Thermodynamics in the fixed scale approach with the shifted boundary conditions

Takashi Umeda (Hiroshima Univ.)

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Fixed scale approach to study QCD thermodynamics

Fixed scale approach

Temperature $T=1/(N_t a)$ is varied by N_t at fixed a

a : lattice spacing
N_t : lattice size in t-direction

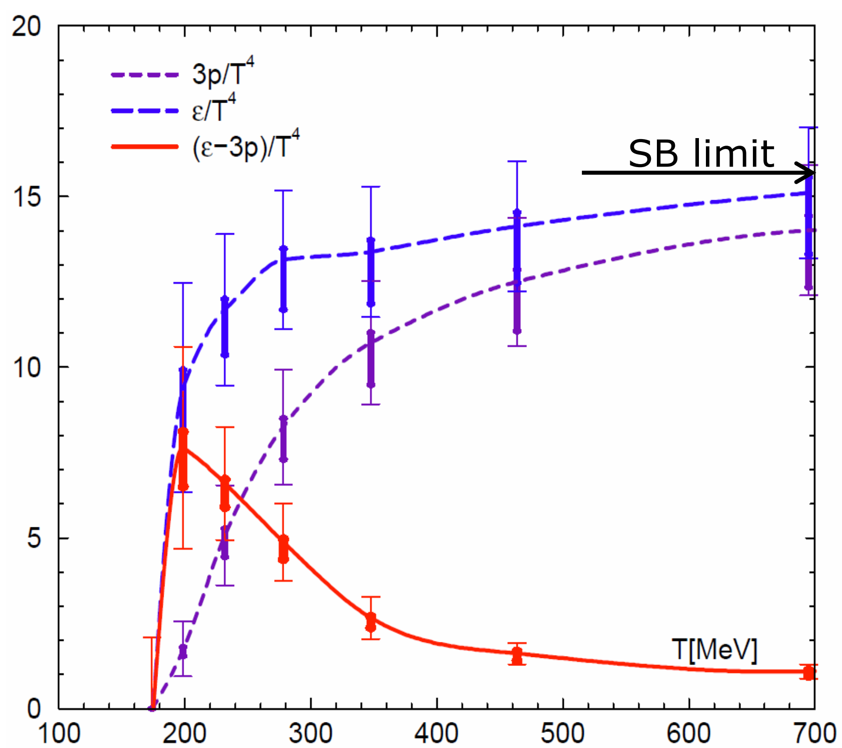
T. Umeda et al. (WHOT-QCD) Phys.Rev.D79 (2009) 051501.

- Coupling parameters are common at each T

  To study Equation of States
  - T=0 subtractions are common
  - beta-functions are common
  - Line of Constant Physics is automatically satisfied

⇨ Cost for T=0 simulations can be largely reduced

Equation of State in N_f=2+1 QCD



T. Umeda et al. (WHOT-QCD)
Phys. Rev. D85 (2012) 094508

EOS is obtained by

temperature integration

in the Fixed scale approach

$$\frac{p}{T^4} = \int_0^T dt \frac{\epsilon - 3p}{t^5}$$

Some groups adopted the approach

- tmfT, arXiv:1311.1631
- Wuppertal, JHEP08(2012)126.

However possible temperatures are restricted by integer N_t

Shifted boundary conditions

L. Giusti and H. B. Meyer, Phys. Rev. Lett. 106 (2011) 131601.
Thermal momentum distribution from path integrals with shifted boundary conditions

New method to calculate thermodynamic potentials
(entropy density, specific heat, etc.)

The method is based on the partition function

$$Z(\vec{z}) = Tr\{e^{-L_0 \hat{H}} e^{i\hat{p}\vec{z}}\}$$

which can be expressed by Path-integral with shifted boundary condition

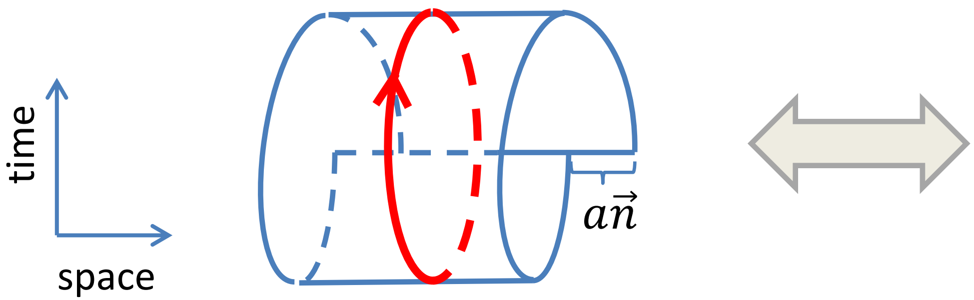
$$\phi(L_0, \vec{x}) = \pm\phi(0, \vec{x} + \vec{z})$$

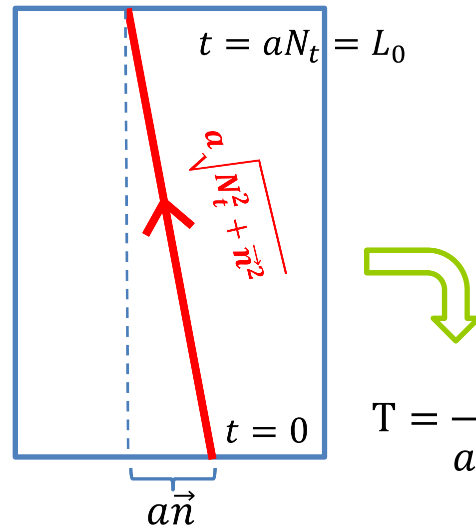
- L. Giusti and H. B. Meyer, JHEP 11 (2011) 087
- L. Giusti and H. B. Meyer, JHEP 01 (2013) 140
- L. Giusti and M. Pepe, arXiv:1403.0360.

Shifted boundary conditions

$$\phi(L_0, \vec{x}) = \pm\phi(0, \vec{x} + \vec{z})$$

$$\vec{z} = a\vec{n}$$





$$T = \frac{1}{a\sqrt{N_t^2 + \vec{n}^2}}$$

By using the shifted boundary
various T's are realized with the same lattice spacing

T resolution is largely improved
while keeping advantages of the fixed scale approach

Test in quenched QCD

Simulation setup

- quenched QCD
- β=6.0
  a ~ 0.1fm
- 32^3 x N_t lattices, N_t = 3, 4, 5, 6, 7, 8, 9 24^4 (T=0)
- boundary condition
  - spatial : periodic boundary condition
  - temporal: shifted boundary condition

$$U_\mu(L_0, \vec{x}) = U_\mu(0, \vec{x} + \vec{z})$$

- heat-bath algorithm (on SX-8R)
  only "even-shift" to keep even-odd structure
  e.g. $\vec{z}/a = (0,0,0), (1,1,0), (2,0,0), (2,1,1), (2,2,0), (3,1,0), \ldots$

Test in quenched QCD

Choice of boundary shifts

$$U_\mu(L_0, \vec{x}) = U_\mu(0, \vec{x} + \vec{z}) \qquad \vec{z} = a\vec{n}$$

					Nt							
n^2	n_1	n_2	n_3	e/o	10	9	8	7	6	5	4	3
0	0	0	0	0	10.00	9.00	8.00	7.00	6.00	5.00	4.00	3.00
2	1	1	0	0	10.10	9.11	8.12	7.14	6.16	5.20	4.24	3.32
4	2	0	0	0	10.20	9.22	8.25	7.28	6.32	5.39	4.47	3.61
6	2	1	1	0	10.30	9.33	8.37	7.42	6.48	5.57	4.69	3.87
8	2	2	0	0	10.39	9.43	8.49	7.55	6.63	5.74	4.90	4.12
10	3	1	0	0	10.49	9.54	8.60	7.68	6.78	5.92	5.10	4.36
12	2	2	2	0	10.58	9.64	8.72	7.81	6.93	6.08	5.29	4.58
14	3	2	1	0	10.68	9.75	8.83	7.94	7.07	6.24	5.48	4.80
16	4	0	0	0	10.77	9.85	8.94	8.06	7.21	6.40	5.66	5.00
18	3	3	0	0	10.86	9.95	9.06	8.19	7.35	6.56	5.83	5.20
18	4	1	1	0	10.86	9.95	9.06	8.19	7.35	6.56	5.83	5.20
20	4	2	0	0	10.95	10.05	9.17	8.31	7.48	6.71	6.00	5.39
22	3	3	2	0	11.05	10.15	9.27	8.43	7.62	6.86	6.16	5.57
24	4	2	2	0	11.14	10.25	9.38	8.54	7.75	7.00	6.32	5.74
26	4	3	1	0	11.22	10.34	9.49	8.66	7.87	7.14	6.48	5.92
26	5	1	0	0	11.22	10.34	9.49	8.66	7.87	7.14	6.48	5.92
30	5	2	1	0	11.40	10.54	9.70	8.89	8.12	7.42	6.78	6.24
32	4	4	0	0	11.49	10.63	9.80	9.00	8.25	7.55	6.93	6.40
34	4	3	3	0	11.58	10.72	9.90	9.11	8.37	7.68	7.07	6.56

Trace anomaly（e-3p）/T^4

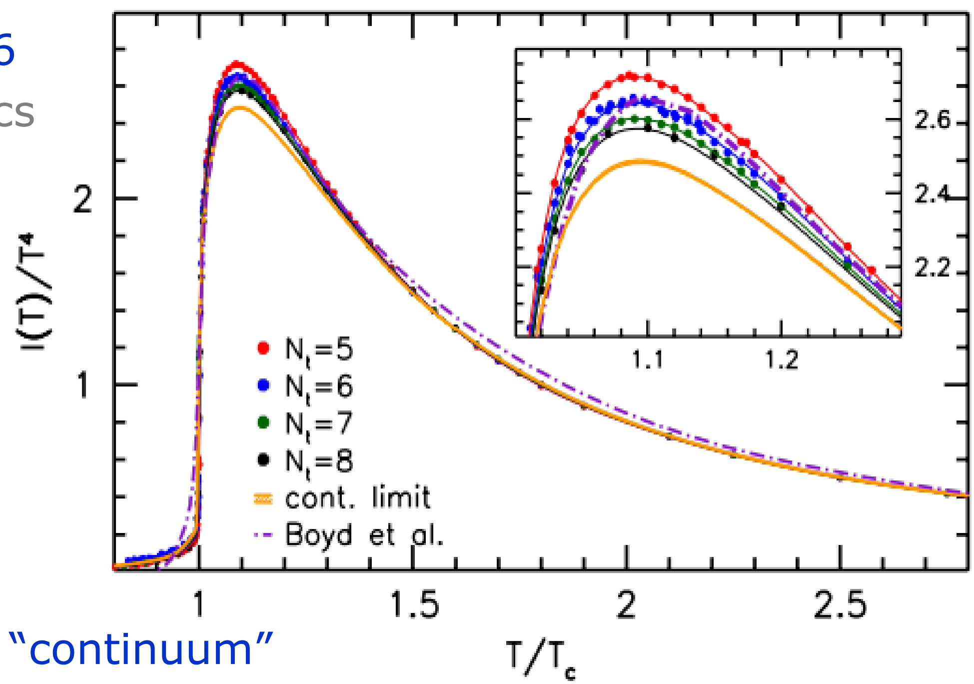
$$\frac{\epsilon - 3p}{T^4} = \left(\frac{1}{VT^3}\right) a\frac{d\beta}{da} \left\langle \frac{dS}{d\beta} \right\rangle_{sub}$$

Reference data

S. Borsanyi et al., JHEP 07 (2012) 056

Precision SU(3) lattice thermodynamics for a large temperature range

- N_s/N_t = 8 near T_c
- small N_t dependence at T>1.3Tc
- peak height at Nt=6 is about 7% higher than continuum value
- assuming T_c=294MeV

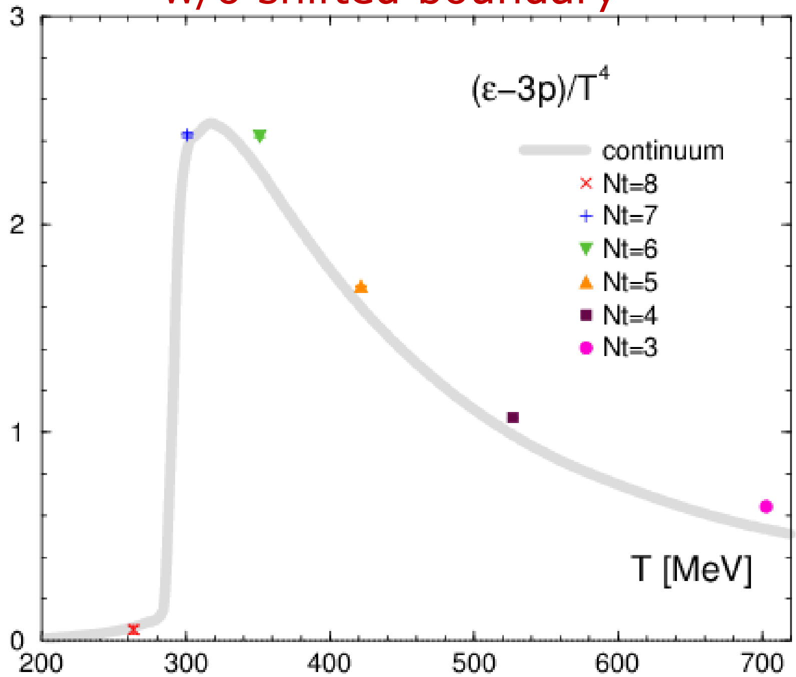


The continuum values are referred as "continuum"

Trace anomaly (e-3p)/T^4

$$\frac{\epsilon - 3p}{T^4} = \left(\frac{1}{VT^3}\right) a\frac{d\beta}{da}\left\langle\frac{dS}{d\beta}\right\rangle_{sub}$$

w/o shifted boundary



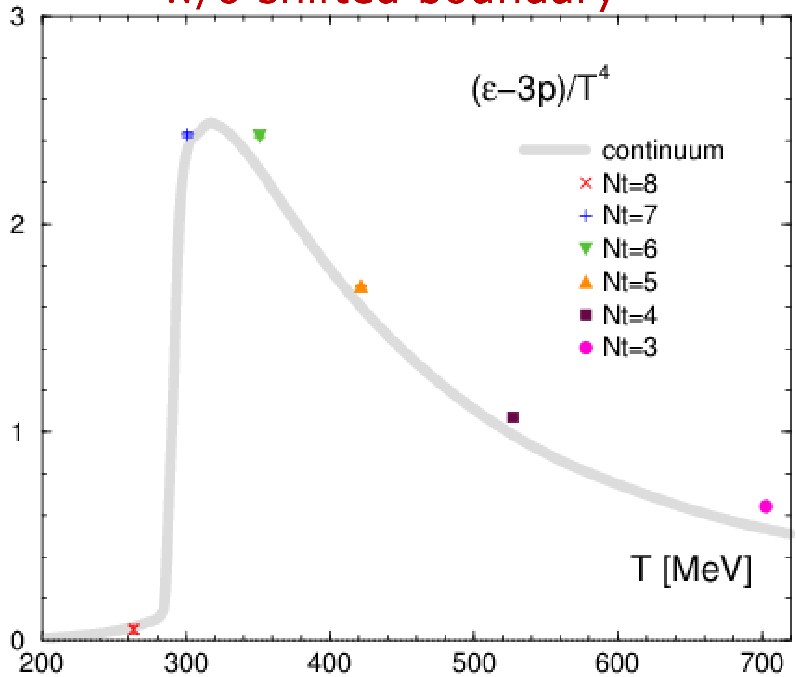
beta-function: Boyd et al. (1998)

Trace anomaly (e-3p)/T^4

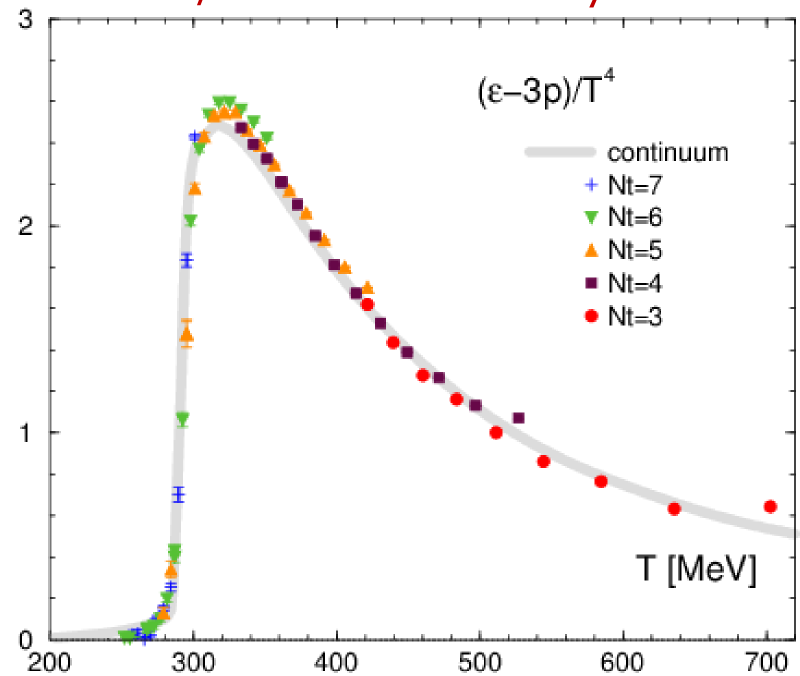
$$\frac{\epsilon - 3p}{T^4} = \left(\frac{1}{VT^3}\right) a\frac{d\beta}{da}\left\langle\frac{dS}{d\beta}\right\rangle_{sub}$$

$$\mathrm{T} = \frac{1}{a\sqrt{N_t^2 + \vec{n}^2}} \qquad V = \prod_{i=1}^{3} \frac{aN_s}{\sqrt{1+(\frac{n_i}{N_t})^2}}$$

w/o shifted boundary

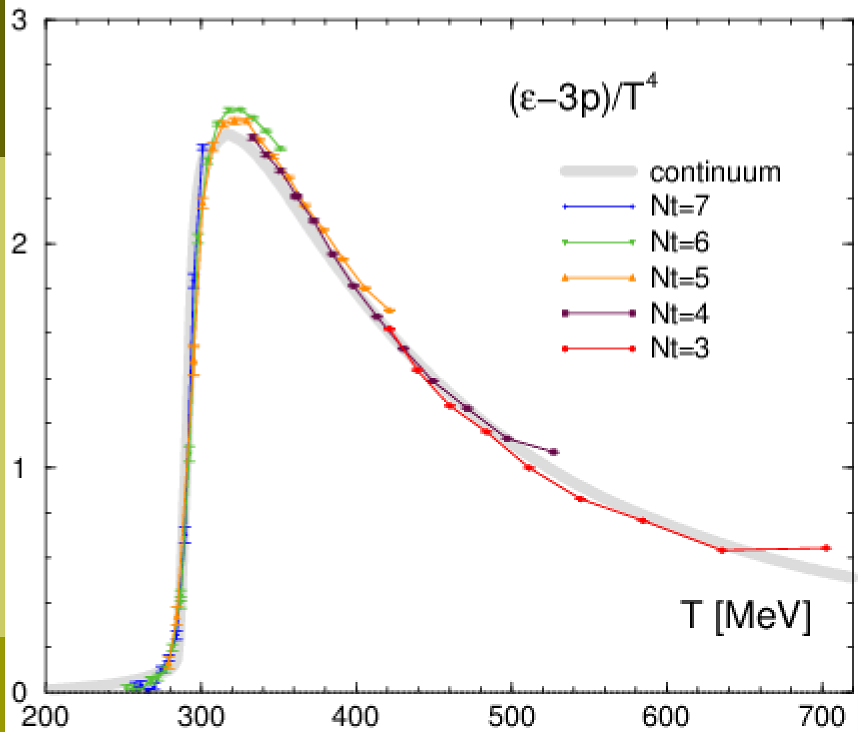


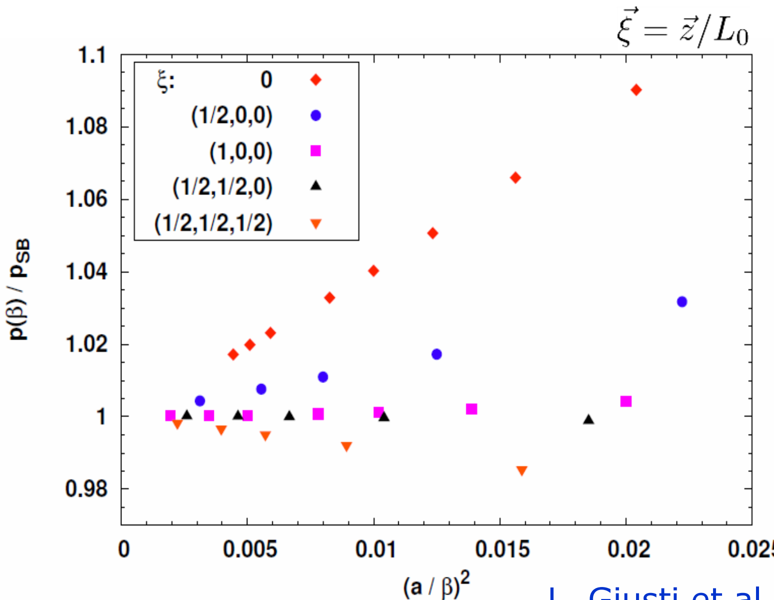
w/ shifted boundary



beta-function: Boyd et al. (1998)

Lattice artifacts from shifted boundaries





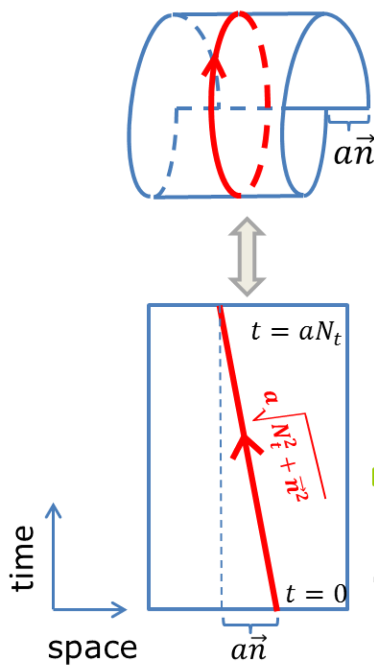
L. Giusti et al. (2011)

Figure 2: Pressure at finite lattice spacing for the SU(N) Yang–Mills theory in the non-interacting limit. The discretization used is the Wilson action and the 'clover' form of the lattice field strength tensor. The inverse temperature is given by $\beta = L_0\sqrt{1+\xi^2}$, and a is the lattice spacing.

- Shifted boundary reduces lattice artifacts of EOS in the non-interacting limit
- We confirmed that the shifted boundary reduces lattice artifacts even in the interacting case numerically.

Critical temperature T_c

Polyakov loop is difficult to be defined because the compact direction has an angle to the temporal direction



$$T = \frac{1}{a\sqrt{N_t^2+\vec{n}^2}}$$

Dressed Polyakov loop

E. Bilgici et al.,

Phys. Rev. D77 (2008) 094007

is defined with light quarks

$$\Sigma_n(m,V) = \int_0^{2\pi} \frac{d\phi}{2\pi} \frac{e^{-i\phi n}}{V} \langle Tr[(m+D_\phi)^{-1}]\rangle_G$$

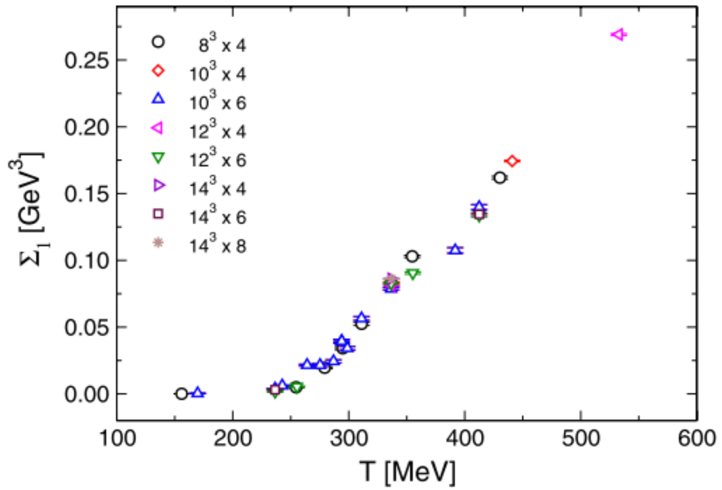
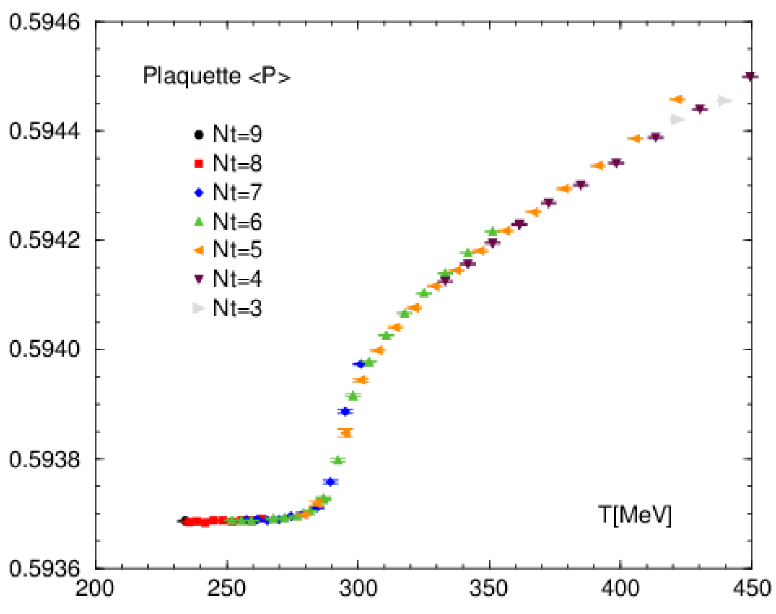


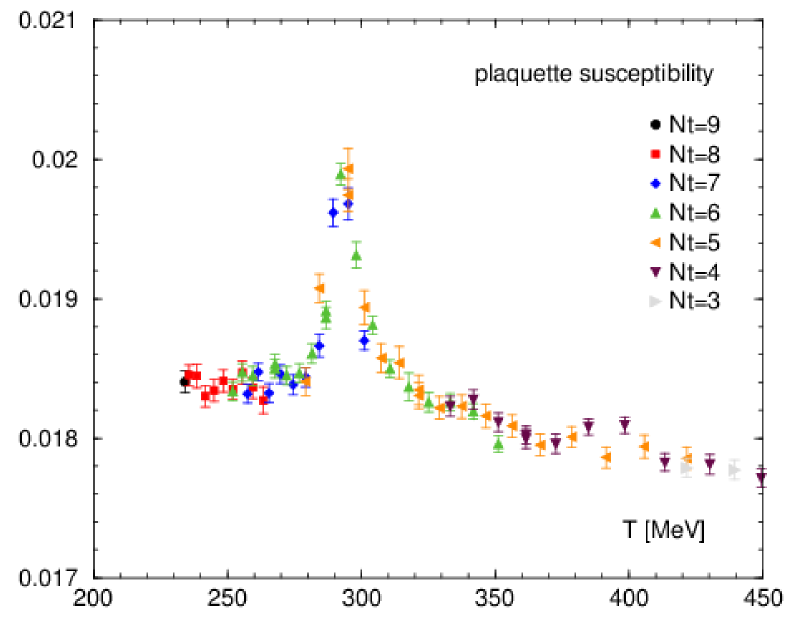
FIG. 2 (color online). The dressed Polyakov loop at $m = 100$ MeV in units of GeV^3 as a function of the temperature T in MeV.

Critical temperature Tc

Plaquette value $\langle P \rangle = \frac{1}{6N_s^3 N_t} \sum_P \langle 1 - \frac{1}{3} ReTrU_P \rangle$

Plaquette susceptibility $\chi_P = 6N_s^3 N_t \left(\langle P^2 \rangle - \langle P \rangle^2 \right)$

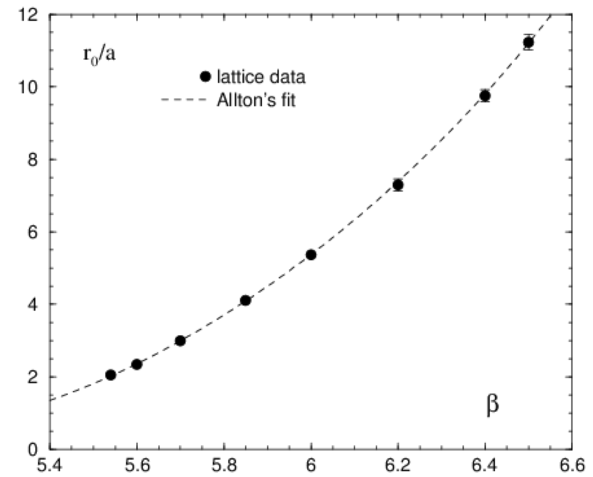




Plaq. suscep. has a peak around T = 294 MeV

Beta-functions (in case of quenched QCD)

$$\frac{\epsilon - 3p}{T^4} = \left(\frac{1}{VT^3}\right) a\frac{d\beta}{da} \left\langle \frac{dS}{d\beta} \right\rangle_{sub}$$



In order to calculate the beta-function
additional T=0 simulations near
the simulation point are necessary

We are looking for
new methods to calculate beta-function

- Reweighting method
- Shifted boundary / Gradient flow

entropy density is calculated from
only finite temperature configs.

L. Giusti, H.B.Meyer, PRL106(2011)131601.

M. Asakawa et al. [FlowQCD Collab.], arXiv:1312.7492

Entropy density from shifted boundaries

- Entropy density at a temperature (T_0)
  by the new method

$$s(T_0)$$

- Entropy density w/o beta-function
  by the T-integration

$$s(T)/a\frac{d\beta}{da}$$

$$\frac{\epsilon-3p}{T^4} = \left(\frac{N_t^3}{N_s^3}\right) a\frac{d\beta}{da}\left\langle\frac{dS}{d\beta}\right\rangle_{sub}$$

$$\frac{p}{T^4} = \int_0^T dT' \, \frac{\epsilon-3p}{T'^5}$$

$$\Rightarrow \quad Ts = \epsilon + p$$

Beta-func is determined by
matching of entropy densities at T_0

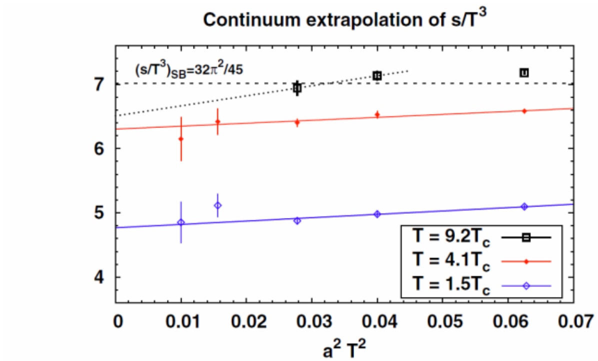
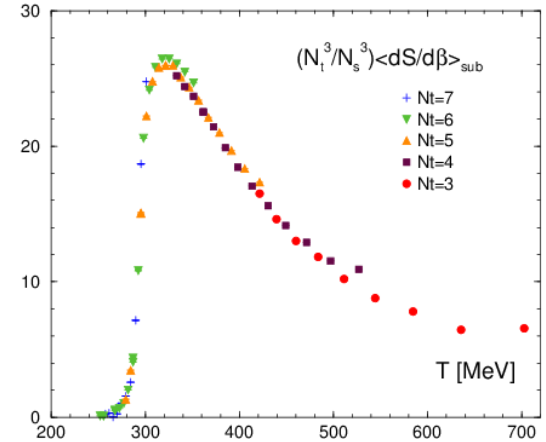


FIG. 1 (color online). Scaling behavior of s/T^3; see Eq. (15). The Stefan-Boltzmann value reached in the high-T limit is also displayed.

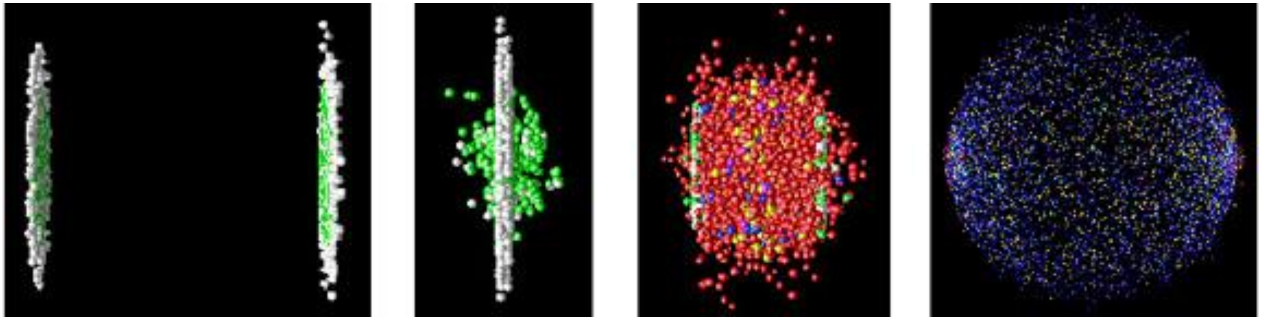


Summary & outlook

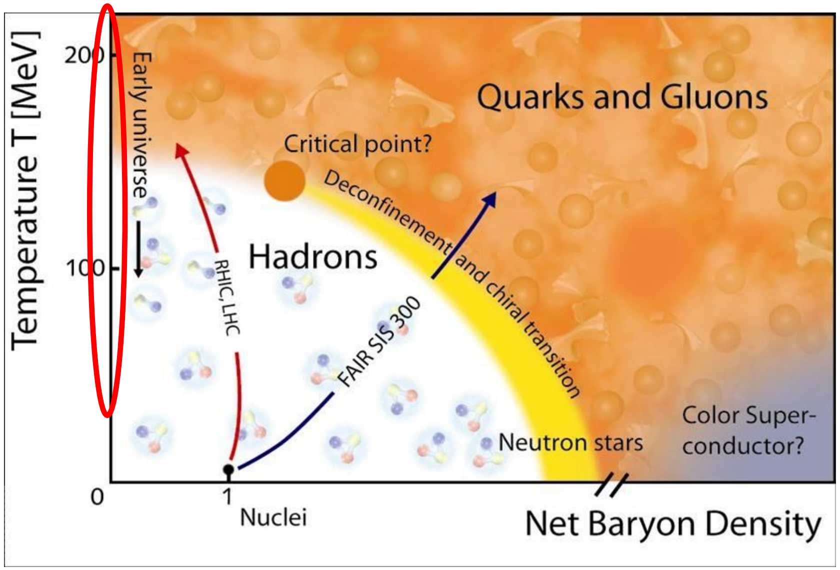
We presented our study of the QCD Thermodynamics
by using Fixed scale approach
and Shifted boundary conditions

- Fixed scale approach
  - Cost for T=0 simulations can be largely reduced
  - first calculation in N_f=2+1 QCD with Wilson-type quarks
- Shifted boundary conditions are promising tool
  to improve the fixed scale approach
  - fine temperature scan
  - suppression of lattice artifacts at larger shifts
  - Tc determination could be possible
  - New method to calculate beta-functions
- Test in full QCD → Nf=2+1 QCD at the physical point

Quark Gluon Plasma in Lattice QCD



from the Phenix group web-site



http://www.gsi.de/fair/experiments/

Observables in Lattice QCD

- Phase diagram in (T, μ, m_{ud}, m_s)
- Critical temperature
- Equation of state (ε/T^4, p/T^4,...)
- Hadronic excitations
- Transport coefficients
- Finite chemical potential
- etc...

Entropy density from shifted boundaries

Entropy density s/T^3

from the cumulant of the momentum distribution

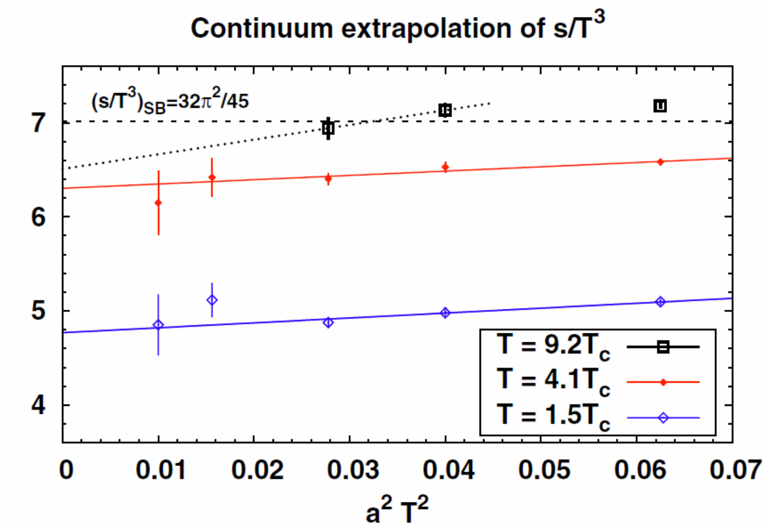
L. Giusti and H. B. Meyer, Phys. Rev. Lett. 106 (2011) 131601

$$\frac{s(T)}{T^3} = \lim_{a\to 0} \frac{2K(T,\vec{z},a)}{|\vec{z}|^2 T^5 V}$$

$$K(T,\vec{z},a) = -\ln \frac{Z(T,\vec{z},a)}{Z(T,\vec{0},a)}$$

$Z(T,\vec{z},a)$: partition function with shifted boundary

where $\vec{z} = (0,0,n_z a)$, n_z being kept fixed when a→0



L. Giusti et al. (2011)

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