Thermodynamics in 2+1 flavor QCD with improved Wilson quarks by the fixed scale approach

Takashi Umeda (Hiroshima Univ.)
for WHOT-QCD Collaboration

HIROSHIMA UNIVERSITY

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QCD Thermodynamics with Wilson quarks

Most (T, $\mu \neq 0$) studies done with staggerd-type quarks

4th-root trick to remove unphysical "tastes"

→ non-locality "Validity is not guaranteed"

It is important to cross-check with

theoretically sound lattice quarks like Wilson-type quarks

Aim of WHOT-QCD collaboration is

finite T & μ calculations using Wilson-type quarks

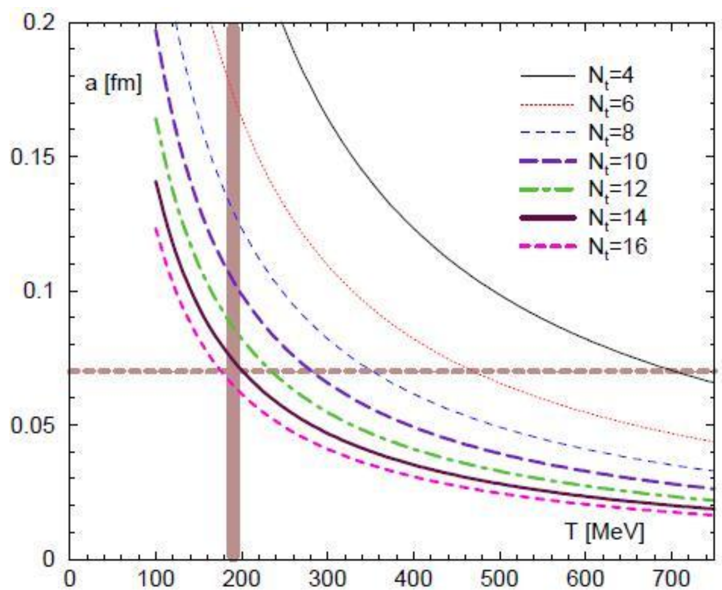
- Phys. Rev. D85 094508 (2012) [arXiv:1202.4719]
  T. Umeda et al. (WHOT-QCD Collaboration)
- PTEP in press [arXiv: 1205.5347 (hep-lat)]
  S. Ejiri, K. Kanaya, T. Umeda for WHOT-QCD Collaboration

Fixed scale approach to study QCD thermodynamics

Temperature $T=1/(N_t a)$ is varied by N_t at fixed a

a : lattice spacing
N_t : lattice size in temporal direction

lattice spacing at fixed N_t



← fixed scale

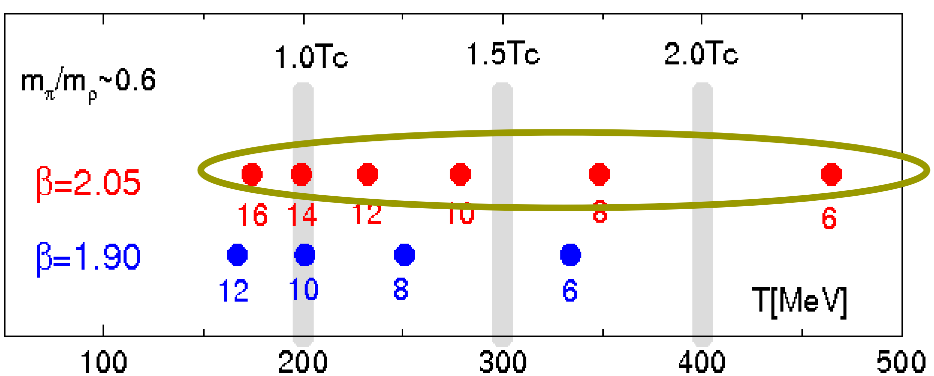
- **Advantages**
  - Line of Constant Physics
  - T=0 subtraction for renorm. (spectrum study at T=0)
  - Lattice spacing at lower T
  - Finite volume effects
- **Disadvantages**
  - T resolution due to integer N_t
  - UV cutoff eff. at high T's

Lattice setup

- T=0 simulation: on $28^3 \times 56$ by CP-PACS/JLQCD *Phys. Rev. D78 (2008) 011502*
  - RG-improved Iwasaki glue + NP-improved Wilson quarks
  - β=2.05, κ_{ud}=0.1356, κ_s=0.1351
  - V~(2 fm)^3 , a~0.07 fm, $\left(m_\pi \sim 636\mathrm{MeV},\ \frac{m_\pi}{m_\rho} \sim 0.63,\ \frac{m_{\eta_{ss}}}{m_\phi} \sim 0.74 \right)$
  - configurations available on the ILDG/JLDG

- T>0 simulations: on $32^3 \times N_t$ (N_t=4, 6, ..., 14, 16) lattices

  RHMC algorithm, same parameters as T=0 simulation



Trace anomaly for N_f=2+1 improved Wilson quarks

$$S = S_g + S_q$$

$$S_g = -\beta \left\{ \sum_{x,\mu>\nu} c_0 W^{1\times1}_{\mu\nu}(x) + \sum_{x,\mu,\nu} c_1 W^{1\times2}_{\mu\nu}(x) \right\} \qquad \beta = \frac{6}{g^2}$$

$$S_q = \sum_{f=u,d,s} \sum_{x,y} \bar{q}^f_x D_{x,y} q^f_y$$

$$D_{x,y} = \delta_{x,y} - \kappa_f \sum_\mu \{(1-\gamma_\mu)U_{x,\mu}\delta_{x+\hat{\mu},y} + (1+\gamma_\mu)U^\dagger_{x-\hat{\mu},\mu}\delta_{x-\hat{\mu},y}\} - \delta_{x,y} c_{SW} \kappa_f \sum_{\mu>\nu} \sigma_{\mu\nu}F_{\mu\nu}$$

$$c_{SW}(\beta) = 1 + 0.113g^2 + 0.0209g^4 + 0.0049g^6$$

Phys. Rev. D73, 034501
CP-PACS/JLQCD

$$\frac{\epsilon - 3p}{T^4} = \frac{N_t^3}{N_s^3}\left(a\frac{\partial\beta}{\partial a}\left\langle \frac{\partial S}{\partial\beta} \right\rangle_{sub} + a\frac{\partial\kappa_{ud}}{\partial a}\left\langle \frac{\partial S}{\partial\kappa_{ud}} \right\rangle_{sub} + a\frac{\partial\kappa_s}{\partial a}\left\langle \frac{\partial S}{\partial\kappa_s} \right\rangle_{sub} \right)$$

$$\left\langle \frac{\partial S}{\partial\beta} \right\rangle = N_s^3 N_t \left(-\left\langle \sum_{x,\mu>\nu} c_0 W^{1\times1}_{\mu\nu}(x) + \sum_{x,\mu,\nu} c_1 W^{1\times2}_{\mu\nu}(x) \right\rangle + N_f \frac{\partial c_{SW}}{\partial\beta}\kappa_f \left\langle \sum_{x,\mu>\nu} \mathrm{Tr}^{(c,s)} \sigma_{\mu\nu}F_{\mu\nu}(D^{-1})_{x,x} \right\rangle \right)$$

$$\left\langle \frac{\partial S}{\partial\kappa_f} \right\rangle = N_f N_s^3 N_t \left(\left\langle \sum_{x,\mu} \mathrm{Tr}^{(c,s)}\{(1-\gamma_\mu)U_{x,\mu}(D^{-1})_{x+\hat{\mu},x} + (1+\gamma_\mu)U^\dagger_{x-\hat{\mu},\mu}(D^{-1})_{x-\hat{\mu},x}\} \right\rangle + c_{SW}\left\langle \sum_{x,\mu>\nu} \mathrm{Tr}^{(c,s)}\sigma_{\mu\nu}F_{\mu\nu}(D^{-1})_{x,x} \right\rangle \right)$$

Noise method (#noise = 1 for each color & spin indices)

Beta-functions from CP-PACS/JLQCD results

Meson spectrum by CP-PACS/JLQCD *Phys. Rev. D78 (2008) 011502.*

5 κ_{ud} x 2 κ_s for each 3 β = 30 data points

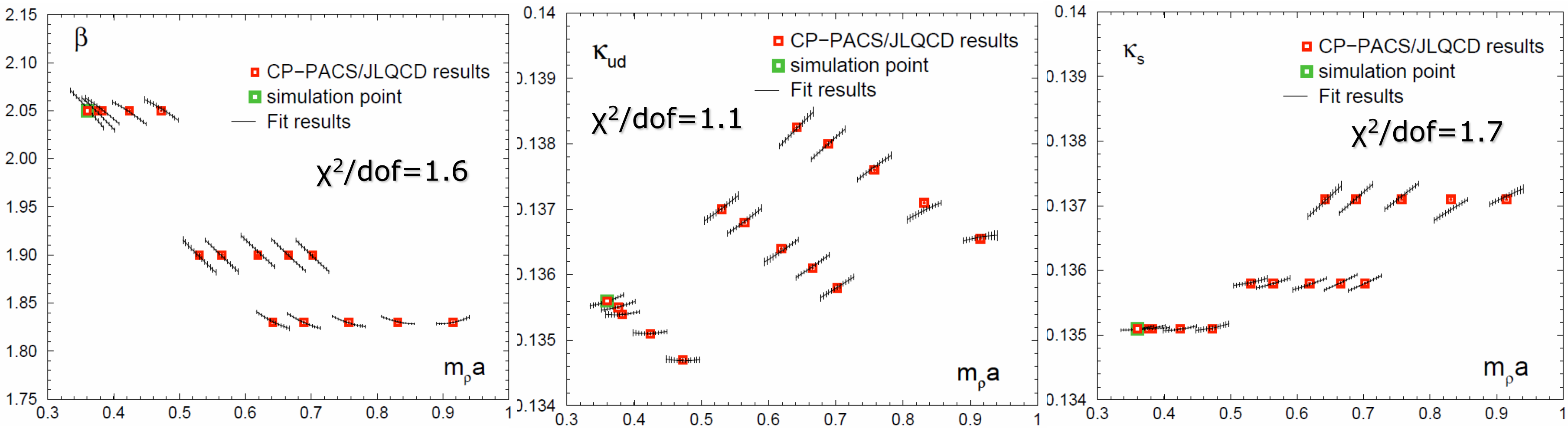
Direct fit method

fit $\beta, \kappa_{ud}, \kappa_s$ as functions of scale (am_ρ), LCP $\left(\frac{m_\pi}{m_\rho}\right), \left(\frac{m_{\eta ss}}{m_\phi}\right)$

$$\begin{pmatrix} \beta \\ \kappa_{ud} \\ \kappa_s \end{pmatrix} = \vec{c}_0 + \vec{c}_1 (am_\rho) + \vec{c}_2 (am_\rho)^2 + \vec{c}_3 \left(\frac{m_\pi}{m_\rho}\right) + \vec{c}_4 \left(\frac{m_\pi}{m_\rho}\right)^2 + \vec{c}_5 (am_\rho) \left(\frac{m_\pi}{m_\rho}\right)$$
$$+ \vec{c}_6 \left(\frac{m_{\eta_{ss}}}{m_\phi}\right) + \vec{c}_7 \left(\frac{m_{\eta_{ss}}}{m_\phi}\right)^2 + \vec{c}_8 (am_\rho) \left(\frac{m_{\eta_{ss}}}{m_\phi}\right) + \vec{c}_9 \left(\frac{m_\pi}{m_\rho}\right) \left(\frac{m_{\eta_{ss}}}{m_\phi}\right)$$
$$+ \vec{c}_{10} (am_\rho)^3 + \vec{c}_{11} \left(\frac{m_\pi}{m_\rho}\right)^3 + \vec{c}_{12} \left(\frac{m_{\eta_{ss}}}{m_\phi}\right)^3 + \vec{c}_{13} (am_\rho) \left(\frac{m_\pi}{m_\rho}\right)^2$$
$$+ \vec{c}_{14} (am_\rho)^2 \left(\frac{m_\pi}{m_\rho}\right) + \vec{c}_{15} (am_\rho) \left(\frac{m_{\eta_{ss}}}{m_\phi}\right)^2 + \vec{c}_{16} (am_\rho)^2 \left(\frac{m_{\eta_{ss}}}{m_\phi}\right)$$
$$+ \vec{c}_{17} \left(\frac{m_\pi}{m_\rho}\right) \left(\frac{m_{\eta_{ss}}}{m_\phi}\right)^2 + \vec{c}_{18} \left(\frac{m_\pi}{m_\rho}\right)^2 \left(\frac{m_{\eta_{ss}}}{m_\phi}\right) + \vec{c}_{19} (am_\rho) \left(\frac{m_\pi}{m_\rho}\right) \left(\frac{m_{\eta_{ss}}}{m_\phi}\right).$$

Beta-functions from CP-PACS/JLQCD results

fit β,κ_{ud},κ_s as functions of $(am_\rho), \left(\frac{m_\pi}{m_\rho}\right), \left(\frac{m_{\eta_{ss}}}{m_\phi}\right)$

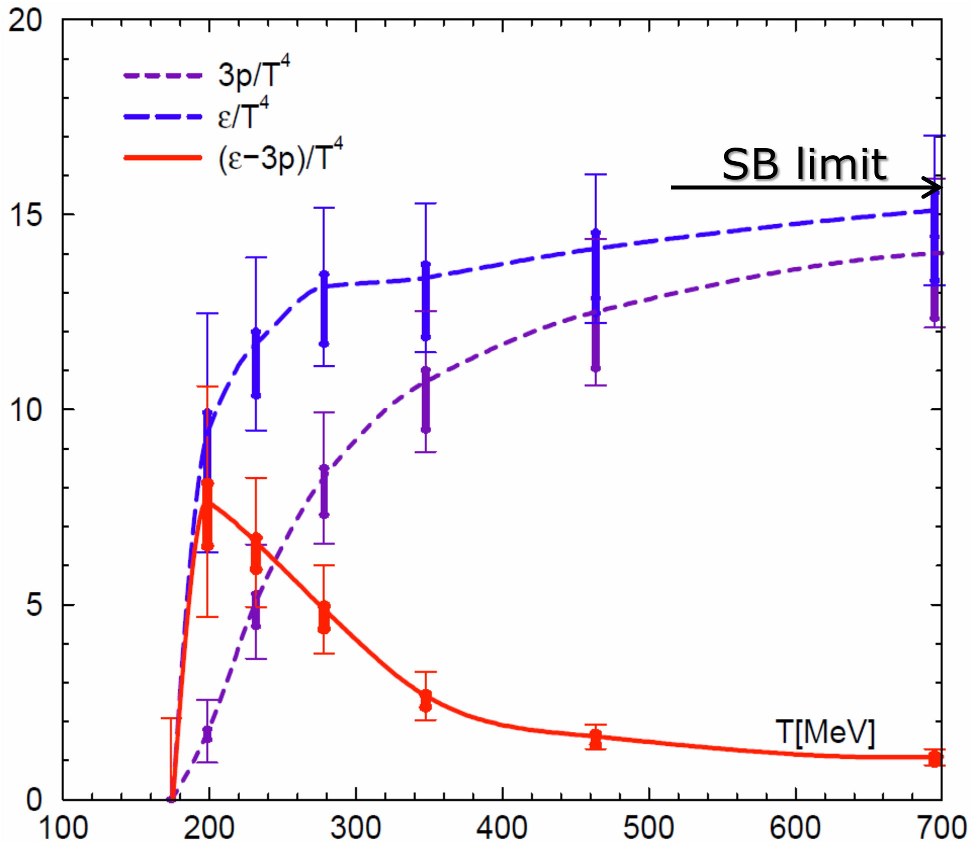


$(X = \beta, \kappa_{ud}, \kappa_s)$ $\quad a\frac{\partial X}{\partial a} = (m_\rho a)\frac{\partial X}{\partial (m_\rho a)}$ with fixed $\left(\frac{m_\pi}{m_\rho}\right), \left(\frac{m_{\eta_{ss}}}{m_\phi}\right)$

$$\left(a\frac{\partial \beta}{\partial a},\ a\frac{\partial \kappa_{ud}}{\partial a},\ a\frac{\partial \kappa_s}{\partial a}\right)_{\text{simulation point}} = \left(-0.279(24)(^{+40}_{-64}), 0.00123(41)(^{+56}_{-68}), 0.00046(26)(^{+42}_{-44})\right)$$

systematic error → $(am_\rho), (am_\pi), (am_K), (am_{K^*})$ for scale dependence

Equation of State in N_f=2+1 QCD



- T-integration

$$\frac{p}{T^4} = \int_0^T dT' \frac{\epsilon - 3p}{T'^5}$$

  is performed by Akima Spline interpolation.

- ε/T^4 is calculated from

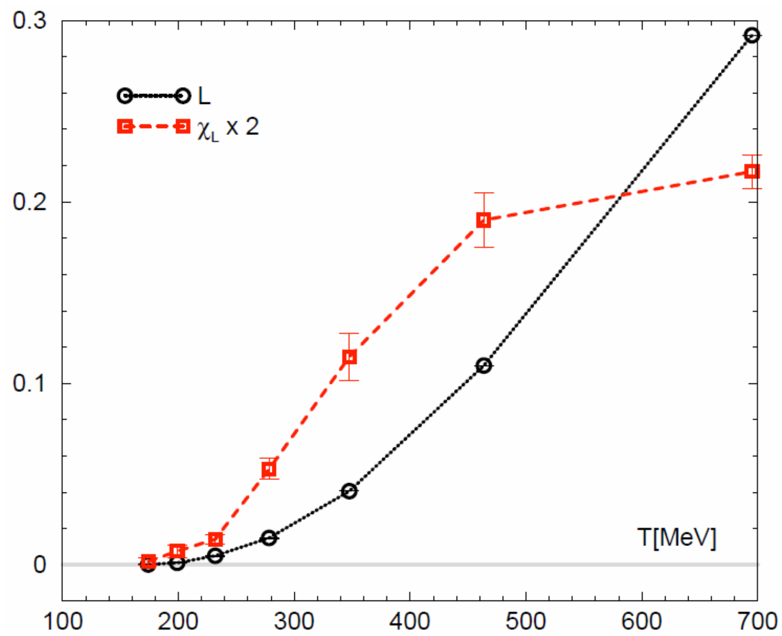
$$\frac{\epsilon - 3p}{T^4} + \frac{3p}{T^4}$$

- A systematic error for beta-functions

Polyakov loop in the fixed scale approach

$$L = \frac{1}{V} \sum_{\vec{x}} \prod_{t=1}^{N_t} U_0(\vec{x}, t)$$

$$\chi_L = V \left(\langle L^2 \rangle - \langle L \rangle^2 \right)$$



- Polyakov loop requires
  T dependent renormalization

$$L_{\rm ren} = \exp\left(-\frac{F_{\bar{q}q}(r=\infty, T)}{2T} \right) = \exp\left(-\frac{c(T)}{2} \right) \langle L \rangle$$

c(T) : additive normalization factor
of heavy quark free energy F_{qq}

- matching of V(r) to $V_{string}(r)$ at $r=1.5r_0$
  Cheng et al. PRD77(2008)014511

$$V_{\rm string}(r) = -\frac{\pi}{12} + \sigma r + c_{\rm m}$$

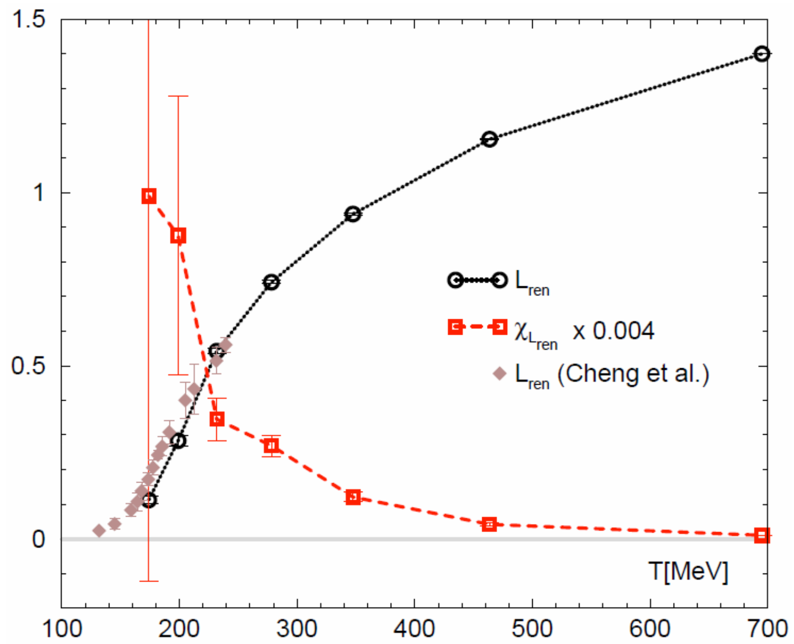
$$L_{\rm ren} = \exp\left(\frac{c_{\rm m} N_t}{2} \right) \langle L \rangle$$

(c_m is a constant at all temperatures)

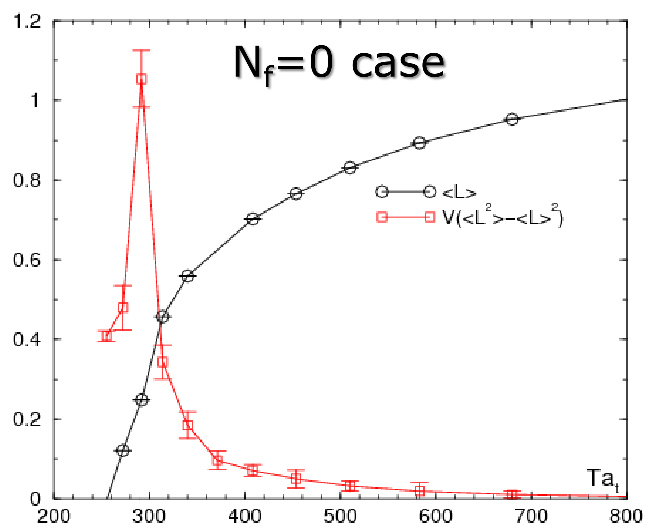
Renormalized Polyakov loop and Susceptibility

$$L_{\rm ren} = \exp\left(\frac{c_{\rm m} N_t}{2}\right)\langle L\rangle$$

(c_m is a constant at all temperatures)



- Roughly consistent with the Staggered result
- χ_L increases around T~200MeV
- Similar behavior to N_f=0 case



Chiral condensate in the fixed scale approach

Chiral condensate by the Wilson quarks requires
additive & multiplicative renormalizations

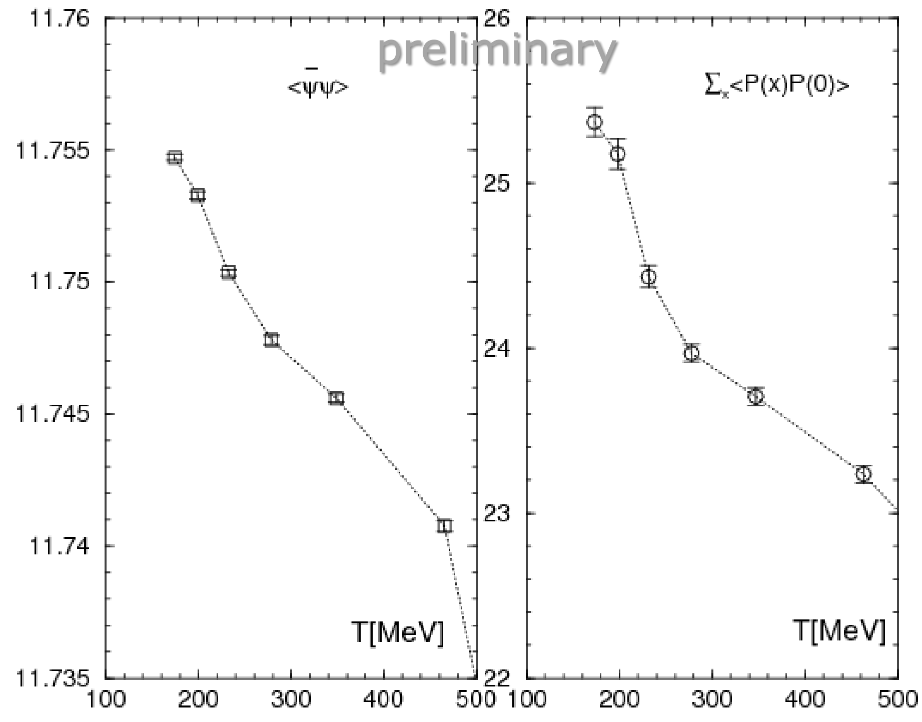
$$\langle\bar{\psi}\psi\rangle(T) = \frac{T}{V}\langle TrD^{-1}\rangle = \frac{\sigma_R(T)}{Z_{\bar{\psi}\psi}} + c_{\bar{\psi}\psi}$$

$$\left\langle \sum_x P(x)P(0) \right\rangle (T) = \frac{\sigma_R(T)}{Z_{PP}} + c_{PP}$$

Renormalization factors

$Z_{\bar{\psi}\psi}, c_{\bar{\psi}\psi}, Z_{PP}, c_{PP}$

are constants at all temperatures



Chiral susceptibility in the fixed scale approach

$$\chi_x = \left\langle \left(\frac{\sigma_R(T)}{Z_X} + c_X \right)^2 \right\rangle - \left\langle \frac{\sigma_R(T)}{Z_X} + c_X \right\rangle^2$$

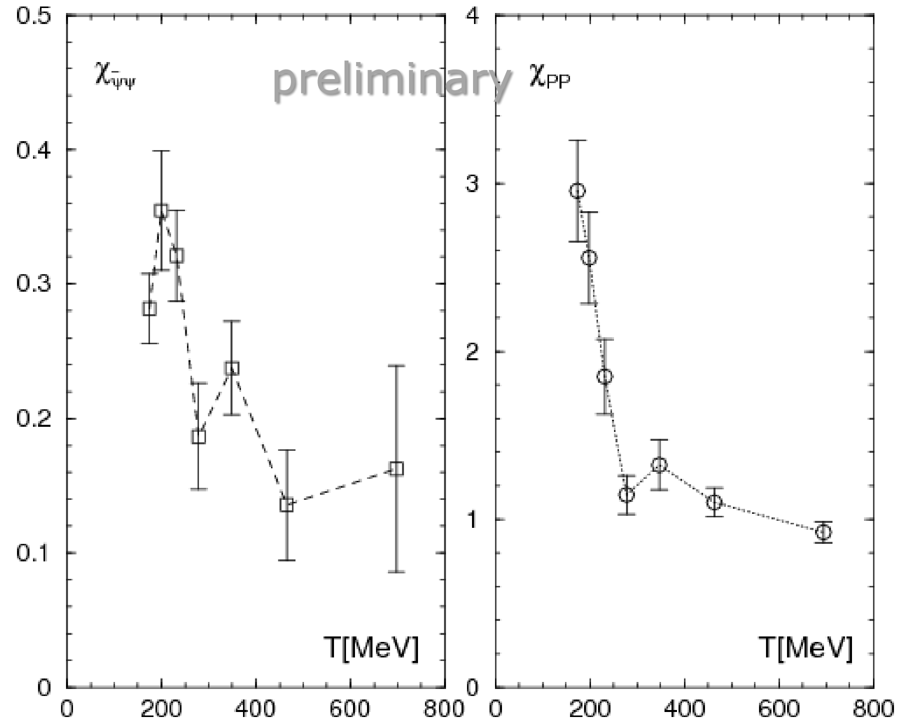
$$= \frac{1}{Z_X^2} \left(\langle \sigma_R^2(T) \rangle - \langle \sigma_R(T) \rangle^2 \right)$$

$(X = \bar{\psi}\psi \;\; \text{or} \;\; PP)$

Renormalization factors $Z_{\bar{\psi}\psi}, Z_{PP}$
are constants at all temperatures

Peak positions in $\chi_{\bar{\psi}\psi}, \chi_{PP}$ are
identical to that in renormalized suscep.

susceptibility peak around 200 MeV (?)
→ more statistics is needed



Summary & outlook

We presented the EOS, Polyakov loop, chiral condensate
in N_f=2+1 QCD using improved Wilson quarks

- Equation of state
  first result in N_f=2+1 QCD with Wilson-type quarks

  Thanks to the fixed scale approach, systematic errors
  coming from lattice artifacts are well under control

- Renormalizations are common at all temperatures

- N_f=2+1 QCD just at the physical point
  the physical point (pion mass ~ 140MeV) by PACS-CS

- Finite density
  Taylor expansion method to explore EOS at $\mu \neq 0$

Renormalized Polyakov loop and Susceptibility

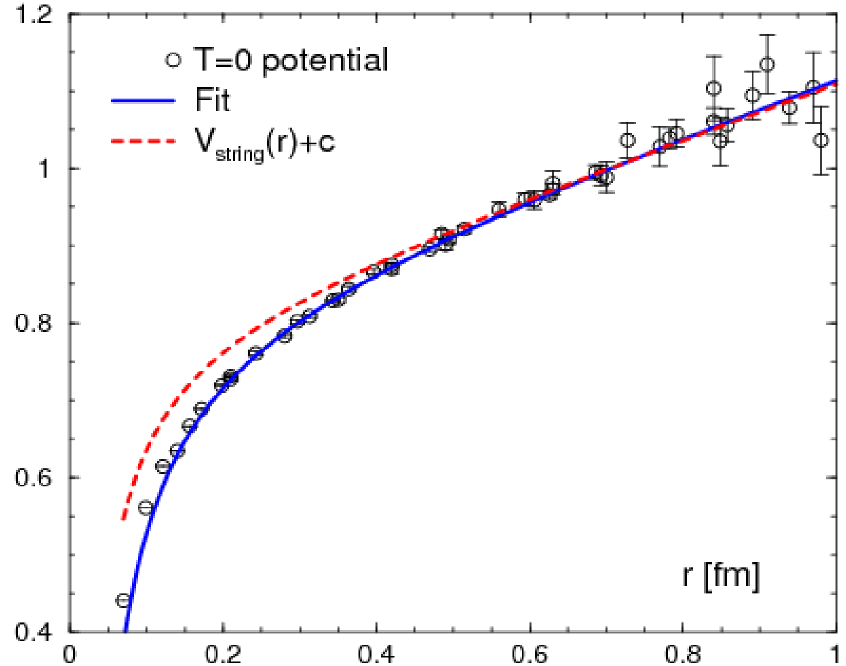
Cheng et al.'s renormalization
[PRD77(2008)014511]

matching of V(r) to $V_{string}(r)$ at $r=1.5r_0$

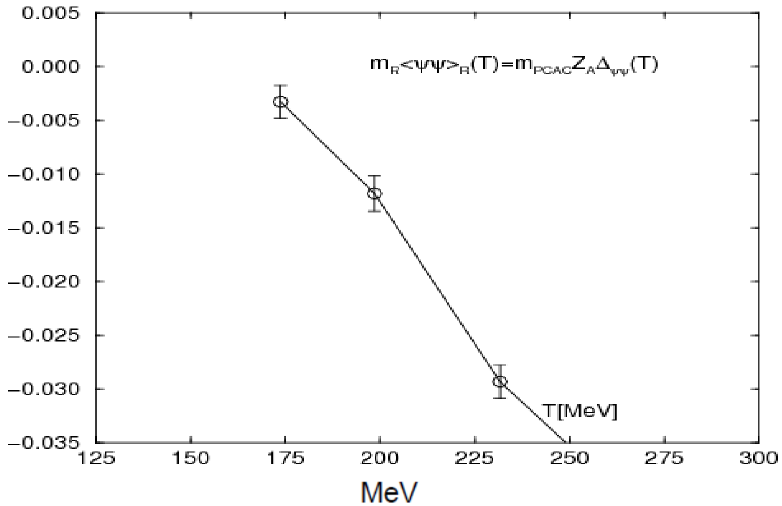
$$V(r) = A - \alpha/r + \sigma r$$

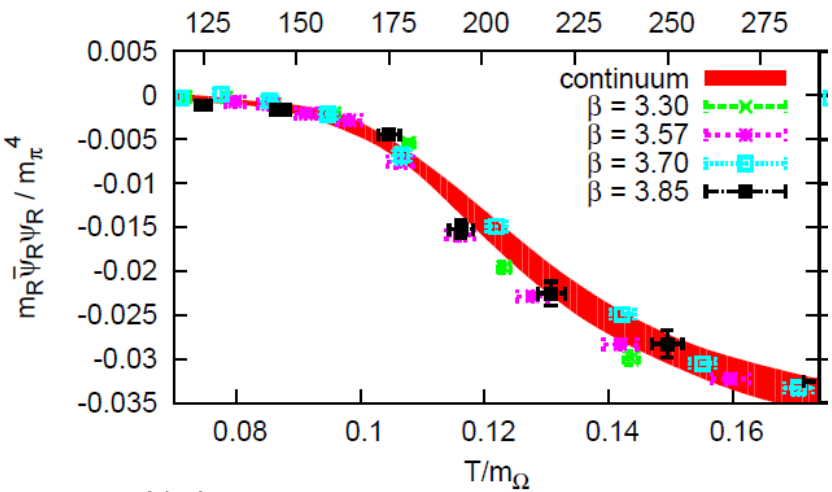
$$V_{string}(r) = c_m - \pi/12r + \sigma r$$

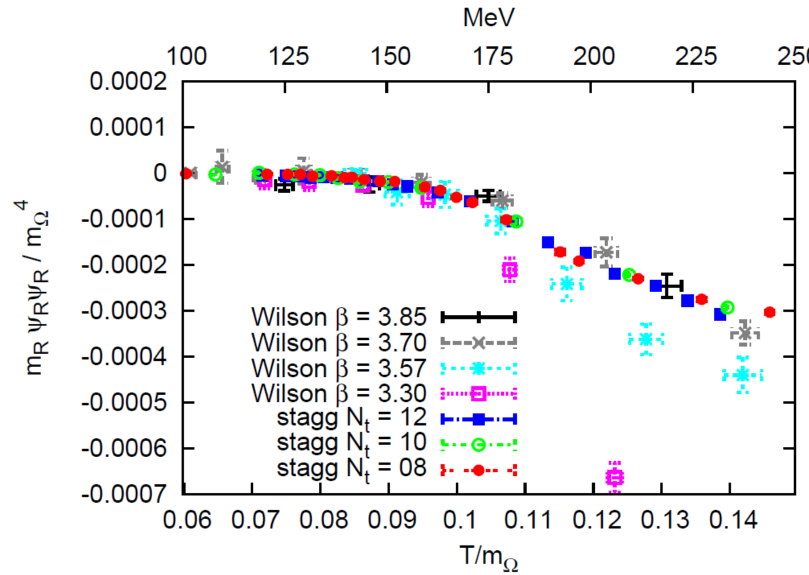
$$L_{ren} = \exp(c_m N_t/2)\langle L \rangle$$



Chiral condensate 3







Lattice2011 Borsanyi et al.

$$m_R\langle\bar\psi\psi\rangle_R(T) = \frac{\Delta^2_{\bar\psi\psi}(T)}{2N_f\Delta_{PP}(T)} + O(a)\ .$$

Fixed scale approach to study QCD thermodynamics

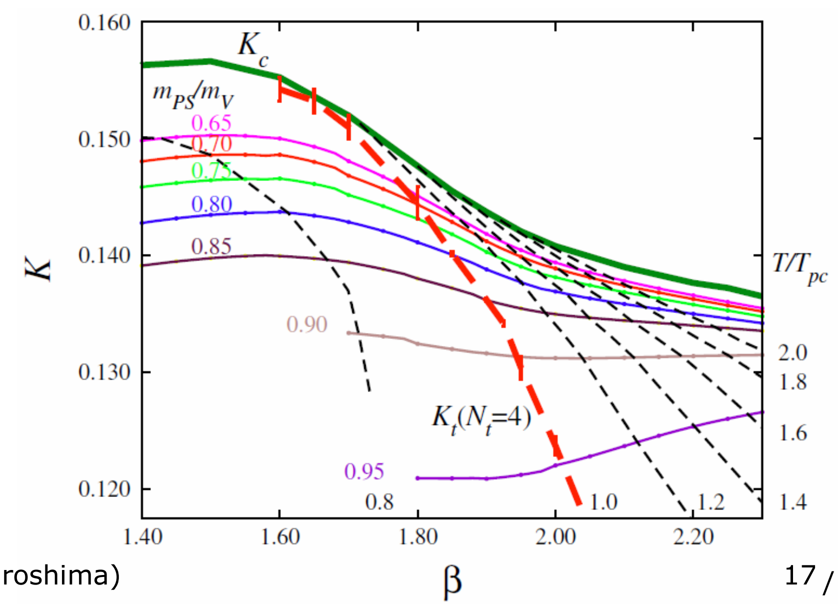
$$T = \frac{1}{N_t a}$$

Temperature $T=1/(N_t a)$ is varied by N_t at fixed a

a : lattice spacing
N_t : lattice size in temporal direction

- Advantages
  - Line of Constant Physics
  - T=0 subtraction for renorm. (spectrum study at T=0)
  - Lattice spacing at lower T
  - Finite volume effects
- Disadvantages
  - T resolution
  - High T region

LCP's in fixed N_t approach
(N_f=2 Wilson quarks at N_t=4)



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spatial volume at fixed N_t