Hot QCD at fixed lattice scales

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Contents of our poster

Our aim is to investigate
QCD Thermodynamics with Wilson-type quarks

- Brief review on Lattice QCD at finite T (zero μ)
- Why do we need "Hot QCD with Wilson-type quarks" ?
- Why is "Hot QCD with Wilson-type quarks" difficult ?
- How do we overcome the difficulties ?
  - We propose "T-integration method"
  - Test with the SU(3) gauge theory
- Summary and Outlook

Introduction

Physics in Lattice QCD at finite temperature
- Phase diagram in (T, μ, m_{ud}, m_s)
- Transition temperature
- Equation of state (e, p, s,...)

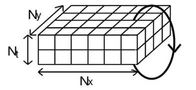
} quantitative studies

- Hadronic excitations
- Transport coefficients (shear/bulk viscosity)
- Finite chemical potential
- etc...

} qualitative studies

These are important to study
- Quark Gluon Plasma in Heavy Ion Collision exp.
- Early universe
- Neutron star
- etc...

Hot QCD on the lattice



Finite T Field Theory on the lattice
- 4dim. Euclidean lattice
- gauge field $U_\mu(x)$ → periodic B.C.
- quark field q(x) → anti-periodic B.C.
- Temperature $T=1/(N_t a)$

Input parameters : (Nf=2+1 QCD)
- β (=6/g²) (lattice gauge coupling)
- am_{ud} (light (up & down) quark masses)
- am_s (strange quark mass)
- N_t (temperature)

(*) lattice spacing "a" is not an input parameter
$a=a(\beta, am_{ud}, am_s)$

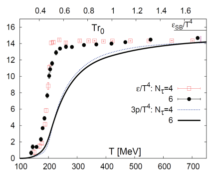
Temperature $T=1/(N_t a)$ is varied by a at fixed N_t

Problems in QCD Thermo. with KS fermions

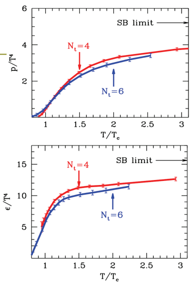
Many QCD thermo. calc. were done with KS fermions.

- Phase diagram
  - N_f=2 massless QCD → O(4) critical exponets
  - KS fermion does not show expected O(4) scaling
  - (Wilson fermion shows O(4), but at rather heavy masses)
- Transition temperature (crossover transition in KS studies)
  - KS results are not consistent with each other
  - MILC : 169(12)(4)MeV(*) *Phys. Rev. D71 (2005) 034504*
  - RBC-Bi : 192(7)(4)MeV *Phys. Rev. D74 (2006) 054507*
  - Wuppertal : 151(3)(3)MeV *Phys. Lett. B643 (2006) 46*
  - (*)T_c at m_q=0
- EOS
  - KS results are not consistent with each other
  - MILC & RBC-Bi are consistent (N_t=4,6,8)
  - Wuppertal (N_t=4,6)

EOS with KS fermions



M.Chen et al. (RBC-Bielefeld)
Phys. Rev. D77 (2008) 014511.



Y.Aoki et al. (Wuppertal)
JHEP 0601 (2006) 089.

Integral method to calculate pressure p/T⁴

$p = \frac{T}{V}\ln Z$ for large volume system

Lattice QCD can not directly calculate the partition function $\ln Z$

however its derivative is possible $\frac{\partial}{\partial\beta}\ln Z = -\left\langle\frac{\partial S_{QCD}}{\partial\beta}\right\rangle$

One can obtain p as the integral of derivative of p

high temp.

$$\left.\frac{p}{T^4}\right|_{\beta_0}^{\beta} = \frac{1}{VT^3}\int_{\beta_0}^{\beta} d\beta' \frac{\partial}{\partial\beta'}\ln Z$$

low temp. with p=0

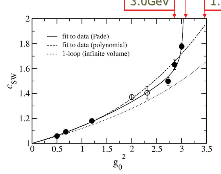
$$= -N_t^4 \int_{\beta_0}^{\beta} d\beta' \frac{1}{N_s^3 N_t}\left(\left\langle\frac{\partial S_{QCD}}{\partial\beta}\right\rangle_{T>0} - \left\langle\frac{\partial S_{QCD}}{\partial\beta}\right\rangle_{T=0}\right)$$

T=0 subtraction

Problems in coarse lattices

Nonperturbative improvement of Wilson fermions :
clover coefficient c_{sw} by the Schrodinger functional method

lattice cutoff 2.0GeV 3.0GeV 1.5GeV

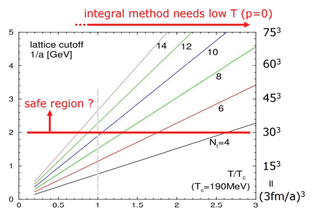


Large uncertainty of c_{sw} at 1/a < 2GeV

CP-PACS,
Phys. Rev. D73 (2006) 034501

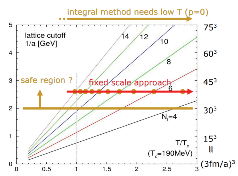
How large Nt is safe ?

T vs 1/a at various Nt



Fixed scale approach to study QCD thermodynamics

Temperature $T=1/(N_t a)$ is varied by N_t at fixed $a(\beta, m_{ud}, m_s)$



- Advantages
  - LCP is trivial
  - T=0 subtraction is done with a common T=0 sim. (T=0 high. stat. spectrum)
  - easy to keep large 1/a at whole T region
  - easy to study T effect without V, 1/a effects
- Disadvantages
  - T resolution by integer N_t
  - QCD code for odd N_t
  - (1/a)/T = const. is not suited for high T limit study

T-integration method to calculate the EOS

We propose a new method ("T-integration method")
to calculate the EOS at fixed scales

T.Umeda et al. (WHOT-QCD) arXiv:0809.2842 [hep-lat]

Our method is based on the trace anomaly (interaction measure),

$$\frac{\epsilon-3p}{T^4} = \left(\frac{N_t^3}{N_s^3}\right) a\frac{d\beta}{da}\left\langle\frac{dS}{d\beta}\right\rangle_{sub}$$

and the thermodynamic relation.

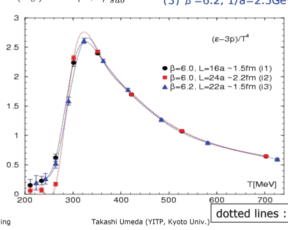
$$\frac{\epsilon-3p}{T^4} = T\frac{\partial(p/T^4)}{\partial T}$$

$$\Rightarrow \frac{p}{T^4} = \int_0^T dT' \frac{\epsilon-3p}{T'^5}$$

Trace anomaly (e - 3p)/T⁴ on isotropic lattices

$$\frac{\epsilon-3p}{T^4} = \left(\frac{N_t^3}{N_s^3}\right) a\frac{\partial\beta}{\partial a}\left\langle\frac{\partial S_g}{\partial\beta}\right\rangle_{sub}$$

(1) β=6.0, 1/a=2.1GeV, V=(1.5fm)³
(2) β=6.0, 1/a=2.1GeV, V=(2.2fm)³
(3) β=6.2, 1/a=2.5GeV, V=(1.5fm)³

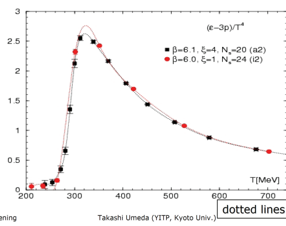


dotted lines : cubic spline

Trace anomaly (e - 3p)/T⁴ on aniso. lattice

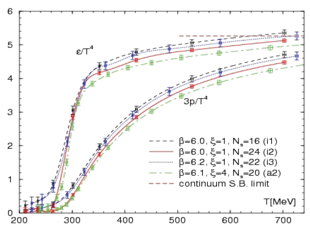
$$\frac{\epsilon-3p}{T^4} = \left(\frac{N_t^3}{N_s^3\xi^3}\right) a_s\left.\frac{\partial\beta}{\partial a_s}\right|_\xi \left\langle\left.\frac{\partial S_g}{\partial\beta}\right|_\xi\right\rangle$$

(1) ξ=4, $1/a_s$=2.0GeV, V=(2.0fm)³
(2) ξ=1, 1/a=2.1GeV, V=(2.2fm)³



dotted lines : cubic spline

Pressure & Energy density



Our fixed scale approach with "T-integration method" works well !!

Toward full QCD calculations and new ideas at T>0 & μ >0

- There are many projects on high statistics full QCD at T=0.
  PACS-CS, JLQCD, MILC, etc...
  - some basic quantities at T=0 are studied
  - T=0 config. are open to the public (by ILDG)

  our method requires no additional T=0 simulation !!
- We have already generated T>0 configurations
  using CP-PACS/JLQCD parameter
  (N_f=2+1 Clover+RG, 1/a=3GeV, pion mass ~ 500MeV)
- Our final goal is to study thermodynamics on
  the physical point (pion mass ~ 140MeV)
  with N_f=2+1 Wilson quarks (PACS-CS)
  or exact chiral symmetry with N_f=2+1 Overlap quarks (JLQCD)