偏移境界条件を用いた有限温度格子QCDの研究

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Fixed scale approach to study QCD thermodynamics

Fixed scale approach

Temperature $T=1/(N_t a)$ is varied by N_t at fixed a

a : lattice spacing
N_t : lattice size in t-direction

- Coupling constants are common at each T

To study Equation of States

- T=0 subtractions are common
- beta-functions are common
- Line of Constant Physics is automatically satisfied

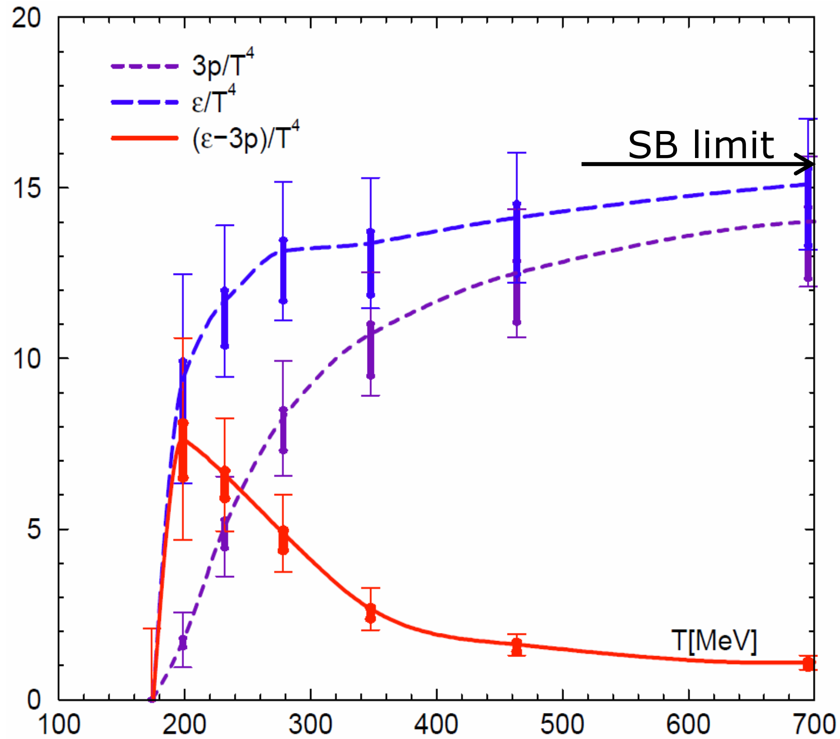
⇨ Cost for T=0 simulations can be largely reduced

However possible temperatures are restricted by integer N_t

△ critical temperature T_c

○ EOS

Equation of State in N_f=2+1 QCD



T. Umeda et al. (WHOT-QCD)
Phys. Rev. D85 (2012) 094508

Fixed scale approach for EOS

- EOS by T-integral method
- Small cost for T=0 simulation
- restricted T's by integer N_t
- beta-functions

Some groups adopted the approach

- tmfT, arXiv:1311.1631
- Wuppertal, JHEP08(2012)126.

Physical point simulation with Wilson quarks is on going

Shifted boundary conditions

L. Giusti and H. B. Meyer, Phys. Rev. Lett. 106 (2011) 131601.
Thermal momentum distribution from path integrals with shifted boundary conditions

New method to calculate thermodynamic potentials
(entropy density, specific heat, etc.)

The method is based on the partition function

$$Z(\vec{z}) = Tr\{e^{-L_0 \hat{H}} e^{i\hat{p}\vec{z}}\}$$

which can be expressed by Path-integral with shifted boundary condition

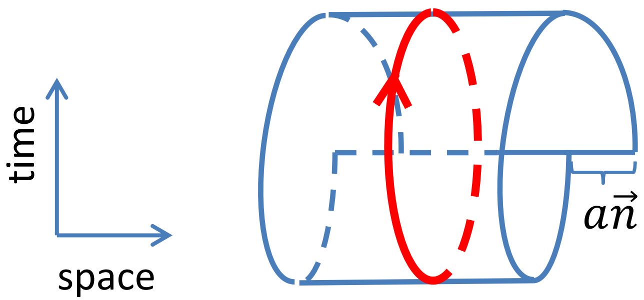
$$\phi(L_0, \vec{x}) = \pm\phi(0, \vec{x} + \vec{z})$$

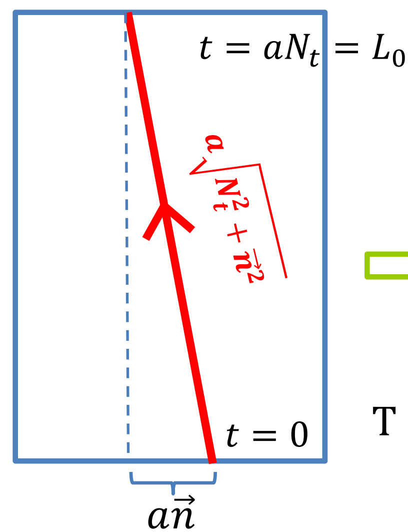
- L. Giusti and H. B. Meyer, JHEP 11 (2011) 087
- L. Giusti and H. B. Meyer, JHEP 01 (2013) 140

Shifted boundary conditions

$$\phi(L_0, \vec{x}) = \pm\phi(0, \vec{x} + \vec{z})$$

$$\vec{z} = a\vec{n}$$





$$\mathrm{T} = \frac{1}{a\sqrt{N_t^2 + \vec{n}^2}}$$

By using the shifted boundary
various T's are realized with the same lattice spacing

T resolution is largely improved
while keeping advantages of the fixed scale approach

Test in quenched QCD

Simulation setup

- quenched QCD
- β=6.0

  a ~ 0.1fm
- 32^3 x N_t lattices, N_t = 3, 4, 5, 6, 7, 8, 9 and 32 (T=0)

  $T_c(N_f=0) \sim 2 \times T_c(N_f=2+1, m_{phys})$
- boundary condition
  - spatial : periodic boundary condition
  - temporal: shifted boundary condition

$$U_\mu(L_0, \vec{x}) = U_\mu(0, \vec{x} + \vec{z})$$

- heat-bath algorithm (code for SX-8R)

  only "even-shift" to keep even-odd structure

  e.g. $\vec{z}/a = (0,0,0), (1,1,0), (2,0,0), (2,1,1), (2,2,0), (3,1,0), \ldots$

Test in quenched QCD

Choice of boundary shifts

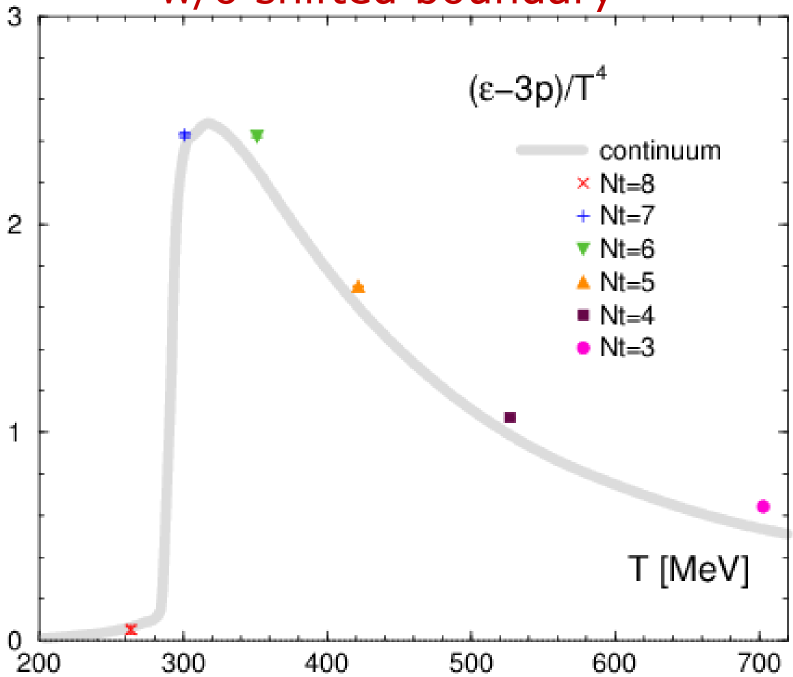
$$U_\mu(L_0, \vec{x}) = U_\mu(0, \vec{x} + \vec{z}) \qquad \vec{z} = a\vec{n}$$

					Nt							
n^2	n_1	n_2	n_3	e/o	10	9	8	7	6	5	4	3
0	0	0	0	0	10.00	9.00	8.00	7.00	6.00	5.00	4.00	3.00
2	1	1	0	0	10.10	9.11	8.12	7.14	6.16	5.20	4.24	3.32
4	2	0	0	0	10.20	9.22	8.25	7.28	6.32	5.39	4.47	3.61
6	2	1	1	0	10.30	9.33	8.37	7.42	6.48	5.57	4.69	3.87
8	2	2	0	0	10.39	9.43	8.49	7.55	6.63	5.74	4.90	4.12
10	3	1	0	0	10.49	9.54	8.60	7.68	6.78	5.92	5.10	4.36
12	2	2	2	0	10.58	9.64	8.72	7.81	6.93	6.08	5.29	4.58
14	3	2	1	0	10.68	9.75	8.83	7.94	7.07	6.24	5.48	4.80
16	4	0	0	0	10.77	9.85	8.94	8.06	7.21	6.40	5.66	5.00
18	3	3	0	0	10.86	9.95	9.06	8.19	7.35	6.56	5.83	5.20
18	4	1	1	0	10.86	9.95	9.06	8.19	7.35	6.56	5.83	5.20
20	4	2	0	0	10.95	10.05	9.17	8.31	7.48	6.71	6.00	5.39
22	3	3	2	0	11.05	10.15	9.27	8.43	7.62	6.86	6.16	5.57
24	4	2	2	0	11.14	10.25	9.38	8.54	7.75	7.00	6.32	5.74
26	4	3	1	0	11.22	10.34	9.49	8.66	7.87	7.14	6.48	5.92
26	5	1	0	0	11.22	10.34	9.49	8.66	7.87	7.14	6.48	5.92
30	5	2	1	0	11.40	10.54	9.70	8.89	8.12	7.42	6.78	6.24
32	4	4	0	0	11.49	10.63	9.80	9.00	8.25	7.55	6.93	6.40
34	4	3	3	0	11.58	10.72	9.90	9.11	8.37	7.68	7.07	6.56

Trace anomaly (e-3p)/T^4

$$\frac{\epsilon - 3p}{T^4} = \left(\frac{1}{VT^3}\right) a\frac{d\beta}{da}\left\langle\frac{dS}{d\beta}\right\rangle_{sub}$$

w/o shifted boundary



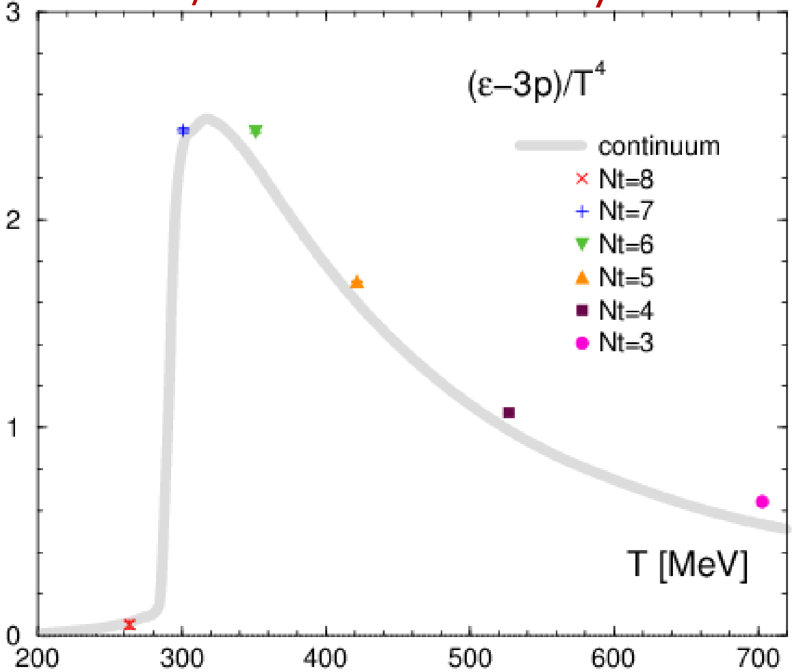
beta-function: Boyd et al. (1998)

Trace anomaly (e-3p)/T^4

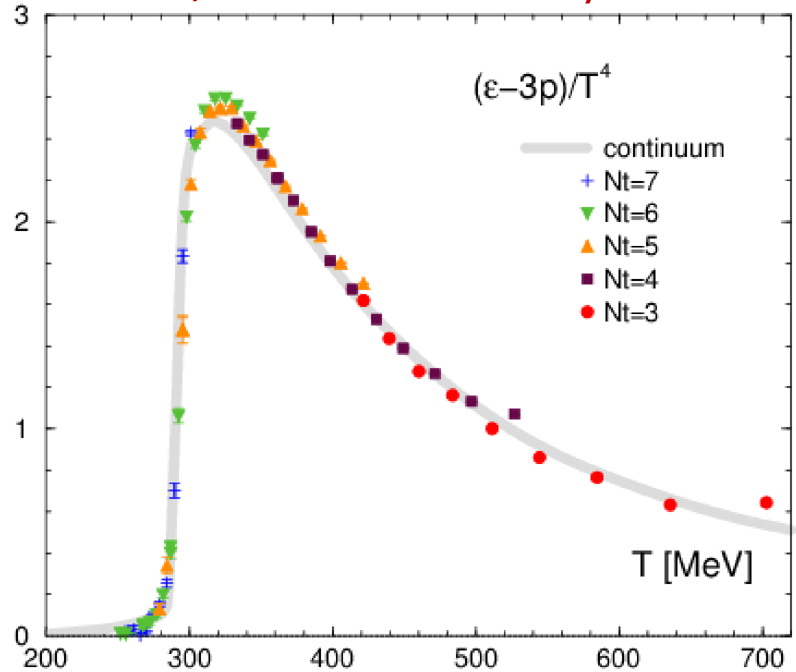
$$\frac{\epsilon - 3p}{T^4} = \left(\frac{1}{VT^3}\right) a \frac{d\beta}{da} \left\langle \frac{dS}{d\beta} \right\rangle_{sub}$$

$$T = \frac{1}{a\sqrt{N_t^2 + \vec{n}^2}} \qquad V = \prod_{i=1}^{3} \frac{aN_s}{\sqrt{1 + (\frac{n_i}{N_t})^2}}$$

w/o shifted boundary

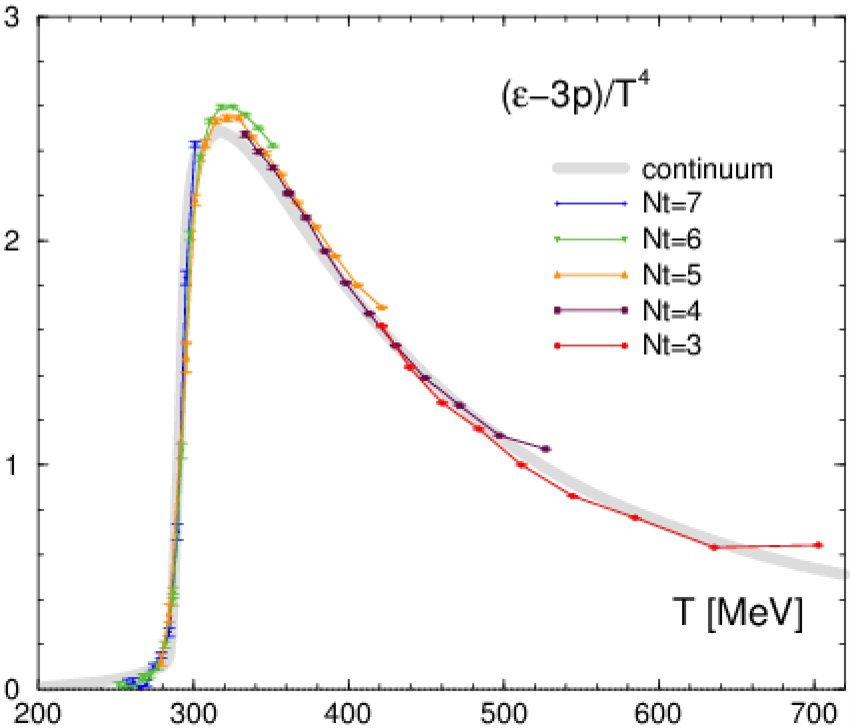


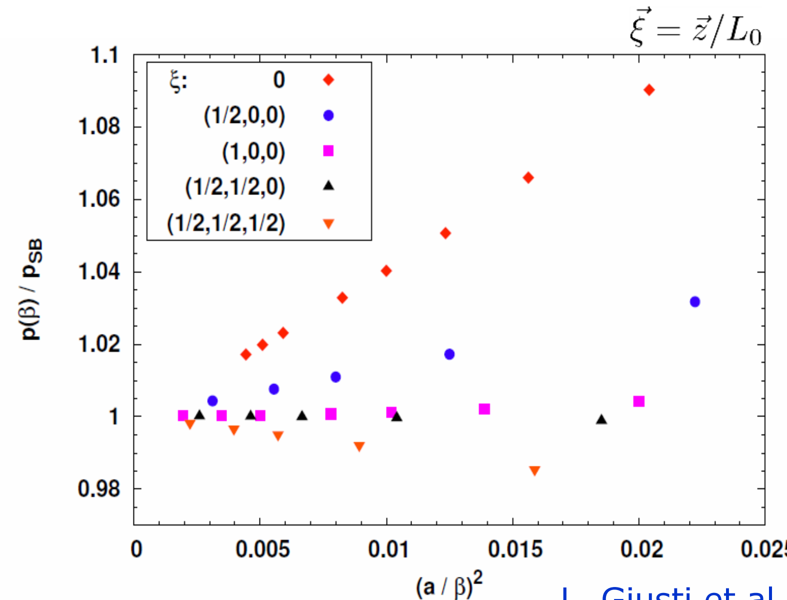
w/ shifted boundary



beta-function: Boyd et al. (1998)

Lattice artifacts from shifted boundaries





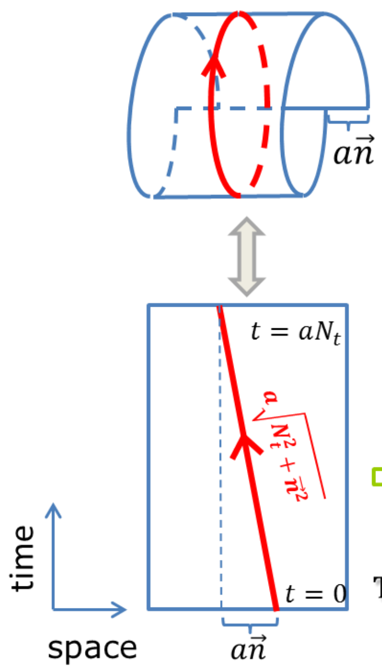
L. Giusti et al. (2011)

Figure 2: Pressure at finite lattice spacing for the SU(N) Yang–Mills theory in the non-interacting limit. The discretization used is the Wilson action and the 'clover' form of the lattice field strength tensor. The inverse temperature is given by $\beta = L_0\sqrt{1+\xi^2}$, and a is the lattice spacing.

- Lattice artifacts are suppressed at larger shifts
- Non-interacting limit with fermions should be checked

Critical temperature T_c

Polyakov loop is difficult to be defined because of misalignment of time and compact directions



$$T = \frac{1}{a\sqrt{N_t^2 + \vec{n}^2}}$$

Dressed Polyakov loop
E. Bilgici et al.,
Phys. Rev. D77 (2008) 094007

Polyakov loop defined with light quarks

$$\Sigma_n(m, V) = \int_0^{2\pi} \frac{d\phi}{2\pi} \frac{e^{-i\phi n}}{V} \langle Tr[(m + D_\phi)^{-1}] \rangle_G$$

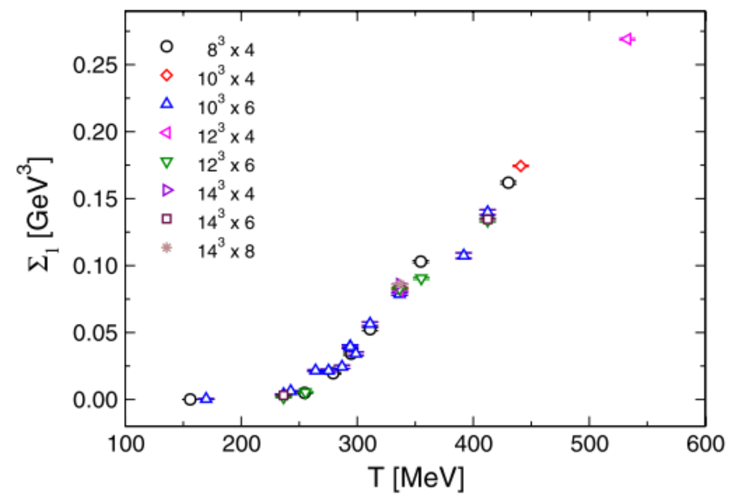
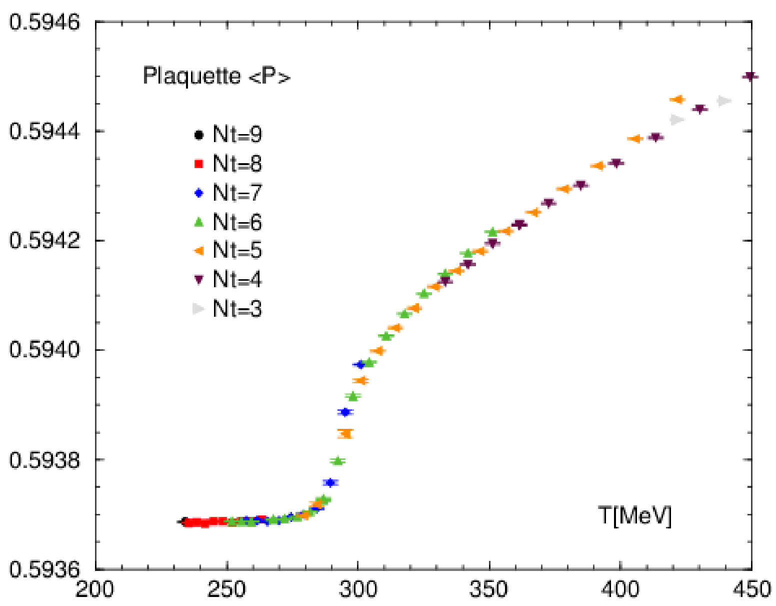


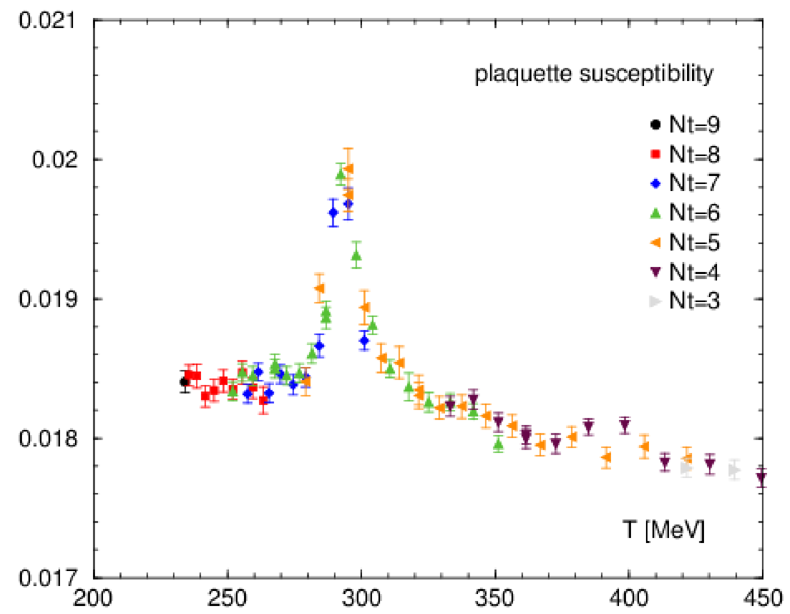
FIG. 2 (color online). The dressed Polyakov loop at $m = 100$ MeV in units of GeV^3 as a function of the temperature T in MeV.

Critical temperature Tc

Plaquette value $\langle P \rangle = \frac{1}{6N_s^3 N_t} \sum_P \langle 1 - \frac{1}{3} ReTrU_P \rangle$

Plaquette susceptibility $\chi_P = N_s^3 N_t \left(\langle P^2 \rangle - \langle P \rangle^2 \right)$





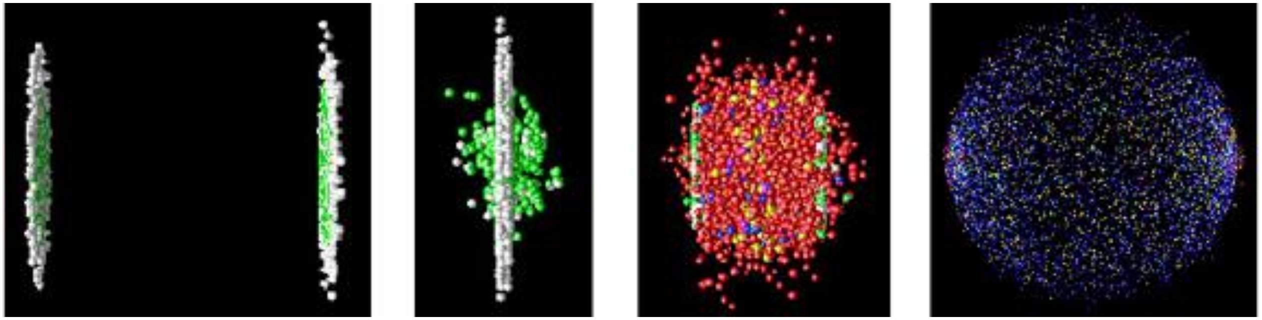
Plaq. suscep. has a peak
around T = 293 MeV

Summary & outlook

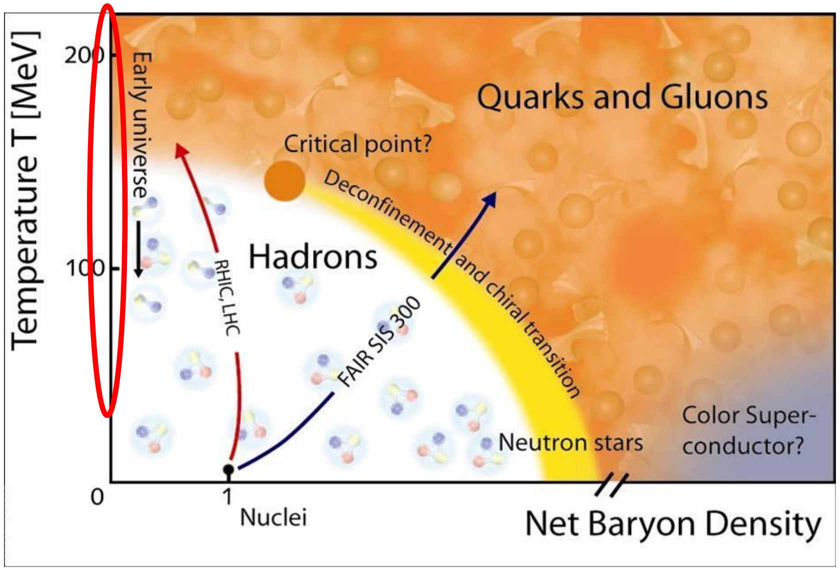
We presented our study of the QCD Thermodynamics
by using Fixed scale approach
and Shifted boundary conditions

- Fixed scale approach
  - Cost for T=0 simulations can be largely reduced
  - first result in N_f=2+1 QCD with Wilson-type quarks
- Shifted boundary conditions are promising tool
  to improve the fixed scale approach
  - fine resolution in Temperature
  - suppression of lattice artifacts at larger shifts
  - Tc determination could be possible
  - New method to estimate beta-functions
- Test in full QCD → Nf=2+1 QCD at the physical point

Quark Gluon Plasma in Lattice QCD



from the Phenix group web-site



http://www.gsi.de/fair/experiments/

Observables in Lattice QCD

- Phase diagram in (T, μ, m_{ud}, m_s)
- Critical temperature
- Equation of state (ε/T^4, p/T^4,...)
- Hadronic excitations
- Transport coefficients
- Finite chemical potential
- etc...