

EOS in 2+1 flavor QCD with improved Wilson quarks by the fixed scale approach

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QCD Thermodynamics on the lattice

Most studies done with staggered-type quarks

- less computational costs
- a part of chiral sym. preserved ...
 - $N_f=2+1$, almost physical quark mass, $\mu \neq 0$
- 4th-root trick to remove unphysical "tastes"
 - non-locality "universality is not guaranteed"

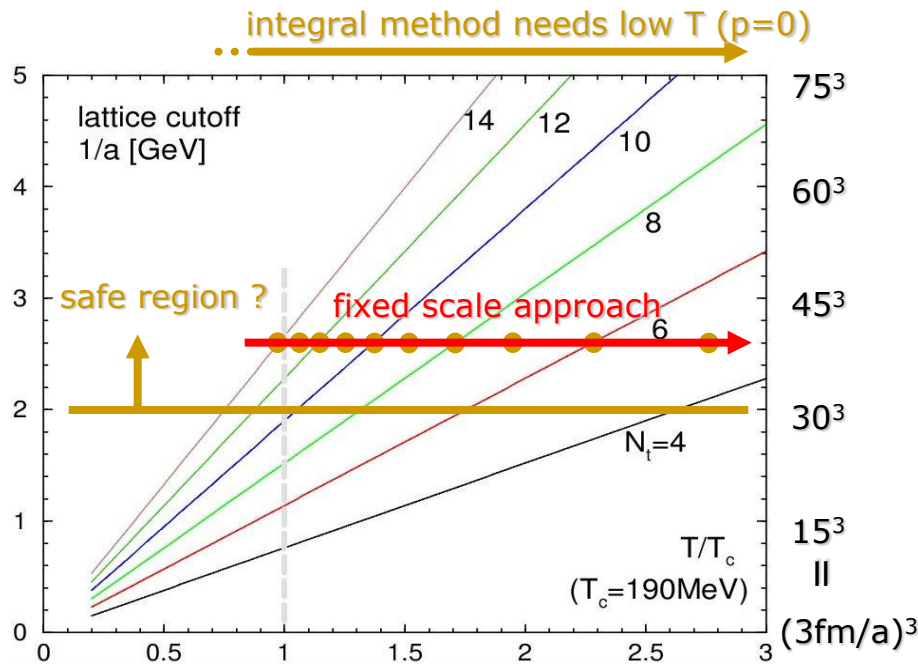
It is important to cross-check with
theoretically sound lattice quarks like Wilson-type quarks

Our aim is to investigate
QCD Thermodynamics with Wilson-type quarks

→ WHOT-QCD Collaboration

Fixed scale approach to study QCD thermodynamics

Temperature $T=1/(N_t a)$ is varied by N_t at fixed a



Advantages

- Line of Constant Physics
- $T=0$ subtraction for renorm. (spectrum study at $T=0$)
- larger $1/a$ in whole T region

Disadvantages

- T resolution by integer N_t
- Statistics in lower T region
- coding for odd N_t

T-integration method to calculate the EOS

We propose a new method (“**T-integration method**”)
to calculate the EOS at fixed scales

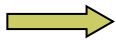
T.Umeda et al. (WHOT-QCD), Phys.Rev.D79 (2009) 051501(R)

Our method is based on **the trace anomaly (interaction measure)**,

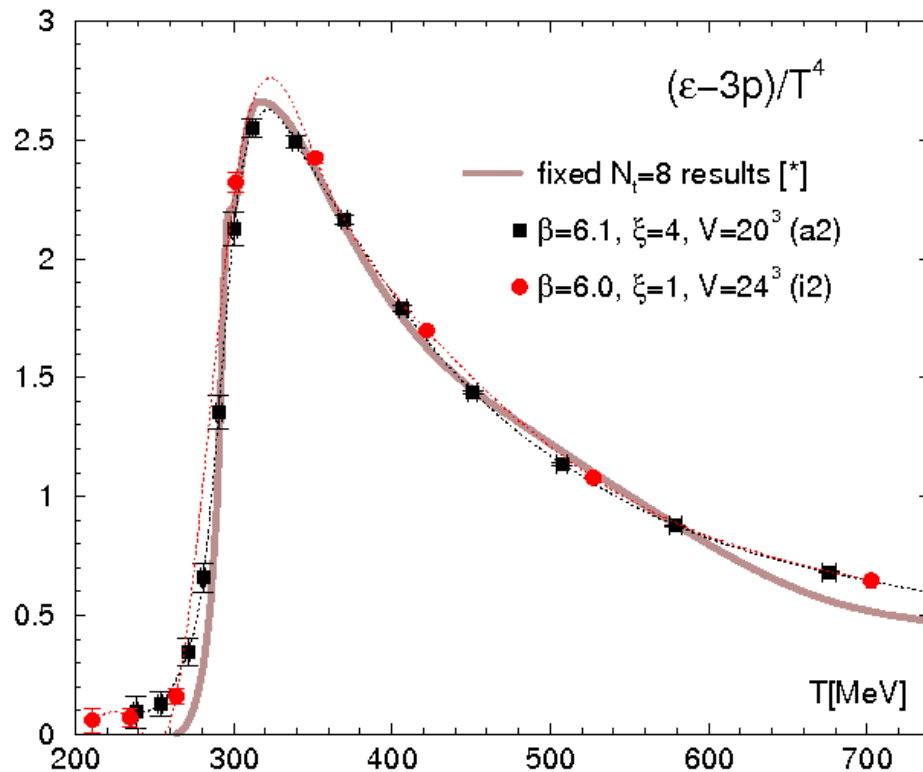
$$\frac{\epsilon - 3p}{T^4} = \left(\frac{N_t^3}{N_s^3} \right) a \frac{d\beta}{da} \left\langle \frac{dS}{d\beta} \right\rangle_{sub}$$

and **the thermodynamic relation**.

$$\frac{\epsilon - 3p}{T^4} = T \frac{\partial(p/T^4)}{\partial T}$$

 $\frac{p}{T^4} = \int_0^T dT' \frac{\epsilon - 3p}{T'^5}$

Test in quenched QCD



[*] G. Boyd et al., NPB469, 419 (1996)

- Our results are roughly consistent with previous results.
- Our results deviate from the fixed $N_t=8$ results [*] at higher T ($aT \sim 0.3$ or higher)
- Trace anomaly is sensitive to spatial volume at lower T (below T_c).
 $V > (2\text{fm})^3$ is necessary.

T=0 & T>0 configurations for $N_f=2+1$ QCD

■ T=0 simulation: on $28^3 \times 56$ by CP-PACS/JLQCD *Phys. Rev. D78 (2008) 011502*

- RG-improved Iwasaki glue + NP-improved Wilson quarks

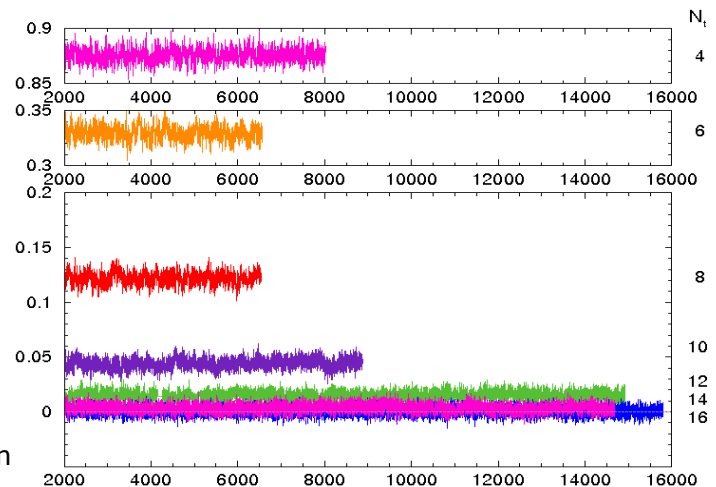
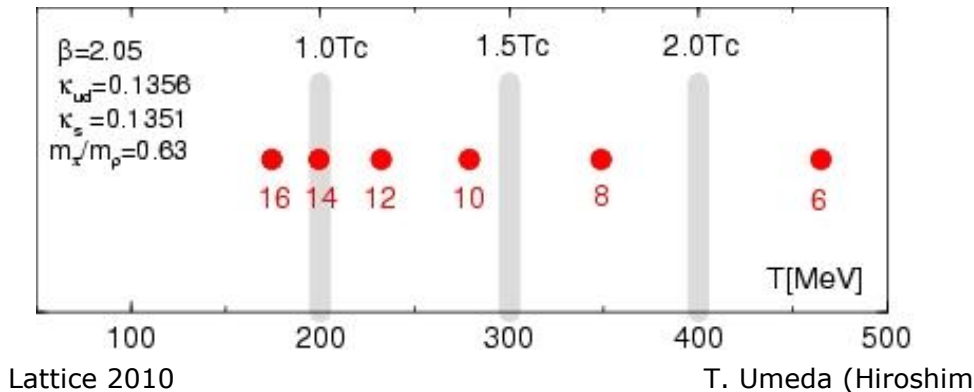
- $\beta=2.05$, $\kappa_{ud}=0.1356$, $\kappa_s=0.1351$

- $V \sim (2 \text{ fm})^3$, $a=0.07 \text{ fm}$, ($m_\pi \sim 634 \text{ MeV}$, $\frac{m_\pi}{m_\rho} = 0.63$, $\frac{m_{\eta's}}{m_\phi} = 0.74$)

- configurations available on the ILDG/JLDG

■ T>0 simulations: on $32^3 \times N_t$ ($N_t=4, 6, \dots, 14, 16$) lattices

RHMC algorithm, same parameters as T=0 simulation



Formulation for $N_f=2+1$ improved Wilson quarks

$$S = S_g + S_q \quad \begin{aligned} S_g &= -\beta \left\{ \sum_{x,\mu>\nu} c_0 W_{\mu\nu}^{1\times 1}(x) + \sum_{x,\mu,\nu} c_1 W_{\mu\nu}^{1\times 2}(x) \right\} \\ S_q &= \sum_{f=u,d,s} \sum_{x,y} \bar{q}_x^f D_{x,y} q_y^f \end{aligned} \quad \beta = \frac{6}{g^2}$$

$$D_{x,y} = \delta_{x,y} - \kappa_f \sum_{\mu} \left\{ (1 - \gamma_{\mu}) U_{x,\mu} \delta_{x+\hat{\mu},y} + (1 + \gamma_{\mu}) U_{x-\hat{\mu},\mu}^{\dagger} \delta_{x-\hat{\mu},y} \right\} - \delta_{x,y} c_{SW} \kappa_f \sum_{\mu>\nu} \sigma_{\mu\nu} F_{\mu\nu}$$

$$c_{SW}(\beta) = 1 + 0.113g^2 + 0.0209g^4 + 0.0049g^6$$

Phys. Rev. D73, 034501
CP-PACS/JLQCD

$$\frac{\epsilon - 3p}{T^4} = \frac{N_t^3}{N_s^3} \left(a \frac{\partial \beta}{\partial a} \left\langle \frac{\partial S}{\partial \beta} \right\rangle_{sub} + a \frac{\partial \kappa_{ud}}{\partial a} \left\langle \frac{\partial S}{\partial \kappa_{ud}} \right\rangle_{sub} + a \frac{\partial \kappa_s}{\partial a} \left\langle \frac{\partial S}{\partial \kappa_s} \right\rangle_{sub} \right)$$

$$\left\langle \frac{\partial S}{\partial \beta} \right\rangle = N_s^3 N_t \left(- \left\langle \sum_{x,\mu>\nu} c_0 W_{\mu\nu}^{1\times 1}(x) + \sum_{x,\mu,\nu} c_1 W_{\mu\nu}^{1\times 2}(x) \right\rangle + N_f \frac{\partial c_{SW}}{\partial \beta} \kappa_f \left\langle \sum_{x,\mu>\nu} \text{Tr}^{(c,s)} \sigma_{\mu\nu} F_{\mu\nu} (D^{-1})_{x,x} \right\rangle \right)$$

$$\begin{aligned} \left\langle \frac{\partial S}{\partial \kappa_f} \right\rangle &= N_f N_s^3 N_t \left(\left\langle \sum_{x,\mu} \text{Tr}^{(c,s)} \left\{ (1 - \gamma_{\mu}) U_{x,\mu} (D^{-1})_{x+\hat{\mu},x} + (1 + \gamma_{\mu}) U_{x-\hat{\mu},\mu}^{\dagger} (D^{-1})_{x-\hat{\mu},x} \right\} \right\rangle \right. \\ &\quad \left. + c_{SW} \left\langle \sum_{x,\mu>\nu} \text{Tr}^{(c,s)} \sigma_{\mu\nu} F_{\mu\nu} (D^{-1})_{x,x} \right\rangle \right) \end{aligned}$$

← Noise method (#noise = 1 for each color & spin indices)

Beta-functions from CP-PACS/JLQCD results

Trace anomaly needs **Beta-functions** in $N_f=2+1$ QCD

$$\frac{\epsilon - 3p}{T^4} = \frac{N_t^3}{N_s^3} \left(a \frac{\partial \beta}{\partial a} \left\langle \frac{\partial S}{\partial \beta} \right\rangle_{sub} + a \frac{\partial \kappa_{ud}}{\partial a} \left\langle \frac{\partial S}{\partial \kappa_{ud}} \right\rangle_{sub} + a \frac{\partial \kappa_s}{\partial a} \left\langle \frac{\partial S}{\partial \kappa_s} \right\rangle_{sub} \right)$$

Direct fit method Phys. Rev. D64 (2001) 074510

fit $\beta, \kappa_{ud}, \kappa_s$ as functions of $(am_\rho), \left(\frac{m_\pi}{m_\rho}\right), \left(\frac{m_{\eta_{ss}}}{m_\phi}\right)$

$$\begin{pmatrix} \beta \\ \kappa_L \\ \kappa_S \end{pmatrix} = \vec{c}_1 + \vec{c}_2(am_\rho) + \vec{c}_3(am_\rho)^2 + \vec{c}_4\left(\frac{m_\pi}{m_\rho}\right) + \vec{c}_5\left(\frac{m_\pi}{m_\rho}\right)^2 + \vec{c}_6(am_\rho)\left(\frac{m_\pi}{m_\rho}\right) \\ + \vec{c}_7\left(\frac{m_{\eta_{ss}}}{m_\phi}\right) + \vec{c}_8\left(\frac{m_{\eta_{ss}}}{m_\phi}\right)^2 + \vec{c}_9(am_\rho)\left(\frac{m_{\eta_{ss}}}{m_\phi}\right) + \vec{c}_{10}\left(\frac{m_\pi}{m_\rho}\right)\left(\frac{m_{\eta_{ss}}}{m_\phi}\right)$$

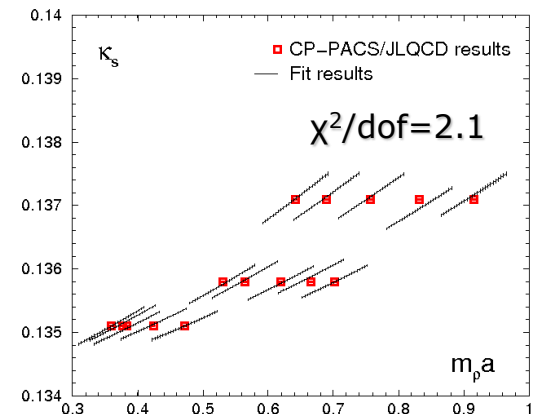
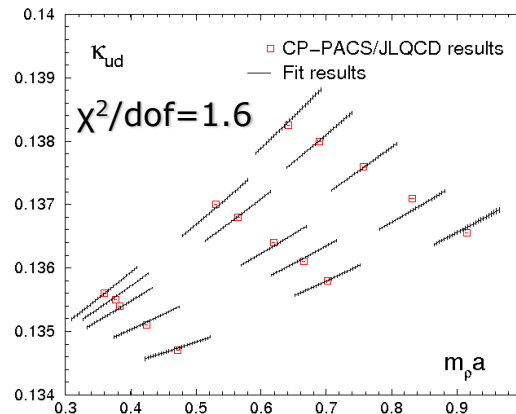
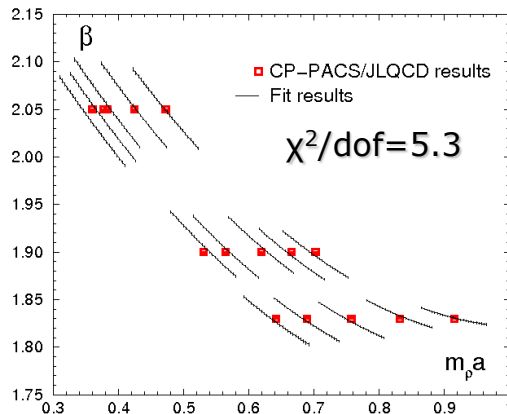
$am_\rho \frac{\partial X}{\partial (am_\rho)}$ with fixed $\left(\frac{m_\pi}{m_\rho}\right), \left(\frac{m_{\eta_{ss}}}{m_\phi}\right)$ ($X = \beta, \kappa_{ud}, \kappa_s$)

Beta-functions from CP-PACS/JLQCD results

Meson spectrum by CP-PACS/JLQCD *Phys. Rev. D78 (2008) 011502*.

3 (β) \times 5 (κ_{ud}) \times 2 (κ_s) = 30 data points

fit $\beta, \kappa_{ud}, \kappa_s$ as functions of $(am_\rho), \left(\frac{m_\pi}{m_\rho}\right), \left(\frac{m_{\eta_{ss}}}{m_\phi}\right)$



$$\left(a \frac{\partial \beta}{\partial a}, a \frac{\partial \kappa_{ud}}{\partial a}, a \frac{\partial \kappa_s}{\partial a} \right) \text{simulation point}$$

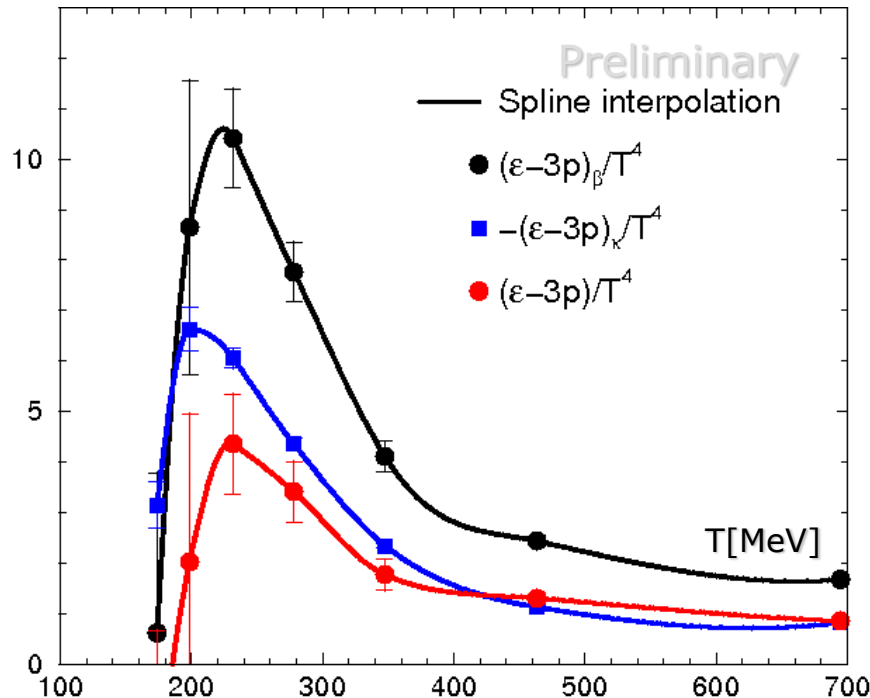
$$= (-0.334(4), 0.00289(6), 0.00203(5))$$

only statistical error

Trace anomaly in $N_f=2+1$ QCD

$$\frac{\epsilon - 3p}{T^4} = \frac{N_t^3}{N_s^3} \left(a \frac{\partial \beta}{\partial a} \left\langle \frac{\partial S}{\partial \beta} \right\rangle_{sub} + a \frac{\partial \kappa_{ud}}{\partial a} \left\langle \frac{\partial S}{\partial \kappa_{ud}} \right\rangle_{sub} + a \frac{\partial \kappa_s}{\partial a} \left\langle \frac{\partial S}{\partial \kappa_s} \right\rangle_{sub} \right) \quad S = S_g + S_q$$

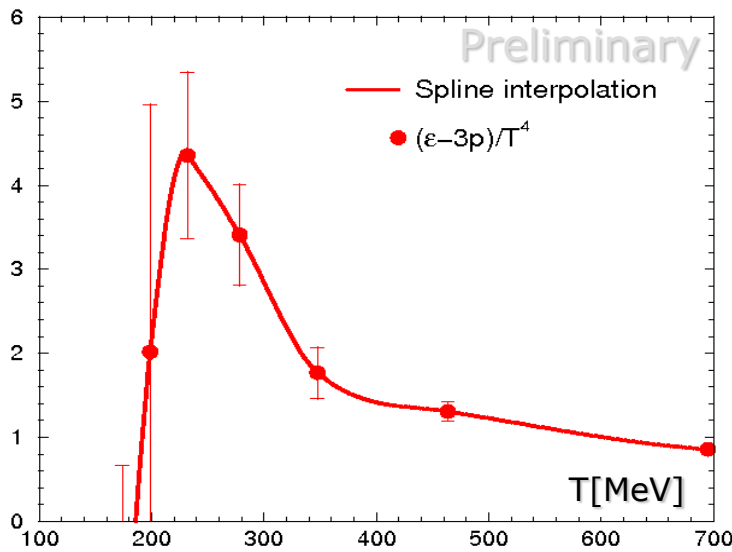
$$\underbrace{\hspace{10em}}_{(\epsilon - 3p)_\beta / T^4} \quad \underbrace{\hspace{10em}}_{(\epsilon - 3p)_\kappa / T^4}$$



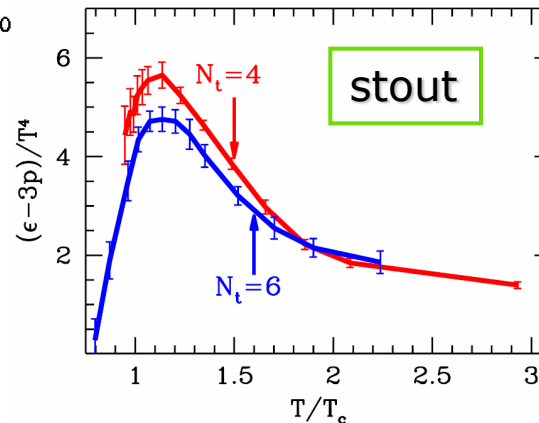
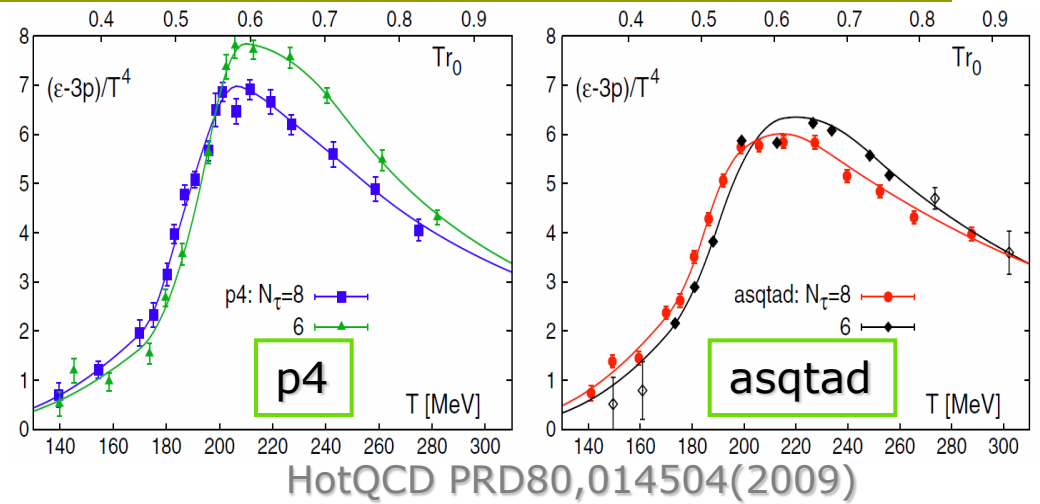
Nt	config. (x 5MD traj.)	
	S_g	S_q
56	1300(*)	143
16	1542	208
14	1448	192
12	1492	336
10	863	240
8	628	320
6	657	160
4	802	143

(*) $T=0$ (Nt=56) by CP-PACS/JLQCD
 S_g calculated with 6500traj.

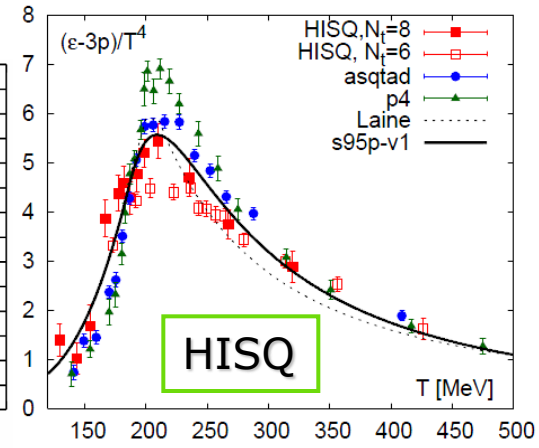
Trace anomaly in $N_f=2+1$ QCD



- peak height $\sim 5-7$ in KS results ($m_q \sim m_q^{\text{phys.}}$)
- peak height \rightarrow small as $N_t \rightarrow$ large

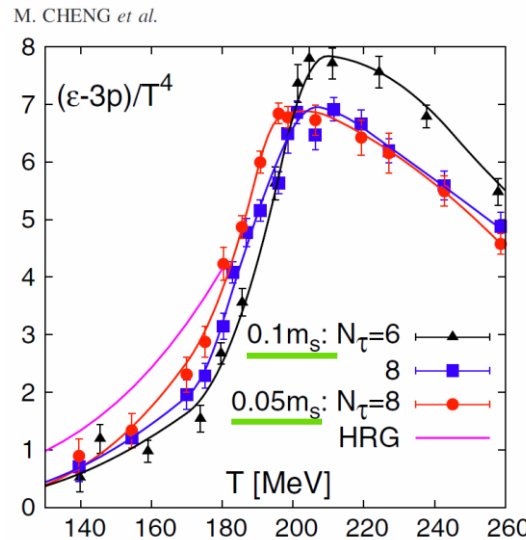
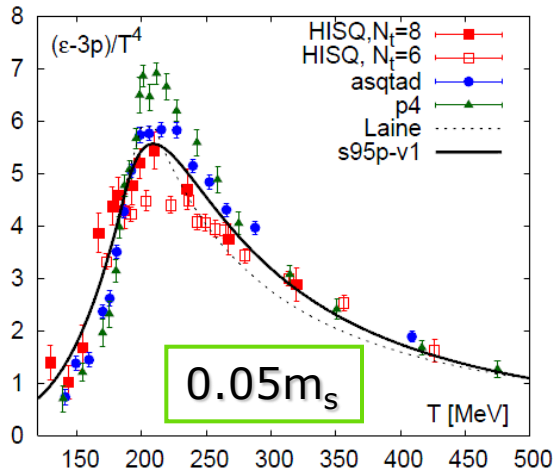


Aoki et al. JHEP01,089(2006)
T. Umeda (Hiroshima)

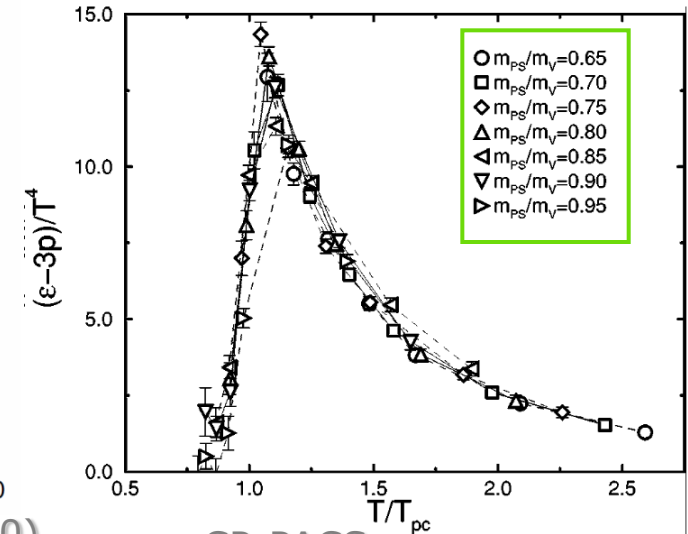


HotQCD arXiv1005.1131
11 / 14

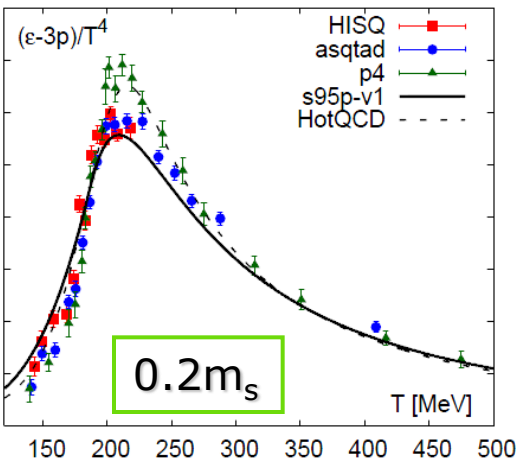
Quark mass dependence of Trace anomaly



HotQCD PRD91,054504(2010)



CP-PACS
PRD64,074510 (2001)

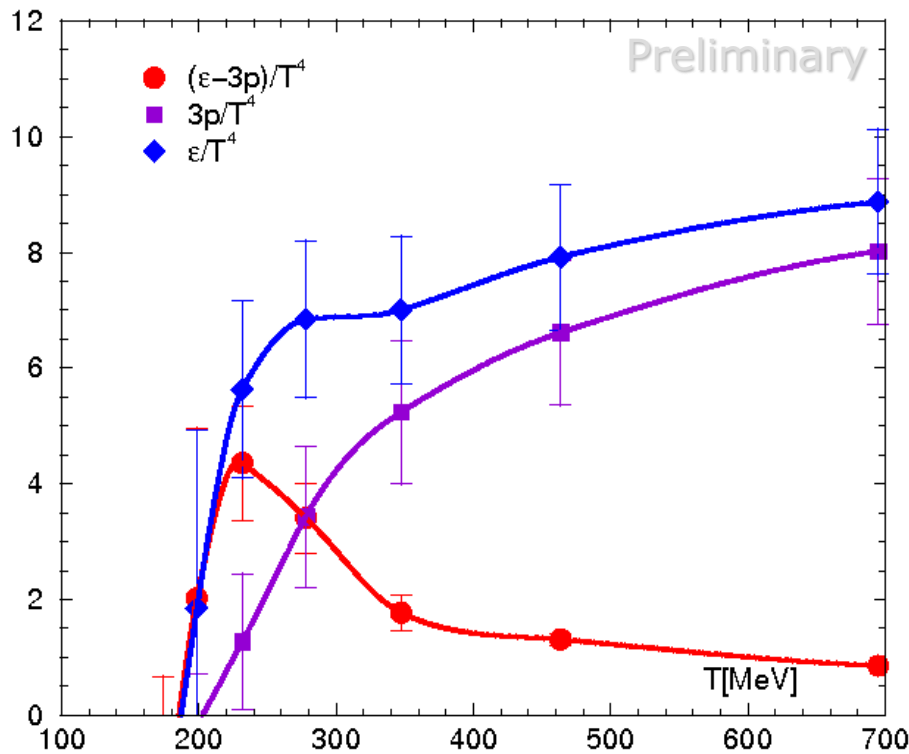


HotQCD arXiv1005.1131

Lattice 2010

- peak height of the Trace anomaly
→ small quark mass dependence
- peak position shifts slightly.
- Our result seems to be reasonable !

Equation of State in Nf=2+1 QCD



■ T-integration

$$\frac{p}{T^4} = \int_0^T dT' \frac{\epsilon - 3p}{T'^5}$$

is performed by the trapezoidal rule.

■ ϵ/T^4 is calculated from

$$\frac{\epsilon - 3p}{T^4} + \frac{3p}{T^4}$$

■ Large error in whole T region

Summary & outlook

We presented the EOS in $N_f=2+1$ QCD using improve Wilson quarks

- Beta functions

 - More work needed

 - Reweighting method ?

- Equation of state

 - More statistics needed in the lower temperature region

- $N_f=2+1$ QCD just at the physical point

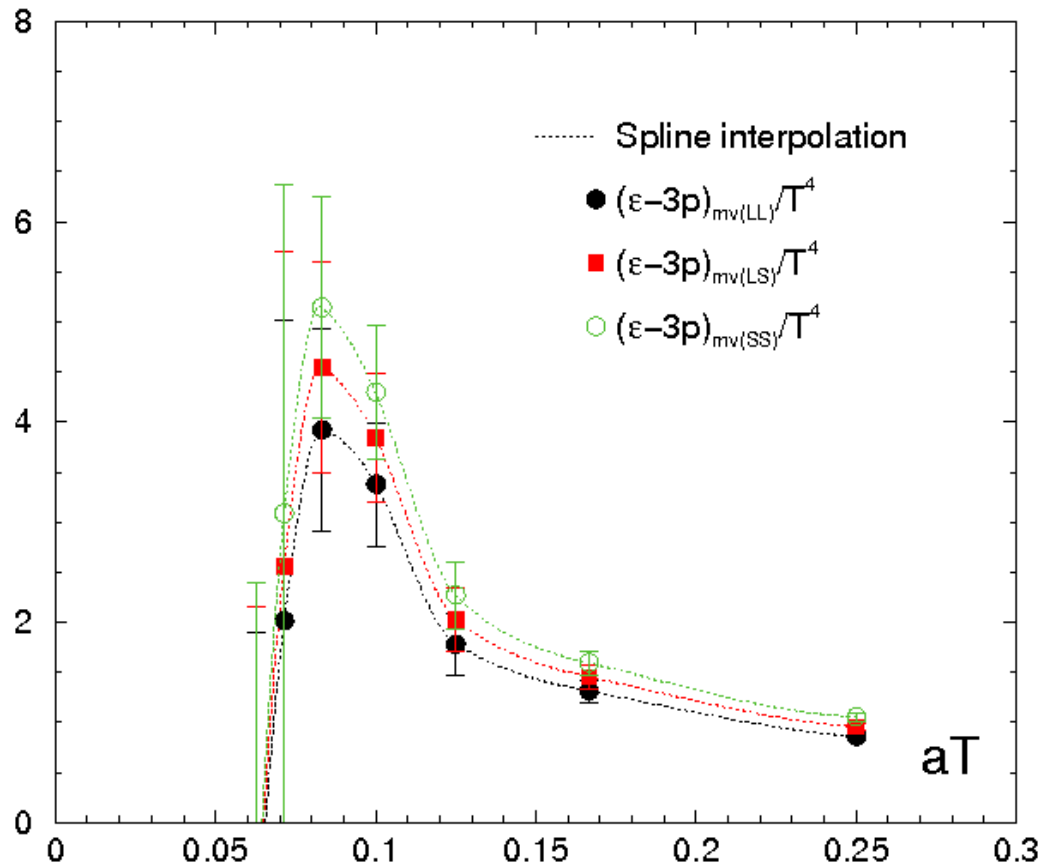
 - the physical point (pion mass $\sim 140\text{MeV}$) by PACS-CS

- Finite density

 - We can combine our approach with the Taylor expansion method, to explore EOS at $\mu \neq 0$

Back-up slides

A systematic error



To calc. Beta-functions

fit $\beta, \kappa_{ud}, \kappa_s$ as functions of

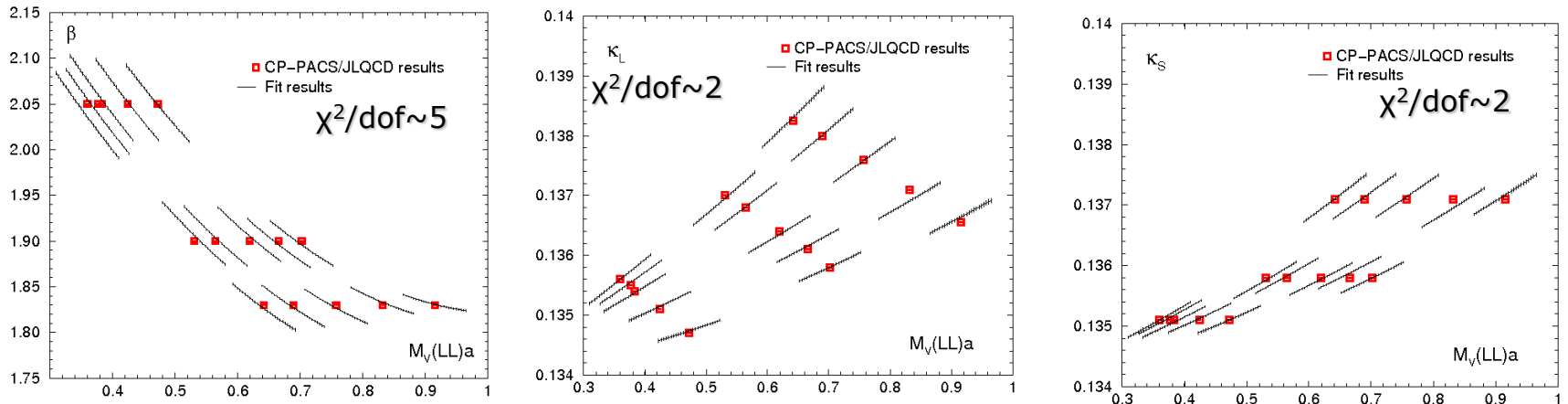
$$(am_\rho), \left(\frac{m_\pi}{m_\rho}\right), \left(\frac{m_{\eta_{ss}}}{m_\phi}\right) \Rightarrow am_\rho \frac{\partial X}{\partial(am_\rho)}$$

$$(am_{K^*}), \left(\frac{m_\pi}{m_\rho}\right), \left(\frac{m_{\eta_{ss}}}{m_\phi}\right) \Rightarrow am_{K^*} \frac{\partial X}{\partial(am_{K^*})}$$

$$(am_\phi), \left(\frac{m_\pi}{m_\rho}\right), \left(\frac{m_{\eta_{ss}}}{m_\phi}\right) \Rightarrow am_\phi \frac{\partial X}{\partial(am_\phi)}$$

Beta-functions from CP-PACS/JLQCD results

Meson spectrum by CP-PACS/JLQCD *Phys. Rev. D78 (2008) 011502.*



fit $\beta, \kappa_{ud}, \kappa_S$ as functions of $(am_\rho), \left(\frac{m_\pi}{m_\rho}\right), \left(\frac{m_{\eta_{SS}}}{m_\phi}\right)$

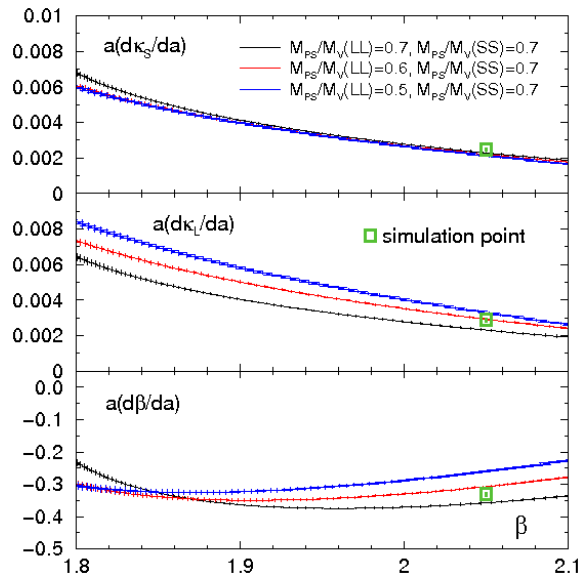
$\left(a \frac{\partial \beta}{\partial a}, a \frac{\partial \kappa_{ud}}{\partial a}, a \frac{\partial \kappa_S}{\partial a}\right)$ simulation point

$$= (-0.330(3), 0.00288(5), 0.00247(5)) \quad m_\rho$$

$$= (-0.340(3), 0.00286(5), 0.00242(5)) \quad m_{K^*}$$

$$= (-0.345(3), 0.00285(5), 0.00242(5)) \quad m_\phi$$

Beta-functions from CP-PACS/JLQCD results



$$\left(a \frac{\partial \beta}{\partial a}, a \frac{\partial \kappa_{ud}}{\partial a}, a \frac{\partial \kappa_s}{\partial a} \right) \text{simulation point}$$

$$= (-0.330(3), 0.00288(5), 0.00247(5)) \quad m_\rho$$

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$$= (-0.345(3), 0.00285(5), 0.00242(5)) \quad m_\phi$$

Trace anomaly (β & κ derivative part)

$$\frac{\epsilon - 3p}{T^4} = \frac{N_t^3}{N_s^3} \left(a \frac{\partial \beta}{\partial a} \left\langle \frac{\partial S}{\partial \beta} \right\rangle_{sub} + a \frac{\partial \kappa_{ud}}{\partial a} \left\langle \frac{\partial S}{\partial \kappa_{ud}} \right\rangle_{sub} + a \frac{\partial \kappa_s}{\partial a} \left\langle \frac{\partial S}{\partial \kappa_s} \right\rangle_{sub} \right)$$

$$(\epsilon - 3p)_\beta / T^4 = (\epsilon - 3p)_{glue} / T^4 + (\epsilon - 3p)_{clover} / T^4$$

$$(\epsilon - 3p)_\kappa / T^4 = (\epsilon - 3p)_{\kappa_{ud}} / T^4 + (\epsilon - 3p)_{\kappa_s} / T^4$$

