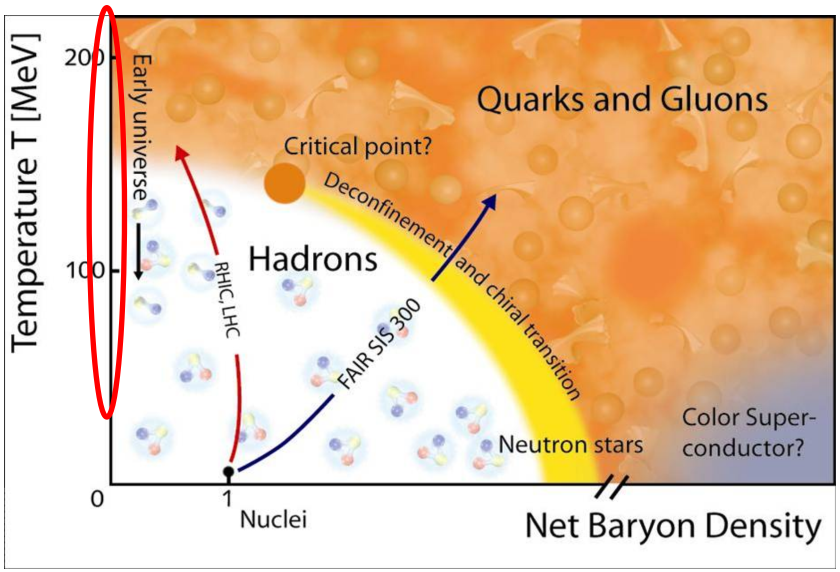
固定格子間隔での有限温度格子QCDの研究

Takashi Umeda (Hiroshima Univ.)
for WHOT-QCD Collaboration

HIROSHIMA UNIVERSITY

JPS meeting, Kwansei-gakuin, Hyogo, 26 Mar. 2012

Quark Gluon Plasma in Lattice QCD



http://www.gsi.de/fair/experiments/

Quark Gluon Plasma will be understood with theoretical models (e.g. hydrodynamic models) which require "physical inputs"

Lattice QCD : first principle calculation

- Phase diagram in (T, μ, m_{ud}, m_s)
- Transition temperature
- Equation of state (ε/T^4, p/T^4,...)
- etc...

Choice of quark actions on the lattice

Most (T, μ≠0) studies done with staggerd-type quarks

→ N_f=2+1, physical quark mass, (μ≠0)

- 4th-root trick to remove unphysical "tastes"

  → non-locality "Validity is not guaranteed"

It is important to cross-check with

theoretically sound lattice quarks like Wilson-type quarks

The objective of WHOT-QCD collaboration is
finite T & μ calculations using Wilson-type quarks

Equation of state in 2+1 flavor QCD with improved Wilson quarks by the fixed scale approach
T. Umeda et al. (WHOT-QCD Collab.) [arXiv:1202.4719]

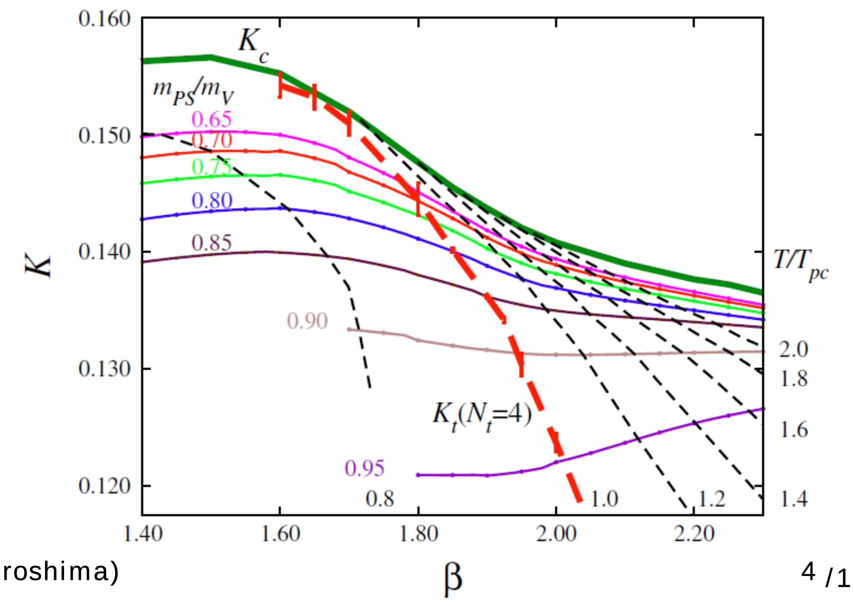
Fixed scale approach to study QCD thermodynamics

Temperature $T=1/(N_t a)$ is varied by N_t at fixed a

a : lattice spacing
N_t : lattice size in temporal direction

- **Advantages**
  - Line of Constant Physics
  - T=0 subtraction for renorm. (spectrum study at T=0)
  - Lattice spacing at lower T
  - Finite volume effects
- **Disadvantages**
  - T resolution
  - High T region

LCP's in fixed N_t approach
(N_f=2 Wilson quarks at N_t=4)



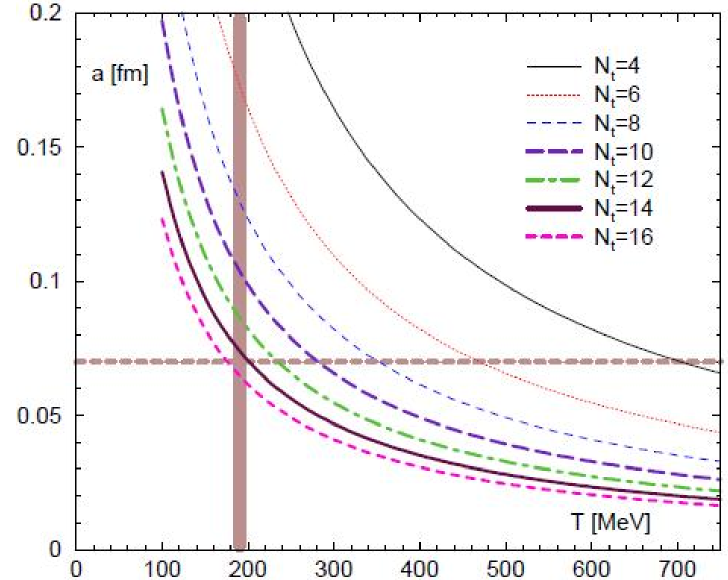
Fixed scale approach to study QCD thermodynamics

Temperature $T=1/(N_t a)$ is varied by N_t at fixed a

a : lattice spacing
N_t : lattice size in temporal direction

- **Advantages**
  - Line of Constant Physics
  - T=0 subtraction for renorm. (spectrum study at T=0)
  - Lattice spacing at lower T
  - Finite volume effects
- **Disadvantages**
  - T resolution
  - High T region

lattice spacing at fixed N_t



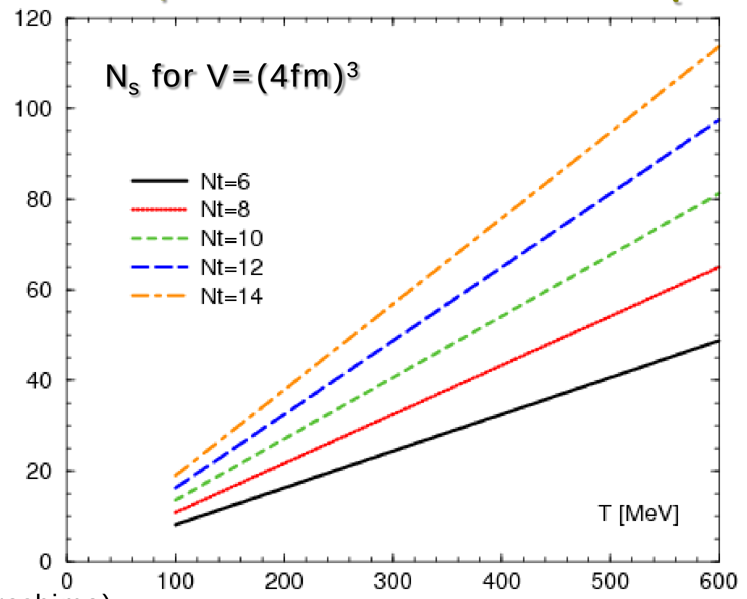
Fixed scale approach to study QCD thermodynamics

Temperature $T=1/(N_t a)$ is varied by N_t at fixed a

a : lattice spacing
N_t : lattice size in temporal direction

- **Advantages**
  - Line of Constant Physics
  - T=0 subtraction for renorm. (spectrum study at T=0)
  - Lattice spacing at lower T
  - Finite volume effects
- **Disadvantages**
  - T resolution
  - High T region

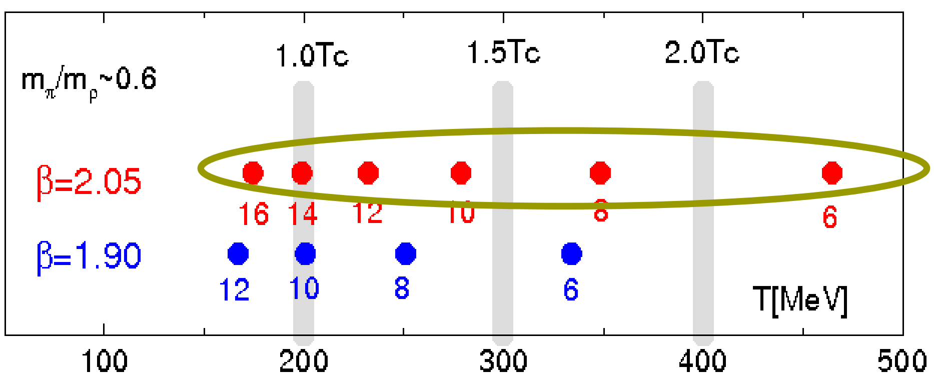
spatial volume at fixed N_t



Lattice setup

- T=0 simulation: on 28^3 x 56 by CP-PACS/JLQCD *Phys. Rev. D78 (2008) 011502*
  - RG-improved Iwasaki glue + NP-improved Wilson quarks
  - β=2.05, κ_{ud}=0.1356, κ_s=0.1351
  - V~(2 fm)^3 , a~0.07 fm, ($m_\pi \sim 634\text{MeV}, \ \dfrac{m_\pi}{m_\rho} = 0.63, \ \dfrac{m_{\eta_{ss}}}{m_\phi} = 0.74$)
  - configurations available on the ILDG/JLDG
- T>0 simulations: on 32^3 x N_t (N_t=4, 6, ..., 14, 16) lattices

RHMC algorithm, same parameters as T=0 simulation



Beta-functions from CP-PACS+JLQCD results

Direct fit method

fit $\beta, \kappa_{ud}, \kappa_s$ as functions of

scale: (am_ρ), LCP: $\left(\frac{m_\pi}{m_\rho}\right), \left(\frac{m_{\eta_{ss}}}{m_\phi}\right)$

$$
\begin{aligned}
\begin{pmatrix} \beta \\ \kappa_{ud} \\ \kappa_s \end{pmatrix} = {} & \vec{c}_0 + \vec{c}_1 (am_\rho) + \vec{c}_2 (am_\rho)^2 + \vec{c}_3 \left(\frac{m_\pi}{m_\rho}\right) + \vec{c}_4 \left(\frac{m_\pi}{m_\rho}\right)^2 + \vec{c}_5 (am_\rho) \left(\frac{m_\pi}{m_\rho}\right) \\
& + \vec{c}_6 \left(\frac{m_{\eta_{ss}}}{m_\phi}\right) + \vec{c}_7 \left(\frac{m_{\eta_{ss}}}{m_\phi}\right)^2 + \vec{c}_8 (am_\rho) \left(\frac{m_{\eta_{ss}}}{m_\phi}\right) + \vec{c}_9 \left(\frac{m_\pi}{m_\rho}\right) \left(\frac{m_{\eta_{ss}}}{m_\phi}\right) \\
& + \vec{c}_{10} (am_\rho)^3 + \vec{c}_{11} \left(\frac{m_\pi}{m_\rho}\right)^3 + \vec{c}_{12} \left(\frac{m_{\eta_{ss}}}{m_\phi}\right)^3 + \vec{c}_{13} (am_\rho) \left(\frac{m_\pi}{m_\rho}\right)^2 \\
& + \vec{c}_{14} (am_\rho)^2 \left(\frac{m_\pi}{m_\rho}\right) + \vec{c}_{15} (am_\rho) \left(\frac{m_{\eta_{ss}}}{m_\phi}\right)^2 + \vec{c}_{16} (am_\rho)^2 \left(\frac{m_{\eta_{ss}}}{m_\phi}\right) \\
& + \vec{c}_{17} \left(\frac{m_\pi}{m_\rho}\right) \left(\frac{m_{\eta_{ss}}}{m_\phi}\right)^2 + \vec{c}_{18} \left(\frac{m_\pi}{m_\rho}\right)^2 \left(\frac{m_{\eta_{ss}}}{m_\phi}\right) + \vec{c}_{19} (am_\rho) \left(\frac{m_\pi}{m_\rho}\right) \left(\frac{m_{\eta_{ss}}}{m_\phi}\right).
\end{aligned}
$$

→ $a\frac{\partial X}{\partial a} = (m_\rho a)\frac{\partial X}{\partial (m_\rho a)}$ at a LCP $\quad (X = \beta,\ \kappa_{ud},\ \kappa_s)$

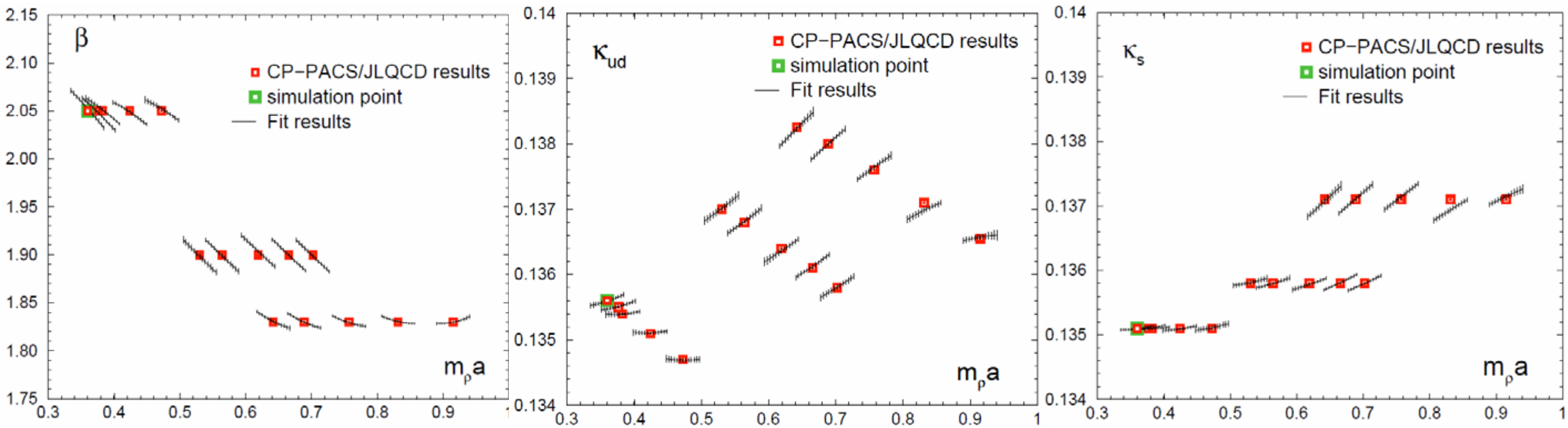
We estimate a systematic error using
$(am_\rho), (am_\pi), (am_K), (am_{K^*})$ for scale dependence

Beta-functions from CP-PACS/JLQCD results

Meson spectrum by CP-PACS/JLQCD *Phys. Rev. D78 (2008) 011502.*

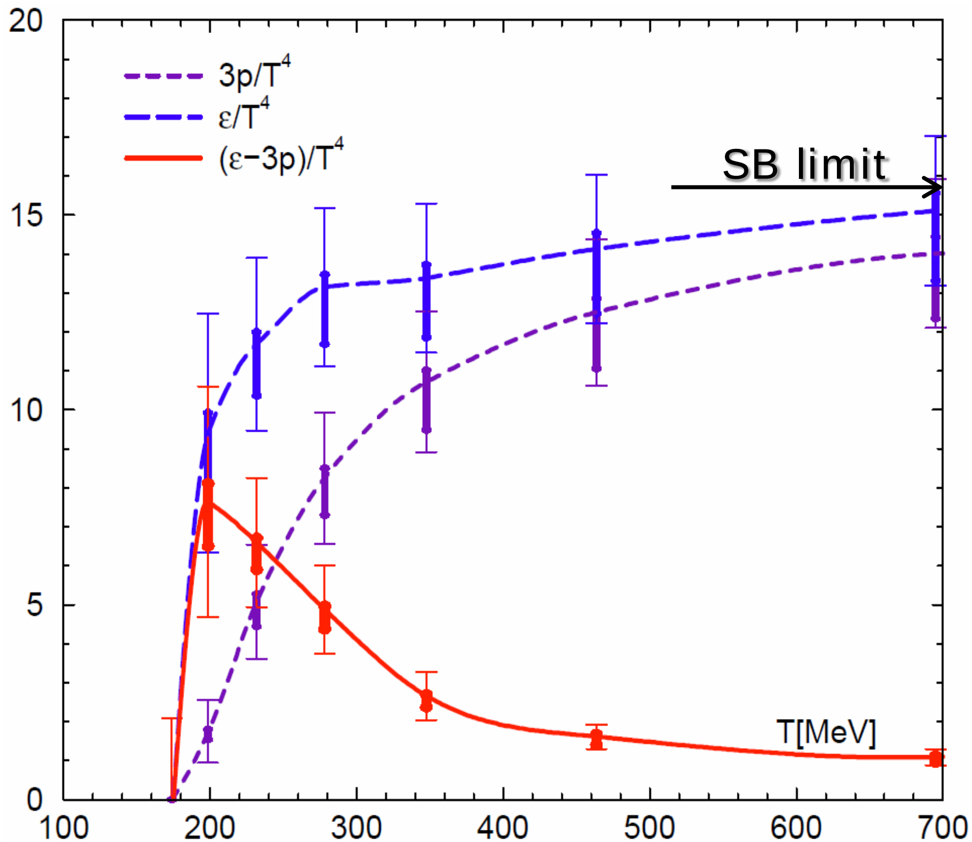
3 (β) x 5 (κ_{ud}) x 2 (κ_s) = 30 data points

fit β, κ_{ud}, κ_s as functions of $(am_\rho), \left(\frac{m_\pi}{m_\rho}\right), \left(\frac{m_{\eta ss}}{m_\phi}\right)$



$$\left(a\frac{\partial\beta}{\partial a},\ a\frac{\partial\kappa_{ud}}{\partial a},\ a\frac{\partial\kappa_s}{\partial a} \right)_{\text{simulation point}} = \left(-0.279(24)(^{+40}_{-64}), 0.00123(41)(^{+56}_{-68}), 0.00046(26)(^{+42}_{-44})\right)$$

Equation of State in Nf=2+1 QCD



- T-integration

$$\frac{p}{T^4} = \int_0^T dT' \frac{\epsilon - 3p}{T'^5}$$

is performed by Akima Spline interpolation.

- ε/T^4 is calculated from

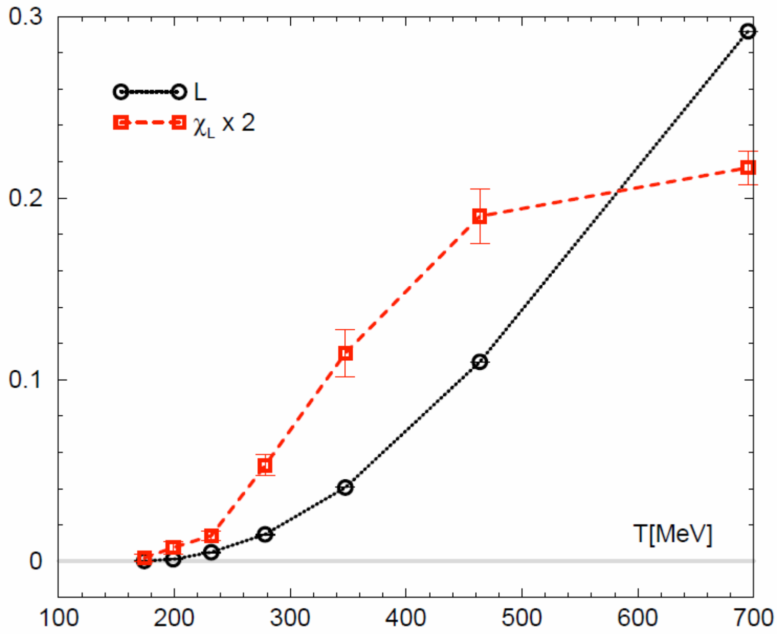
$$\frac{\epsilon - 3p}{T^4} + \frac{3p}{T^4}$$

- Large error in whole T region
- A systematic error due to beta-functions

Polyakov loop and Susceptibility

$$L = \frac{1}{V}\sum_{\vec{x}}\prod_{t=1}^{N_t} U_0(\vec{x},t)$$

$$\chi_L = V\left(\langle L^2\rangle - \langle L\rangle^2\right)$$



Polyakov loop requires
T dependent renormalization

$$\frac{F_{\bar{q}q}(r,T)}{T} = -\ln\left(\langle TrL(\vec{x})TrL(\vec{y})^\dagger\rangle\right) + c(T)$$

$F_{\bar{q}q}(r,T)$: heavy quark free energy

$$r = |\vec{x}-\vec{y}|$$

c(T) : additive normalization constant
(self-energy of the (anti-)quark sources)

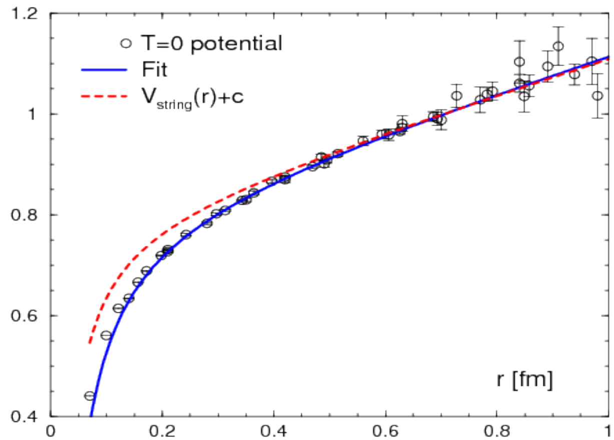
$$\begin{aligned} L_{\text{ren}} &= \exp\left(-\frac{F_{\bar{q}q}(r=\infty,T)}{2T}\right) \\ &= \exp\left(-\frac{c(T)}{2}\right)\langle L\rangle \end{aligned}$$

Renormalized Polyakov loop and Susceptibility

Cheng et al.'s renormalization
[PRD77(2008)014511]

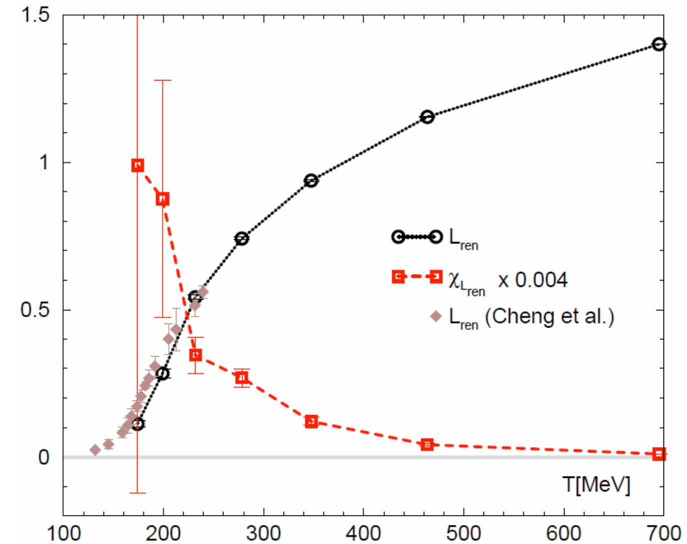
matching of V(r) to V_{string}(r) at r=1.5r_0

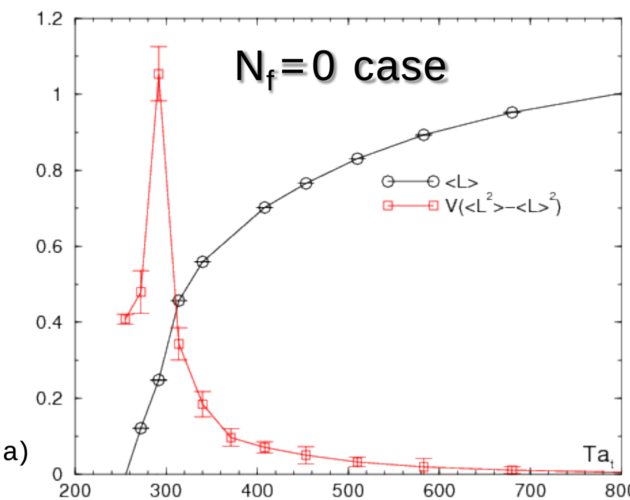
$$V_{\rm string}(r) = -\frac{\pi}{12} + \sigma r + c_{\rm m}$$



$$L_{\rm ren} = \exp\left(\frac{c_{\rm m} N_t}{2}\right) \langle L \rangle$$

(c_m is identical at a fixed scale)





Summary & outlook

We presented the EOS and renormalized Polyakov loop
in N_f=2+1 QCD using improve Wilson quarks

- Equation of state
  - More statistics are needed in the lower temperature region
  - Results at different scales (β=1.90 by CP-PACS/JLQCD)
- N_f=2+1 QCD just at the physical point
  - the physical point (pion mass ~ 140MeV) by PACS-CS
  - beta-functions using reweighting method
- Finite density
  - Taylor expansion method to explore EOS at $\mu \neq 0$

Renormalized Polyakov loop and Susceptibility

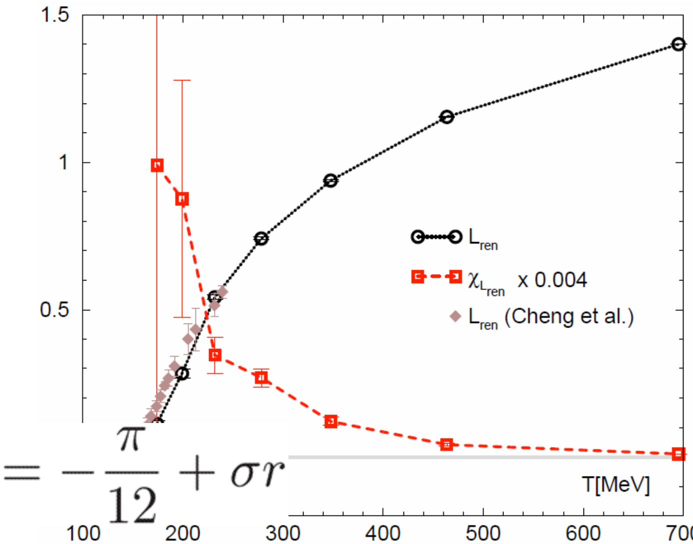
Cheng et al.'s renormalization
[PRD77(2008)014511]

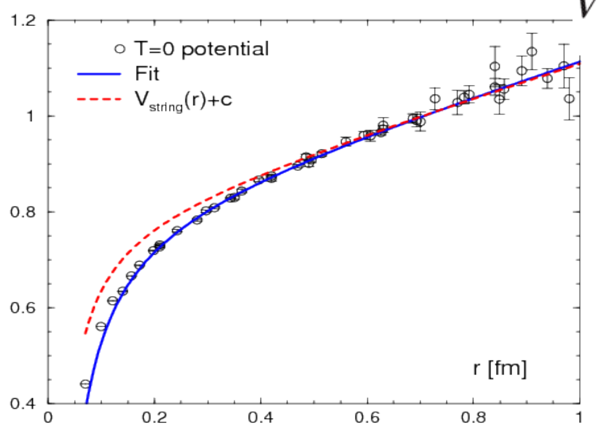
matching of $V(r)$ to $V_{string}(r)$ at $r=1.5r_0$

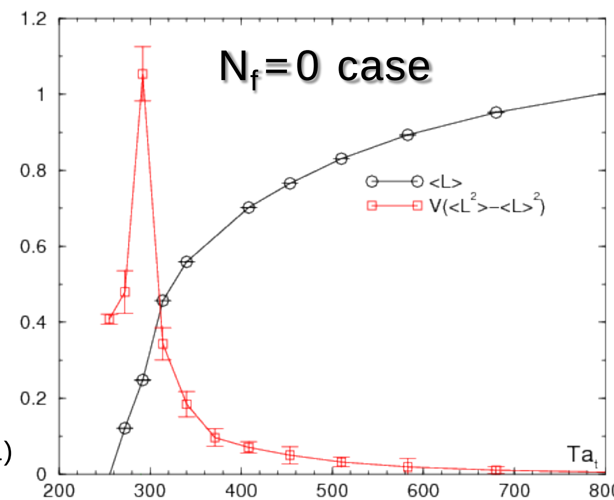
$V(r) = A - \alpha/r + \sigma r$

$V_{string}(r) = c_{diff} - \pi/12r + \sigma r$

$$V_{\text{string}}(r) = -\frac{\pi}{12} + \sigma r$$







$L_{ren} = \exp(c_{diff} N_t/2)\langle L \rangle$