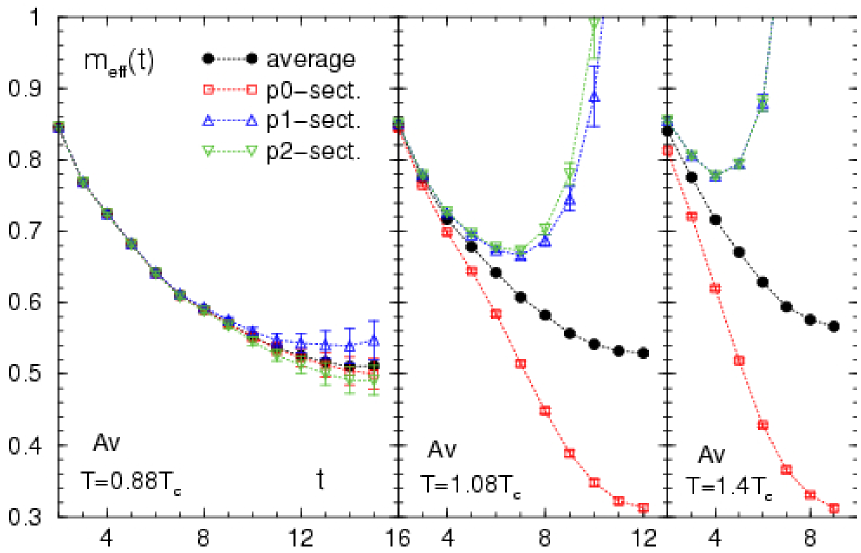
- We discuss a constant contribution to meson correlators at $T>0$
- The constant mode is important to study temporal correlators in deconfined phase

As a result of the study, we find that drastic changes in charmonium correlators for χ_c states just above the T_c are due to the constant contribution. The other differences in the χ_c states are small.
It may indicate the survival of χ_c states above T_c, at least $T=1.4T_c$.

→ The result may affect the scenario of J/ψ suppression.
It is important to take the constant effects into account for studies of the dissociation temp. of χ_c & ψ'.

Discussion

Polyakov loop sector dependence



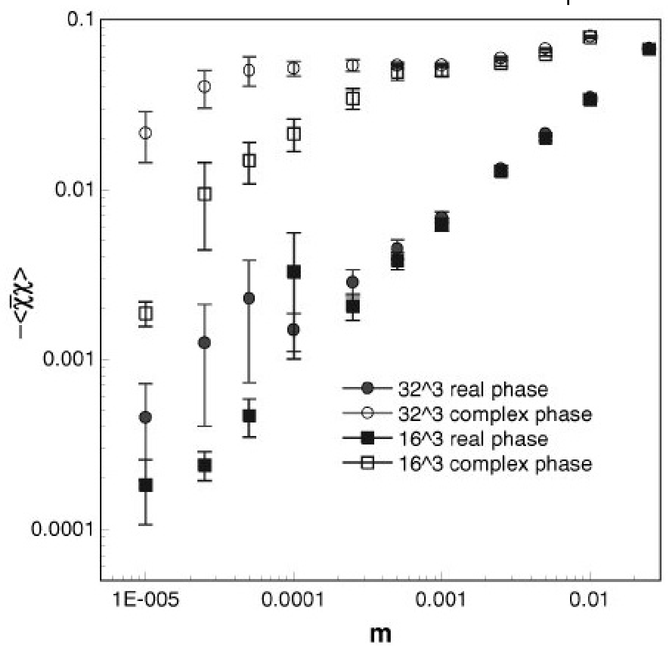
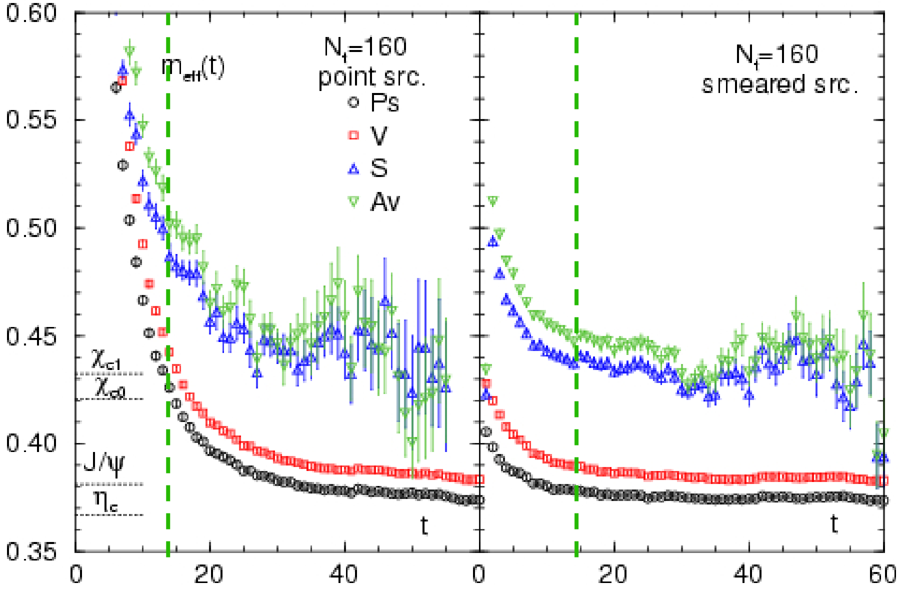


FIG. 1. The chiral condensate $\langle\bar{\chi}\chi\rangle$ plotted as a function of quark mass for a pure gauge calculation on $16^3 \times 4$ and $32^3 \times 4$ lattices. The real phase (closed points) is the most physical [$\det(D-m)$ is largest for this phase]. No evidence is seen for the expected anomalous behavior, $\langle\bar{\chi}\chi\rangle \sim m^{-1}$ as $m \to 0$.

(*) These is no Z3 symmetry
in full QCD.

S. Chandrasekharan et al., PRL82, 2463, (1999).

Results with extended op.



Spatially extended operators:

$$O_\Gamma(\vec{x},t) = \sum_{\vec{y}} \phi(\vec{y})\bar{q}(\vec{x}-\vec{y},t)\Gamma q(\vec{x},t)$$

with a smearing func. $\phi(x)$
in Coulomb gauge

$$\phi(\vec{x}) = \exp\left(-A|\vec{x}|^P\right)$$

A, P are tuned by matching with charmonium wave func.

momentum distribution of quark propagator
is given by Fourier transformation of $\phi(x)$

our extended op. enhances lower momentum of quark prop.

→ The extended op. yields large overlap with lowest states

(※) smearing func. changes only overlap with each state