Scaling properties of the chiral phase transition in the low density region of two-flavor QCD with improved Wilson fermions

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Abstract:

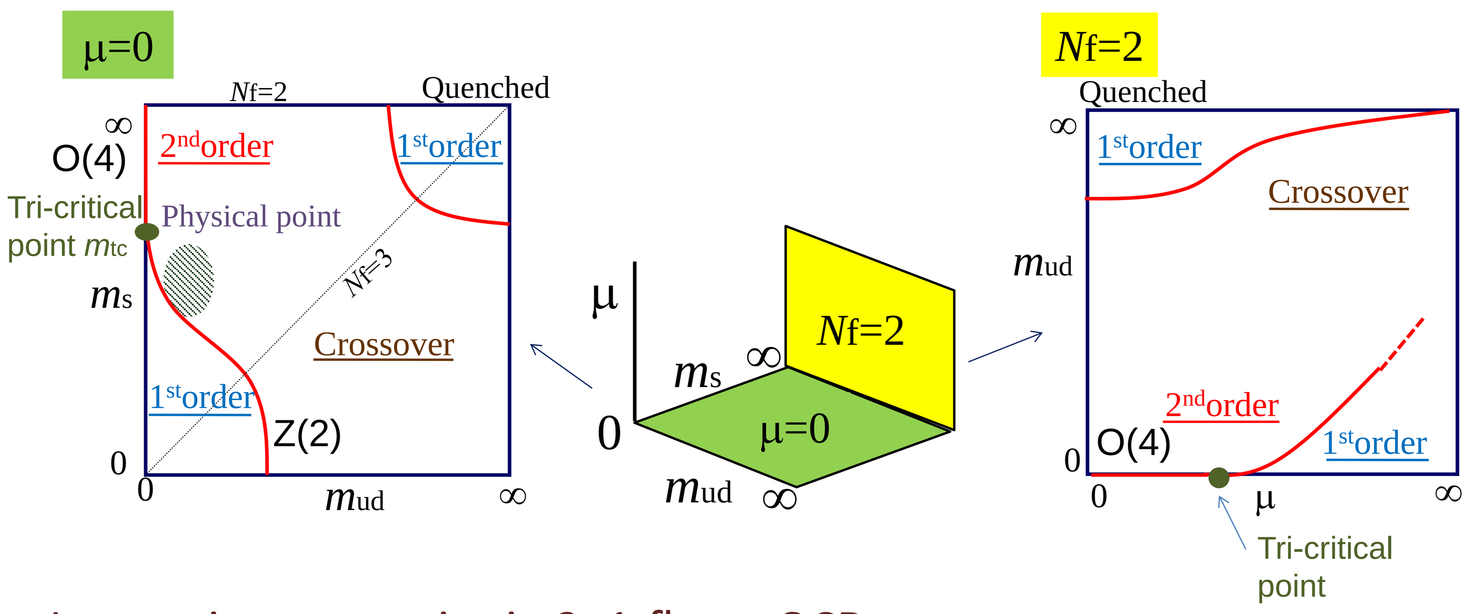
We study scaling behavior of a chiral order parameter in the low density region, performing a simulation of two-flavor QCD with improved Wilson quarks. The scaling behavior of the chiral order parameter defined by a Ward-Takahashi identity agrees with the scaling function of the three-dimensional O(4) spin model at zero chemical potential. We extend the scaling study to finite density QCD. Applying the reweighting method and calculating derivatives of the chiral order parameter with respect to the chemical potential, the scaling properties of the chiral phase transition are discussed in the low density region. We moreover calculate the curvature of the phase boundary of the chiral phase transition in the temperature and chemical potential plane assuming the O(4) scaling relation.

1. Introduction and Motivation

Quark mass dependence of QCD phase transition

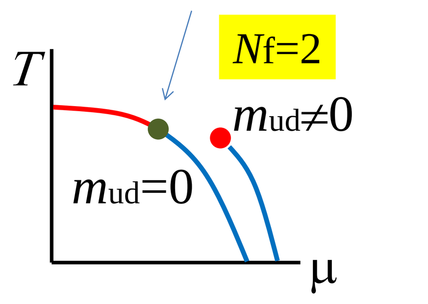
Expected nature of phase transitions

(2-flavor QCD and O(4) spin model: same universality class?)



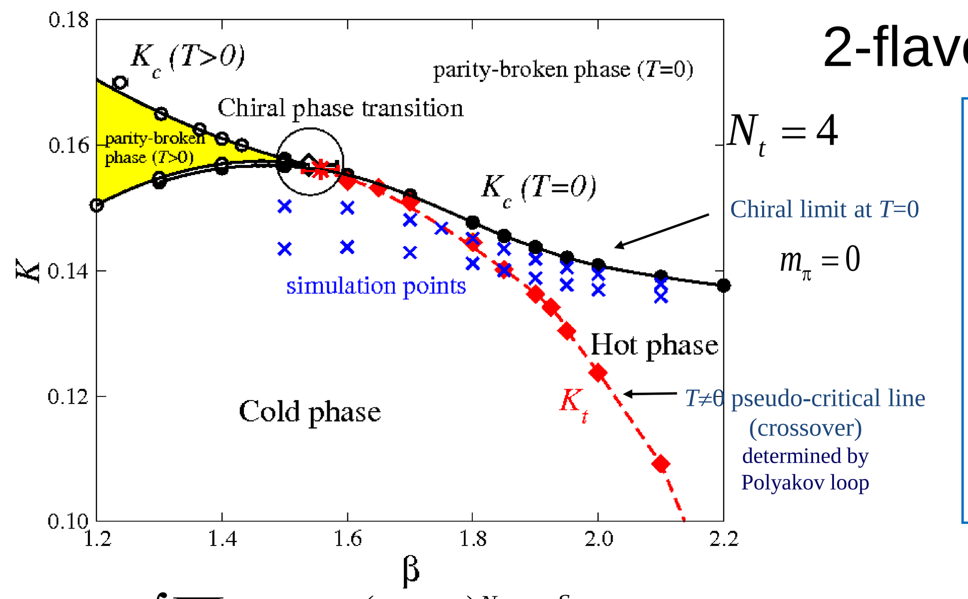
Interesting properties in 2+1-flavor QCD

- O(4) scaling for $m_s >> m_{tc}$
- Tri-critical scaling at m_{tc}
- First order transition at $m_s < m_{tc}$
- O(4) scaring at small μ ⟵ This study
- Tri-critical point at finite μ



2. Wilson quark simulations

Iwasaki improved gauge + Clover improved Wilson quark action



2-flavor QCD $16^3 \times 4$ lattice

m_{PS}/m_V=0.65			m_{PS}/m_V=0.80		
β	K	T/T_{pc}	β	K	T/T_{pc}
1.50	0.150290	0.82(3)	1.50	0.143480	0.76(4)
1.60	0.150030	0.86(3)	1.60	0.143749	0.80(4)
1.70	0.148086	0.94(3)	1.70	0.142871	0.84(4)
1.75	0.146763	1.00(4)	1.80	0.141139	0.93(5)
1.80	0.145127	1.07(4)	1.85	0.140070	0.99(5)
1.85	0.143502	1.18(4)	1.90	0.138817	1.08(5)
1.90	0.141849	1.32(5)	1.95	0.137716	1.20(6)
1.95	0.140472	1.48(5)	2.00	0.136931	1.35(7)
2.00	0.139411	1.67(6)	2.10	0.135860	1.69(8)
2.10	0.137833	2.09(7)			

#conf=500-2600

#noise=100-150 for each color and spin indices

$$Z = \int \prod_{x,\mu} dU_\mu(x) (\det M)^{N_f} e^{-S_g},$$

$$S_g = -\beta \left\{ c_0 \sum_{x,\mu>\nu} W_{\mu\nu}^{1\times1}(x) + c_1 \sum_{x,\mu\neq\nu} W_{\mu\nu}^{1\times2}(x) \right\}$$

$$M_{x,y} = \delta_{x,y} - K\sum_{i=1}^{3}\left[(1-\gamma_i)U_i\delta_{x+\hat{i},y} + (1+\gamma_i)U_i^+\delta_{x-\hat{i},y}\right]$$
$$- K\left[e^{\mu_q a}(1-\gamma_4)U_4\delta_{x+\hat{4},y} + e^{-\mu_q a}(1+\gamma_4)U_4^+\delta_{x-\hat{4},y}\right] + \delta_{x,y} c_{SW} K \sum_{\mu<\nu}\sigma_{\mu\nu}F_{\mu\nu}$$

μq: quark chemical potential

$W_{\mu\nu}^{n\times m}$: $n\times m$ wilson loop

Clover term

$\beta = 6/g^2$,

$c_1 = -0.331,\ c_0 = 1-8c_1,$

$c_{SW} = (1-0.8412\beta^{-1})^{-3/4}$

3. Scaling behavior of chiral order parameter at finite μ

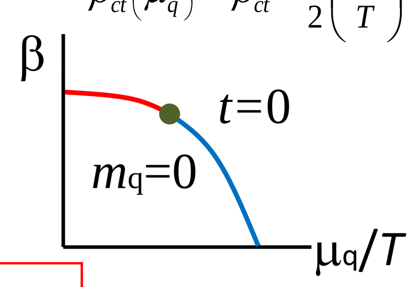
3-1. O(4) scaling test for 2-flavor QCD

3-D O(4) spin model: magnetization (M), external field (h), reduced temperature (t)

2-flavor QCD: chiral order parameter $\langle\bar{\psi}\psi\rangle$, ud-quark mass ($m_q$), critical β ($\beta_{ct}$), ($c$)

$$M = \langle\bar{\psi}\psi\rangle, \qquad h = 2m_q a, \qquad t = \beta - \beta_{ct} + \frac{c}{2}\left(\frac{\mu_q}{T}\right)^2$$

$$\beta_{ct}(\mu_q) = \beta_{ct} - \frac{c}{2}\left(\frac{\mu_q}{T}\right)^2$$



Scaling function: $$M/h^{1/\delta} = f\left(t/h^{1/\beta\delta}\right)$$

The second derivative:

$$\left.\frac{d^2M}{d(\mu_q/T)^2}\right|_{\mu_q=0} = c\left.\frac{dM}{dt}\right|_{\mu_q=0}, \qquad \frac{dM/dt}{h^{1/\delta-1/\beta\delta}} = \left.\frac{df(x)}{dx}\right|_{x=t/h^{1/\beta\delta}}$$

Note: c is the curvature of the critical line in the (β, μ_q/T) plane.

We compare the scaling functions of 2-flavor QCD and O(4) spin model.

3-2. Chiral order parameter defined by Ward-Takahashi identities

Ward-Takahashi identities in the continuum limit

$$\langle\partial_\mu A_\mu(x)P(y)\rangle - 2m_q a\langle P(x)P(y)\rangle = \delta(x-y)\langle\bar{\psi}\psi(x)\rangle \qquad P = \bar{\psi}\gamma_5\psi,\ A_\mu = \bar{\psi}\gamma_\mu\gamma_5\psi$$

$$\Rightarrow \quad 2m_q a = -m_\pi\frac{\langle A_4(t)P(0)\rangle}{\langle P(t)P(0)\rangle} \qquad \langle\bar{\psi}\psi\rangle^{WI} = \frac{2m_q a(2K)^2}{N_s^3 N_t}\sum_{x,y}\langle P(x)P(y)\rangle$$

Because the chiral symmetry is explicitly broken for Wilson quarks, we define $m_q a$ and $\langle\bar{\psi}\psi\rangle^{WI}$ by the Ward-Takahashi identities [Bochichio et al., NPB262, 331 (1985)].

3-3. Reweighting method for the chiral order parameter at finite μ

$$\langle\bar{\psi}\psi\rangle^{WI} = \frac{2m_q a(2K)^2}{N_s^3 N_t}\sum_{x,y}\langle P(x)P(y)\rangle = \frac{2m_q a(2K)^2}{N_s^3 N_t}\left\langle \mathrm{tr}\left(M^{-1}\gamma_5 M^{-1}\gamma_5\right)\right\rangle$$

The expectation value at finite μ is computed by the reweighting method with O(μ³) error.

$$\left\langle \mathrm{tr}\left(M^{-1}\gamma_5 M^{-1}\gamma_5\right)\right\rangle_{(\beta,\mu)} = \frac{1}{Z}\int DU\,\mathrm{tr}\left(M^{-1}\gamma_5 M^{-1}\gamma_5\right)(\det M(\mu))^{N_f} e^{-S_g(\beta)} = \frac{\left\langle \mathrm{tr}\left(M^{-1}\gamma_5 M^{-1}\gamma_5\right)e^{N_f(\ln\det M(\mu)-\ln\det M(0))}\right\rangle_{(\beta_0,0)}}{\left\langle e^{N_f(\ln\det M(\mu)-\ln\det M(0))}\right\rangle_{(\beta_0,0)}}$$

$$\ln\det M(\mu) - \ln\det M(0) = \mu\frac{d\ln\det M}{d\mu} + \frac{\mu^2}{2}\frac{d^2\ln\det M}{d\mu^2} + O(\mu^3)$$

We neglect O(μ³).

$$\mathrm{tr}\left(M^{-1}\gamma_5 M^{-1}\gamma_5\right)(\mu) = \mathrm{tr}\left(M^{-1}\gamma_5 M^{-1}\gamma_5\right)(0) + \mu\frac{d\,\mathrm{tr}\left(M^{-1}\gamma_5 M^{-1}\gamma_5\right)}{d\mu} + \frac{\mu^2}{2}\frac{d^2\mathrm{tr}\left(M^{-1}\gamma_5 M^{-1}\gamma_5\right)}{d\mu^2} + O(\mu^3)$$

$$\frac{\partial\ln\det M}{\partial\mu} = \mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}\right), \quad \frac{\partial^2\ln\det M}{\partial\mu^2} = \mathrm{tr}\left(M^{-1}\frac{\partial^2 M}{\partial\mu^2}\right) - \mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}\right), \quad \frac{\partial\mathrm{tr}\left(M^{-1}\gamma_5 M^{-1}\gamma_5\right)}{\partial\mu^2} = -2\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\gamma_5 M^{-1}\gamma_5\right),$$

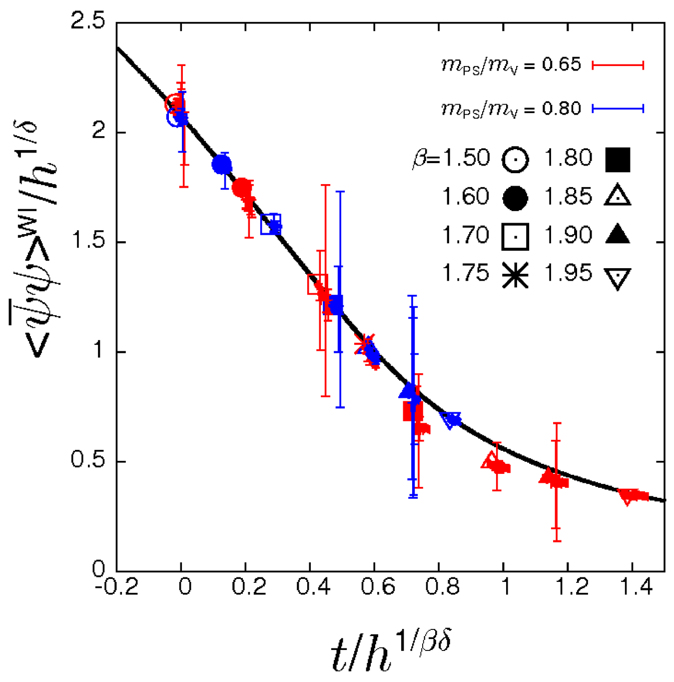
$$\frac{\partial^2\mathrm{tr}\left(M^{-1}\gamma_5 M^{-1}\gamma_5\right)}{\partial\mu^2} = -2\mathrm{tr}\left(M^{-1}\frac{\partial^2 M}{\partial\mu^2}M^{-1}\gamma_5 M^{-1}\gamma_5\right) + 4\mathrm{tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\gamma_5 M^{-1}\gamma_5\right) + 2\mathrm{Tr}\left(M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\gamma_5 M^{-1}\frac{\partial M}{\partial\mu}M^{-1}\gamma_5\right),$$

$$\left.\frac{\partial^n M}{\partial\mu^n}\right|_{x,y} = \begin{cases} -K\left[(1-\gamma_4)U_4\delta_{x+\hat{4},y} - (1+\gamma_4)U_4^+\delta_{x-\hat{4},y}\right] & \text{for } n:\text{odd} \\ -K\left[(1-\gamma_4)U_4\delta_{x+\hat{4},y} + (1+\gamma_4)U_4^+\delta_{x-\hat{4},y}\right] & \text{for } n:\text{even} \end{cases}$$

- These operators can be calculated by the random noise method.

3-4. Scaling plot at finite μ and O(4) scaling function

We fit the data of $\langle\bar{\psi}\psi\rangle^{WI}$ by the O(4) scaling function adjusting the parameter β_{ct} and c in t.



$$M/h^{1/\delta} = f\left(t/h^{1/\beta\delta}\right) \qquad t = \beta - \beta_{ct} + \frac{c}{2}\left(\frac{\mu_q}{T}\right)^2$$

Best values: $\beta_{ct} = 1.510,\quad c = 0.029$

Symbols: results at μ=0

Lines on the right of the symbols: result at μ≠0

We omitted data having large error.

O(4) scaling function: J. Engels et al., Nucl. Phys. B 678 (2003) 533

Consistent with O(4) scaling function.

3-5. Derivatives of the chiral order parameter at μ=0

3-5-1. method 1

We fit the data of $\langle\bar{\psi}\psi\rangle^{WI}$ at finite μ by $\langle\bar{\psi}\psi\rangle^{WI}(\mu) = x + y(\mu/T)^2$, where x and y are the fit parameters.

$$x = \langle\bar{\psi}\psi\rangle^{WI}, \quad y = \frac{1}{2}\frac{\partial^2\langle\bar{\psi}\psi\rangle^{WI}}{\partial(\mu_q/T)^2} \quad \text{at } \mu=0$$

(The first derivative is zero due to the symmetry: μ → −μ.)

3-5-2. method 2

We calculate the expectation value of the following operators and obtain the derivatives.

$$\langle\bar{\psi}\psi\rangle^{WI} = \frac{2ma(2K)^2}{N_s^3 N_t}F_0, \quad \frac{\partial^2\langle\bar{\psi}\psi\rangle^{WI}}{\partial(\mu_q/T)^2} = \frac{2ma(2K)^2}{N_s^3 N_t^3}(F_2 - F_0 A_2)$$

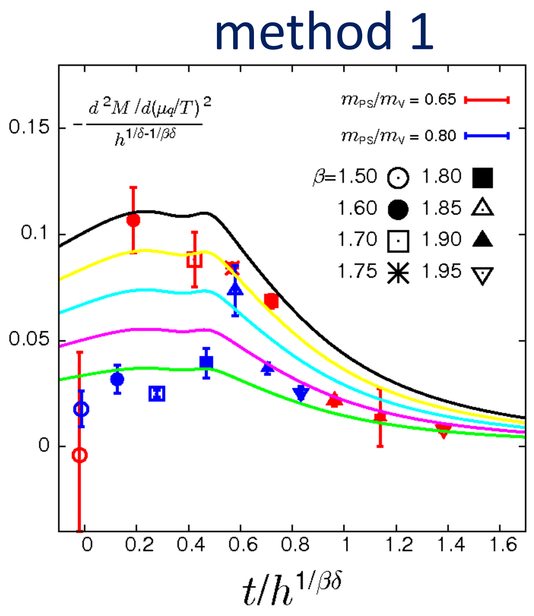
$$F_0 = \left\langle \mathrm{tr}\left(M^{-1}\gamma_5 M^{-1}\gamma_5\right)\right\rangle, \quad A_2 = \left\langle N_f\frac{\partial^2\ln\det M}{\partial\mu^2}\right\rangle + \left\langle\left(N_f\frac{\partial\ln\det M}{\partial\mu}\right)^2\right\rangle,$$

$$F_2 = \left\langle\frac{\partial^2\mathrm{tr}\left(M^{-1}\gamma_5 M^{-1}\gamma_5\right)}{\partial\mu^2}\right\rangle + 2\left\langle\frac{\partial\mathrm{tr}\left(M^{-1}\gamma_5 M^{-1}\gamma_5\right)}{\partial\mu}N_f\frac{\partial\ln\det M}{\partial\mu}\right\rangle + \left\langle\mathrm{tr}\left(M^{-1}\gamma_5 M^{-1}\gamma_5\right)N_f\frac{\partial^2\ln\det M}{\partial\mu^2}\right\rangle + \left\langle\mathrm{tr}\left(M^{-1}\gamma_5 M^{-1}\gamma_5\right)N_f^2\left(\frac{\partial\ln\det M}{\partial\mu}\right)^2\right\rangle \qquad (\mu \equiv \mu_q a = (\mu_q/T)/N_t)$$

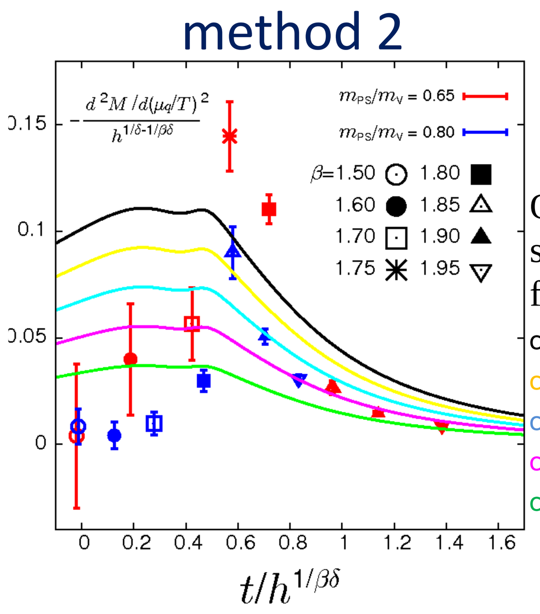
Second derivative of $\langle\bar{\psi}\psi\rangle^{WI}$ for 2-flavor QCD

$$\frac{d^2\langle\bar{\psi}\psi\rangle/d(\mu_q/T)^2}{h^{1/\delta-1/\beta\delta}} = c\left.\frac{df(x)}{dx}\right|_{x=t/h^{1/\beta\delta}}$$

method 1



method 2



O(4) scaling function

c=0.06

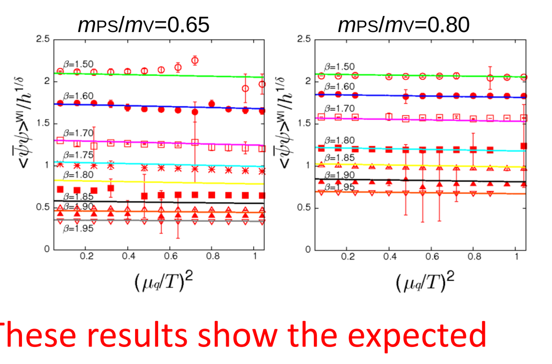
c=0.05

c=0.04

c=0.03

c=0.02

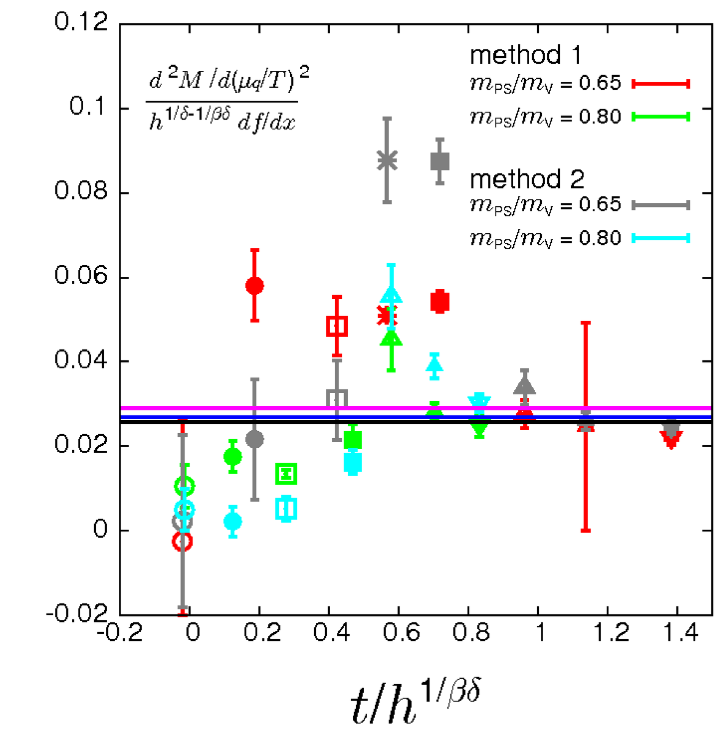
In method 1, we fit the data in μ/T<1.0



These results show the expected scaling behavior although the error is still large.

3-6. Curvature of the critical line in the ciral limit

$$\frac{d^2\beta_{ct}}{d(\mu_q/T)^2} = -c$$



We estimate the curvature c by three method.

1) Global fit in the scaling plot (Sec. 3-4).
   c = 0.0290 (magenta line)
2) From the ratio of the 2nd derivative and df/dx.
   (1-1) Method1: c = 0.0273 (42) (Blue line)
   (1-2) Method2: c = 0.0257 (43) (Black line)

cf) Result by an improved staggered:
Keczmarek et al., Phys. Rev. D 83 (2011) 014504

$$-\frac{1}{2T_c}\frac{d^2T_c}{d(\mu_q/T)^2} = 0.059(2)(4)$$

To calculate the curvature of $T_c(\mu_q)$, we need $a(d\beta/da)$ in the chiral limit. $\frac{1}{T_c}\frac{d^2T_c}{d(\mu_q/T)^2} = -\frac{d^2\beta_{ct}}{d(\mu_q/T)^2}\Big/\frac{a\,d\beta}{da}$

4. Summary

1) We discussed the scaling property of the chiral phase transition in the low density region.
2) We calculated the chiral order parameter and its second derivative performing a simulation of 2-flavor QCD with Wilson quarks and compared the results with an expected scaling function.
3) The scaling behavior is roughly consistent with our expectation.
4) The curvature of the critical line in the (β, μ) plane was estimated.