有限温度格子ＱＣＤの
新しいアプローチの可能性

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for WHOT-QCD Collaboration

This talk is (partly) based on Phys. Rev. D 79, 051501(R) (2009)

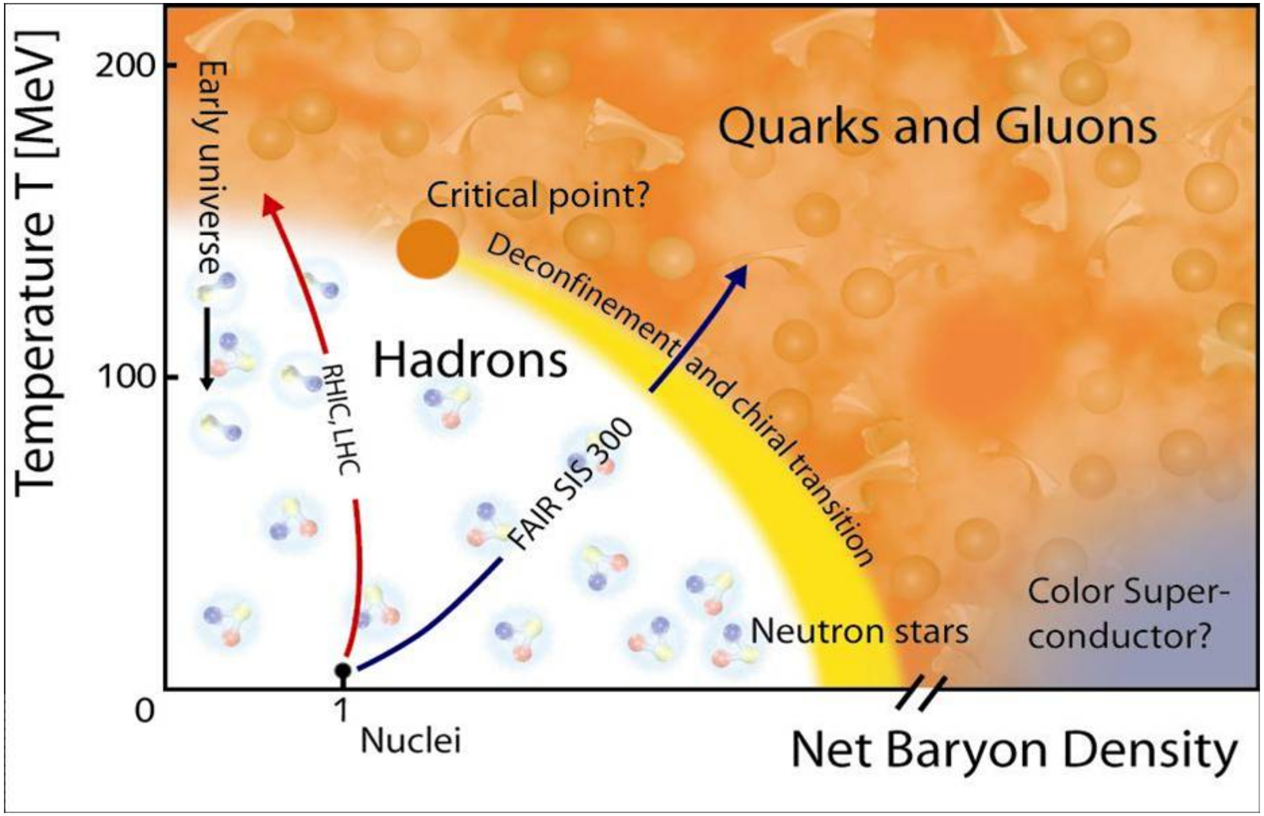
S. Ejiri, S. Aoki, T. Hatsuda, N. Ishii, K. Kanaya,
Y. Maezawa, H. Ohno, T.U. (WHOT-QCD Collaboration)

GCOE-PD seminar, Kyoto, Japan, 18 Mar. 2009

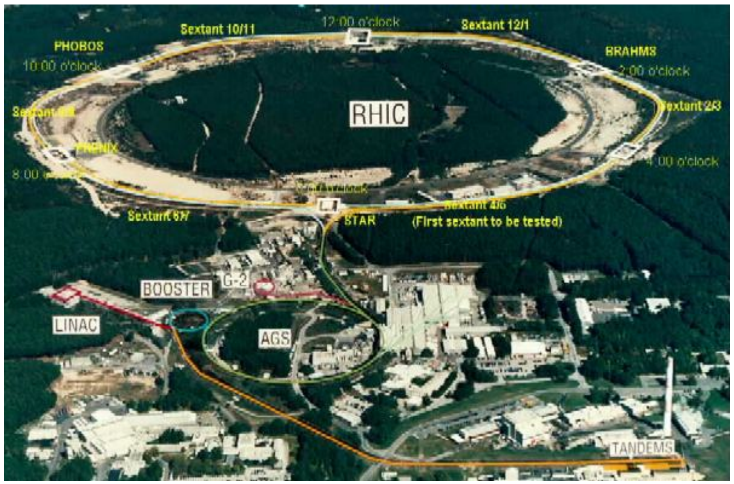
Study of Quark-Gluon Plasma

QCD phase diagram in (Temperature, density)

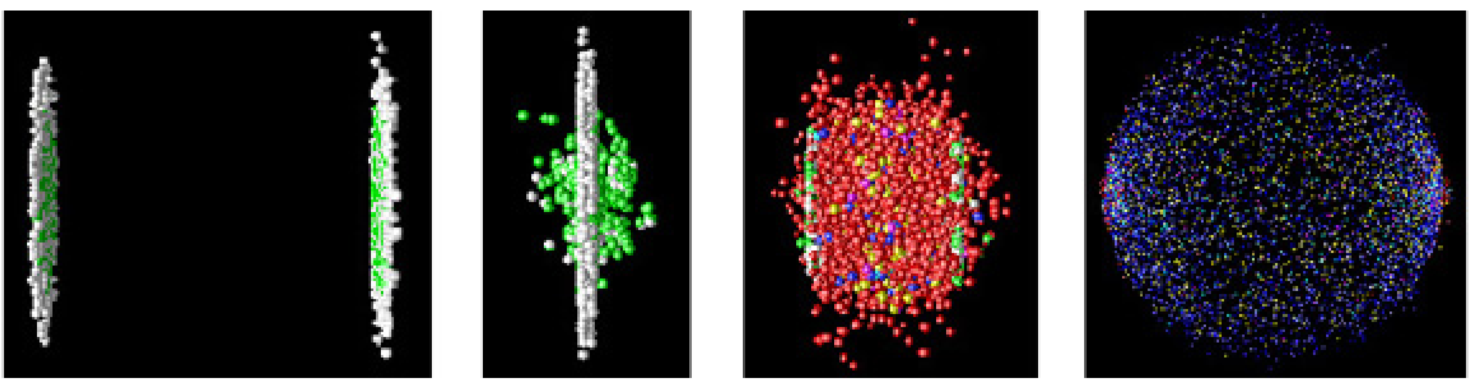
http://www.gsi.de/fair/experiments/



Heavy Ion Collision experiments



- SPS : CERN (– 2005)
  Super Proton Synchrotron
- RHIC: BNL (2000 –)
  Relativistic Heavy Ion Collider
- LHC : CERN (2009 -)
  Large Hadron Collider

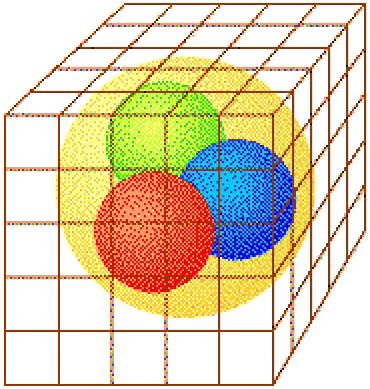


from the Phenix group web-site

Lattice QCD simulations

Lattice QCD

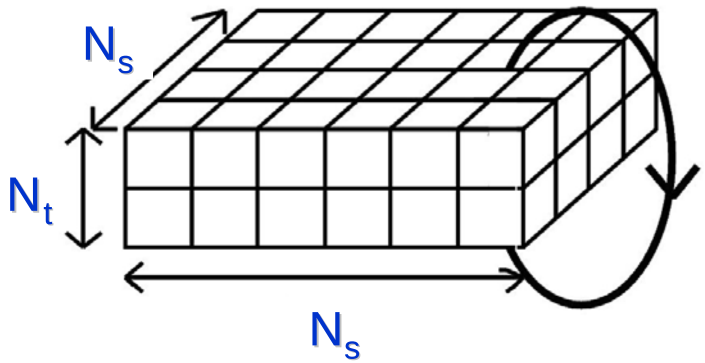
- First principle (nonperturbative) calculation of QCD
- QCD action is defined on the lattice (discretized space-time)
- Path integral is carried out by Monte Carlo Integration



Contents of this talk

- Introduction
- Problems in the conventional approach
- A new approach for QCD Thermodynamics on lattices
- The EOS calculation by "T-integration method"
- Summary

Hot QCD on the lattice



Finite T Field Theory on the lattice

- 4dim. Euclidean lattice ($N_s^3 x N_t$)
- gauge field $U_\mu(x)$ → periodic B.C.
- quark field $q(x)$ → anti-periodic B.C.
- Temperature $T=1/(N_t a)$

Input parameters :	β (=6/g^2)	(lattice gauge coupling)
(Nf=2+1 QCD)	am_{ud}	(light (up & down) quark masses)
	am_s	(strange quark mass)
	N_t	(temperature)

(*) lattice spacing "a" is not an input parameter

$a=a(\beta, am_{ud}, am_s)$

e.g. $(am_\rho)_{lat}/(0.77\text{GeV})=a[\text{GeV}^{-1}]$

Temperature $T=1/(N_t a)$ is varied by a at fixed N_t

Fermions on the lattice

Lattice QCD

QCD action is defined on the lattice

Fermion doubling problem

- naive discretization causes 2^4 doublers
- Nielsen-Ninomiya's No-go theorem
  → Doublers appear unless chiral symmetry is broken

- Staggered (KS) fermion → Low cost
  16 doublers = 4 spinors x 4 flavors
  Fourth root trick : still debated
- Wilson fermion → Moderate cost
  adds the Wilson term to kill extra 2^4-1 doublers
- Domain Wall fermion → High cost
- Overlap fermion → High cost
- …

Problems in QCD Thermo. with KS fermions

Many QCD thermo. calc. were done with KS fermions.

- Phase diagram
  N_f=2 massless QCD → O(4) critical exponets
  KS fermion does not exhibit expected O(4) scaling
  (Wilson fermion shows O(4), but at rather heavy masses)

- Transition temperature (crossover transition in KS studies)
  Equation of State (p/T^4, e/T^4, s/T^4, ...)
  KS results are not consistent with each other

N_f=2, 2+1 is not 4 !!!

Integral method to calculate pressure p/T^4

$$p = \frac{T}{V} \ln Z$$ for large volume system

Lattice QCD can not directly calculate the partition function $\ln Z$

however its derivative is possible $\frac{\partial}{\partial \beta} \ln Z = -\left\langle \frac{\partial S_{QCD}}{\partial \beta} \right\rangle$

One can obtain p as the integral of derivative of p

high temp.

low temp. with p≃0

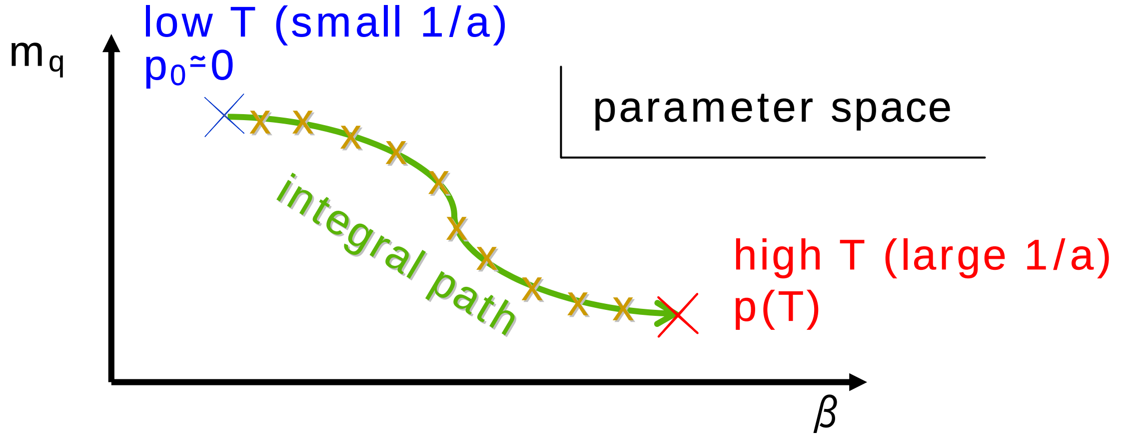
$$\left.\frac{p}{T^4}\right|_{\beta_0}^{\beta} = \frac{1}{VT^3} \int_{\beta_0}^{\beta} d\beta' \frac{\partial}{\partial \beta'} \ln Z$$

$$= -N_t^4 \int_{\beta_0}^{\beta} d\beta' \frac{1}{N_s^3 N_t} \left(\langle \frac{\partial S_{QCD}}{\partial \beta} \rangle_{T>0} - \langle \frac{\partial S_{QCD}}{\partial \beta} \rangle_{T=0} \right)$$

T=0 subtraction

Line of constant physics (LCP)

In case of $N_f=2+1$ QCD
there are three (bare) parameters: β, (am_{ud}) and (am_s)



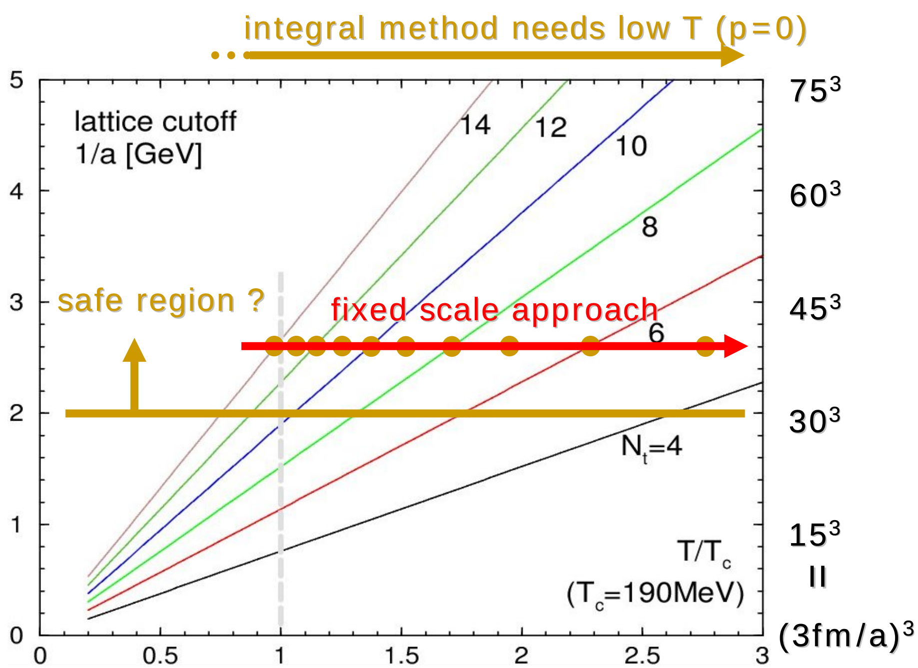
Line of Constant Physics (LCP) defined at T=0

QCD Thermodynamics requires huge computational cost !!

Most group adopts KS fermion to study the QCD Thermodynamics.

Fixed scale approach to study QCD thermodynamics

Temperature $T=1/(N_t a)$ is varied by N_t at fixed $a(\beta,\ m_{ud},\ m_s)$



- Advantages
  - LCP is trivially exact
  - T=0 subtraction is done with a common T=0 sim. (T=0 high. stat. spectrum)
  - easy to keep large 1/a at whole T region
  - easy to study T effect without V, 1/a effects
- Disadvantages
  - T resolution by integer N_t
  - program for odd N_t
  - (1/a)/T = const. is not suited for high T limit study

T-integration method to calculate the EOS

We propose a new method ("T-integration method")
to calculate the EOS at fixed scales

T.Umeda et al. (WHOT-QCD) Phys. Rev. D 79, 051501(R) (2009)

Our method is based on the trace anomaly (interaction measure),

$$\frac{\epsilon - 3p}{T^4} = \left(\frac{N_t^3}{N_s^3}\right) a \frac{d\beta}{da} \left\langle \frac{dS}{d\beta} \right\rangle_{sub}$$

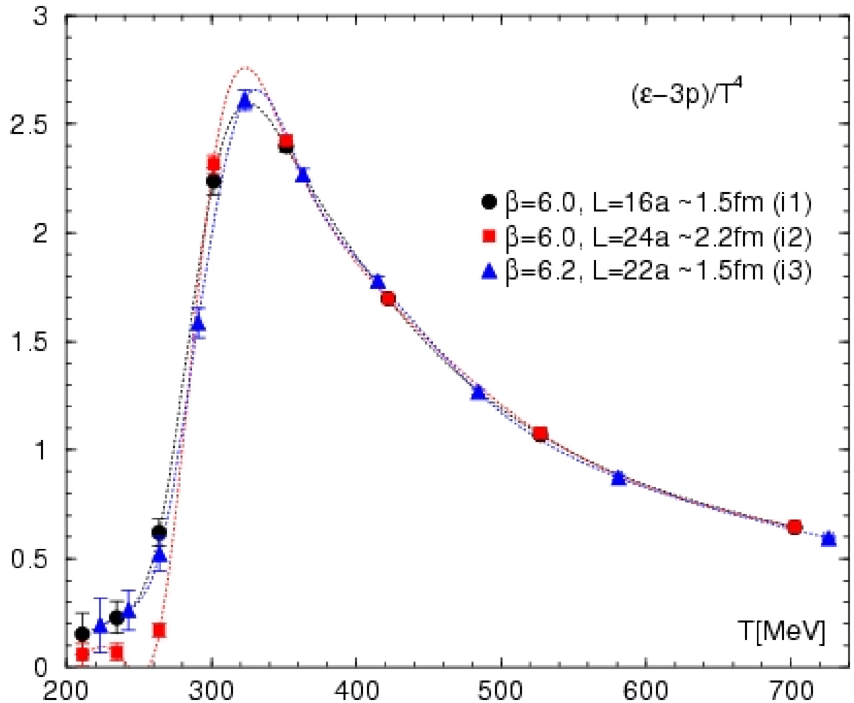
and the thermodynamic relation.

$$\frac{\epsilon - 3p}{T^4} = T \frac{\partial (p/T^4)}{\partial T}$$

$$\Longrightarrow \quad \frac{p}{T^4} = \int_0^T dT' \, \frac{\epsilon - 3p}{T'^5}$$

Trace anomaly (e - 3p)/T^4 in SU(3) gauge theory

We present results from SU(3) gauge theory as a test of our method



dotted lines : cubic spline

(1) β =6.0, 1/a=2.1GeV, V=(1.5fm)^3
(2) β =6.0, 1/a=2.1GeV, V=(2.2fm)^3
(3) β =6.2, 1/a=2.5GeV, V=(1.5fm)^3

beta function : G.Boyd et al. ('96)
lattice scale r_0 : R.Edwards et al. ('98)

$$\frac{\epsilon - 3p}{T^4} = \left(\frac{N_t^3}{N_s^3}\right) a\frac{\partial\beta}{\partial a} \left\langle \frac{\partial S_g}{\partial\beta} \right\rangle_{sub}$$

Trace anomaly (e - 3p)/T^4 in SU(3) gauge theory

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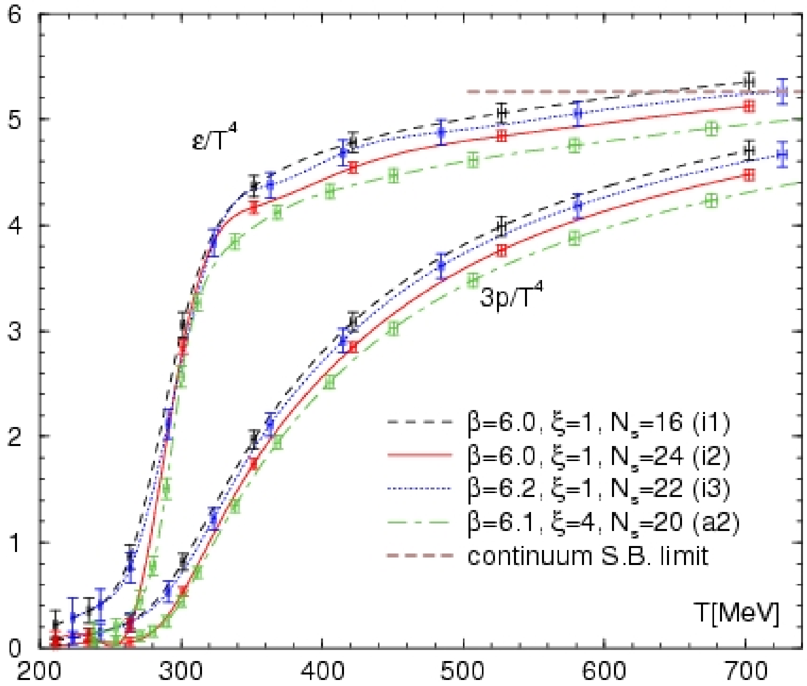

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Integration $\left(\frac{p}{T^4} = \int_0^T dT' \frac{\epsilon - 3p}{T'^5}\right)$

is performed with the cubic spline of (e-3p)/T4

Our fixed scale approach with "T-integration method" works well !!

Summary and future plans

- A new approach to study the QCD Thermodynamics is proposed.
  - "T-integral method" to calculate the EOS works well.
  - There are many advantages in the approach.
- We have already generated T>0 configurations using CP-PACS/JLQCD parameter (N_f=2+1 Clover+RG, 1/a=3GeV, pion mass ~ 500MeV)
- Our final goal is to study thermodynamics on the physical point (pion mass ~ 140MeV) with N_f=2+1 Wilson quarks (PACS-CS) or exact chiral symmetry with N_f=2+1 Overlap quarks (JLQCD)
- We are looking for new ideas to study QGP physics in our approach. (density correlations, J/psi suppression, finite density...)

Backup slides

Recent lattice calculations of EOS

Hot-QCD Collab. (2007～)

- RBC-Bielefeld: $N_t=4,6,8$ Staggered (p4) quark
  pion mass ~ 220MeV, $N_f=2+1$
  Phys. Rev. D77 (2008) 014511
- MILC: $N_t=4,6,8$ Staggered (Asqtad) quark
  pion mass ~ 220MeV, $N_f=2+1$
  Phys. Rev. D75 (2007) 094505

Wuppertal: $N_t=4,6$ Staggered (stout) quark
pion mass ~ 140MeV, $N_f=2+1$
JHEP 0601 (2006) 089

CP-PACS: $N_t=4,6$ Wilson (MFI Clover) quark
pion mass ~ 500MeV, $N_f=2$
Phys. Rev. D64 (2001) 074510

Introduction

Physics in Lattice QCD at finite temperature

- Phase diagram in (T, μ, m_{ud}, m_s)
- Transition temperature
- Equation of state (e, p, s,...)

} quantitative studies

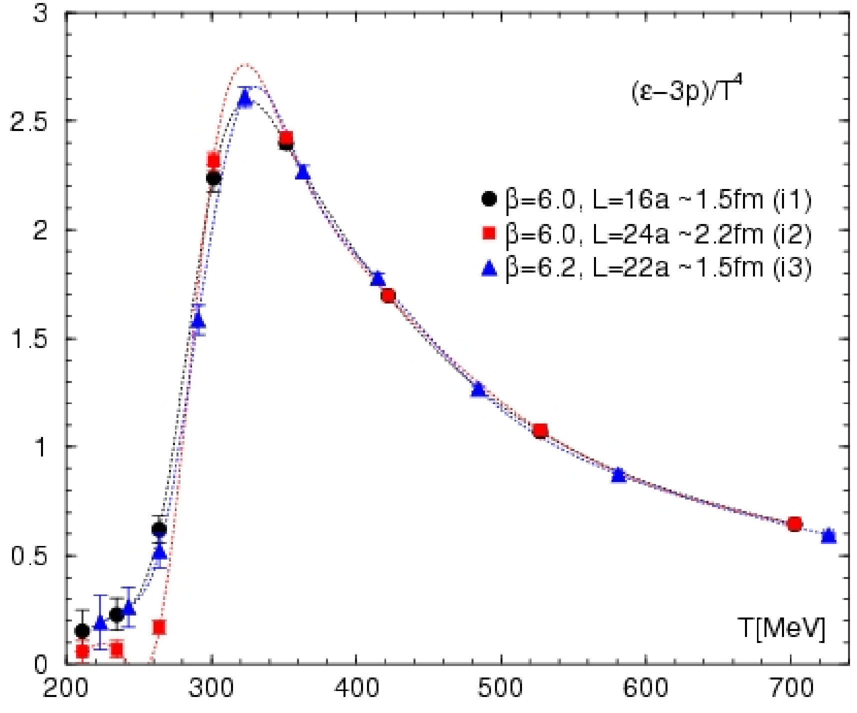
- Excitation spectrum
- Transport coefficients (shear/bulk viscosity)
- Finite chemical potential
- etc...

} qualitative studies

These are important to study
- Quark Gluon Plasma in Heavy Ion Collision exp.
- Early universe
- Neutron star
- etc...

Trace anomaly (e - 3p)/T^4 on isotropic lattices

$$\frac{\epsilon - 3p}{T^4} = \left(\frac{N_t^3}{N_s^3}\right) a \frac{\partial \beta}{\partial a} \left\langle \frac{\partial S_g}{\partial \beta} \right\rangle_{sub}$$



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(1) β =6.0, 1/a=2.1GeV, V=(1.5fm)^3
(2) β =6.0, 1/a=2.1GeV, V=(2.2fm)^3
(3) β =6.2, 1/a=2.5GeV, V=(1.5fm)^3

beta function : G.Boyd et al. ('96)
lattice scale r_0 : R.Edwards et al. ('98)

- Excellent agreement between (1) and (3)
  → scale violation is small
  1/a=2GeV is good

- Finite volume effect appears below & near T_c
  → volume size is important
  V=(2fm)^3 is necessary.

Simulation parameters (isotropic lattices)

We present results from SU(3) gauge theory as a test of our method

- plaquette gauge action on N_s^3 x N_t lattices
- Jackknife analysis with appropriate bin-size

To study scale- & volume-dependence,
we prepare 3-type of lattices.

(1) β =6.0, V=$(16a)^3$
1/a=2.1GeV

β	N_s	N_t	T[MeV]	conf.
6.0	16	16	~ 0	350k
6.0	16	10	210	350k
6.0	16	9	230	250k
6.0	16	8	260	200k
6.0	16	7	300	100k
6.0	16	6	350	50k
6.0	16	5	420	50k
6.0	16	4	530	50k
6.0	16	3	700	50k

(2) β =6.0, V=$(24a)^3$
1/a=2.1GeV

β	N_s	N_t	T[MeV]	conf.
6.0	24	16	~ 0	150k
6.0	24	10	210	250k
6.0	24	9	230	200k
6.0	24	8	260	150k
6.0	24	7	300	100k
6.0	24	6	350	50k
6.0	24	5	420	50k
6.0	24	4	530	50k
6.0	24	3	700	50k

(3) β =6.2, V=$(22a)^3$
1/a=2.5GeV

β	N_s	N_t	T[MeV]	conf.
6.2	22	22	~ 0	250k
6.2	22	13	220	350k
6.2	22	12	240	350k
6.2	22	11	270	350k
6.2	22	10	290	250k
6.2	22	9	320	200k
6.2	22	8	360	200k
6.2	22	7	420	100k
6.2	22	6	490	100k
6.2	22	5	580	50k
6.2	22	4	730	50k