Thermodynamics at fixed lattice spacing

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This talk is based on arXiv:0809.2842 [hep-lat]

JPS meeting, Yamagata Univ., Yamagata , 20-23 Sept. 2008

Introduction

Equation of State (EOS)
is important for phenomenological study of QGP, etc.

Methods to calculate the EOS have been established,
e.g. Integral method J. Engels et al. ('90).

Temperature $T=1/(N_\tau a)$ is varied by $a(\beta)$ at fixed N_τ

The EOS calculation requires huge computational cost,
in which T=0 calculations dominate despite T>0 study.

- Search for a Line of Constant Physics (LCP)
- beta functions at each temperature
- T=0 subtraction at each temperature

T-integration method to calculate the EOS

We propose a new method ("T-integration method")
to calculate the EOS at fixed scales (*)

Temperature $T=1/(N_\tau a)$ is varied by N_τ at fixed $a(\beta)$

Our method is based on the trace anomaly (interaction measure),

$$\frac{\epsilon - 3p}{T^4} = \left(\frac{N_\tau^3}{N_\sigma^3}\right) a\frac{d\beta}{da}\left\langle\frac{dS}{d\beta}\right\rangle_{sub}$$

and the thermodynamic relation.

$$\frac{\epsilon - 3p}{T^4} = T\frac{\partial(p/T^4)}{\partial T} \quad \longrightarrow \quad \frac{p}{T^4} = \int_0^T dT' \, \frac{\epsilon - 3p}{T'^5}$$

(*) fixed scale approach has been adopted in L.Levkova et al. ('06)
whose method is based on the derivative method.

Notable points in T-integration method

Our method can reduce computational cost at T=0 drastically.

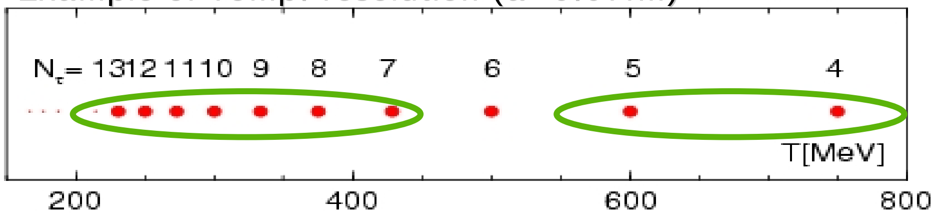
- Zero temperature subtraction is performed using a common T=0 calculation.
- Line of Constant Physics (LCP) is trivially exact (even in full QCD).
- Only the beta functions at the simulation point are required.

However ...

- Temperatures are restricted by integer N_τ.

  → Sufficiently fine lattice is necessary.

Example of Temp. resolution (a=0.07fm)



Integer N_τ provides
- higher resolution at $T \sim T_c$
- lower resolution at high T

$T \sim T_c$ is important for EOS

Simulation parameters (isotropic lattices)

We present results from SU(3) gauge theory as a test of our method

- plaquette gauge action on $N_\sigma^3 \times N_\tau$ lattices
- Jackknife analysis with appropriate bin-size

To study scale- & volume-dependence,
we prepare 3-type of lattices.

(1) β =6.0, V=(16a)^3
a=0.094fm

β	N_σ	N_τ	T[MeV]	conf.
6.0	16	16	~ 0	350k
6.0	16	10	210	350k
6.0	16	9	230	250k
6.0	16	8	260	200k
6.0	16	7	300	100k
6.0	16	6	350	50k
6.0	16	5	420	50k
6.0	16	4	530	50k
6.0	16	3	700	50k

(2) β =6.0, V=(24a)^3
a=0.094fm

β	N_σ	N_τ	T[MeV]	conf.
6.0	24	16	~ 0	150k
6.0	24	10	210	250k
6.0	24	9	230	200k
6.0	24	8	260	150k
6.0	24	7	300	100k
6.0	24	6	350	50k
6.0	24	5	420	50k
6.0	24	4	530	50k
6.0	24	3	700	50k

(3) β =6.2, V=(22a)^3
a=0.078fm

β	N_σ	N_τ	T[MeV]	conf.
6.2	22	22	~ 0	250k
6.2	22	13	220	350k
6.2	22	12	240	350k
6.2	22	11	270	350k
6.2	22	10	290	250k
6.2	22	9	320	200k
6.2	22	8	360	200k
6.2	22	7	420	100k
6.2	22	6	490	100k
6.2	22	5	580	50k
6.2	22	4	730	50k

Simulation parameters (anisotropic lattice)

Anisotropic lattice is useful to increase Temp. resolution,
we also test our method on an anisotropic lattice $a_\sigma \neq a_\tau$

- plaquette gauge action on $N_\sigma^3 \times N_\tau$ lattices
  with anisotropy $\xi = a_\sigma / a_\tau = 4$

β	N_σ	N_τ	T[MeV]	conf.
6.1	20	80	~ 0	220k
6.1	20	32	250	520k
6.1	20	30	270	220k
6.1	20	29	280	220k
6.1	20	28	290	220k
6.1	20	27	300	220k
6.1	20	26	310	220k
6.1	20	24	340	220k
6.1	20	22	370	220k
6.1	20	20	410	220k
6.1	20	18	450	220k
6.1	20	16	510	220k
6.1	20	14	580	220k
6.1	20	12	680	220k
6.1	20	10	810	220k
6.1	20	8	1010	220k

$\beta = 6.1$, $\xi = 4$
$V = (20a_\sigma)^3$
$= (1.95\text{fm})^3$
$a = 0.097\text{fm}$

- EOS calculation
- static quark free energy

β	N_σ	N_τ	T[MeV]	conf.
6.1	20	30	270	220k
6.1	20	29	280	220k
6.1	20	28	290	220k
6.1	20	27	300	220k
6.1	20	26	310	220k
6.1	30	30	270	80k
6.1	30	29	280	100k
6.1	30	28	290	180k
6.1	30	27	300	100k
6.1	30	26	310	80k
6.1	40	30	270	70k
6.1	40	29	280	130k
6.1	40	28	290	300k
6.1	40	27	300	140k
6.1	40	26	310	70k

$V = (20a_\sigma)^3 = (1.95\text{fm})^3$

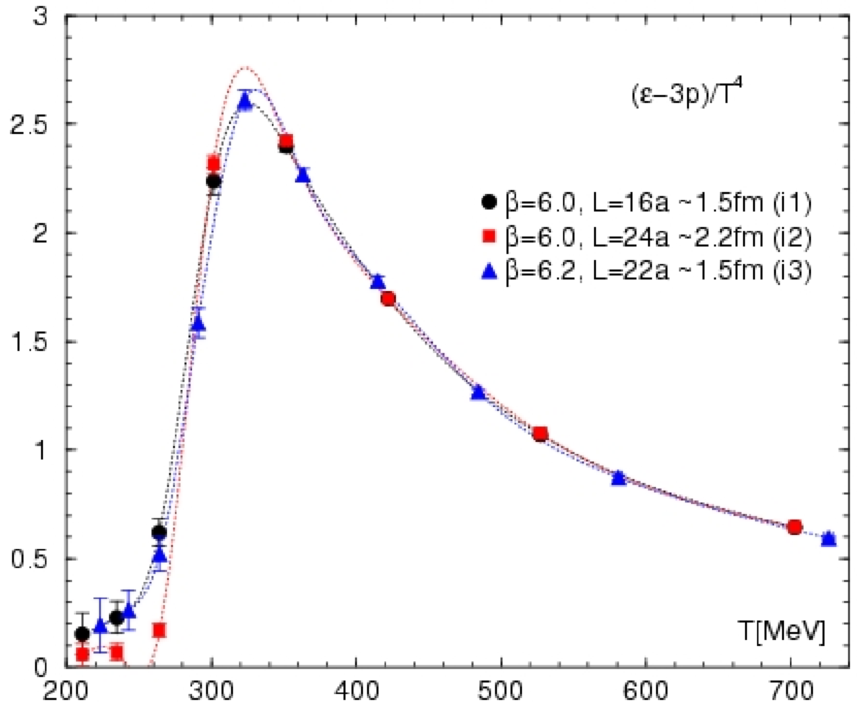
$V = (30a_\sigma)^3 = (2.92\text{fm})^3$

$V = (40a_\sigma)^3 = (3.89\text{fm})^3$

- critical temp.

Trace anomaly $(e - 3p)/T^4$ on isotropic lattices

$$\frac{\epsilon - 3p}{T^4} = \left(\frac{N_\tau^3}{N_\sigma^3}\right) a\frac{\partial\beta}{\partial a}\left\langle\frac{\partial S_g}{\partial\beta}\right\rangle_{sub}$$



dotted lines : cubic spline

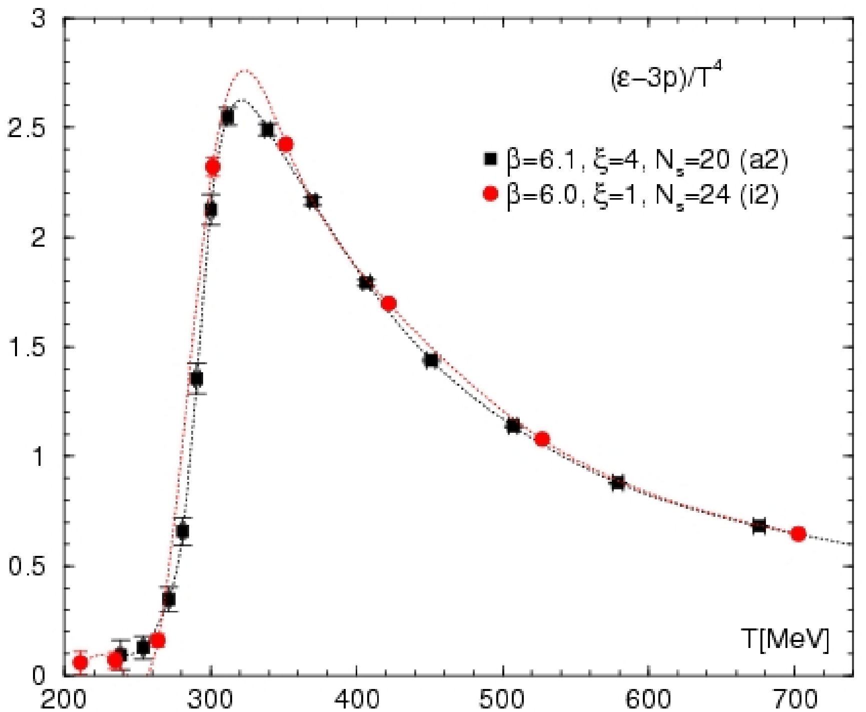
(1) β =6.0, a=0.094fm, V=(1.5fm)^3
(2) β =6.0, a=0.094fm, V=(2.2fm)^3
(3) β =6.2, a=0.068fm, V=(1.5fm)^3

beta function : G.Boyd et al. ('96)
lattice scale r_0 : R.Edwards et al. ('98)

- Excellent agreement between (1) and (3)
  → scale violation is small
  a=0.1fm is good
- Finite volume effect appears below & near T_c
  → volume size is important
  V=(2fm)^3 is necessary.

Trace anomaly $(e - 3p)/T^4$ on aniso. lattice

$$\frac{\epsilon - 3p}{T^4} = \left(\frac{N_\tau^3}{N_\sigma^3 \xi^3}\right) a_\sigma \frac{\partial \beta}{\partial a_\sigma}\bigg|_\xi \left\langle \frac{\partial S_g}{\partial \beta}\bigg|_\xi \right\rangle$$



dotted lines : cubic spline

(1) ξ =4, a=0.097fm, V=(2.0fm)^3
(2) ξ =1, a=0.094fm, V=(2.2fm)^3

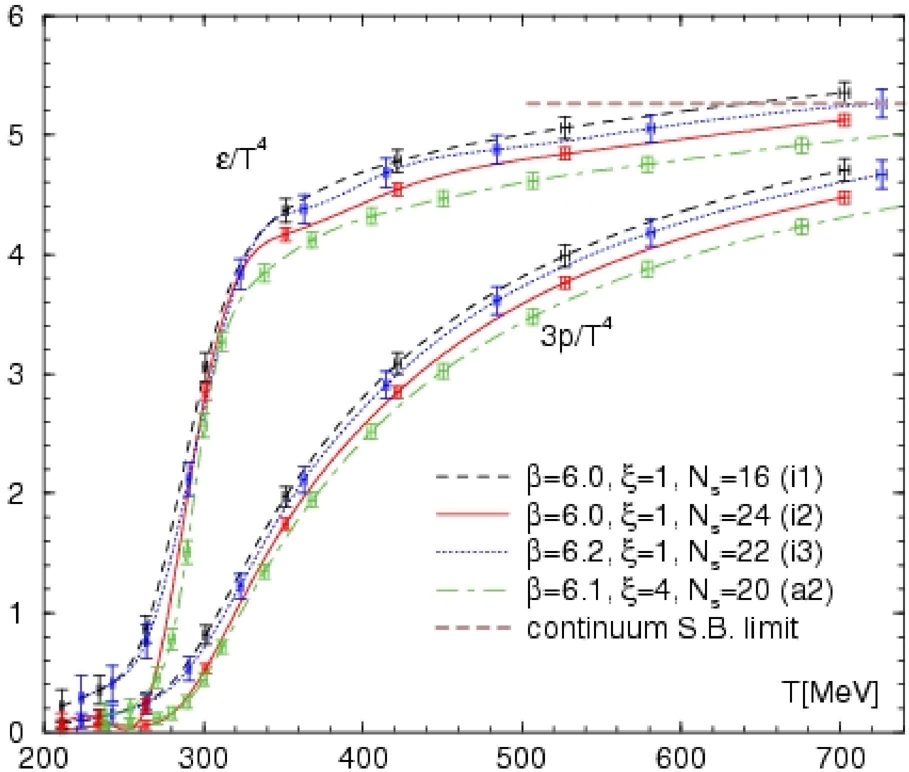
beta function : obtained by r_0/a_σ fit
r_0/a_σ data H.Matsufuru et al. ('01)

$\frac{\partial \xi_0}{\partial \beta}\Big|_\xi$ is required in SU(3) gauge theory.

→ T.R.Klassen ('98)

- Anisotropic lattice is useful to increase Temp. resolution.

Pressure & Energy density



- Integration $\left(\frac{p}{T^4} = \int_0^T dT' \frac{\epsilon - 3p}{T'^5} \right)$

  is performed with the cubic spline of $(e-3p)/T^4$

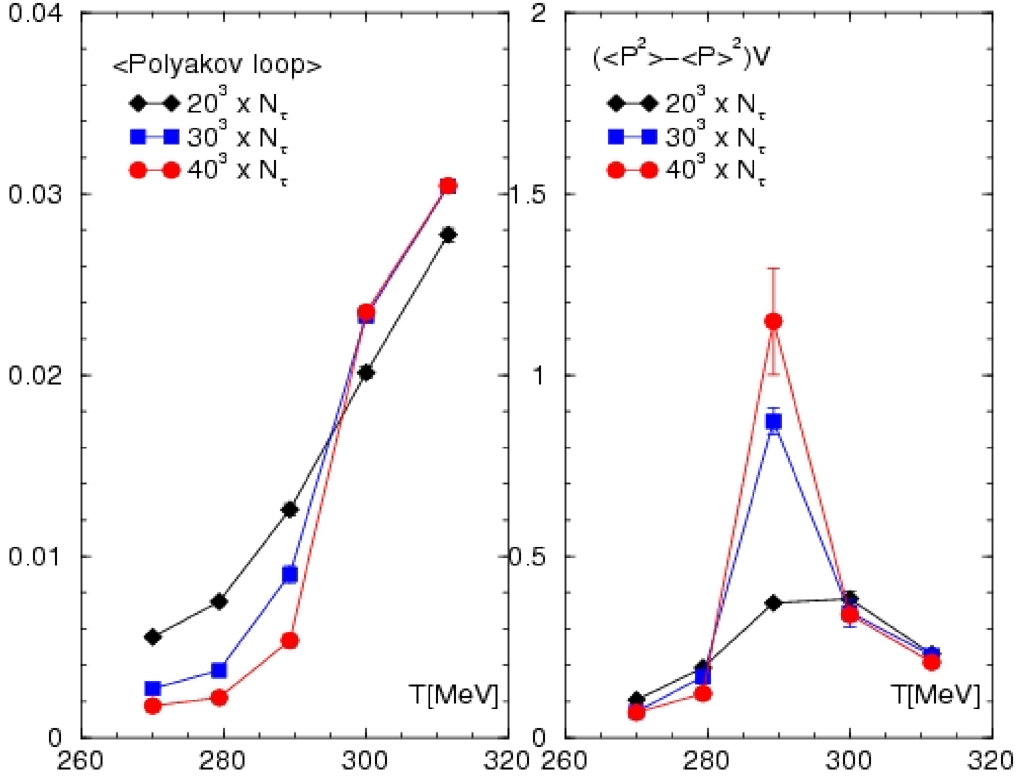
- Cubic spline vs trapezoidal inte. yields small difference ~ 1σ

- Our results are roughly consistent with previous results.

- Unlike the fixed N_τ approach, scale/temp. is not constant.

→ Lattice artifacts increase as temperature increases.

Transition temperature at fixed scale

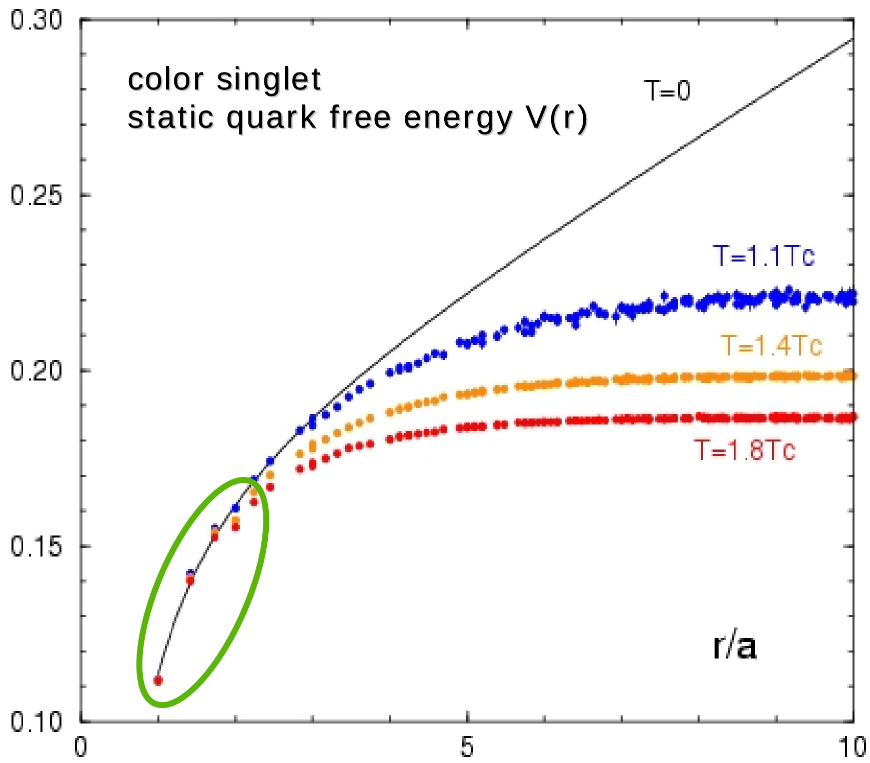


T-dependence of
the (rotated) Polyakov loop
and its susceptibility

- No renormalization is required upto overall factor due to the fixed scale.
- Rough estimation of critical temperature is possible.

T_c = 280~300 MeV
at $\beta = 6.1$, $\xi = 4$
(SU(3) gauge theory)

Static quark free energy at fixed scale



Static quark free energies at fixed scale

- Due to the fixed scale, no renomalization constant is required.

→ small thermal effects in V(r) at short distance (without any matching)

- Easy to distinguish temperature effect of V(r) from scale & volume effects

Conclusion

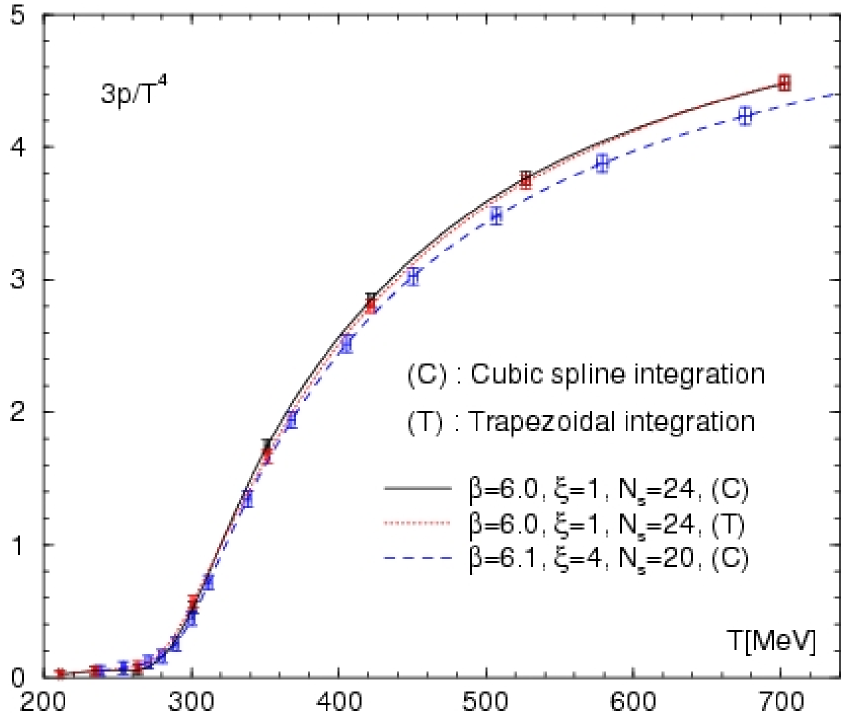
- We studied thermodynamics of SU(3) gauge theory
  at fixed lattice scale
- Our method (T-integration method) works well
  to calculate the EOS
- Fixed scale approach is also useful for
  - critical temperature
  - static quark free energy
  - etc.
- Our method is also available in full QCD !!

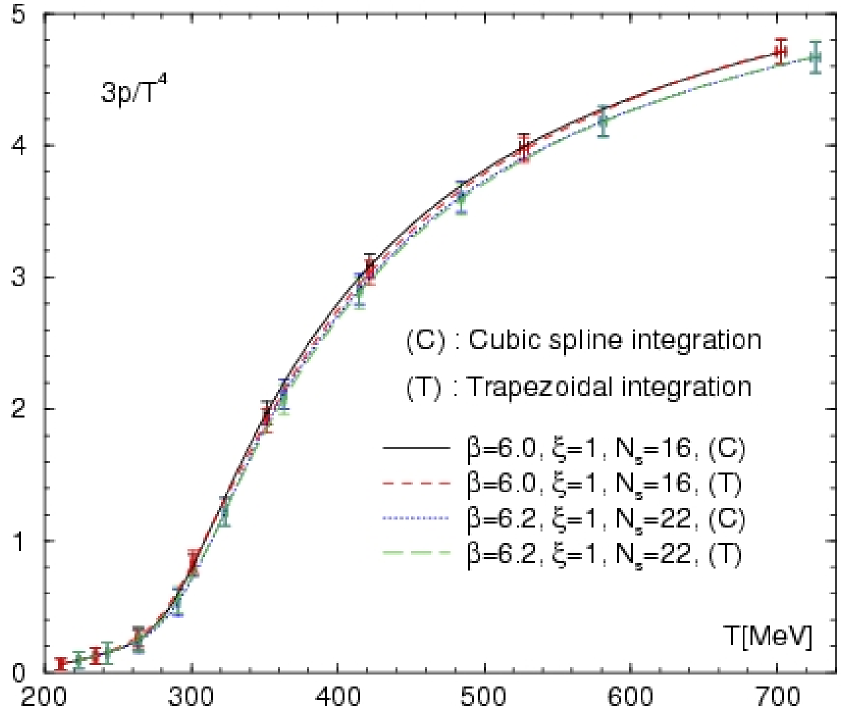
Therefore …

Toward full QCD calculations

- Our method is suited for
  already performed high statistics full QCD results.
- When beta functions are (able to be) known at a simulation point
  and T=0 configurations are open to the public,
  our method requires no additional T=0 simulation !!
- We are pushing forward in this direction
  using CP-PACS/JLQCD results in ILDG
  (N_f=2+1 Clover+RG, a=0.07fm, m_{ps}/m_V=0.6)
- Our final goal is to study
  thermodynamics on the physical point
  with 2+1 flavors of Wilson quarks → see PACS-CS talks

Pressure & Energy density





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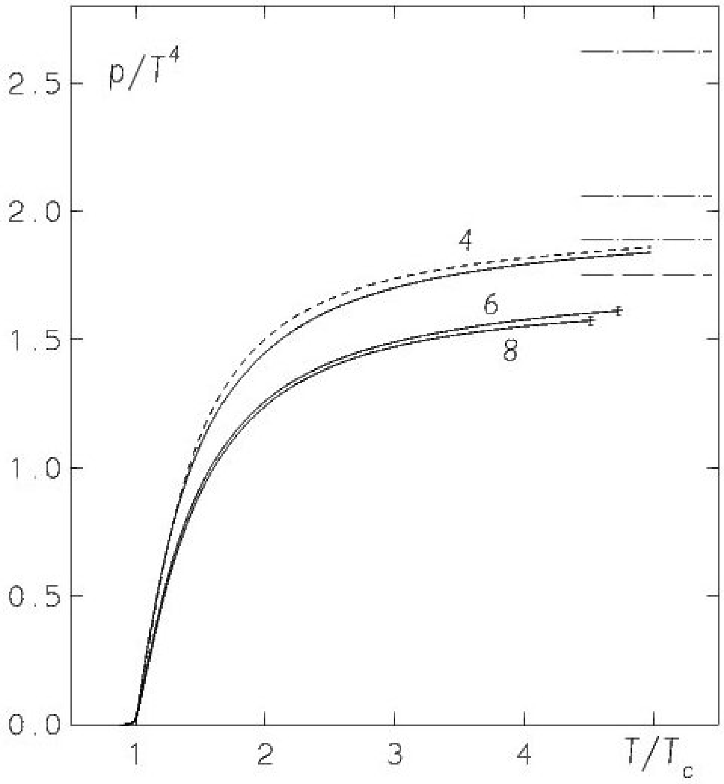
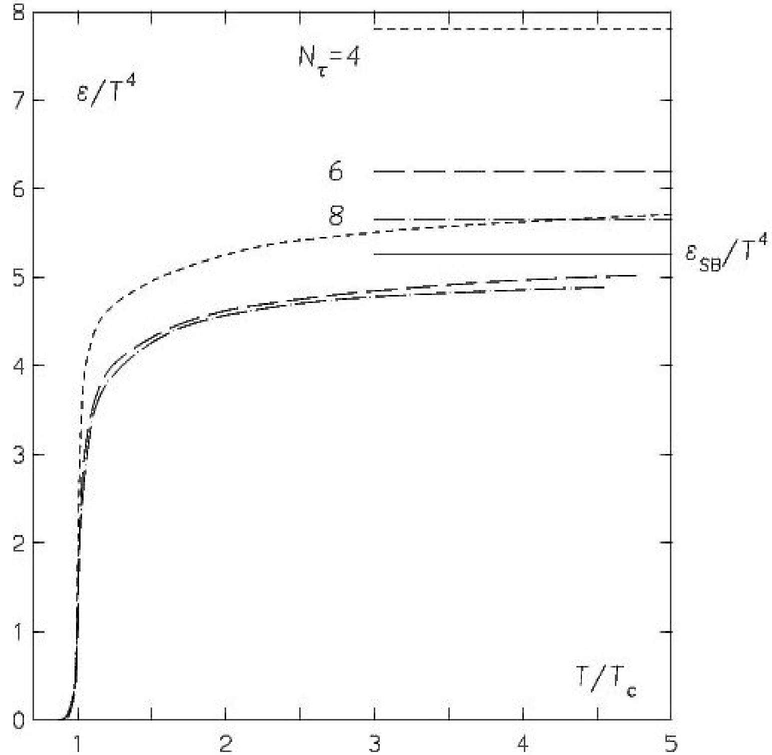




G.Boyd et al. ('96)

Simulation parameters (isotropic lattices)

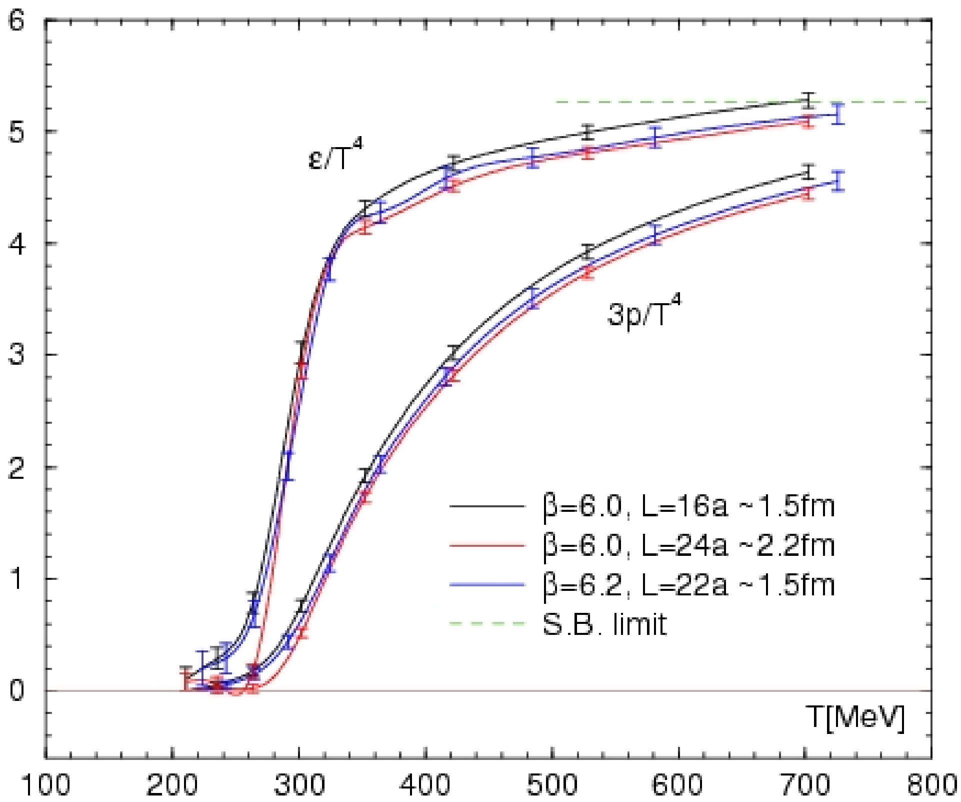
We present results from SU(3) gauge theory as a test of our method

- plaquette gauge action on N_σ^3 x N_τ lattices
- Jackknife analysis with appropriate bin-size

To study scale- & volume-dependence,
we prepare 3-type of lattices.

set	β	ξ	N_s	N_t	r_0/a_s	a_s[fm]	L[fm]	$a(dg^{-2}/da)$
i1	6.0	1	16	3-10	$5.35(^{+2}_{-3})$	0.093	1.5	-0.098172
i2	6.0	1	24	3-10	$5.35(^{+2}_{-3})$	0.093	2.2	-0.098172
i3	6.2	1	22	4-13	7.37(3)	0.068	1.5	-0.112127
a2	6.1	4	20	8-34	5.140(32)	0.097	2.0	-0.10704

Pressure & Energy density



- Integration $\left(\frac{p}{T^4} = \int_0^T dT' \frac{\epsilon - 3p}{T'^5} \right)$

  is performed with the cubic spline of (e-3p)/T^4

- Our results are roughly consistent with previous results.
  -- mild scale violation
  -- Large volume is important

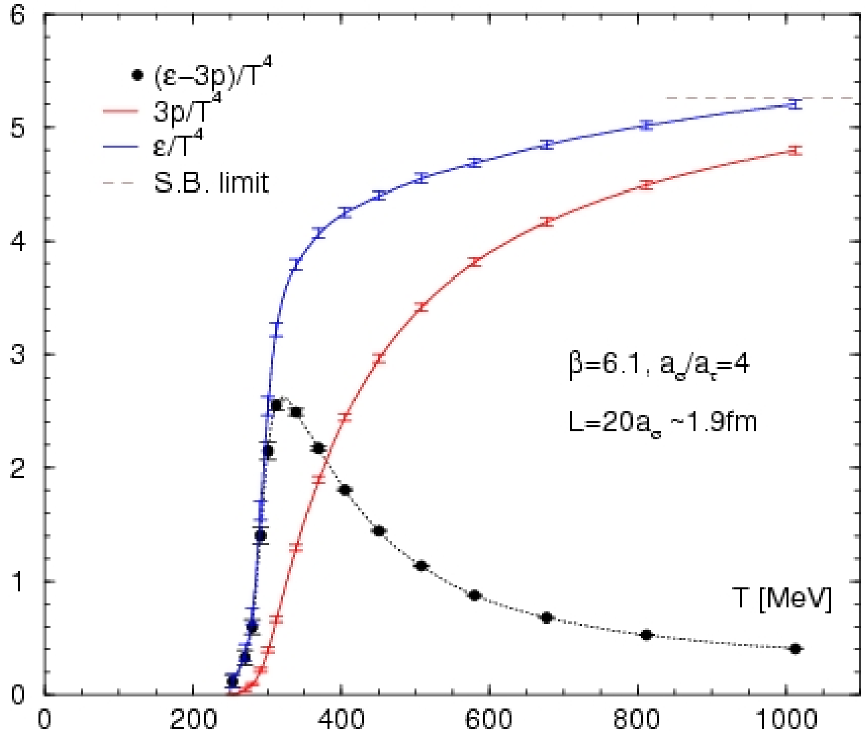
- Unlike the fixed N_τ approach, scale/temp. is not constant.

→ Lattice artifacts increase as temperature increases.

EOS on an anisotropic lattice

$$\frac{\epsilon - 3p}{T^4} = \left(\frac{N_\tau^3}{N_\sigma^3 \xi^3}\right) a_\sigma \frac{\partial \beta}{\partial a_\sigma}\bigg|_\xi \left\langle \frac{\partial S_g}{\partial \beta}\bigg|_\xi \right\rangle$$

beta function : obtained by r_0/a_σ fit
r_0/a_σ data H.Matsufuru et al. ('01)



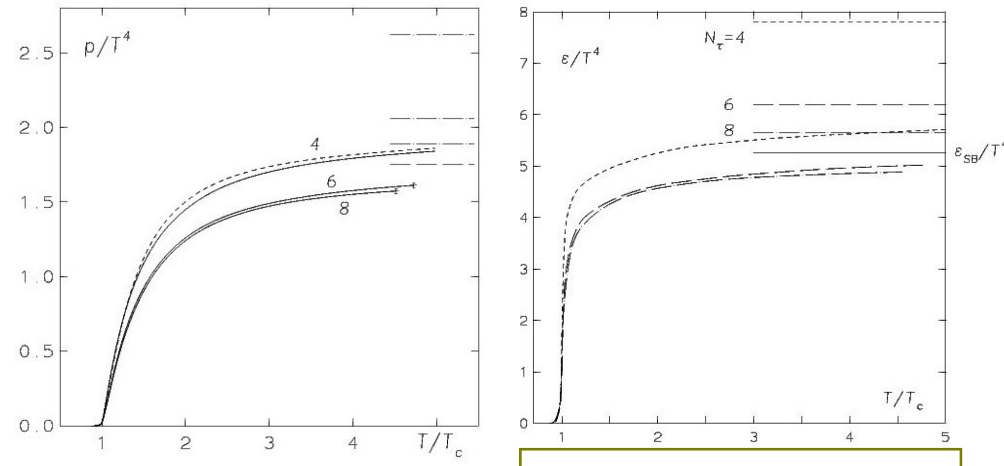
- Anisotropic lattice is useful to increase Temp. resolution.
- Results are roughly consistent with previous & isotropic results
- Additional coefficients are required to calculate $(e-3p)/T^4$

$\frac{\partial \xi_0}{\partial \beta}\Big|_\xi$ is required in SU(3) gauge theory.

→ T.R.Klassen ('98)

EOS on an anisotropic l

$$\frac{\epsilon - 3p}{T^4} = \left(\frac{N_\tau^3}{N_\sigma^3 \xi^3} \right) a_\sigma \frac{\partial \beta}{\partial a_\sigma} \bigg|_\xi \bigg\langle \frac{\partial S}{\partial \ldots}$$



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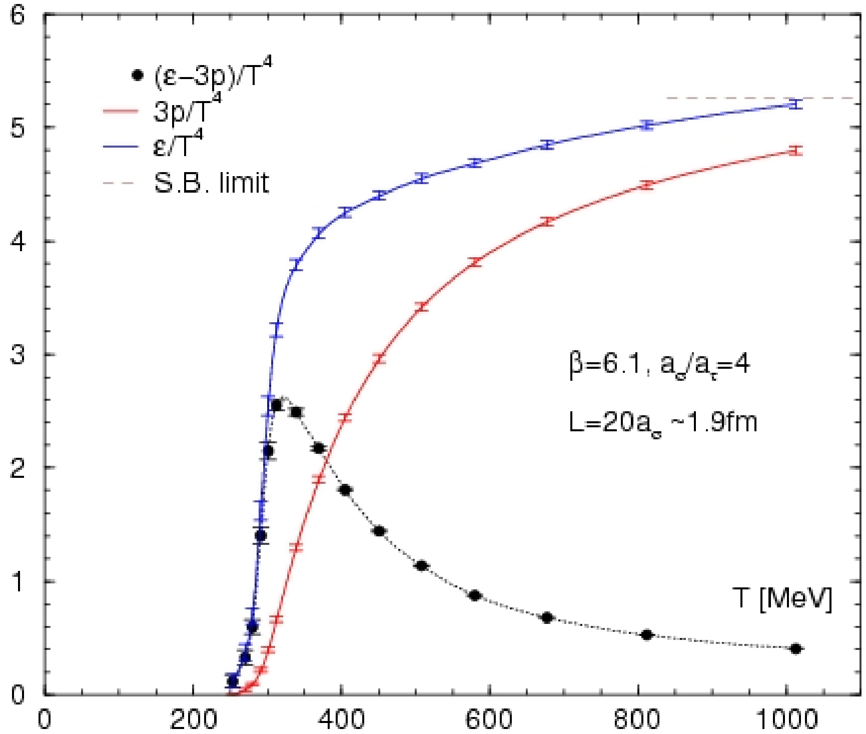

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