Quarkonium correlators on the lattice

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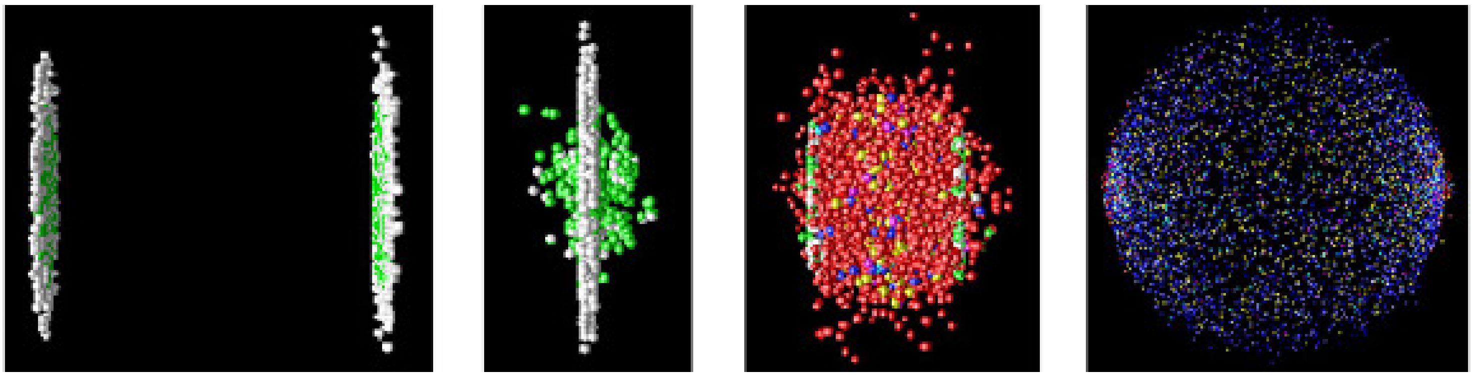
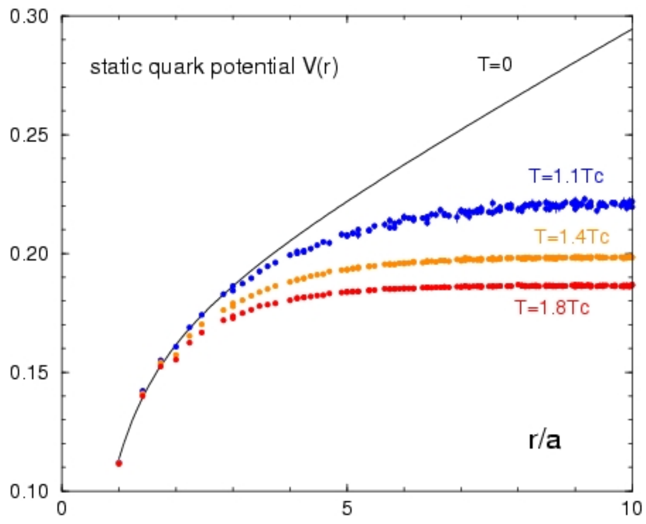
Contents of this talk

from the Phenix group web-site



- Introduction
  - -- Quark Gluon Plasma & J/ψ suppression
  - -- Lattice studies on J/ψ suppression
- Our approach to study charmonium dissociation
- Charmonium wave functions at T>0
- Discussion & Summary

J/ ψ suppression as a signal of QGP



Confined phase:
linear raising potential
→ bound state of c - $\bar{c}$

De-confined phase:
Debye screening
→ scattering state of c - $\bar{c}$

T.Hashimoto et al.('86), Matsui&Satz('86)

<u>Lattice QCD calculations:</u>

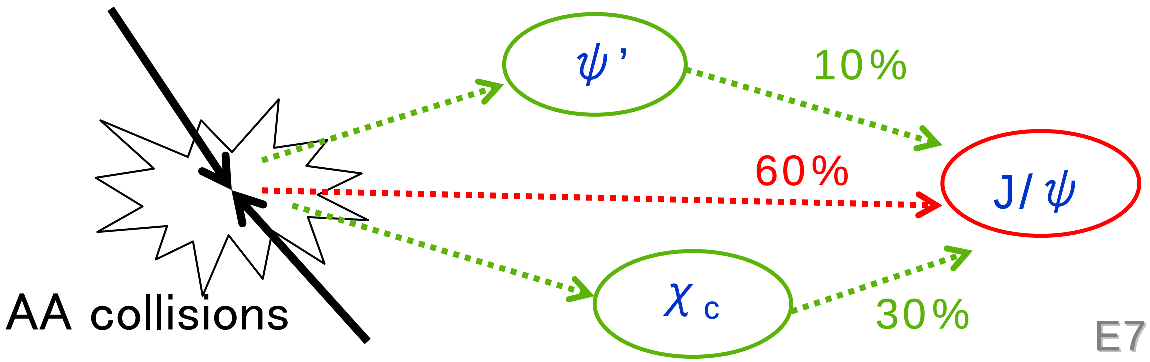
Spectral function by MEM: T.Umeda et al.('02), S.Datta et al.('04), Asakawa&Hatsuda('04), A.Jakovac et al.('07), G.Aatz et al.('06)

Wave func.: T.Umeda et al.('00)

B. C. dep.: H.Iida et al. ('06)

→ all calculations conclude that J/ψ survives till 1.5T_c or higher

Sequential J/ψ suppression scenario



J/ψ (1S)	:	$J^{PC} = 1^{--}$	M=3097MeV	(Vector)
ψ (2S)	:	$J^{PC} = 1^{--}$	M=3686MeV	(Vector)
χ_{c0} (1P)	:	$J^{PC} = 0^{++}$	M=3415MeV	(Scalar)
χ_{c1} (1P)	:	$J^{PC} = 1^{++}$	M=3511MeV	(AxialVector)

PDG('06)

It is important to study dissociation temperatures for not only J/ψ but also ψ(2S), χ_c's

Hot QCD on the lattice

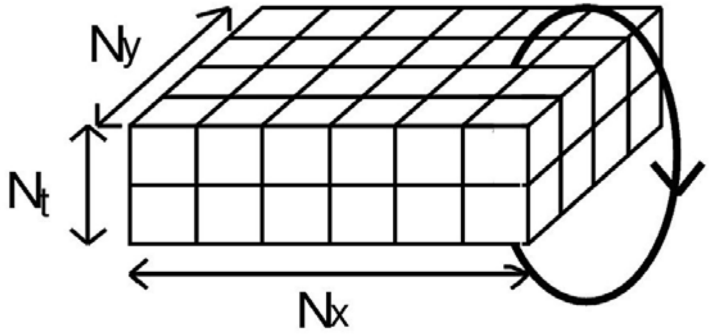
Lattice QCD enables us to perform

nonperturbative calculations of QCD

$$\langle X \rangle = \frac{1}{Z_{QCD}} \int Dq(x) D\bar{q}(x) DA_\mu(x) X(q, \bar{q}, A_\mu) e^{-S_{QCD}}$$

Path integral by Monte Carlo integration

QCD action on a lattice



Finite T Field Theory on the lattice

- 4dim. Euclidean lattice
- gauge field $U_\mu(x)$ → periodic B.C.
- quark field q(x) → anti-periodic B.C.
- Temperature $T=1/(N_t a)$

Spectral function on the lattice

Thermal hadron (charmonium) correlation functions $C_H(\tau,T)$

$$\begin{aligned} C_H(\tau,T) &= \sum_{\vec{r}} \langle J_H(\tau,\vec{r}) J_H^{\dagger}(0,\vec{0}) \rangle \\ &= \int_0^{\infty} d\omega \sigma_H(\omega,T) \frac{\cosh(\omega(\tau - 1/2T))}{\sinh(\omega/2T)} \end{aligned}$$

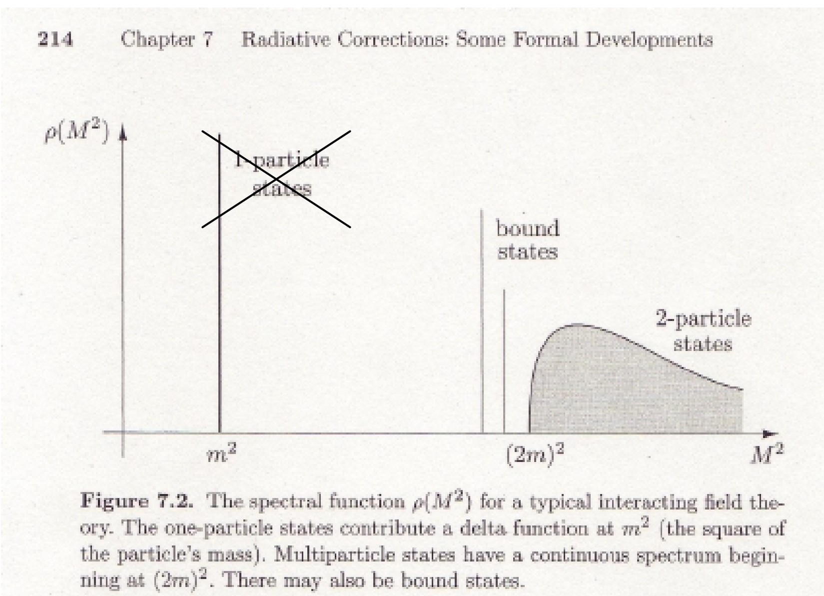

214 Chapter 7 Radiative Corrections: Some Formal Developments

Figure 7.2. The spectral function $\rho(M^2)$ for a typical interacting field theory. The one-particle states contribute a delta function at m^2 (the square of the particle's mass). Multiparticle states have a continuous spectrum beginning at $(2m)^2$. There may also be bound states.

Michael E. Peskin, Perseus books (1995)

Spectral function $\sigma_H(\omega,T)$

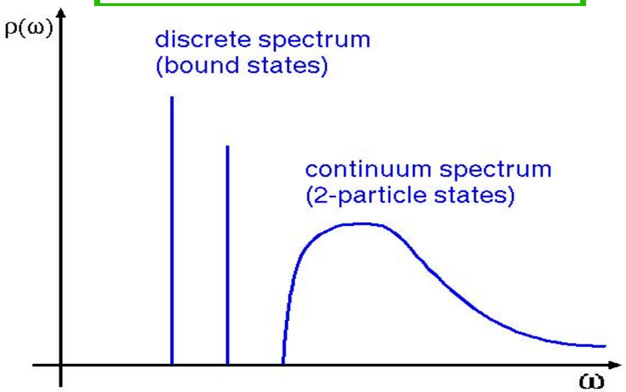
- discrete spectra
  → bound states
  charmonium states
- continuum spectra
  → 2-particle states
  $c\,\bar{c}$ scattering states
  melted charmonium

Spectral functions in a finite volume

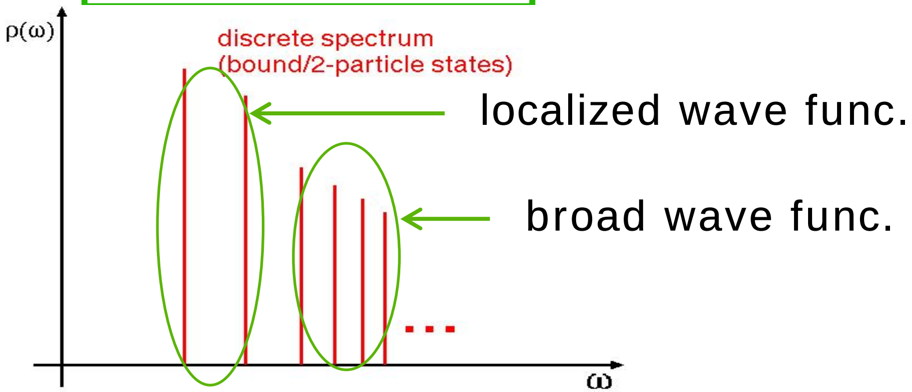
Momenta are discretized in finite ($V=L^3$) volume

$p_i/a = 2n_i\pi/L$ ($n_i=0,\pm1,\pm2,\ldots$) for Periodic boundary condition

infinite volume case



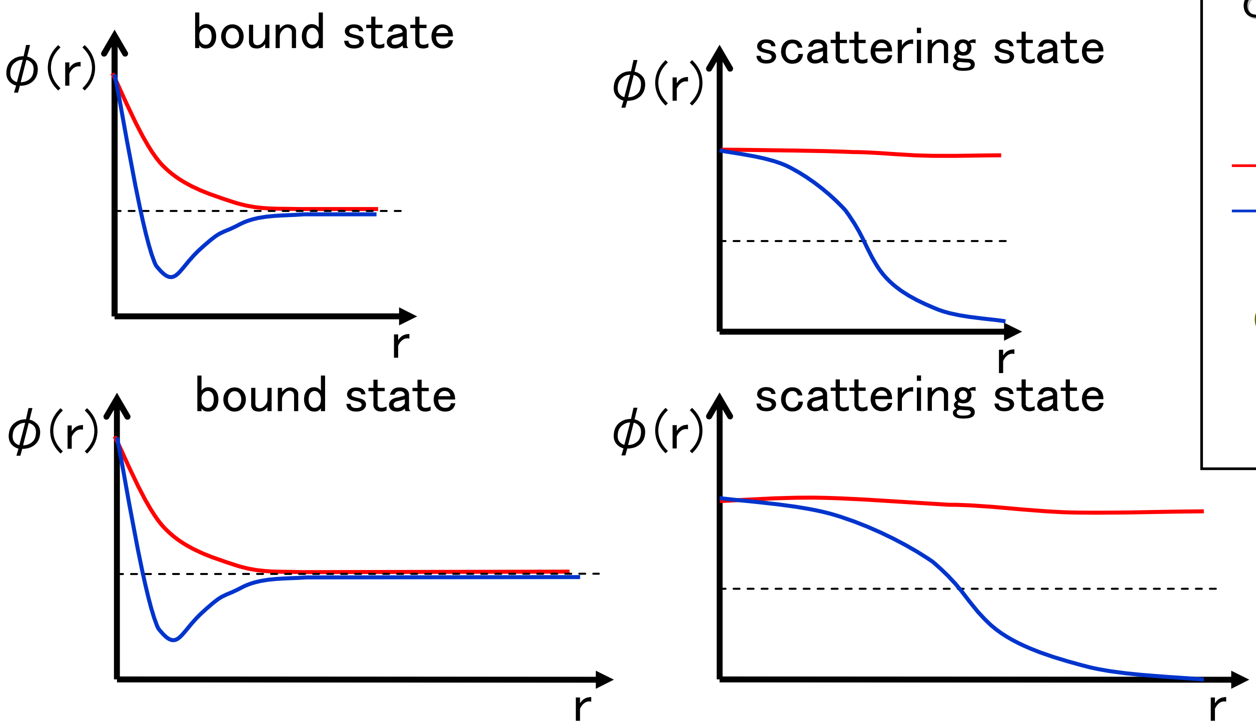
finite volume case



In a finite volume (e.g. Lattice simulations),
discrete spectra does not always indicate bound states !

Shape of wave functions may be good signature
to find out the charmonium melting.

Bound state or scattering state ?



Φ(r): wave function
r : c - c distance

lowest state
next lowest state

examples for
· S-wave
· Periodic B.C

- localized wave function
- small vol. dependence

- broad wave function
- vol. dependence

Wave functions at finite temperature

Temp. dependence of (Bethe-Salpeter) "Wave function"

$$BS(\vec{r},t) = \sum_{\vec{x}} \langle \bar{q}(\vec{x}+\vec{r},t)\Gamma q(\vec{x},t)\bar{q}(\vec{0},0)\Gamma q(\vec{0},0)\rangle$$

$$\Psi(|\vec{r}|,t) = BS(\vec{r},t)/BS(\vec{r}_0,t)$$

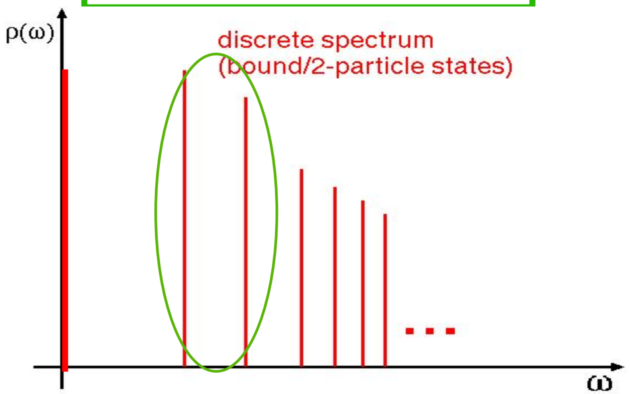
$$\Gamma = \begin{cases} \gamma_5 & \text{(Ps)} \\ \gamma_i & \text{(Ve)} \quad (i=1,2,3) \\ \sum_j \left(\overrightarrow{\partial}_j \gamma_j - \overleftarrow{\partial}_j \gamma_j\right) & \text{(Sc)} \\ \sum_{j,k} \epsilon_{ijk} \left(\overrightarrow{\partial}_j \gamma_k - \overleftarrow{\partial}_j \gamma_k\right) & \text{(Av)} \quad (i=1,2,3) \end{cases}$$

Remarks on wave function of quark-antiquark

- gauge variant → Coulomb gauge fixing
- large or small components of quark/antiqaurk
  → wave func. for large components

Technique to calculate wave function at T>0

finite volume case



It is difficult to extract higher states
from lattice correlators (at T>0)
even if we use MEM !!

It is important to investigate
a few lowest states (at T>0)

Constant mode can be separated
by the Midpoint subtraction

T. Umeda (2007)

In order to study a few lowest states,
the variational analysis is one of the most reliable methods !

N x N correlation matrix : C(t)

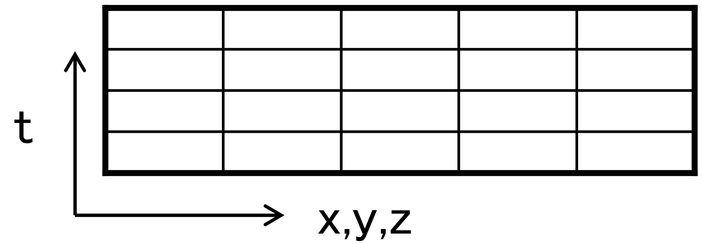
$$C(t)\psi = \lambda(t,t_0)C(t_0)\psi \qquad \lambda_i(t,t_0) = e^{-E_i(t-t_0)}$$

Lattice setup

- Quenched approximation (no dynamical quark effect)
- Anisotropic lattices

  lattice spacing : a_s = 0.0970(5) fm

  anisotropy : $a_s/a_t = 4$



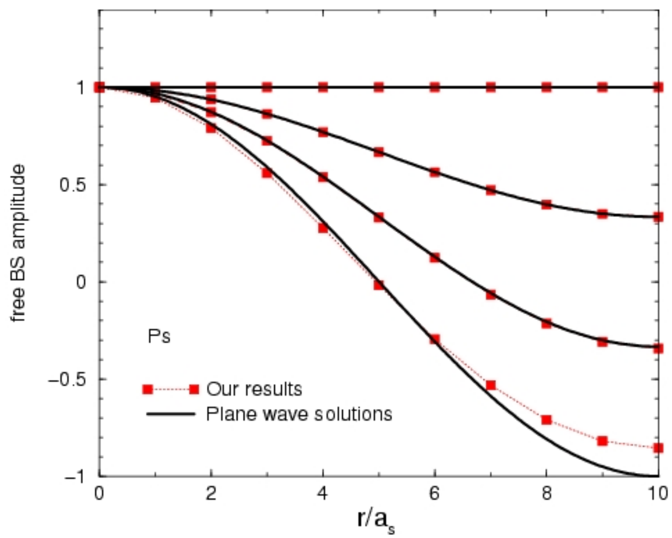
- r_s=1 to suppress doubler effects
- Variational analysis with 4 x 4 correlation matrix

N_t	32	26	20	16	12
T/T_c	0.88	1.08	1.40	1.75	2.33
# of conf.					
$V=16^3$	300	300	300	300	300
$V=20^3$	300	300	300	300	300
$V=32^3$	—	—	—	—	100

Wave functions in free quark case

Test with free quarks ($L_S/a=20$, $ma=0.17$)

in case of S-wave channels



- Free quarks make trivial waves with an allowed momentum in a box

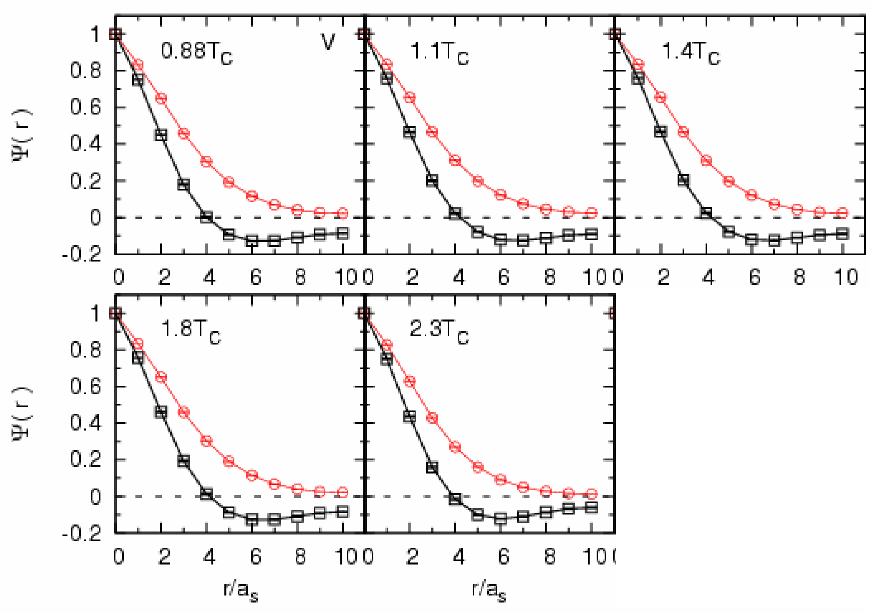
$$\Psi_k(|\vec{r}|,t) = \frac{\sum_{\vec{p}=\vec{k}} \cos(p_1 r_1)\cos(p_2 r_2)\cos(p_3 r_3)}{\sum_{\vec{p}=\vec{k}} 1}$$

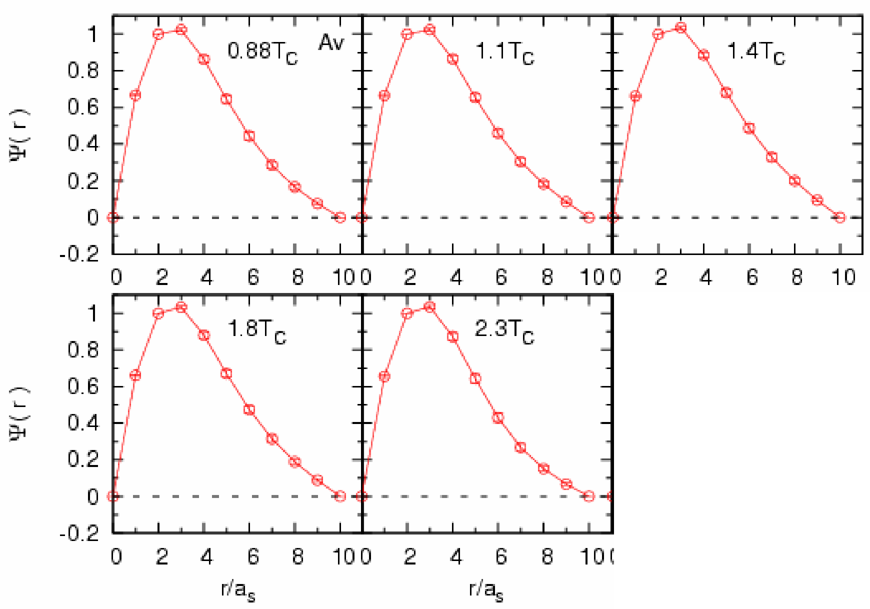
- The wave function is constructed with eigen functions of 6 x 6 correlators

- 6 types of Gaussian smeared operators
  $\phi(x) = \exp(-A|x|^2)$,
  A = 0.02, 0.05, 0.1, 0.15, 0.2, 0.25

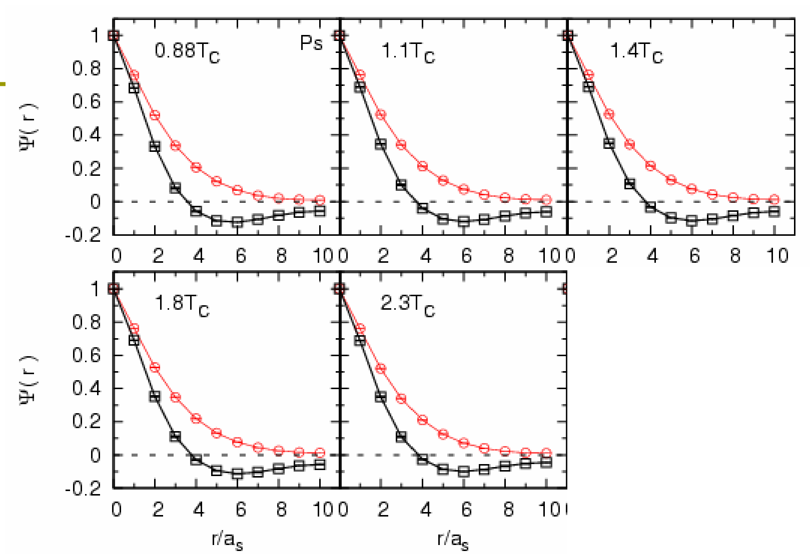
- Our method well reproduces the known result (!)

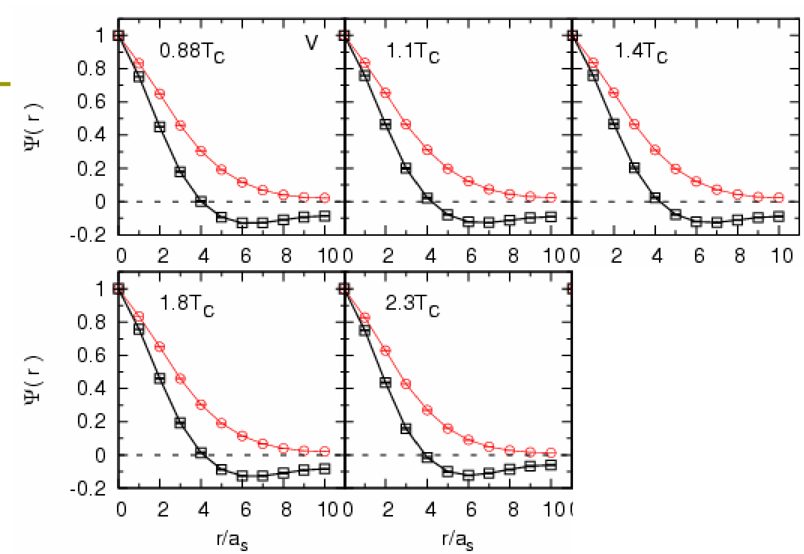
Charmonium wave functions at finite temperatures

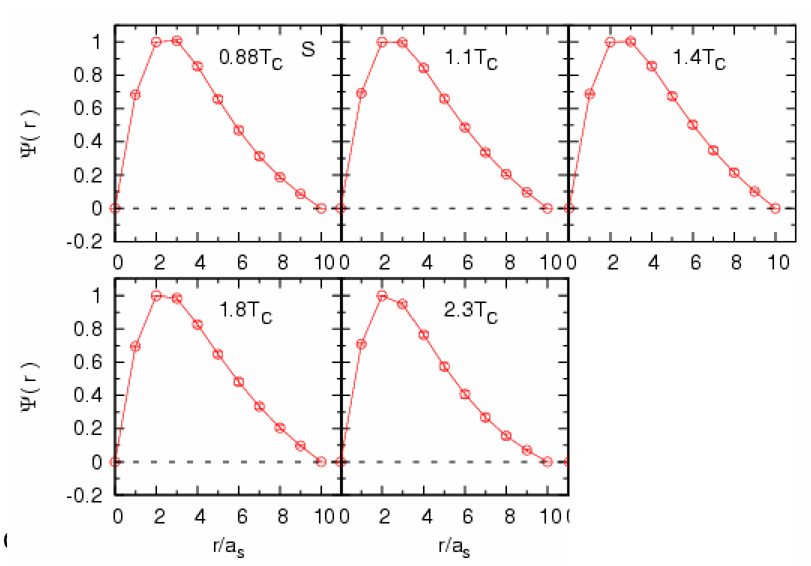


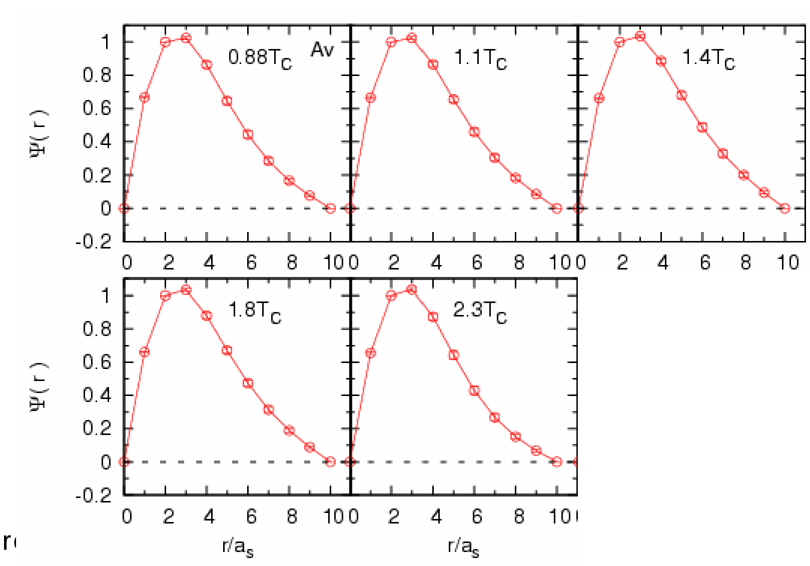


- Small temperature dependence in each channels
- Clear signals of bound states even at T=2.3Tc (!)
- $(2\text{fm})^3$ may be small for P-wave states.

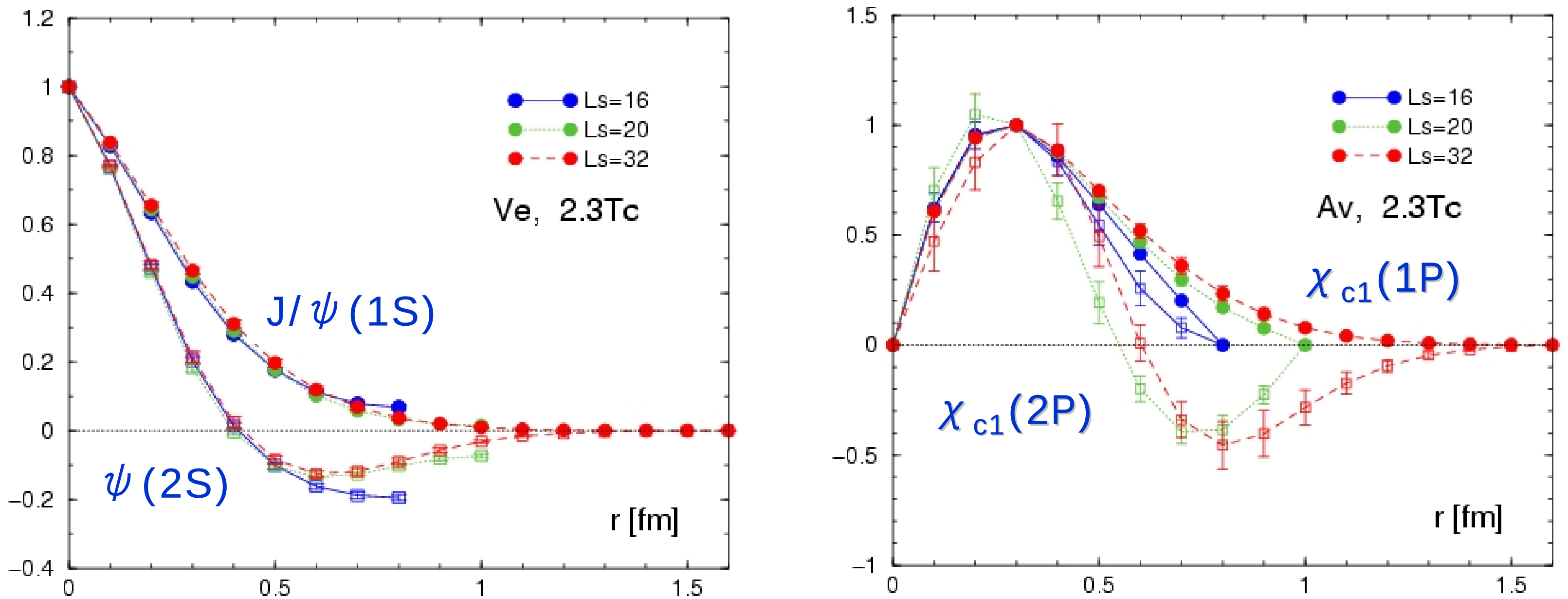

Ps
0.88T_C
1.1T_C
1.4T_C
1.8T_C
2.3T_C
Ψ(r)
r/a_s


V
0.88T_C
1.1T_C
1.4T_C
1.8T_C
2.3T_C
Ψ(r)
r/a_s


S
0.88T_C
1.1T_C
1.4T_C
1.8T_C
2.3T_C
Ψ(r)
r/a_s


Av
0.88T_C
1.1T_C
1.4T_C
1.8T_C
2.3T_C
Ψ(r)
r/a_s

Volume dependence at T=2.3Tc



- Clear signals of bound states even at T=2.3Tc (!)
- Large volume is necessary for P-wave states.

Discussion

We found localized wave functions
up to 2.3Tc for S- & P- wave channels.

(1) Does variational analysis does work well ?

wave function for lowest/next-lowest state

+ contributions from higher states ← contaminations

our results suggest

there are no/small broad wave functions
even in the higher states (!)

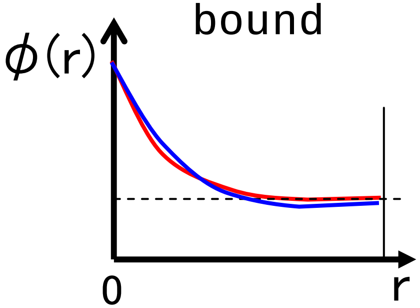
(2) Unbound state with localized wave function ?

anyway,

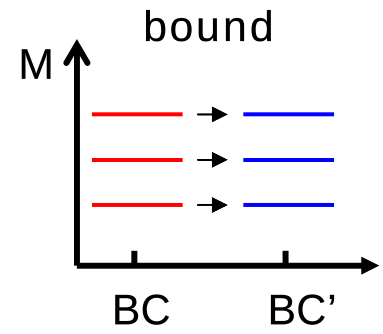
tight wave function is incompatible
with the J/ψ suppression (!)

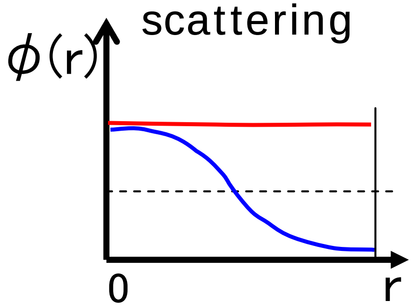
(3) Small changes in wave functions
contradict some potential model results with $T_{dis}(J/\psi) > T_c$ (!)

Boundary condition dependence

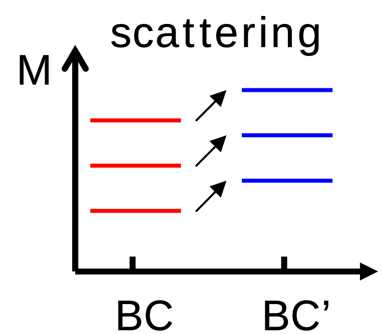


The wave functions are localized,
their energies are insensitive to B.C.





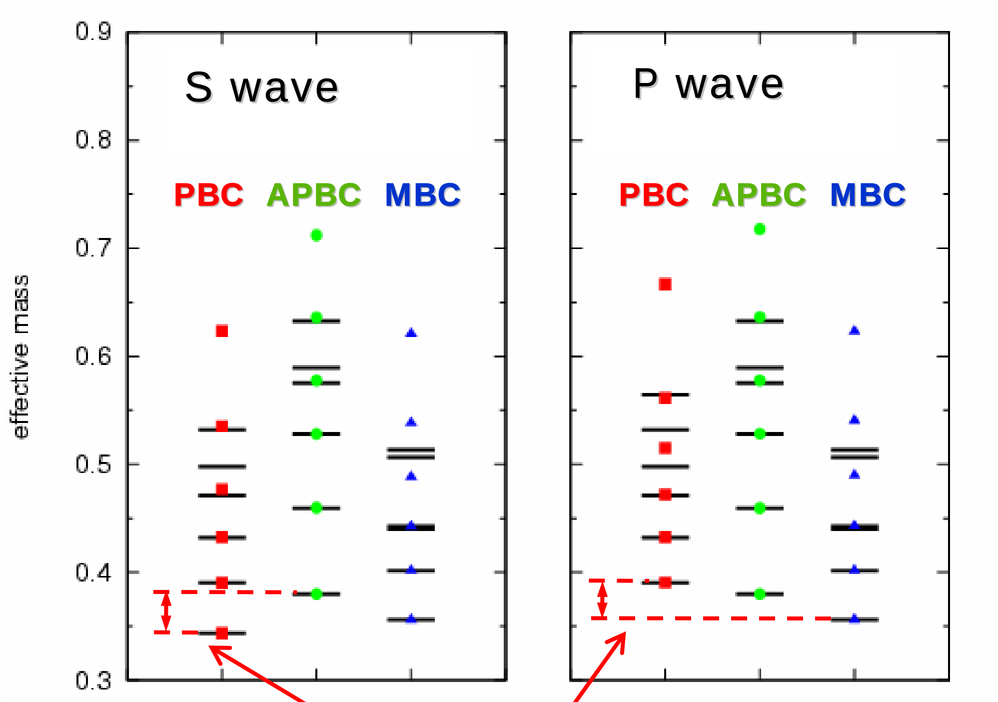
The momenta depends on BC,
the scattering state energies
are sensitive to B.C.



The idea has been originally applied for the charmonium study in H. Iida et al., Phys. Rev. D74 (2006) 074502.

Variational analysis in free quark case

Test with free quarks ($L_s/a=20$, $ma=0.17$)



Mass diff. between the lowest masses in each BC

$$q(x_i + L_i) = b_i q(x_i)$$

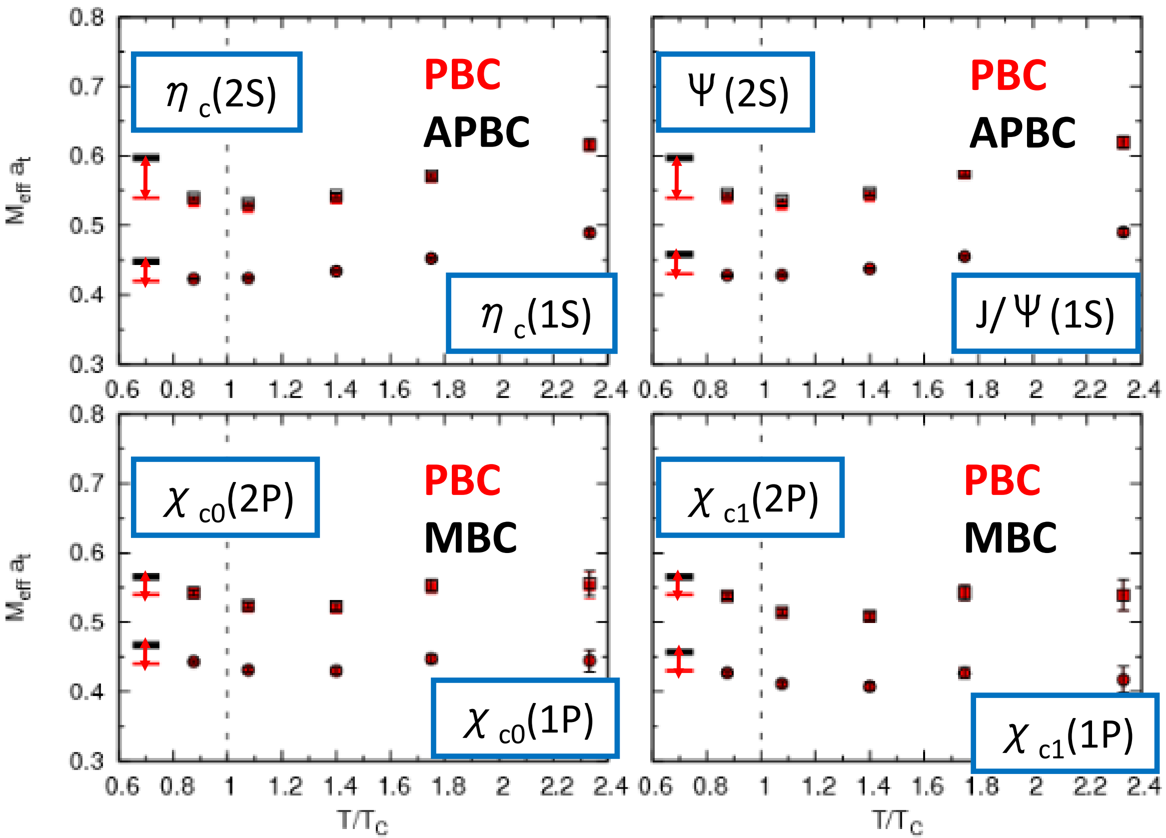
$b_i = 1$: periodic

$b_i = -1$: anti-periodic

PBC : b=(1, 1, 1)
APBC : b=(-1,-1,-1)
MBC : b=(-1, 1, 1)

an expected diff. in $V=(2\text{fm})^3$ (free quark case) ~ 200MeV

Temperature dependence of charmonium spectra



$q(x_i + L_i) = b_i q(x_i)$

$b_i = 1$: periodic

$b_i = -1$: anti-periodic

PBC : b=(1, 1, 1)
APBC : b=(-1,-1,-1)
MBC : b=(-1, 1, 1)

an expected gap in $V=(2\mathrm{fm})^3$ (free quark case) ~ 200MeV

- No significant differences in the different B.C.
- Analysis is difficult at higher temperature ($2T_c$～)

Summary and future plan

We investigated T_{dis} of charmonia from Lattice QCD
without Bayesian (MEM) analysis using...

- Bethe-Salpeter “wave function”
- Volume dependence of the “wave function”
- Boundary condition dependence

No evidence for unbound c-c̄ quarks up to T = 2.3 Tc

→ The result may affect the scenario of J/ψ suppression.

Future plan

- Possible scenarios for the experimental J/ψ suppression
- Higher Temp. calculations (T/Tc=3~5)
- Full QCD calculations (Nf=2+1 Wilson is now in progress)