Constant mode in meson correlators at finite temperature

Takashi Umeda

筑波大学
University of Tsukuba

This talk is based on the hep-lat/0701005.

to appear in Phys. Rev. D

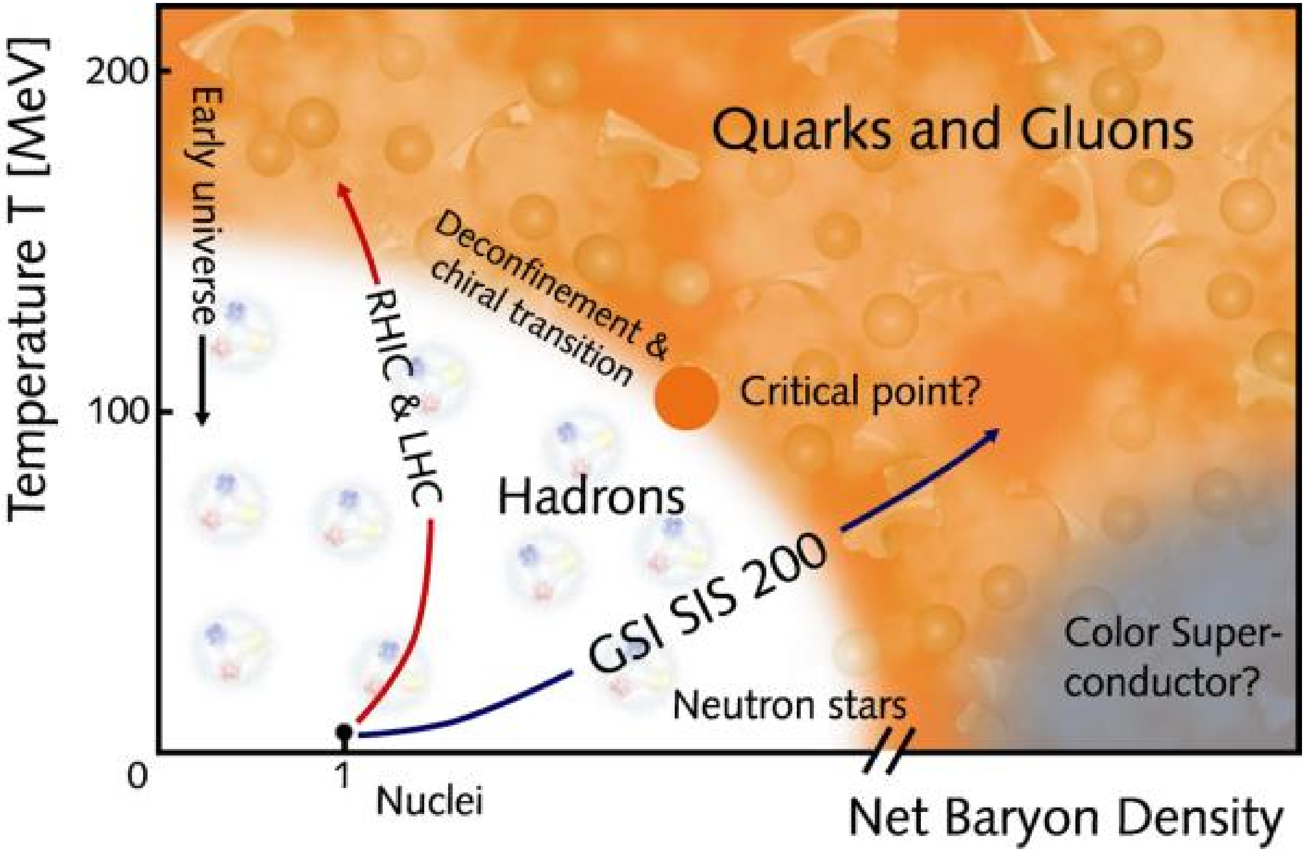
Tsukuba Theory Seminar, Univ. of Tsukuba, Apr. 20, 2007

Contents

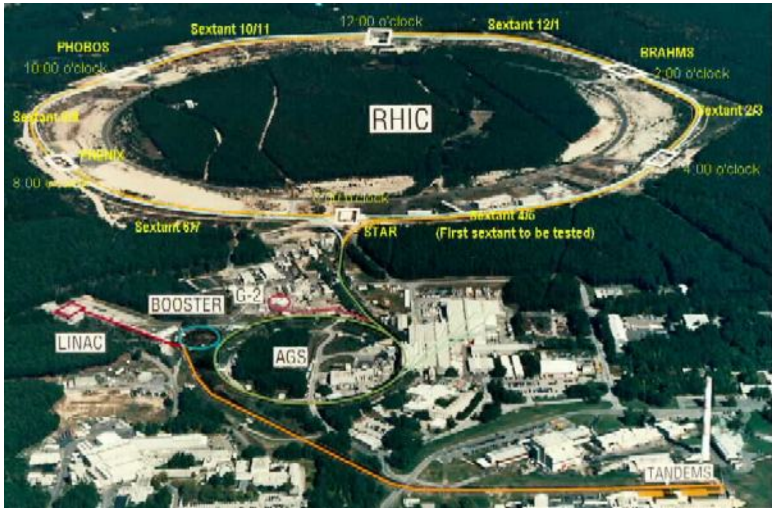
- Introduction
  - Overview
  - Search for Quark Gluon Plasma
  - (Sequential) J/ψ suppression
- Quenched QCD calculations
  - Lattice setup
  - T dependence of charmonium correlators
  - Constant mode in meson correlators
- Discussion & summary

Quark Gluon Plasma (QGP)

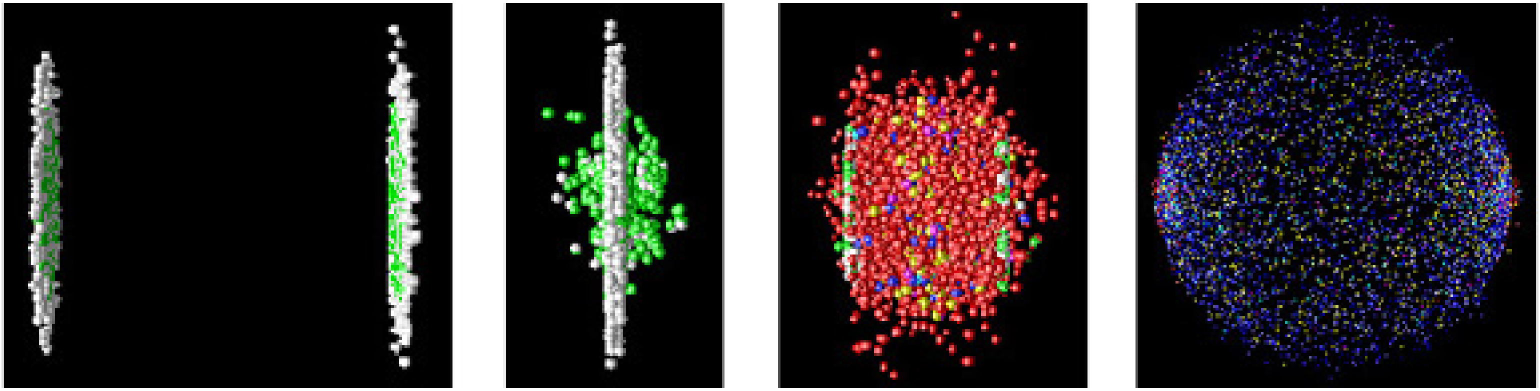
from Munster Univ. web-site



Experiments



- SPS : CERN (– 2005)
  Super Proton Synchrotron
- RHIC: BNL (2000 – now)
  Relativistic Heavy Ion Collider
- LHC : CERN (2009 –)
  Large Hadron Collider

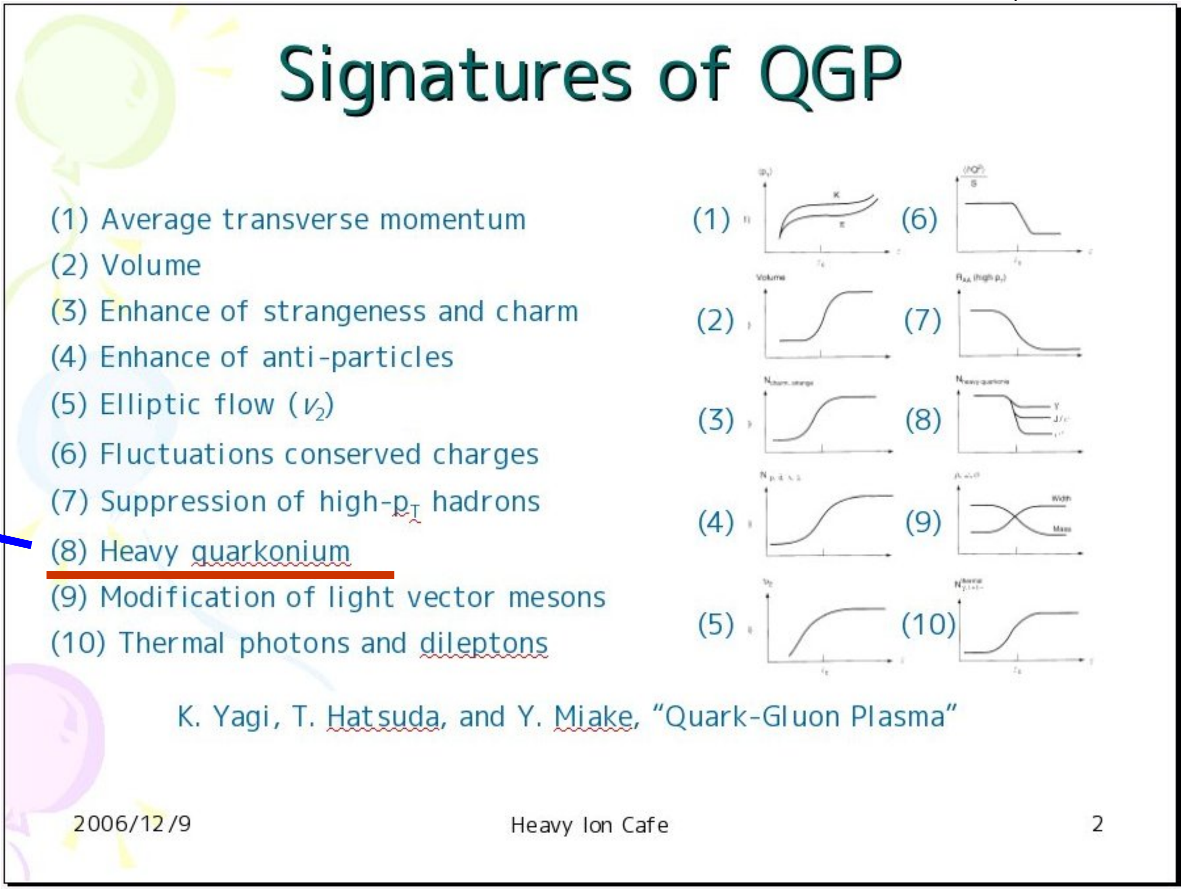


from the Phoenix group web-site

Signatures of QGP

J/ψ suppression

T.Hashimoto et al., PRL57 (1986) 2123.
T.Matsui & H.Satz, PLB178 (1986) 416.



Talk by Ozawa @ Heavy Ion Cafe

First paper on the J/ψ suppression

VOLUME 57, NUMBER 17 PHYSICAL REVIEW LETTERS 27 OCTOBER 1986

Mass Shift of Charmonium near Deconfining Temperature and Possible Detection in Lepton-Pair Production

Takaaki Hashimoto and Osamu Miyamura
Department of Applied Mathematics, Faculty of Engineering Science, Osaka University, Toyonaka, Osaka 560, Japan

Kikuji Hirose
Teikoku Women's University, Moriguchi, Osaka 570, Japan

and

Takeshi Kanki
College of General Education, Osaka University, Toyonaka, Osaka 560, Japan
(Received 27 May 1986)

photo : Prof. Osamu Miyamura

VOLUME 57, NUMBER 17 PHYSICAL REVIEW LETTERS 27 OCTOBER 1986

in Monte Carlo analyses.[7,8] A related question is whether charmoniumlike clusters may still exist in a quark-gluon plasma. We have made tentative calculations by screened Coulombic potential and found that possibility small. Thus, contribution to lepton pair in the J/ψ mass region from the deconfinement phase would be mainly thermal quark-antiquark annihilation.[18] In connection with this point, we make a com-

lin, 1985), p. 1.
[4]R. D. Pisarski, Phys. Lett. **110B**, 155 (1982).
[5]R. D. Pisarski and F. Wilczek, Phys. Rev. D **29**, 338 (1984).
[6]L. McLerran and B. Svetitsky, Phys. Rev. D **24**, 450 (1981).
[7]M. Fukugita, T. Kaneko, and A. Ukawa, Phys. Lett. **154B**, 185 (1985).
[8]C. Dorninger, H. Leeb, and H. Markum, Z. Phys. C **29**

J/ψ suppression in Exp.

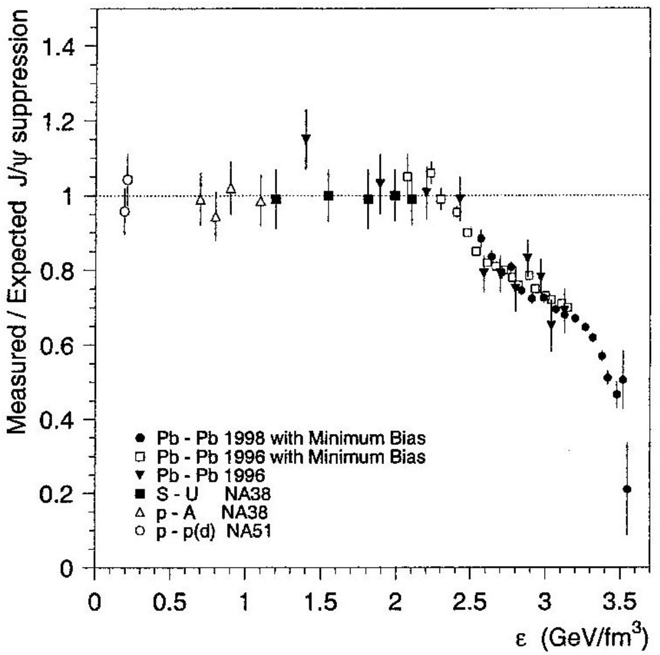


Fig. 7. Measured J/ψ production yields, normalised to the yields expected assuming that the only source of suppression is the ordinary absorption by the nuclear medium. The data is shown as a function of the energy density reached in the several collision systems.

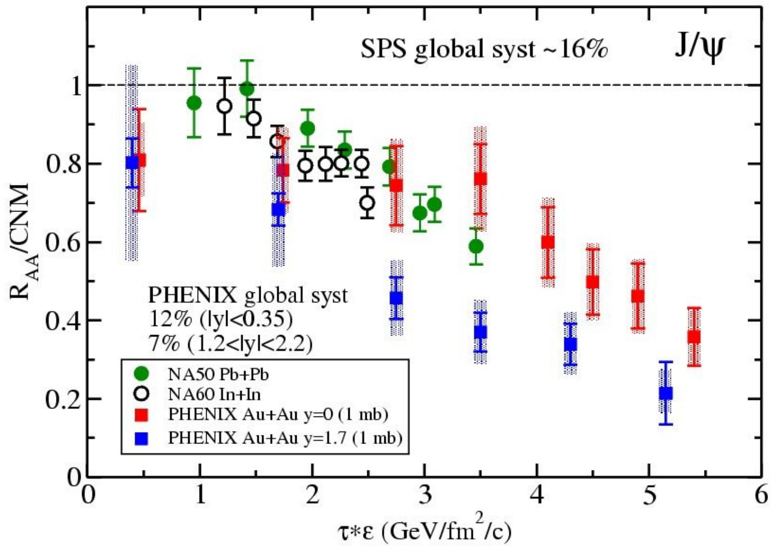


Figure 10: Survival fraction (R_{AA}/CNM) vs energy density comparison of PHENIX Au+Au suppression to that from NA38/50 at CERN.

Phys.Lett.B477(2000)28
NA50 Collaboration

QM2006
PHENIX Collaboration

Spectral Function in lattice QCD

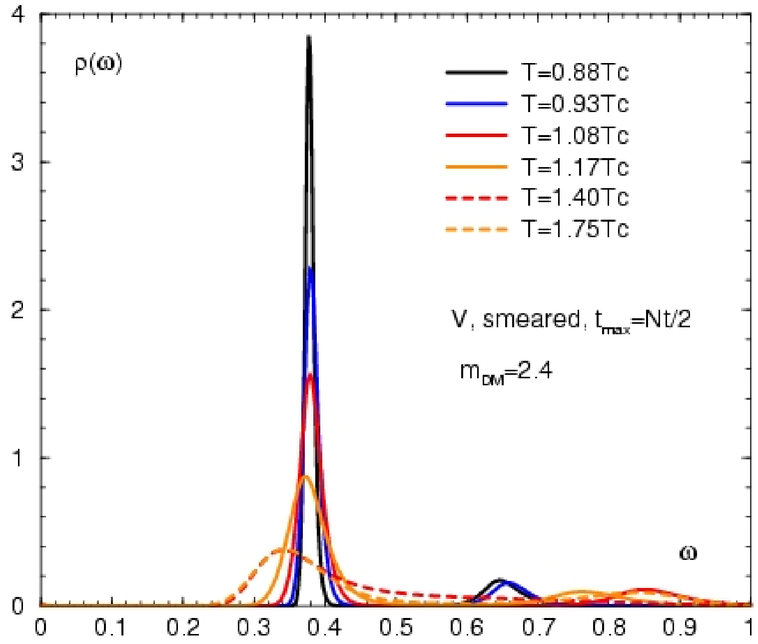
$$C(t) = \int_0^\infty d\omega \rho_\Gamma(\omega) K(\omega,t),$$

$$K(\omega,t) = \frac{\cosh(\omega(N_t/2-t))}{\sinh(\omega N_t/2)}$$

C(t) : meson correlator

Nt = 1/(aT)

T: temperature



Quenched QCD

- *T. Umeda et al., EPJC39S1, 9, (2005).*
- *S. Datta et al., PRD69, 094507, (2004).*
- *T. Hatsuda & M. Asakawa, PRL92, 012001, (2004).*
- *A. Jakovac et al., PRD75, 014506 (2007).*

Full QCD

- *G. Aarts et al., hep-lat/0610065.*

In the studies of SPFs on the lattice,
all studies indicate survival of J/ψ state above T_c ($1.5T_c$?)

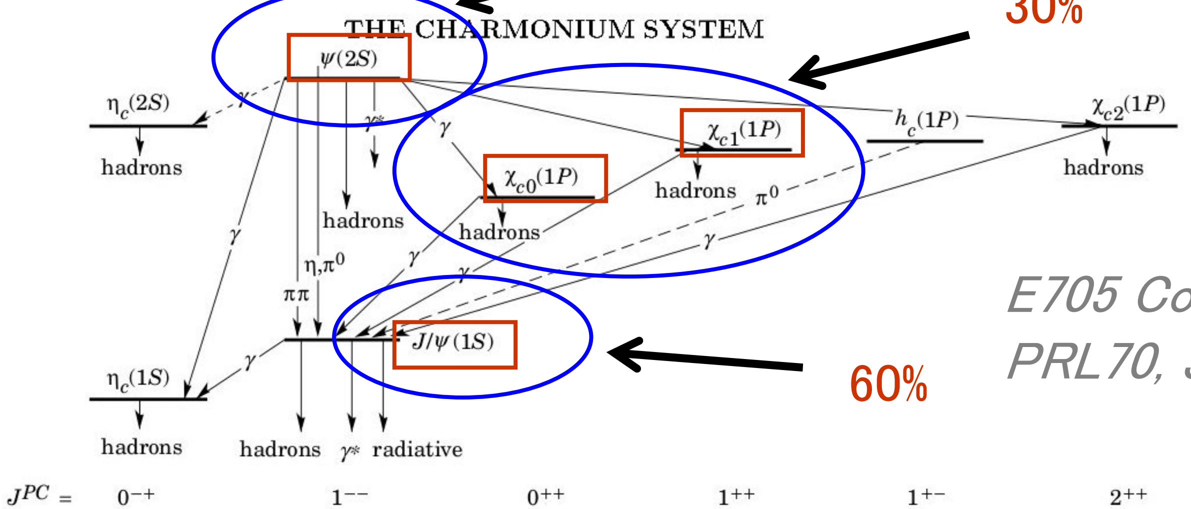
Sequential J/ψ suppression

Particle Data Group (2006)

$c\bar{c}$ MESONS

10%   for the total yield of J/ψ

30%



E705 Collaboration, PRL70, 383, (1993).

60%

Ps   V   S   Av

Dissociation temperatures of J/ψ and ψ' & χ_c are important for QGP phenomenology.

χ_c states on the lattice

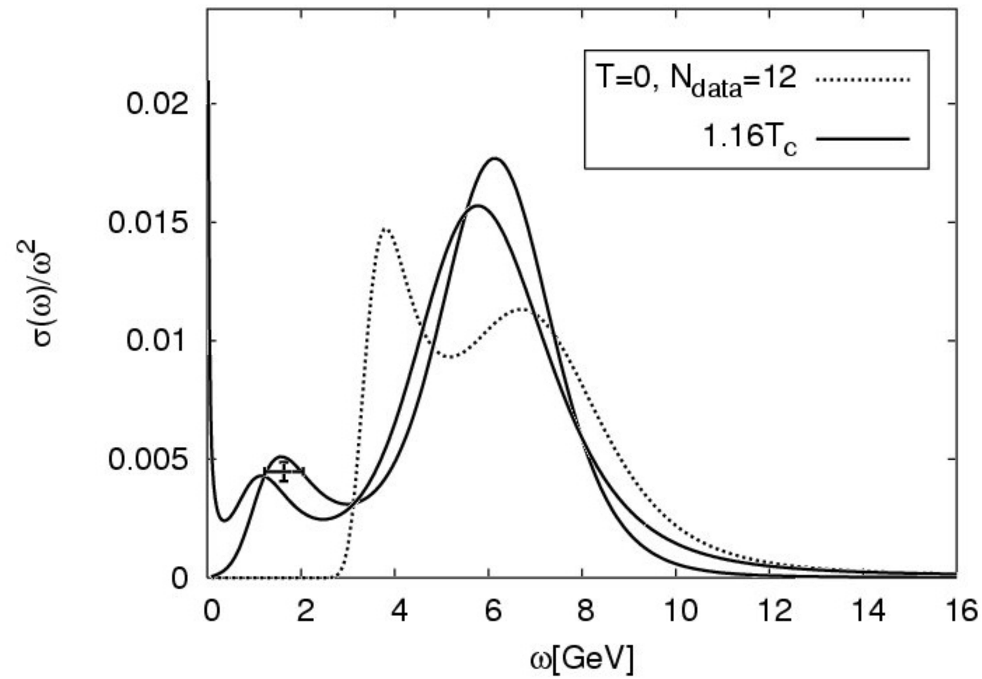


FIG. 19: The scalar spectral function for $\beta = 6.1$ at $T = 1.16T_c$ and at zero temperature reconstructed using $N_{data} = 12$. At finite temperature two default models $m(\omega) = 0.01$ and $m(\omega) = 0.038\omega^2$ have been used.

S. Datta et al.,
PRD69, 094507 (2004).
A.Jakovac et al.,
PRD75, 014506 (2007).

Contents

- Introduction
  - Overview
  - Search for Quark Gluon Plasma
  - (Sequential) J/ψ suppression
- Quenched QCD calculations
  - Lattice setup
  - T dependence of charmonium correlators
  - Constant mode in meson correlators
- Discussion & summary

Quenched QCD at T>0

Lattice setup

anisotropic lattices : $20^3 \times N_t$

$1/a_s = 2.03(1)$ GeV, $a_s/a_t = 4$

Clover quark action with tadpole imp. on anisotropic lattice

H. Matsufuru et al., PRD64, 114503 (2001).

r_s=1 to reduce cutoff effects in higher energy states

F. Karsch et al., PRD68, 014504 (2003).

quark mass is tuned with $M_{J/\psi}$(= 3097MeV)

N_τ	160	32	26	20
T/T_c	~ 0	0.88	1.08	1.4
# of conf.	60	300	300	300

equilib. is 20K sweeps
each config. is separated
by 500 sweeps

Effective mass (local mass)

Definition of effective mass

$$C(t) = A_0 e^{-m_0 t} + A_1 e^{-m_1 t} + \cdots$$
$$\equiv A e^{-m_{eff}(t) t}$$

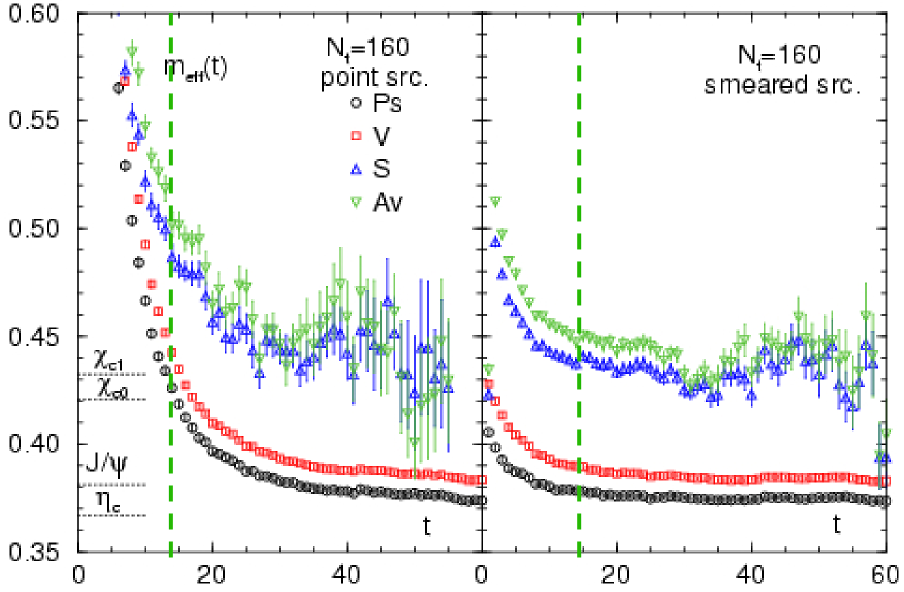
$$\frac{C(t)}{C(t+1)} = e^{-m_{eff}(t)}$$

$$m_{eff}(t) \to m_0 \quad \text{when} \quad (m_1 - m_0)t \gg 1$$

In the (anti) periodic b.c.

$$\frac{C(t)}{C(t+1)} = \frac{\cosh[m_{eff}(t)(N_t/2 - t)]}{\cosh[m_{eff}(t)(N_t/2 - t - 1)]}$$

At zero temperature



(our lattice results)

M_{PS} = 3033(19) MeV

M_V = 3107(19) MeV

(exp. results from PDG06)

$M_{\eta c}$ = 2980 MeV

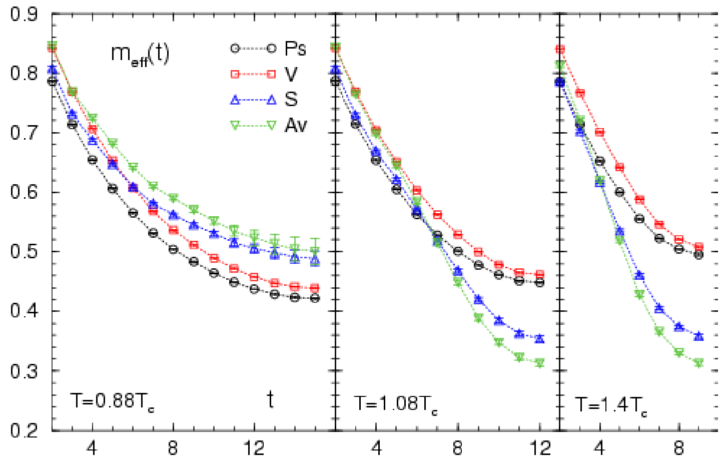
$M_{J/\psi}$ = 3097 MeV

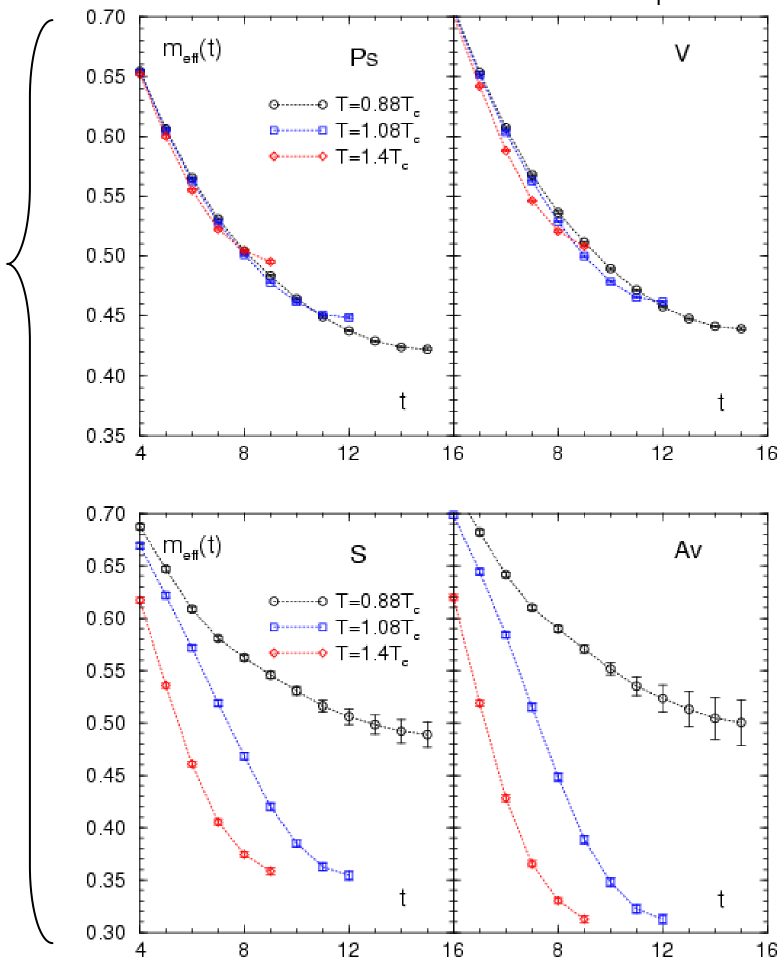
$M_{\chi c0}$ = 3415 MeV

$M_{\chi c1}$ = 3511 MeV

- In our lattice $N_t \simeq 28$ at T_c
  $t = 1 - 14$ is available near T_c
- Spatially extended (smeared) op. is discussed later

Quenched QCD at T>0



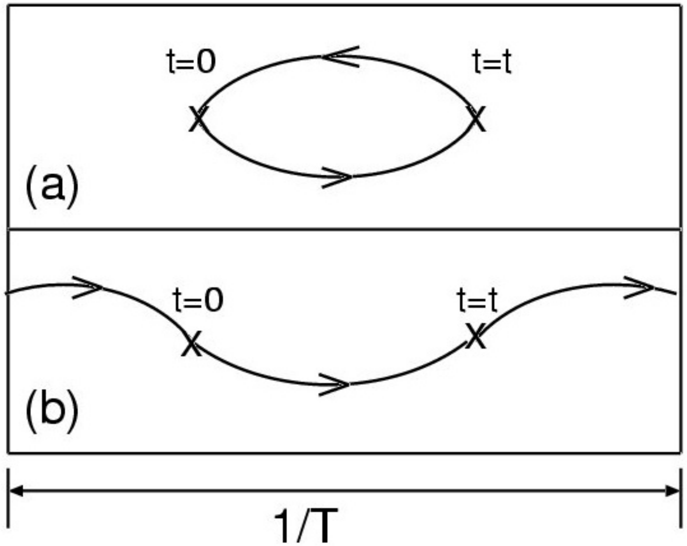


- small change in S−wave states
  → survival of J/ψ & η_c at T>T$_c$
- drastic change in P−wave states
  → dissociation of χ_c just above Tc (?)

S. Datta et al.,
PRD69, 094507 (2004). etc...

Constant mode

Now we consider the meson correlator with p=0 & $m_{q1}=m_{q2}$



$\exp(-m_q t) \times \exp(-m_q t)$
$= \exp(-2m_q t)$

m_q is quark mass
or single quark energy

$\exp(-m_q t) \times \exp(-m_q(L_t-t))$
$= \exp(-m_q L_t)$

L_t = temporal extent

- in imaginary time formalism
  $L_t = 1/\text{Temp.}$
  gauge field : periodic b.c.
  quark field : anti-periodic b.c.
- in confined phase: m_q is infinite

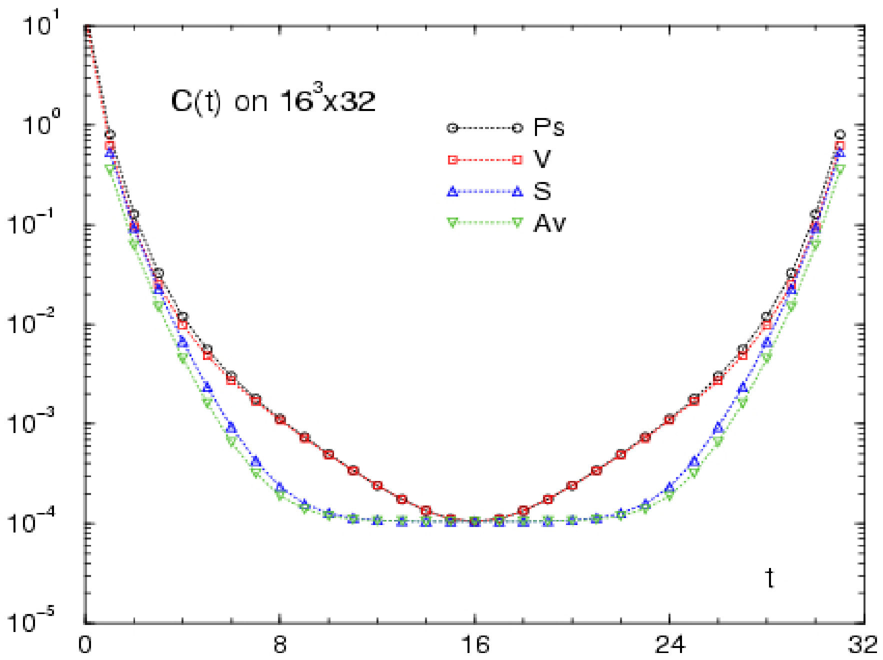
→ the effect appears
only in deconfined phase

Pentaquark (KN state):
two pion state:
→ Dirichlet b.c.
c.f. T.T.Takahashi et al., PRD71, 114509 (2005).

Free quark calculations

$$
\begin{aligned}
C(t) &= \sum_{\vec{x}} \langle O_\Gamma(\vec{x},t) O_\Gamma^\dagger(\vec{0},0)\rangle, \\
& O_\Gamma(\vec{x},t) = \bar{q}(\vec{x},t)\Gamma q(\vec{x},t), \\
& \Gamma = \gamma_5, \gamma_i, 1 \text{ and } \gamma_i\gamma_5
\end{aligned}
$$

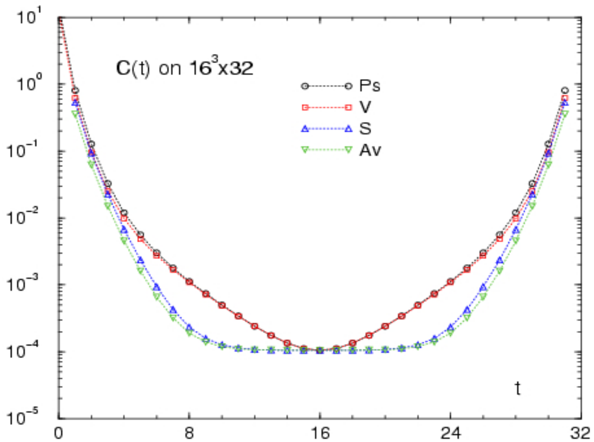
for Ps, V, S and Av channels

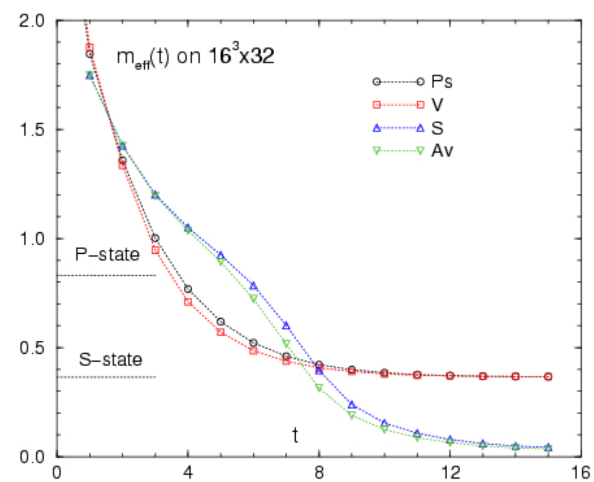


- 16^3 x 32 isotropic lattice
- Wilson quark action
  with $m_q a = 0.2$

Obvious constant contribution in P–wave states

Free quark calculations





Continuum form of the correlators

calculated by S. Sasaki

$$C(t) = \sum_{\vec{p}} \frac{4}{\cosh\left(E_p N_t/2\right)} \times$$

$$\begin{cases} \left(E_p^2 \cosh\left[2E_p(t-N_t/2)\right]\right) & \text{for } \Gamma = \gamma_5 \\ \left((E_p^2 - p_i^2)\cosh\left[2E_p(t-N_t/2)\right] + p_i^2\right) & \text{for } \Gamma = \gamma_i \\ -\left(p^2 \cosh\left[2E_p(t-N_t/2)\right] + (E_p^2 - p^2)\right) & \text{for } \Gamma = 1 \\ -\left((p^2 - p_i^2)\cosh\left[2E_p(t-N_t/2)\right] + (E_p^2 - p^2 + p_i^2)\right) & \text{for } \Gamma = \gamma_i\gamma_5 \end{cases}$$

where

E_p : single quark energy with relative mom. p

$$p^2 = \sum_i p_i^2$$

Spectral representation

Spectral function of the correlator

F. Karsch et al., PRD68, 014504 (2003).
G. Aarts et al., NPB726, 93 (2005).

$$C(t) = \int_0^\infty d\omega \rho_\Gamma(\omega) K(\omega, t),$$

$$K(\omega, t) = \frac{\cosh(\omega(N_t/2 - t))}{\sinh(\omega N_t/2)}$$

$$\begin{aligned}\rho_\Gamma(\omega) &= \Theta(\omega^2 - 4m_q^2)\frac{N_c}{8\pi\omega}\sqrt{\omega^2 - 4m_q^2}[1 - 2n_F(\omega/2)] \\ &\times[\omega^2(a_H^{(1)} - a_H^{(2)}) + 4m^2(a_H^{(2)} - a_H^{(3)})] \\ &+2\pi\omega\delta(\omega)N_c[(a_H^{(1)} + a_H^{(2)})I_1 + (a_H^{(2)} - a_H^{(3)})I_2]\end{aligned}$$

with

$$I_1 = -2\int_{\vec{k}} n'_F(\omega_{\vec{k}})$$

$$I_2 = -2\int_{\vec{k}} \frac{k^2}{\omega_{\vec{k}}^2} n'_F(\omega_{\vec{k}})$$

	Γ	$a_H^{(1)}$	$a_H^{(2)}$	$a_H^{(3)}$	$a_H^{(1)} - a_H^{(2)}$	$a_H^{(2)} - a_H^{(3)}$	$a_H^{(1)} + a_H^{(2)}$	$a_H^{(2)} - a_H^{(3)}$
Ps	γ_5	1	-1	-1	2	0	0	0
V	γ_i	3	-1	-3	4	2	2	2
S	1	1	-1	1	2	-2	0	-2
Av	$\gamma_i\gamma_5$	3	-1	3	4	-4	2	-4

chiral symmetry in massless limit

Physical interpretation

$$
\begin{aligned}
\rho_\Gamma(\omega) \;\; = \;\; & \Theta(\omega^2 - 4m_q^2)\frac{N_c}{8\pi\omega}\sqrt{\omega^2 - 4m_q^2}[1 - 2n_F(\omega/2)] \\
& \times[\omega^2(a_H^{(1)} - a_H^{(2)}) + 4m^2(a_H^{(2)} - a_H^{(3)})] \\
& + 2\pi\omega\delta(\omega)N_c[(a_H^{(1)} + a_H^{(2)})I_1 + (a_H^{(2)} - a_H^{(3)})I_2]
\end{aligned}
$$

constant contribution remains
in the continuum form & infinite volume

The constant term is related to some transport coefficients.

From Kubo–formula, for example, a derivative of the SPF in the V channel is related to the electrical conductivity σ.

$$\sigma = \frac{1}{6}\frac{\partial}{\partial\omega}\rho_V(\omega)\Big|_{\omega=0}$$

Without constant mode

How much does SPF change at the region $\omega \gg T$

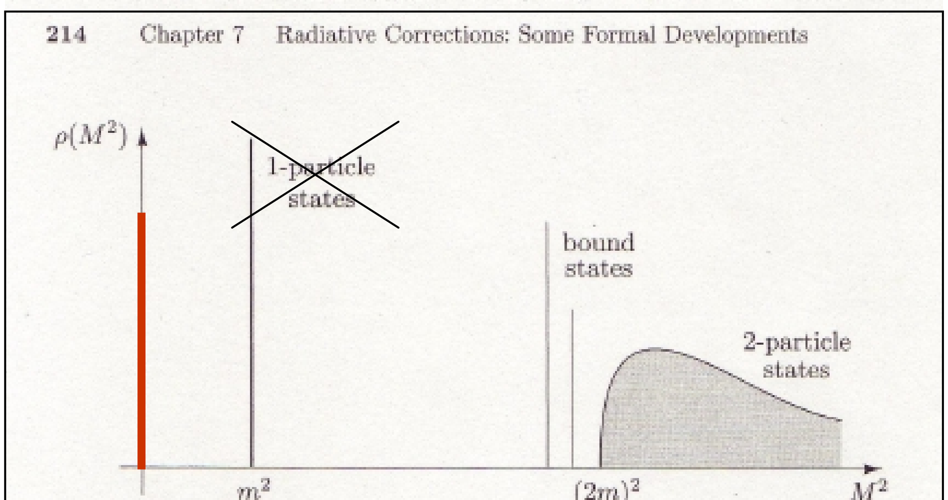

214 Chapter 7 Radiative Corrections: Some Formal Developments

Figure 7.2. The spectral function $\rho(M^2)$ for a typical interacting field theory. The one-particle states contribute a delta function at m^2 (the square of the particle's mass). Multiparticle states have a continuous spectrum beginning at $(2m)^2$. There may also be bound states.

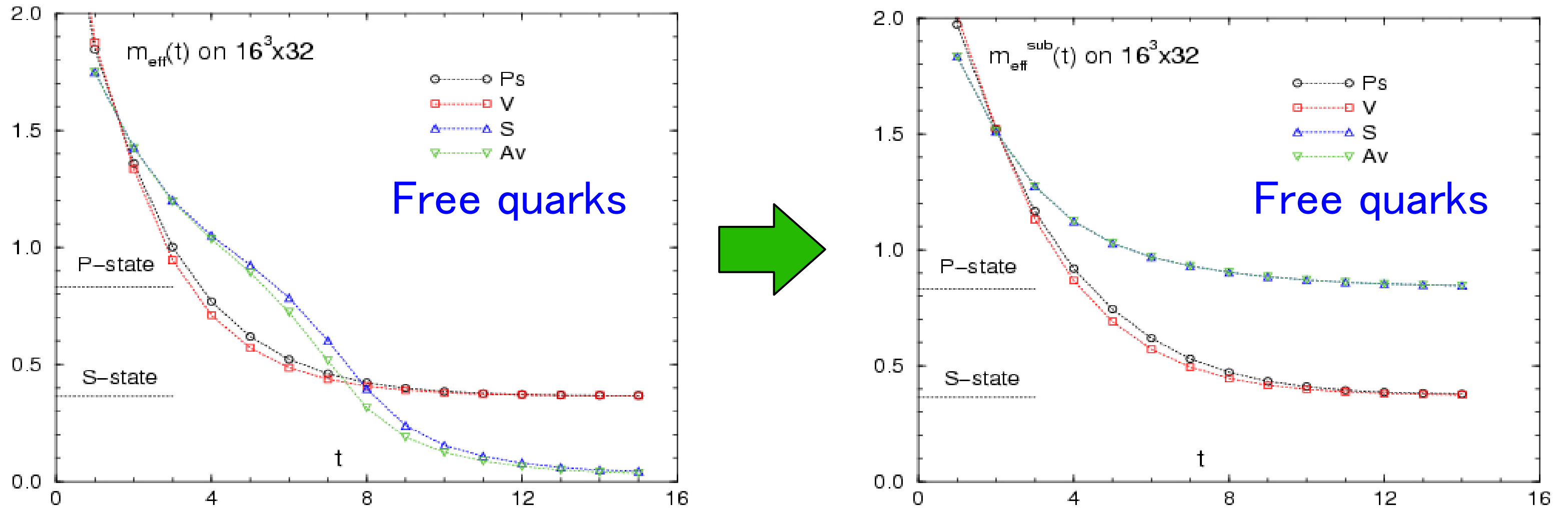
from "An Introduction to Quantum Field Theory"
Michael E. Peskin, Perseus books (1995)

Midpoint subtraction

An analysis to avoid the constant mode

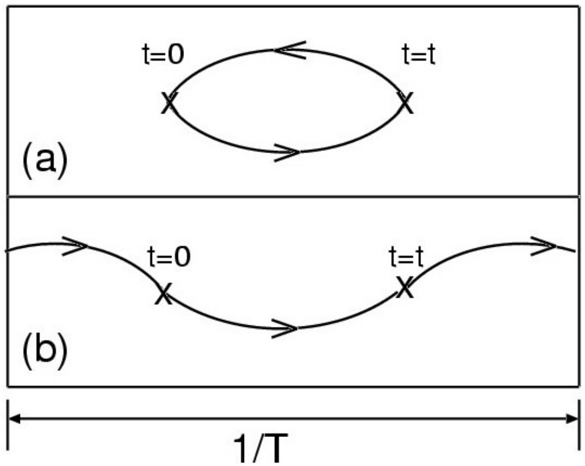
Midpoint subtracted correlator

$$\bar{C}(t) = C(t) - C(N_t/2) \quad \Rightarrow \quad \frac{\bar{C}(t)}{\bar{C}(t+1)} = \frac{\sinh^2\left[\frac{1}{2}m_{eff}^{sub}(t)(N_t/2-t)\right]}{\sinh^2\left[\frac{1}{2}m_{eff}^{sub}(t)(N_t/2-t-1)\right]}$$



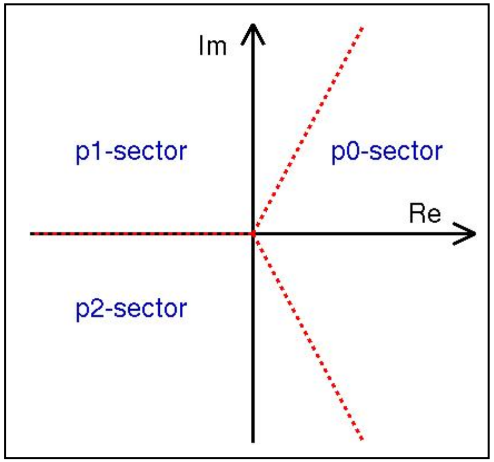
Z_3 symmetrization

Here we consider the Z_3 transformation



Z_3 symmetric

Z_3 asymmetric

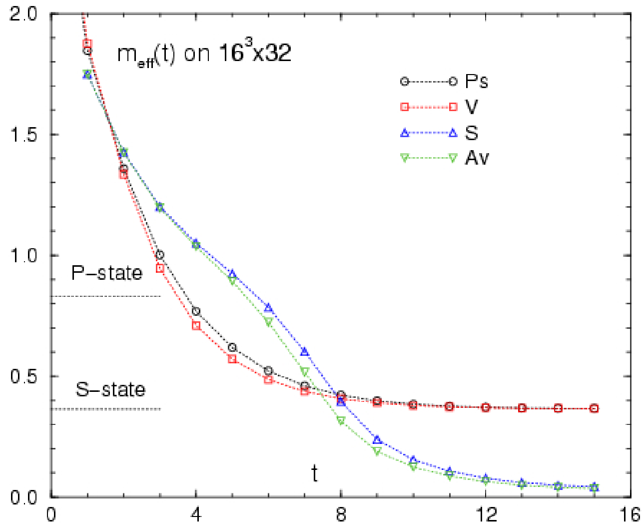


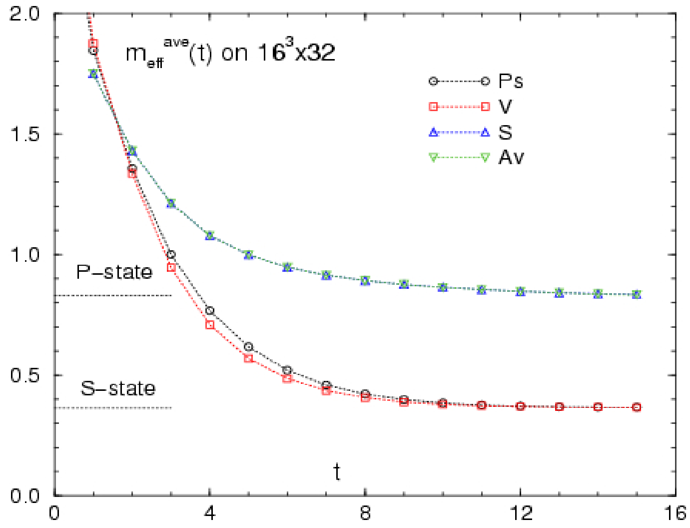
The asymmetry of diag–(b)
is coming from a factor of Re[exp(−i2 π n/3)]

$$C^{ave}(t) = \frac{1}{3}\left(C^{p0}(t) + C^{p1}(t) + C^{p2}(t)\right)$$

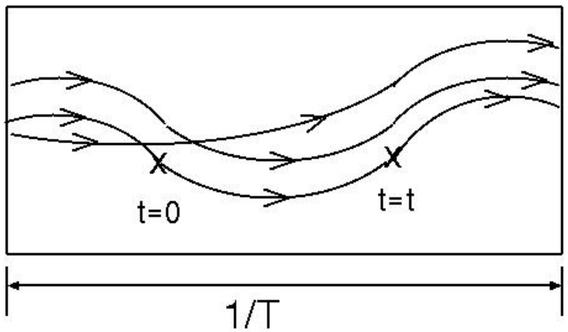
Z_3 asym. terms are removed because $\sum_{n=0}^{2} Re(e^{-i2\pi n/3}) = 0$

Averaged correlators





However, this is not an exact method to avoid the constant contribution.



The 3 times wrapping diagram is also Z_3 symmetric.

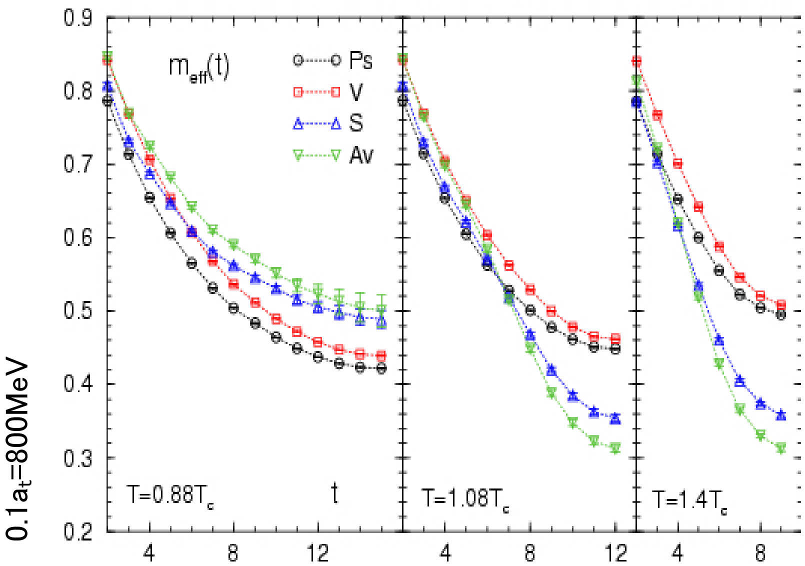
→ the contribution is not canceled.

but, $O(\exp(-m_q N_t)) \gg O(\exp(-3m_q N_t))$

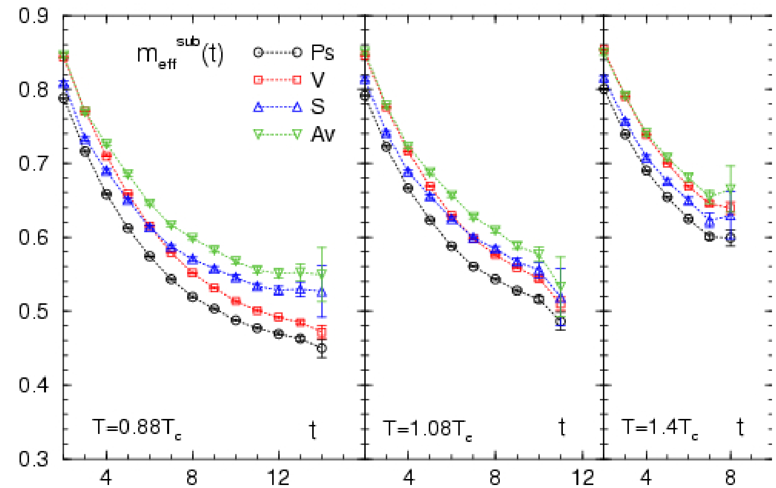
Midpoint subtraction analysis

$$\bar{C}(t) = C(t) - C(N_t/2) \qquad \frac{\bar{C}(t)}{\bar{C}(t+1)} = \frac{\sinh^2\left[\frac{1}{2}m_{eff}^{sub}(t)(N_t/2-t)\right]}{\sinh^2\left[\frac{1}{2}m_{eff}^{sub}(t)(N_t/2-t-1)\right]}$$

usual effective masses at T>0



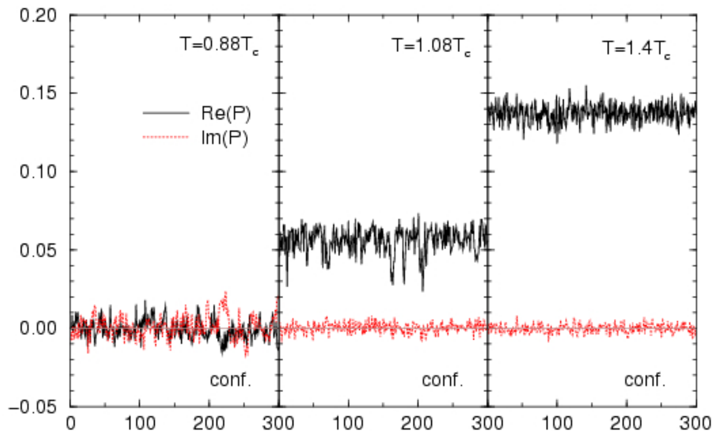
subtracted effective mass



the drastic change in P−wave states disappears in $m_{eff}^{sub}(t)$

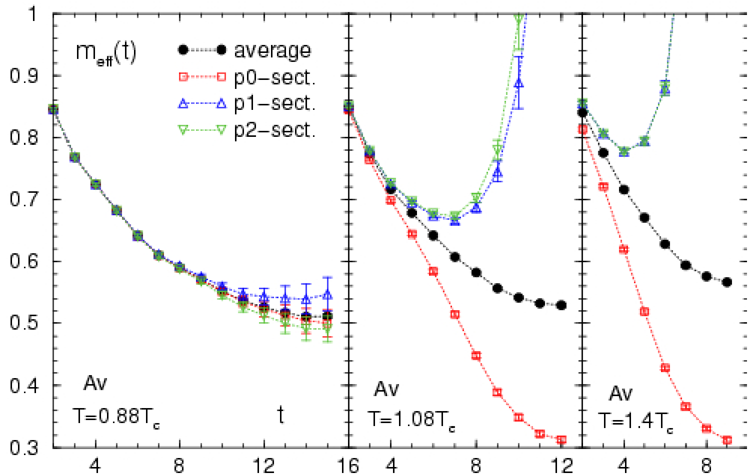
→ the change is due to the constant mode

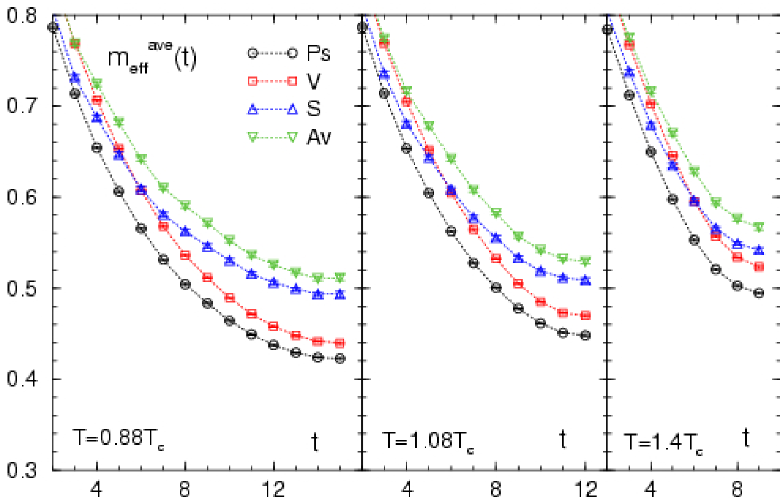
Polyakov loop sector dependence



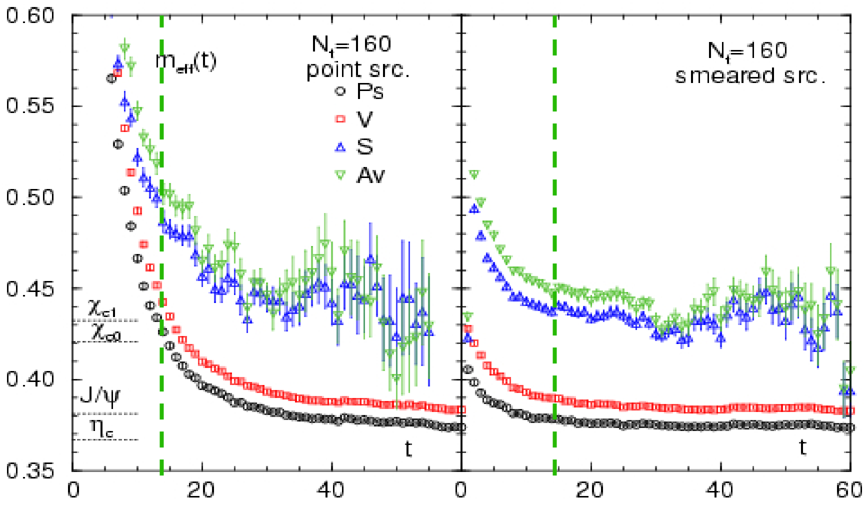
- after Z_3 transformation
  const. → Re(exp(−i2 π n/3))*const.
- even below T_c, small const. effect
  enhances the stat. fluctuation.
- drastic change in P−states disappears.

Results for Av channel





Results with extended op.



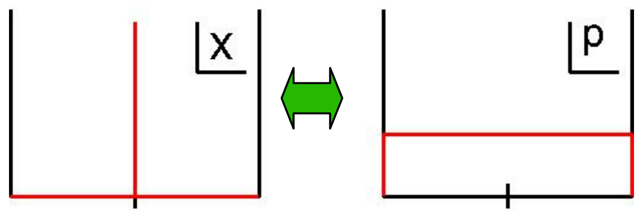
Spatially extended operators:

$$O_\Gamma(\vec{x},t) = \sum_{\vec{y}} \phi(\vec{y})\bar{q}(\vec{x}-\vec{y},t)\Gamma q(\vec{x},t)$$

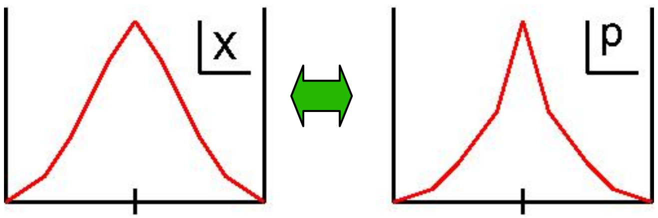
with a smearing func. ϕ(x)

in Coulomb gauge

point source func.



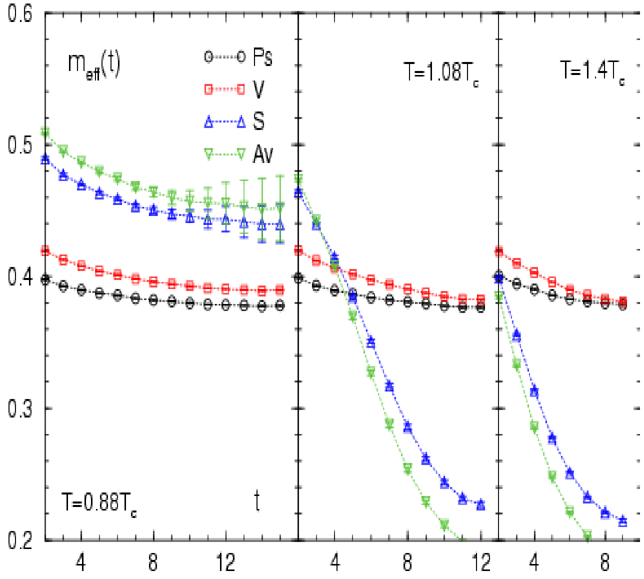
smeared source func.



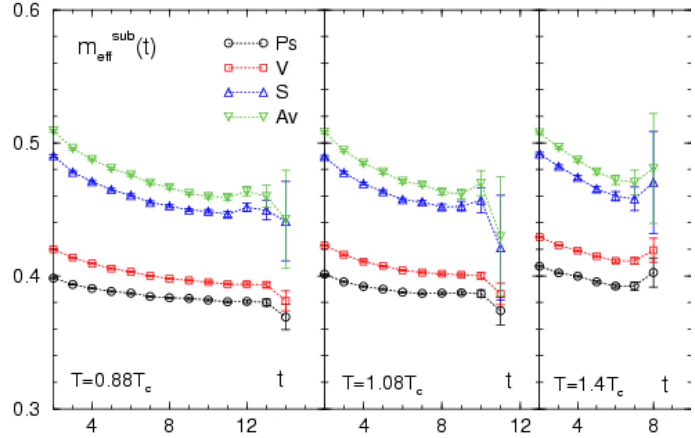
The extended op. yields large overlap with lowest states

Results with extended op.

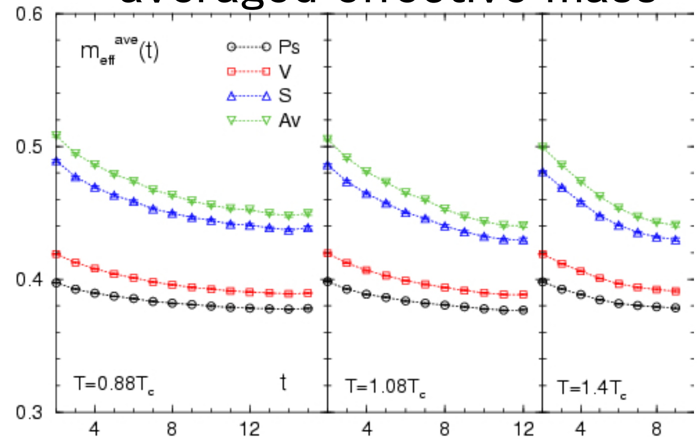
usual effective mass



subtracted effective mass



averaged effective mass

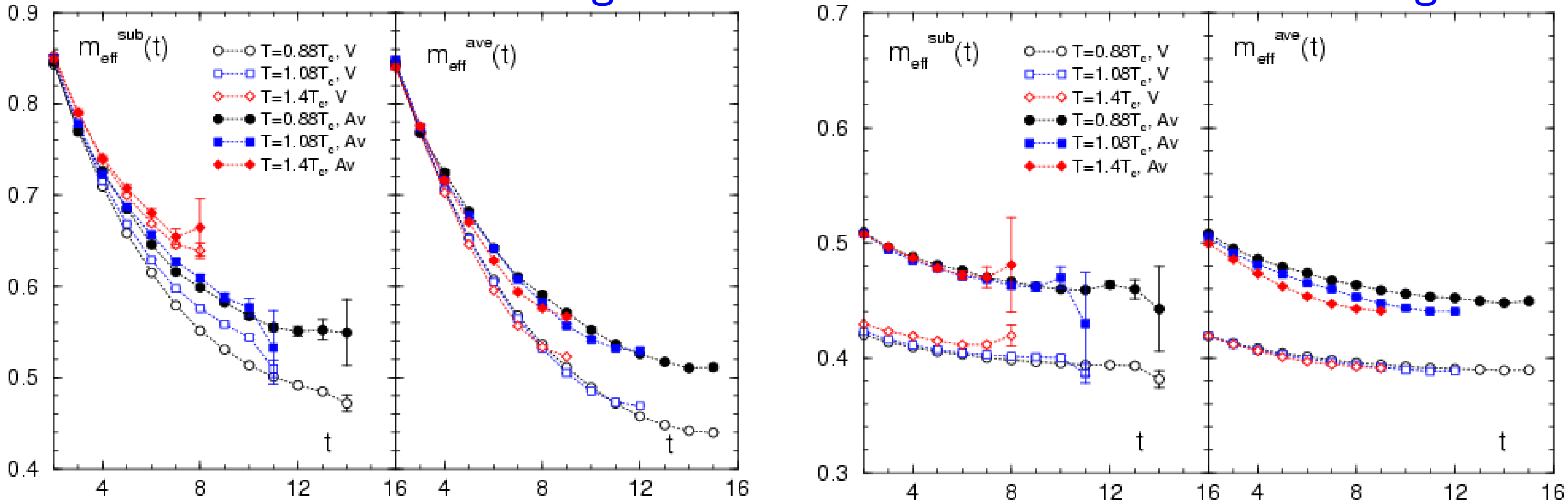


- extended op. enhances overlap with const. mode
- small constant effect is visible in V channel
- no large change above T_c in $m_{eff}^{sub}(t)$ and $m_{eff}^{ave}(t)$

Discussion

point operators — subtracted / averaged

extended operators — subtracted / averaged



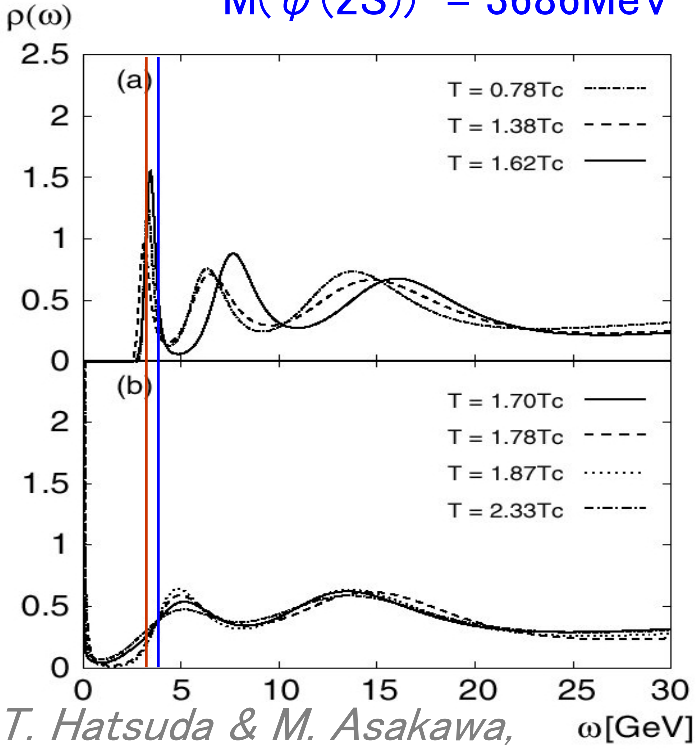
The drastic change of P−wave states is due to the const. contribution.

→ There are small changes in SPFs (except for ω=0).

Why several MEM studies show the dissociation of χ_c states ?

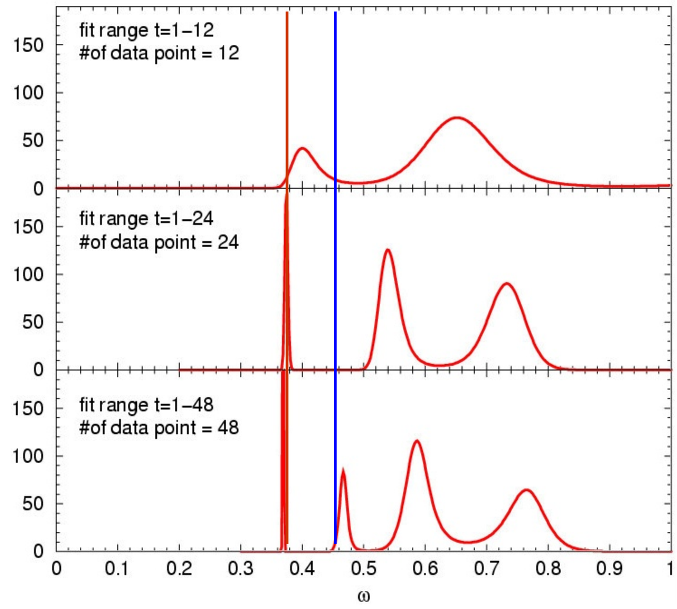
Difficulties on non-lowest states

M(J/ψ(1S)) = 3097MeV
M(ψ(2S)) = 3686MeV



T. Hatsuda & M. Asakawa, PRL92, 012001 (2004).

MEM test using T=0 data



data for T/T_c=1.2

data for T/T_c=0.6

data for T/T_c=0

It is difficult to reproduce the non-lowest states peak at T>0

Furthermore P-wave states have larger noise than S-wave states

English is strange ….

Discussion

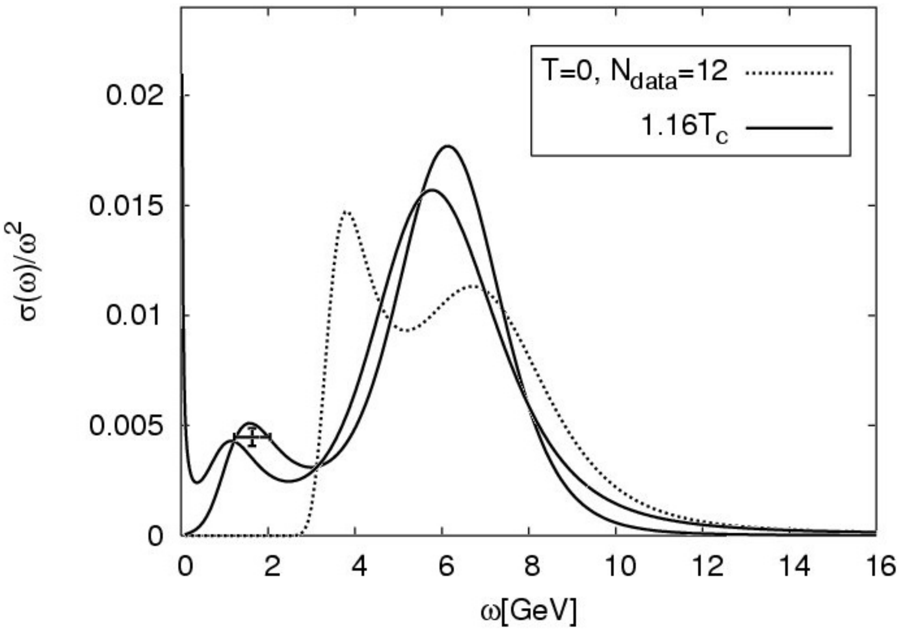


FIG. 19: The scalar spectral function for $\beta = 6.1$ at $T = 1.16T_c$ and at zero temperature reconstructed using $N_{data} = 12$. At finite temperature two default models $m(\omega) = 0.01$ and $m(\omega) = 0.038\omega^2$ have been used.

A.Jakovac et al., hep-lat/0611017.

Most systematic & reliable calc. using MEM for charmonium SPFs

They have concluded that

- the results of SPFs for P-states are not so reliable.
  e.g. large default model dep.
- the drastic change just above Tc is reliable results.

Conclusion

- There is the constant mode in charmonium correlators above T_c
- The drastic change in χ_c states is due to the constant mode

  ➔ the survival of χ_c states above T_c, at least $T=1.4T_c$.

The result may affect the scenario of J/ψ suppression.

In the MEM analysis,
one has to check consistency of the results at $\omega \gg T$
using, e.g., midpoint subtracted correlators.

$$\bar{C}(t) = C(t) - C(N_t/2)$$

$$\bar{C}(t) = \int_0^\infty d\omega \rho_\Gamma(\omega) K^{sub}(\omega, t),$$

$$K^{sub}(\omega, t) = \frac{\sinh^2(\frac{\omega}{2}(N_t/2 - t))}{\sinh(\omega N_t/2)}$$

Discussion

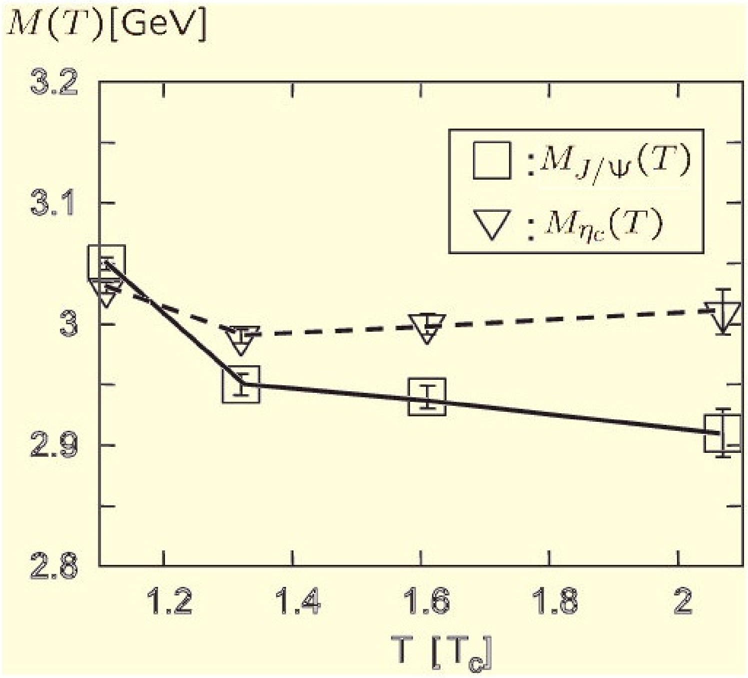


FIG. 8. Temperature dependence of the pole mass (on PBC) of J/Ψ and η_c for $(1.11\text{–}2.07)T_c$. The squares denote $M_{J/\Psi}(T)$ and the inverse triangles denote $M_{\eta_c}(T)$. There occurs the level inversion of J/Ψ and η_c above $1.3T_c$.

H. Iida et al., PRD74, 074502 (2006).

Several groups have presented

- almost no change in Ps channel
- small but visible change
  in V channel

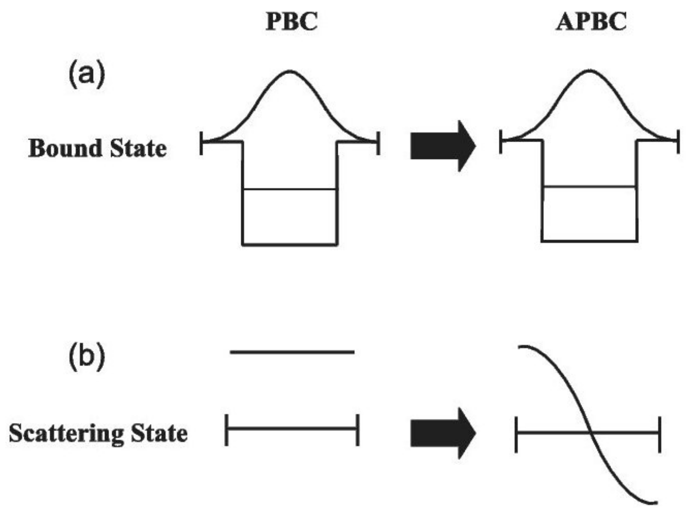
These results can be explained by the constant contribution.

- no constant in Ps channel
- small constant in V channel
  (proportional to p_i^2)

in free quark case

Discussion

H. Iida et al., PRD74, 074502 (2006).



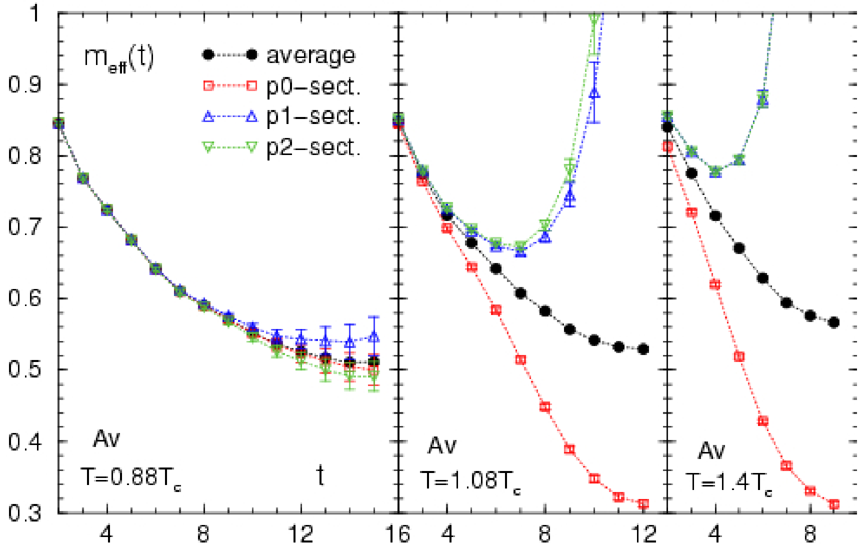
S−wave states : PBC p=(0, 0, 0) ＜ xAPBC p=(π /L, 0, 0)

P−wave states : PBC p=(2π /L, 0, 0) ＞ xAPBC p=(π /L, 0, 0)

P−const. : PBC p=(0, 0, 0) ＜ xAPBC p=(π /L, 0, 0)

Discussion

Polyakov loop sector dependence



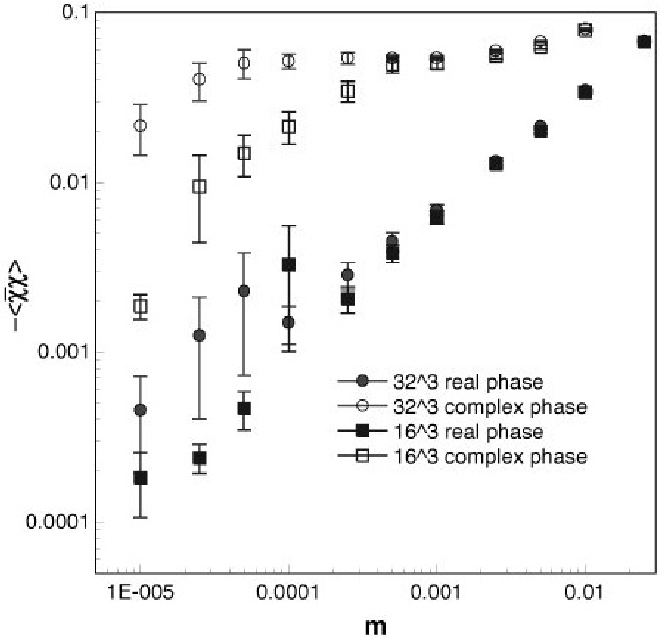


FIG. 1. The chiral condensate $\langle\bar{\chi}\chi\rangle$ plotted as a function of quark mass for a pure gauge calculation on $16^3 \times 4$ and $32^3 \times 4$ lattices. The real phase (closed points) is the most physical [$\det(D - m)$ is largest for this phase]. No evidence is seen for the expected anomalous behavior, $\langle\bar{\chi}\chi\rangle \sim m^{-1}$ as $m \to 0$.

S. Chandrasekharan et al., PRL82, 2463, (1999).

Volume dependence

Size of the constant contribution depends on the volume N_s^3

The dependence is negligible at $N_s/N_t \gtrsim 2$

■ Results on 96^3 x 32

(N_s/N_t=3 ← similar to T>0 quench QCD calculation)