Lattice study of charmonia in Hot QCD

T. Umeda (YITP, Kyoto univ.)
H. Ohno, K. Kanaya (Univ. of Tsukuba)

(WHOT-QCD Collaboration)

Mini-Workshop on "Photons and Leptons in Hot/Dense QCD"
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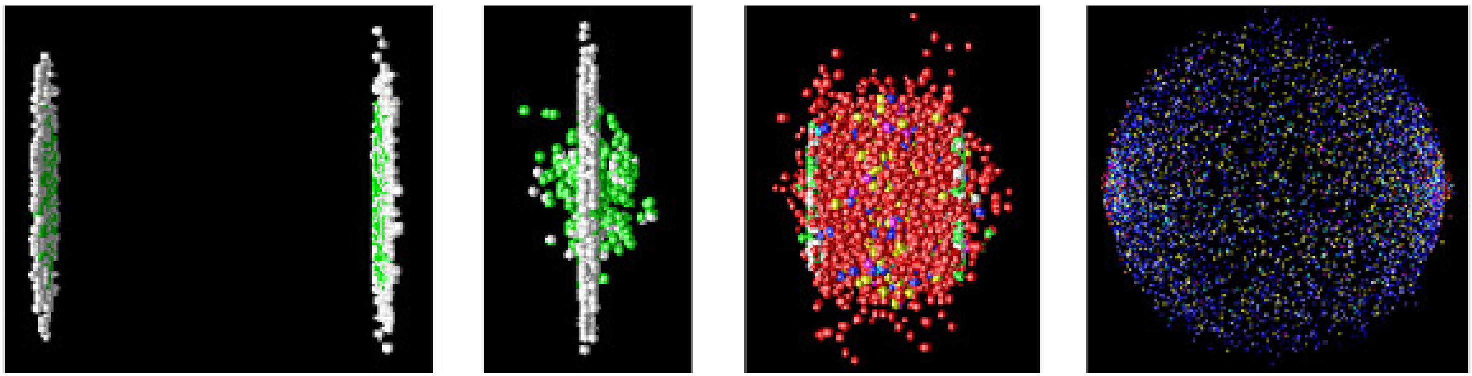
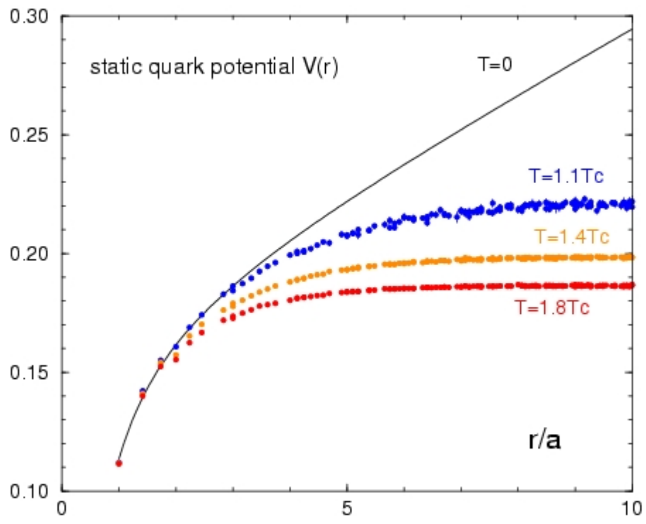
Contents of this talk

from the Phenix group web-site



- Introduction
  - -- Quark Gluon Plasma & J/ψ suppression
  - -- Lattice studies on J/ψ suppression
- Our approach to study charmonium dissociation
- Charmonium spectra & wave functions at T>0
- Summary & future plan

J/ψ suppression as a signal of QGP



Confined phase:

linear raising potential

→ bound state of c - $\bar{c}$

De-confined phase:

Debye screening

→ scattering state of c - $\bar{c}$

T.Hashimoto et al.('86), Matsui&Satz('86)

Lattice QCD calculations:

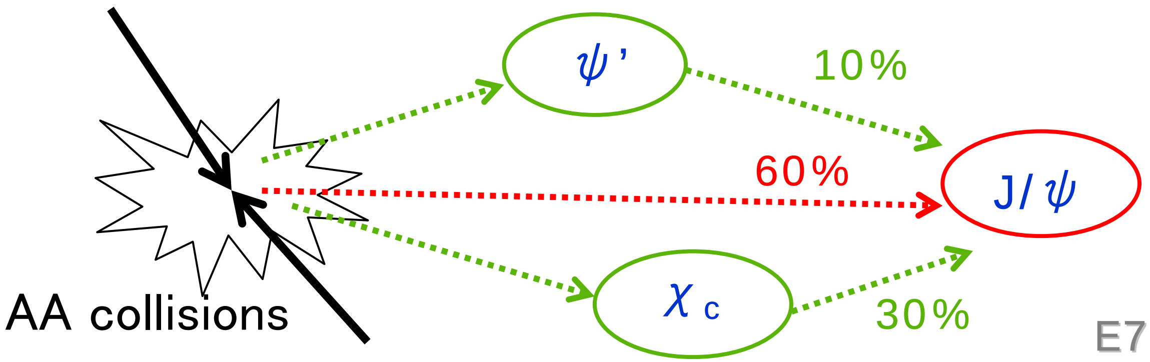
Spectral function by MEM: T.Umeda et al.('02), S.Datta et al.('04), Asakawa&Hatsuda('04), A.Jakovac et al.('07), G.Aatz et al.('06)

Wave func.: T.Umeda et al.('00)

B. C. dep.: H.Iida et al. ('06)

→ all calculations conclude that J/ψ survives till 1.5T_c or higher

Sequential J/ψ suppression scenario



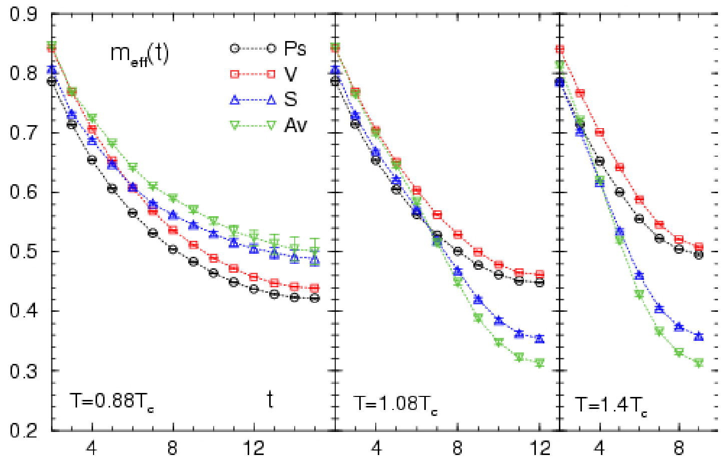
E705 Collab.('93)

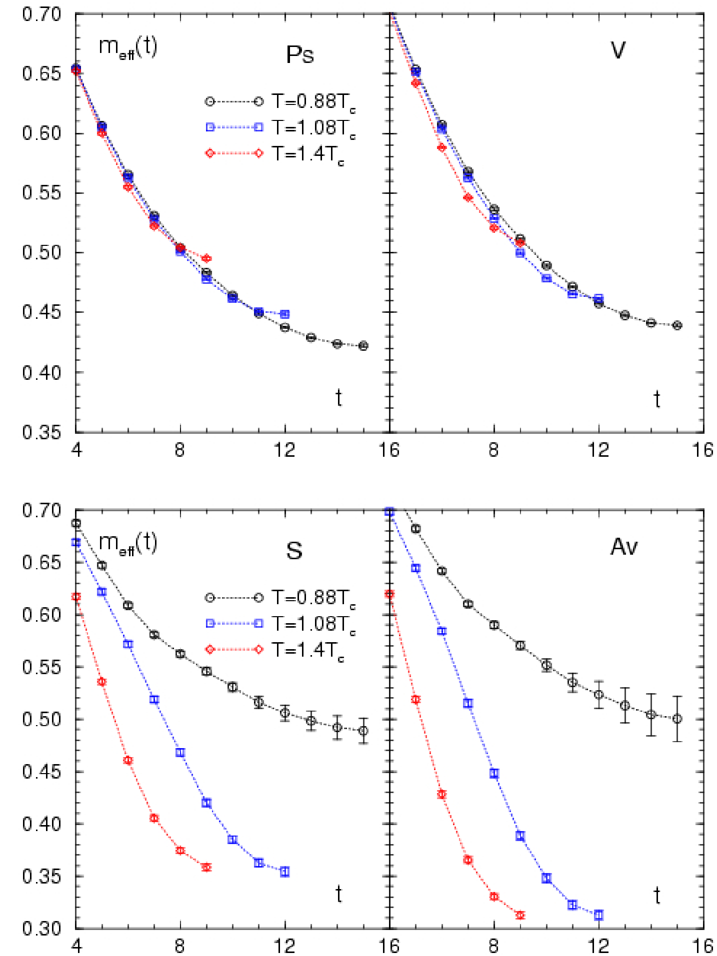
J/ψ (1S)	:	$J^{PC} = 1^{--}$	M=3097MeV	(Vector)
ψ (2S)	:	$J^{PC} = 1^{--}$	M=3686MeV	(Vector)
χ_{c0} (1P)	:	$J^{PC} = 0^{++}$	M=3415MeV	(Scalar)
χ_{c1} (1P)	:	$J^{PC} = 1^{++}$	M=3511MeV	(AxialVector)

PDG('06)

It is important to study dissociation temperatures for not only J/ψ but also ψ(2S), χ_c's

Charmonium correlators at T>0





- small change in S–wave states
  → survival of J/ψ & η_c at T>T$_c$
- drastic change in P–wave states
  → dissociation of χ_c just above Tc (?)

S. Datta et al.,
Phys. Rev. D69 (2004) 094507. etc…

χ_c dissociation just above Tc ?

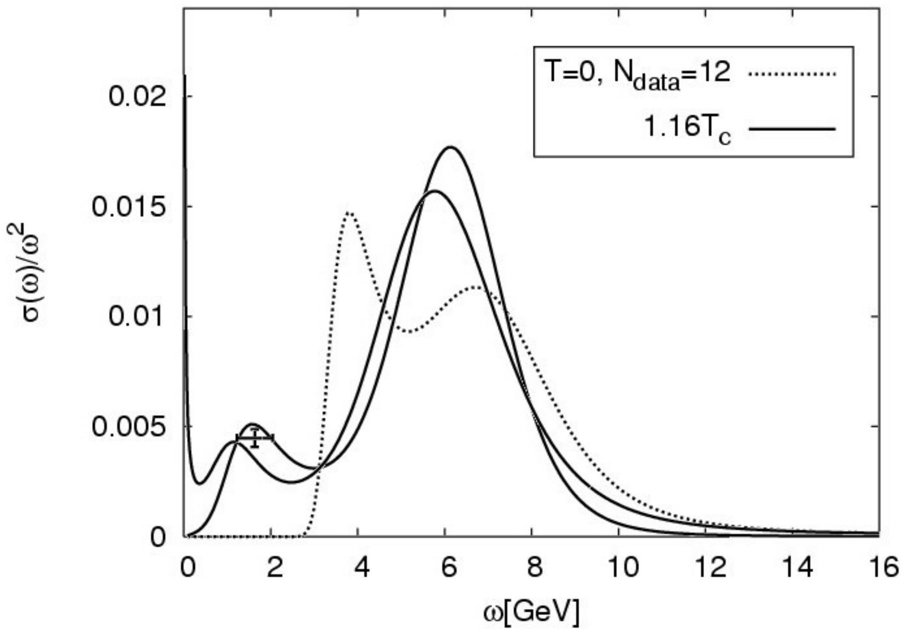


FIG. 19: The scalar spectral function for $\beta = 6.1$ at $T = 1.16T_c$ and at zero temperature reconstructed using $N_{data} = 12$. At finite temperature two default models $m(\omega) = 0.01$ and $m(\omega) = 0.038\omega^2$ have been used.

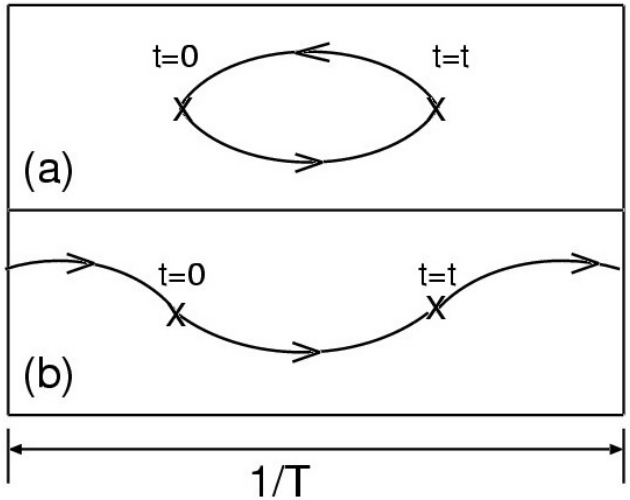
A.Jakovac et al., Phys. Rev. D 75 (2007) 014506.

"χ_c dissociation just above T_c" is consistent with the sequential suppression scenario.

In the paper, they concluded that

- the drastic change of P-states corr. just above T_c is reliable.
- Spectral functions for P-states are not so reliable.
  e.g. large default model dep.

Constant mode in meson correlators



T.Umeda, Phys. Rev. D 75 (2007) 094502.

$\exp(-m_q t) \times \exp(-m_q t)$
$= \exp(-2 m_q t)$

m_q is quark mass
or single quark energy

$\exp(-m_q t) \times \exp(-m_q (L_t - t))$
$= \exp(-m_q L_t)$

L_t = temporal extent

We can separate the constant mode from correlators

usual effective mass

$$\frac{C(t)}{C(t+1)} = \frac{\cosh\left[m_{eff}(t)(N_t/2-t)\right]}{\cosh\left[m_{eff}(t)(N_t/2-t-1)\right]}$$

subtracted effective mass

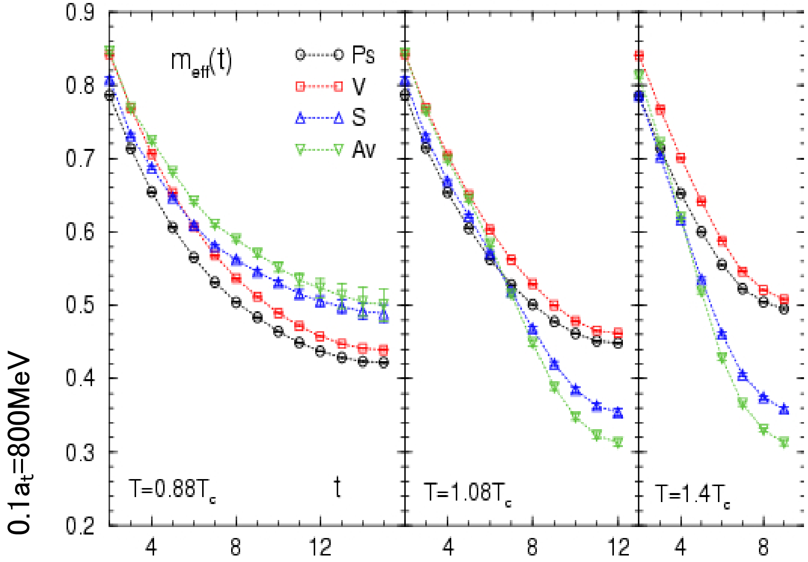
$$\bar{C}(t) = C(t) - C(N_t/2)$$

$$\frac{\bar{C}(t)}{\bar{C}(t+1)} = \frac{\sinh^2\left[\frac{1}{2}m_{eff}^{sub}(t)(N_t/2-t)\right]}{\sinh^2\left[\frac{1}{2}m_{eff}^{sub}(t)(N_t/2-t-1)\right]}$$

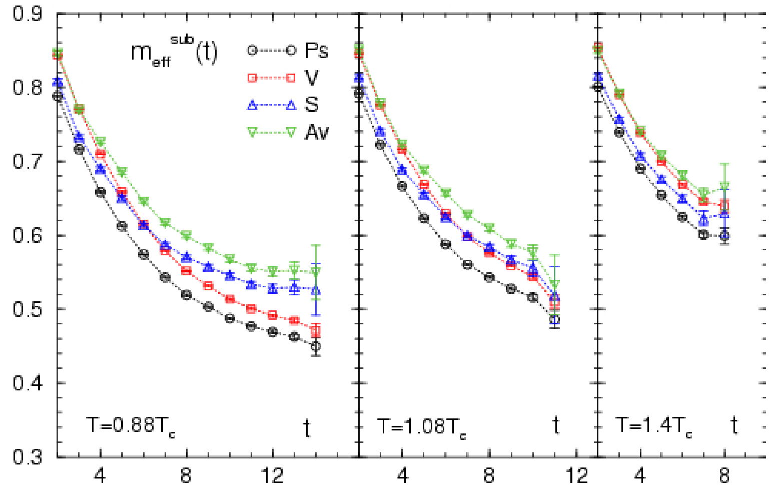
Constant mode effects in charmonia

$$\bar{C}(t) = C(t) - C(N_t/2) \qquad \frac{\bar{C}(t)}{\bar{C}(t+1)} = \frac{\sinh^2\left[\frac{1}{2}m^{sub}_{eff}(t)(N_t/2-t)\right]}{\sinh^2\left[\frac{1}{2}m^{sub}_{eff}(t)(N_t/2-t-1)\right]}$$

usual effective masses at T>0



subtracted effective mass



the drastic change in P–wave states disappears in $m_{eff}^{sub}(t)$

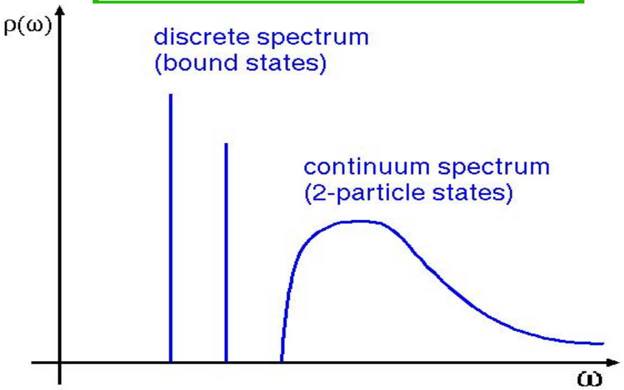
→ the change is due to the constant mode

Spectral functions in a finite volume

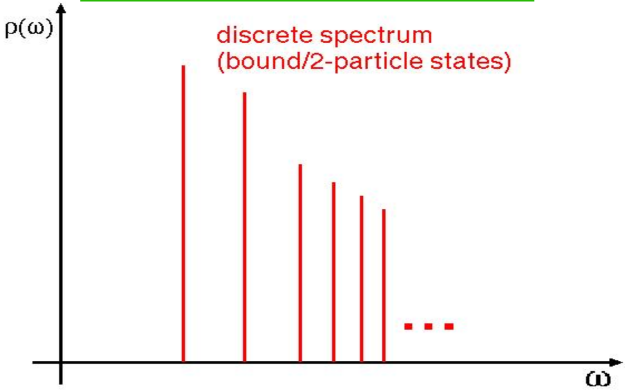
Momenta are discretized in finite ($V=L^3$) volume

$p_i/a = 2n_i\pi/L$ ($n_i=0,\pm1,\pm2,\ldots$) for Periodic boundary condition

infinite volume case



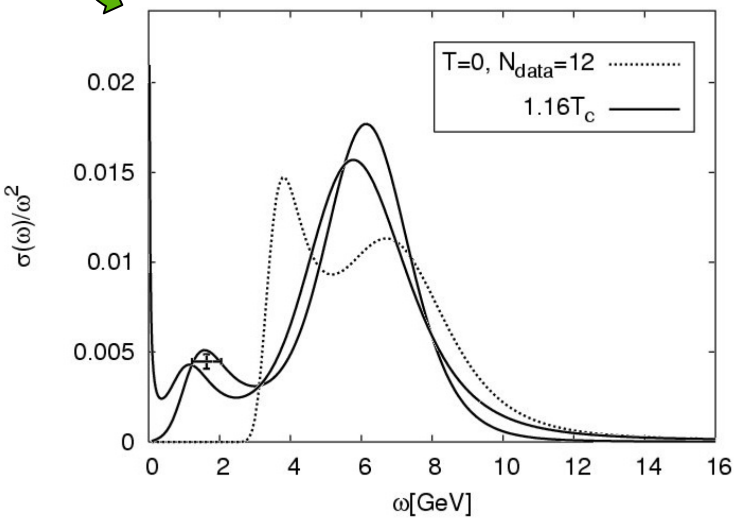
finite volume case



In a finite volume (e.g. Lattice simulations),
discrete spectra does not always indicate bound states !

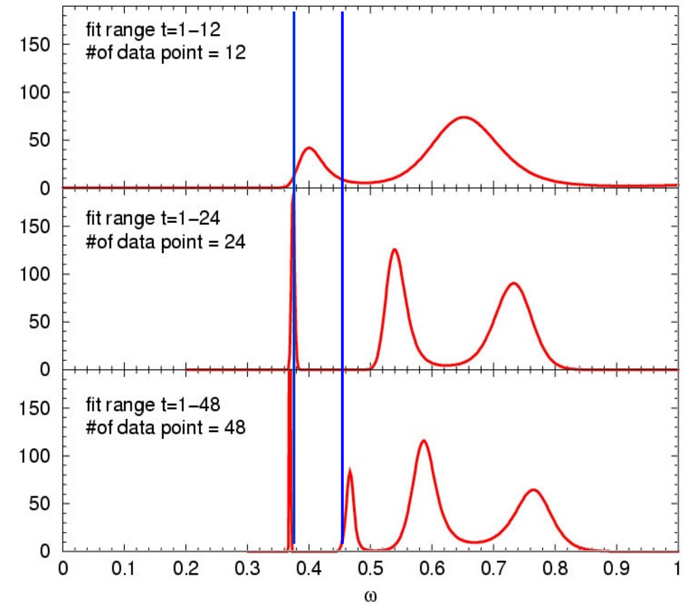
χ_c dissociation just above Tc ?

This result is strange ??



A.Jakovac et al., Phys. Rev. D 75 (2007) 014506.

MEM test using T=0 data



data for T/T_c=1.2

data for T/T_c=0.6

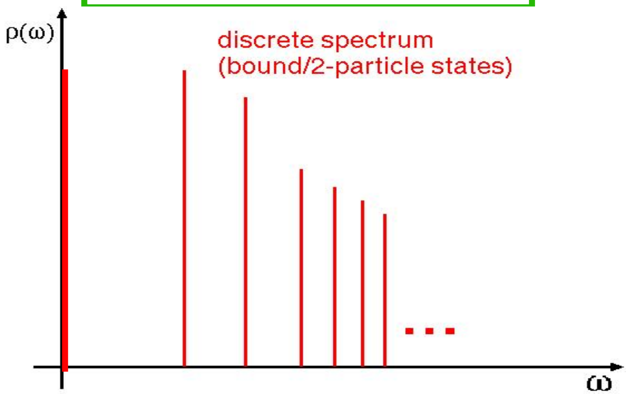
data for T/T_c=0

MEM analysis can fail if data quality is not sufficient.

"P-wave states have larger noise than that of S-wave states"

Another approach to study charmonium at T>0

finite volume case



It is difficult to extract higher states
from lattice correlators (at T>0)
even if we use MEM !!

We want to get information
about a few lowest states (at T>0)

Constant mode can be separated
by the Midpoint subtraction

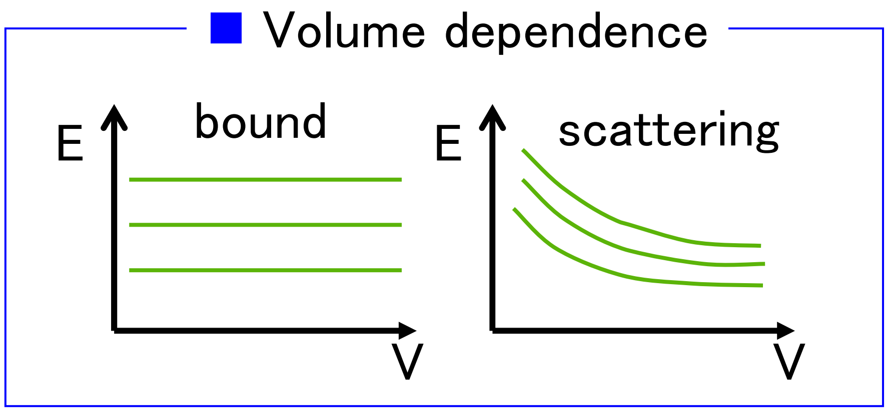
In order to study a few lowest states,
the variational analysis is one of the most reliable approaches !

N x N correlation matrix : C(t)

$$C(t)\psi = \lambda(t,t_0)C(t_0)\psi \qquad \lambda_i(t,t_0) = e^{-E_i(t-t_0)}$$

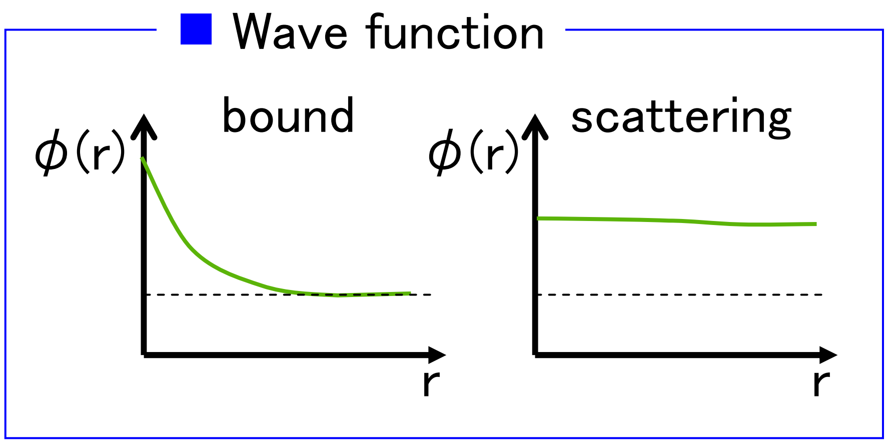
Bound state or scattering state ?

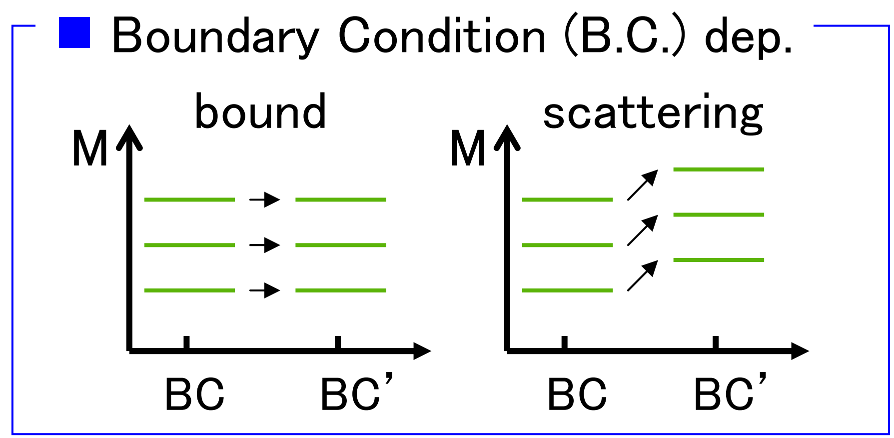
In a finite volume, discrete spectra does not always indicate bound states.



E : energy
V : volume

Φ(r) : wave function
r : c - c̄ distance





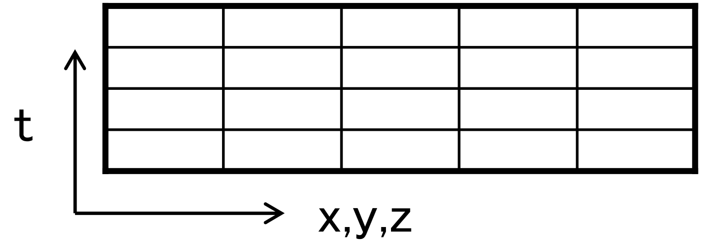
H.Iida et al.('06), N.Ishii et al.('05)

Lattice setup

- Quenched approximation (no dynamical quark effect)
- Anisotropic lattices

  lattice spacing : a_s = 0.0970(5) fm

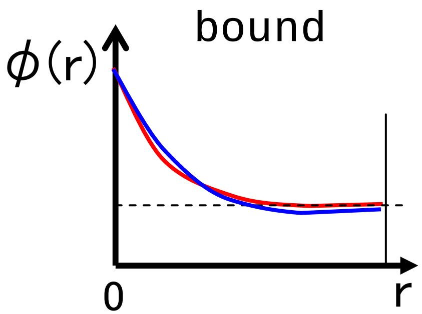
  anisotropy : $a_s/a_t = 4$



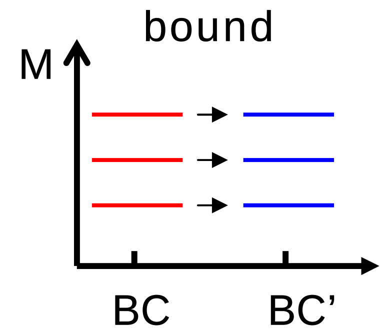
- r_s=1 to suppress doubler effects
- Variational analysis with 4 x 4 correlation matrix

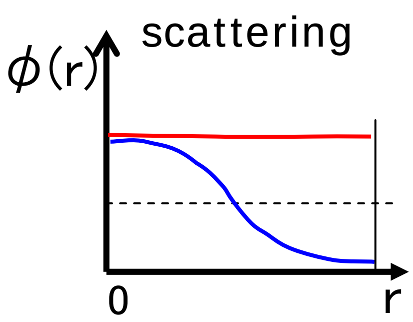
N_t	32	26	20	16	12
T/T_c	0.88	1.08	1.40	1.75	2.33
# of conf.					
$V=16^3$	300	300	300	300	300
$V=20^3$	300	300	300	300	300
$V=32^3$	—	—	—	—	100

Boundary condition dependence

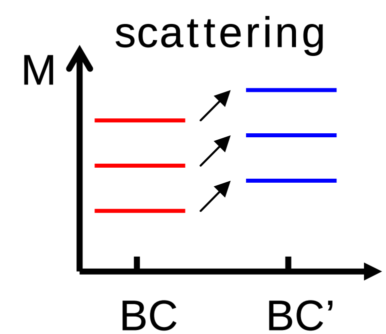


The wave functions are localized,
their energies are insensitive to B.C.





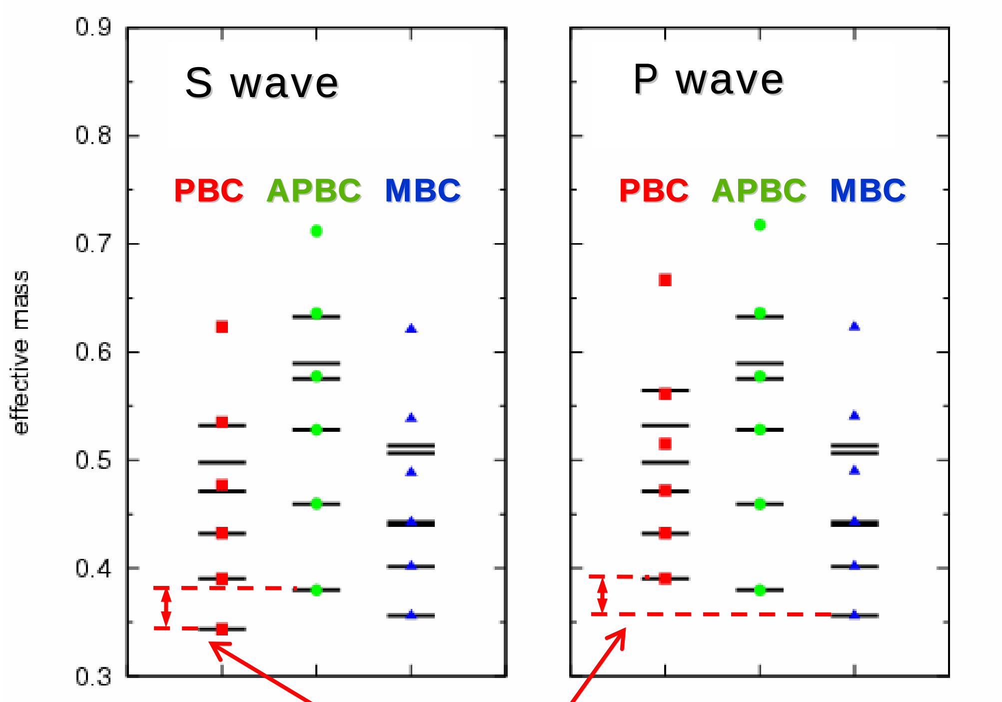
The momenta depends on BC,
the scattering state energies are sensitive to B.C.



The idea has been originally applied for the charmonium study in H. Iida et al., Phys. Rev. D74 (2006) 074502.

Variational analysis in free quark case

Test with free quarks ($L_s/a=20$, $ma=0.17$)



Mass diff. between the lowest masses in each BC

$$q(x_i + L_i) = b_i q(x_i)$$

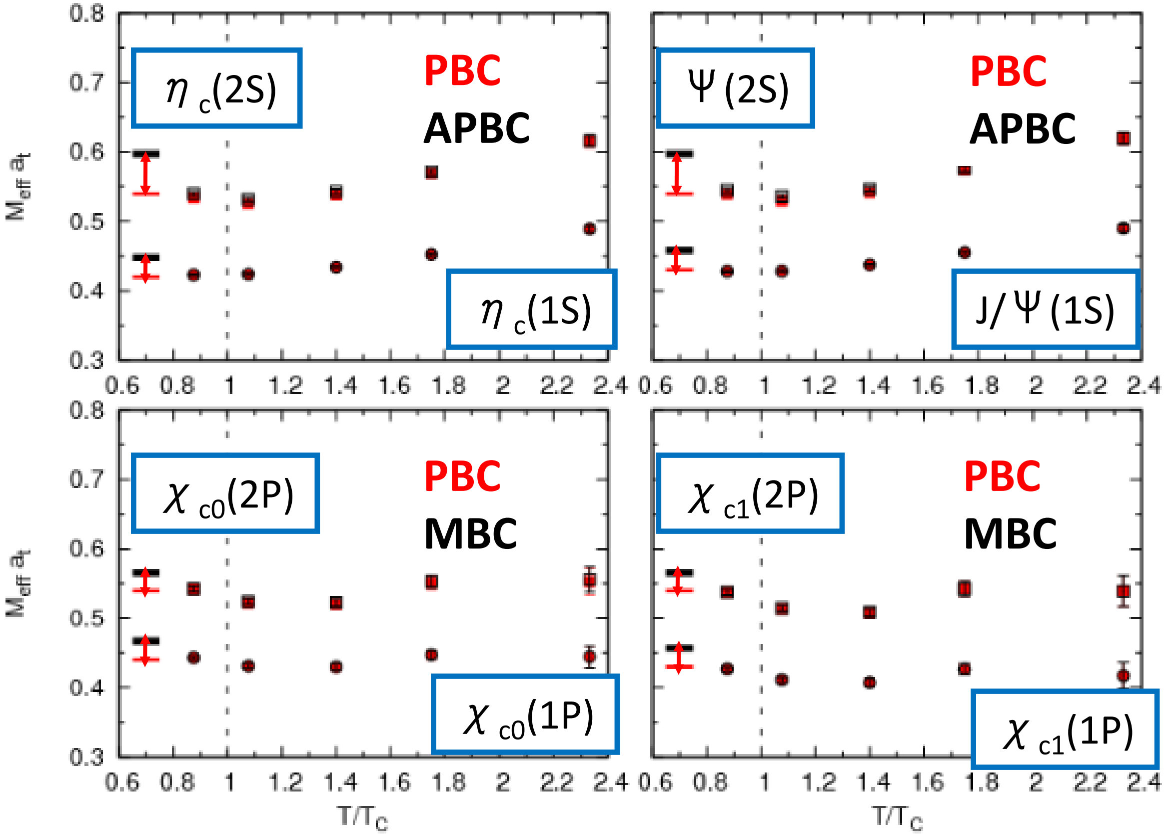
$b_i = 1$: periodic

$b_i = -1$: anti-periodic

PBC : b=(1, 1, 1)
APBC : b=(-1,-1,-1)
MBC : b=(-1, 1, 1)

an expected diff.
in V=(2fm)^3
(free quark case)
~ 200MeV

Temperature dependence of charmonium spectra



$$q(x_i + L_i) = b_i q(x_i)$$

$b_i = 1$: periodic

$b_i = -1$: anti-periodic

PBC : b=(1, 1, 1)
APBC : b=(-1,-1,-1)
MBC : b=(-1, 1, 1)

an expected gap in $V=(2\text{fm})^3$ (free quark case) ~ 200MeV

- No significant differences in the different B.C.
- Analysis is difficult at higher temperature ($2T_c$~)

Wave functions at finite temperature

Temp. dependence of (Bethe-Salpeter) “Wave function”

$$
\begin{aligned}
BS(\vec{r},t) &= \sum_{\vec{x}} \langle \bar{q}(\vec{x}+\vec{r},t)\Gamma q(\vec{x},t)\bar{q}(\vec{0},0)\Gamma q(\vec{0},0)\rangle \\
\Psi(|\vec{r}|,t) &= BS(\vec{r},t)/BS(\vec{r}_0,t)
\end{aligned}
$$

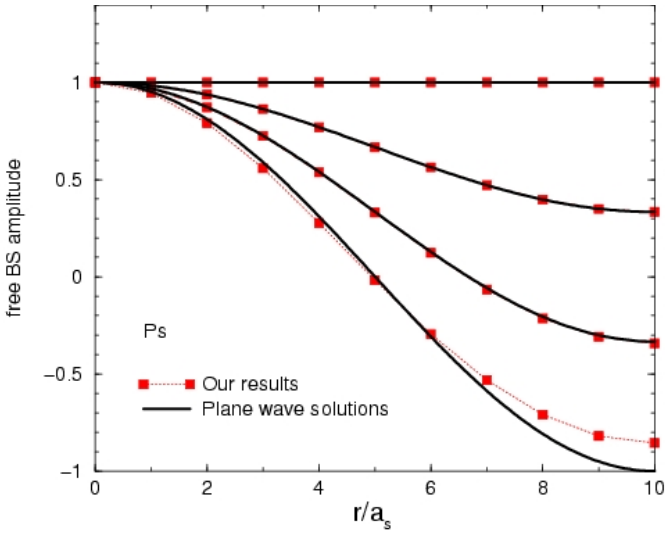
$$
\Gamma = \begin{cases}
\gamma_5 & \text{(Ps)} \\
\gamma_i & \text{(Ve)} \quad (i=1,2,3) \\
\sum_j \left(\overrightarrow{\partial}_j\gamma_j - \overleftarrow{\partial}_j\gamma_j\right) & \text{(Sc)} \\
\sum_{j,k} \epsilon_{ijk} \left(\overrightarrow{\partial}_j\gamma_k - \overleftarrow{\partial}_j\gamma_k\right) & \text{(Av)} \quad (i=1,2,3)
\end{cases}
$$

using the eigen functions of the variational method

→ we can extract the wave functions of each states

Wave functions in free quark case

Test with free quarks ($L_S/a=20$, $ma=0.17$)
in case of S-wave channels



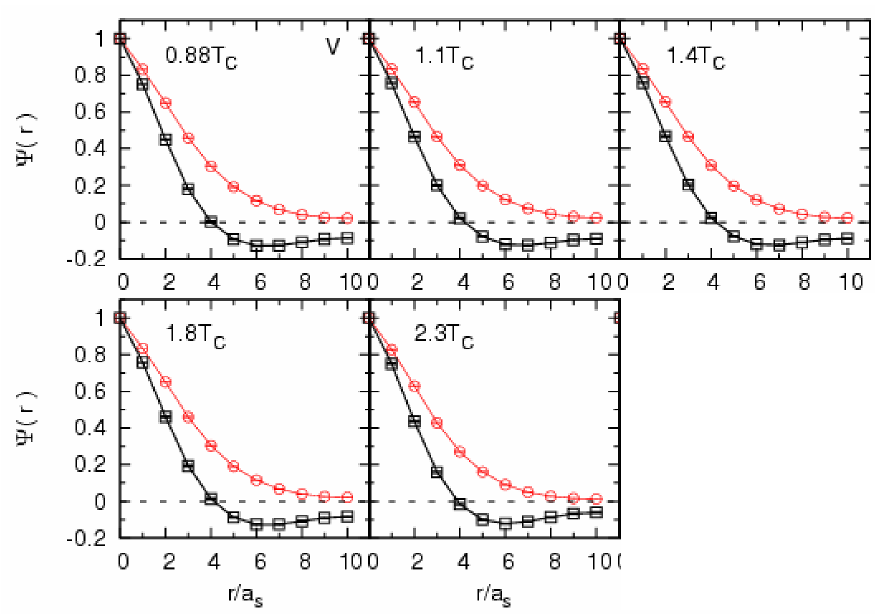
- Free quarks make trivial waves with an allowed momentum in a box

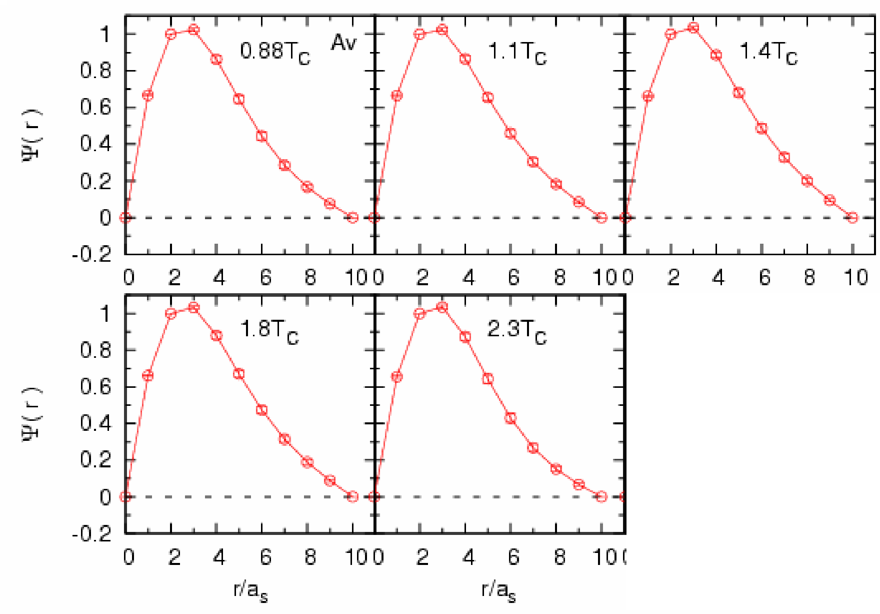
$$\Psi_k(|\vec{r}|, t) = \frac{\sum_{\vec{p}=\vec{k}} \cos(p_1 r_1)\cos(p_2 r_2)\cos(p_3 r_3)}{\sum_{\vec{p}=\vec{k}} 1}$$

- The wave function is constructed with eigen functions of 4 x 4 correlators

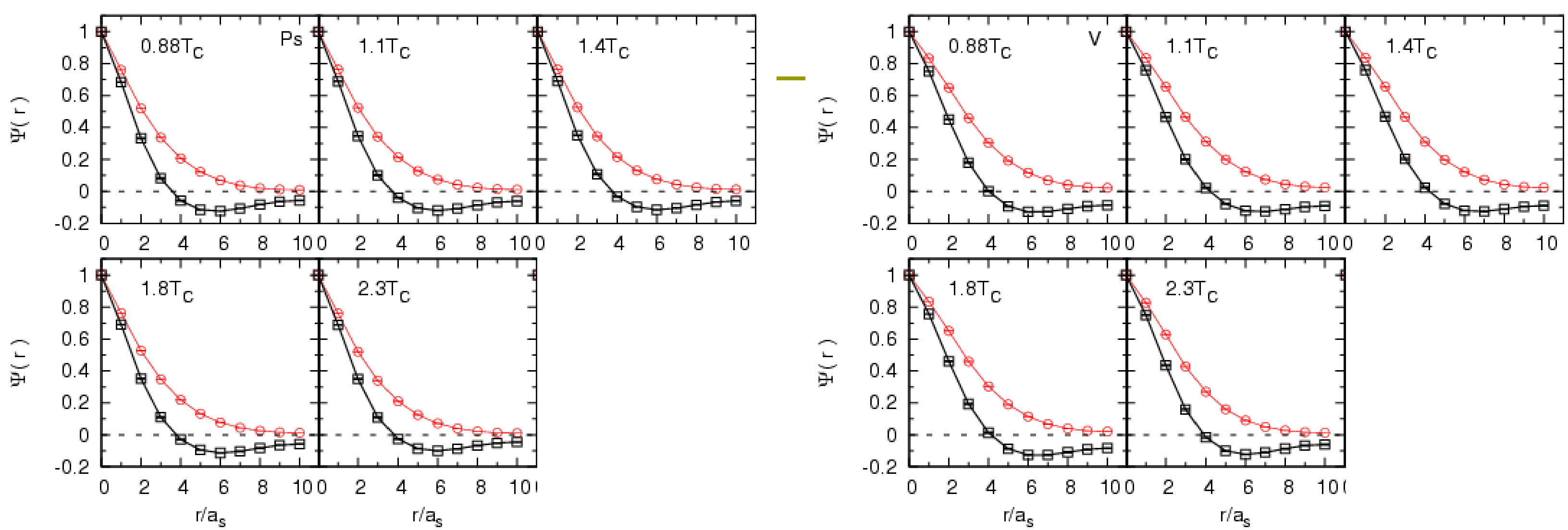
- Our method well reproduces the analytic solutions (!)

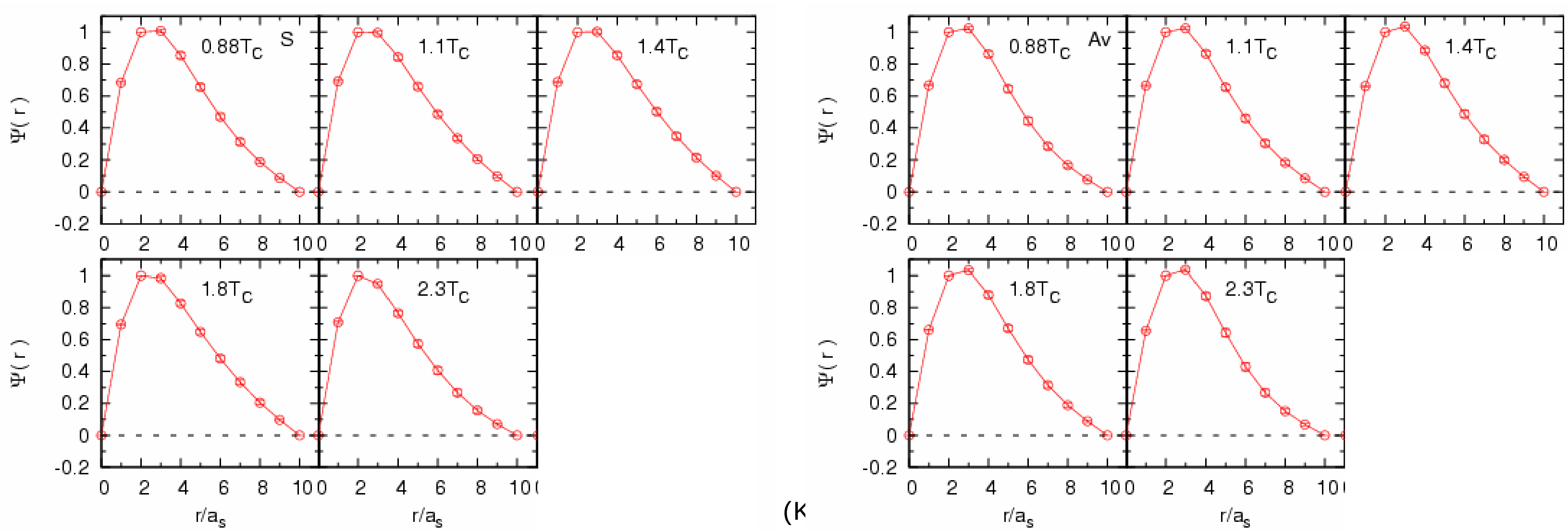
Charmonium wave functions at finite temperatures





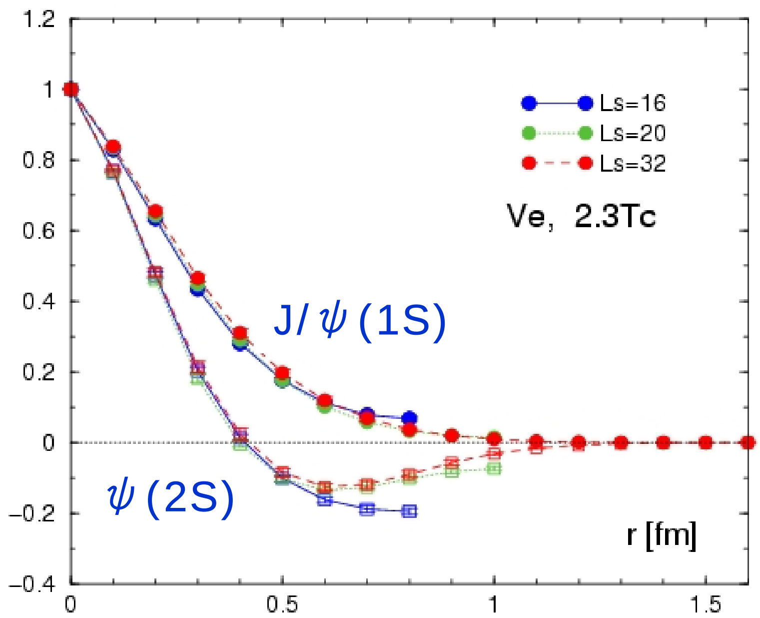
- Small temperature dependence in each channels
- Clear signals of bound states even at T=2.3Tc (!)
- Large volume is necessary for P-wave states.

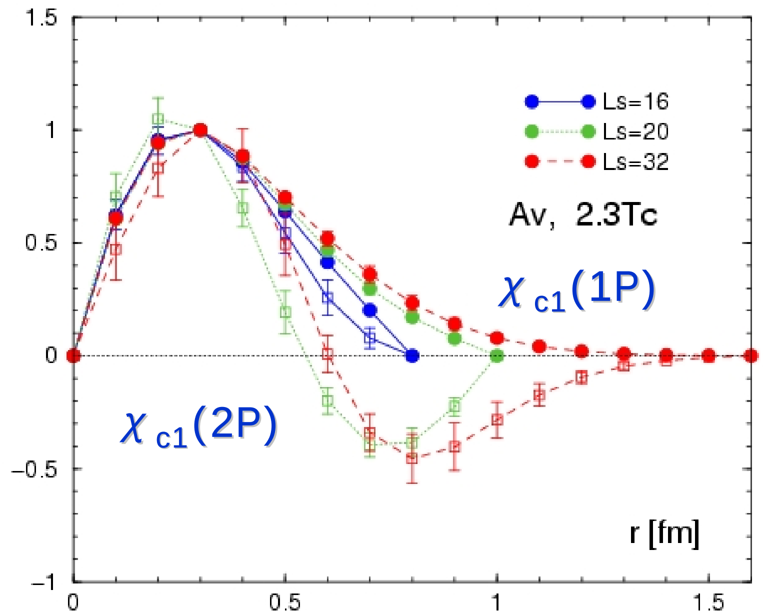

Ps
V
0.88T_C
1.1T_C
1.4T_C
1.8T_C
2.3T_C
Ψ(r)
r/a_s


S
Av
0.88T_C
1.1T_C
1.4T_C
1.8T_C
2.3T_C
Ψ(r)
r/a_s

l (K

Volume dependence at T=2.3Tc





- Clear signals of bound states even at T=2.3Tc (!)
- Large volume is necessary for P-wave states.

Summary and future plan

We investigated T_{dis} of charmonia from Lattice QCD
using another approach to study charmonium at T>0
without Bayesian analysis

- boundary condition dependence
- Wave function (Volume dependence)

No evidence for unbound c-c quarks up to T = 2.3 Tc

→ The result may affect the scenario of J/ψ suppression.

Future plan

- Interpretations of the experimental results on J/ψ suppression
- Higher Temp. calculations (T/Tc=3~5)
- Full QCD calculations (Nf=2+1 Wilson is now in progress)