The hyperfine splitting of charmonium in lattice QCD

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Workshop on quarkonium

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Contents

Introduction (1)

Charmonium spectrum on lattice QCD

lattice cutoff $1/a \gg m_{charm}$ is necessary

→ large computational cost !

- NRQCD
- HQET
- Fermilab

} Recent progress will be presented in the next talk !

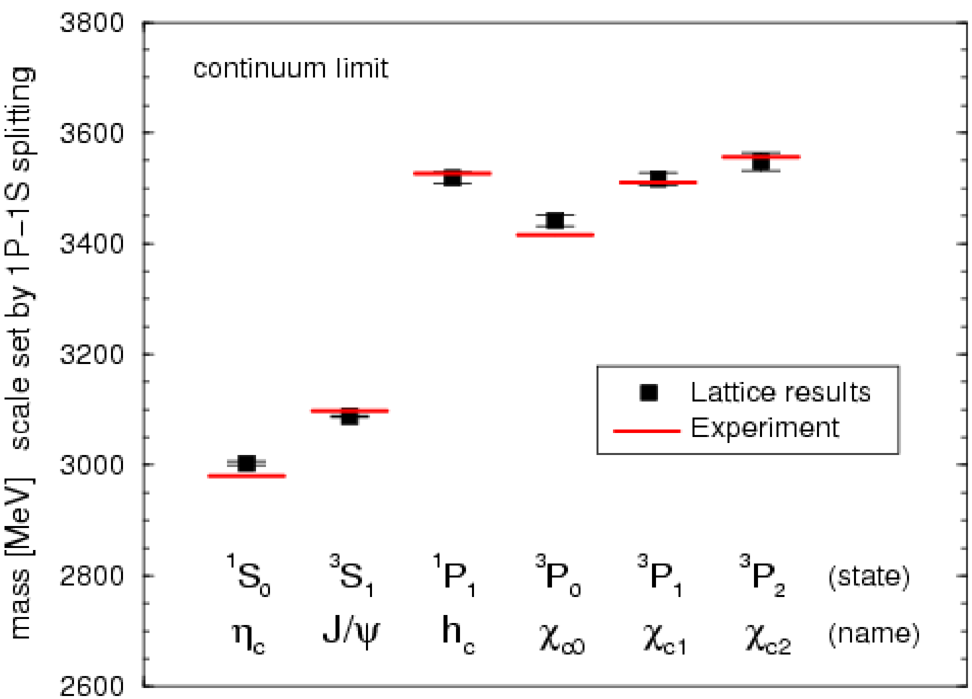
- Anisotropic lattice → next page

where the condition may be relaxed

to $1/a_t \gg m_{charm}$, $1/a_s \sim m_{charm}$?

Introduction (2)

M.Okamoto et al. (CP-PACS), Phys. Rev. D65 (2002)094508



M.Okamoto et al. Phys. Rev. D65 (2002) 094508

- quenched anisotropic lattice
- tree-level clover quark
- continuum extrapolation
  with $1/a_s = 1.0 \sim 2.9$ GeV
  - reliable extrapolation needs $1/a_s \gg m_{charm}$
  - large dep. on Dirac op.

Good agreement with the experimental value

except for the hyperfine splitting

A study by QCD-TARO Collaboration

QCD-TARO Collabration, JHEP08 (2003) 022

- Quenched QCD (without dynamical quarks)
- Isotropic lattices with large cutoff $1/a \gg m_{charm}$
- Nonperturbative improved clover quark

  M.Lüscher et al., Nucl.Phys.B491(1997)323

- Continuum extrapolation

Lattice setup

Gauge field : plaquette gauge

Quark field : nonperturbative improved clover quark

(tree level clover quark, standard Wilson)

beta	$L^3 \times T$	a(fm)	La(fm)	Csw	#conf
6.0	$18^3 \times 48$	0.0931	1.68	1.769	190
6.2	$24^3 \times 72$	0.0677	1.62	1.614	90
6.4	$32^3 \times 96$	0.0513	1.64	1.526	60
6.6	$32^3 \times 96$	0.0397	1.27	1.467	130

scale set by r_0 , quark mass set by m(J/ψ)

finite volume effect is negligible at La > 1.1 fm

Scale determination

- Lattice cutoff

  Sommer scale $r_0 \simeq 0.5$ fm

  r_0 ~ 10 % uncertainty (due to dynamical quark effects)

  Hadron mass (e.g. m(1P)−m(1S))

  for example CP-PACS results ~ 16 % uncertainty

  ▶ discrepancy caused by

  scaling violation & dynamical quark effects

- Quark mass

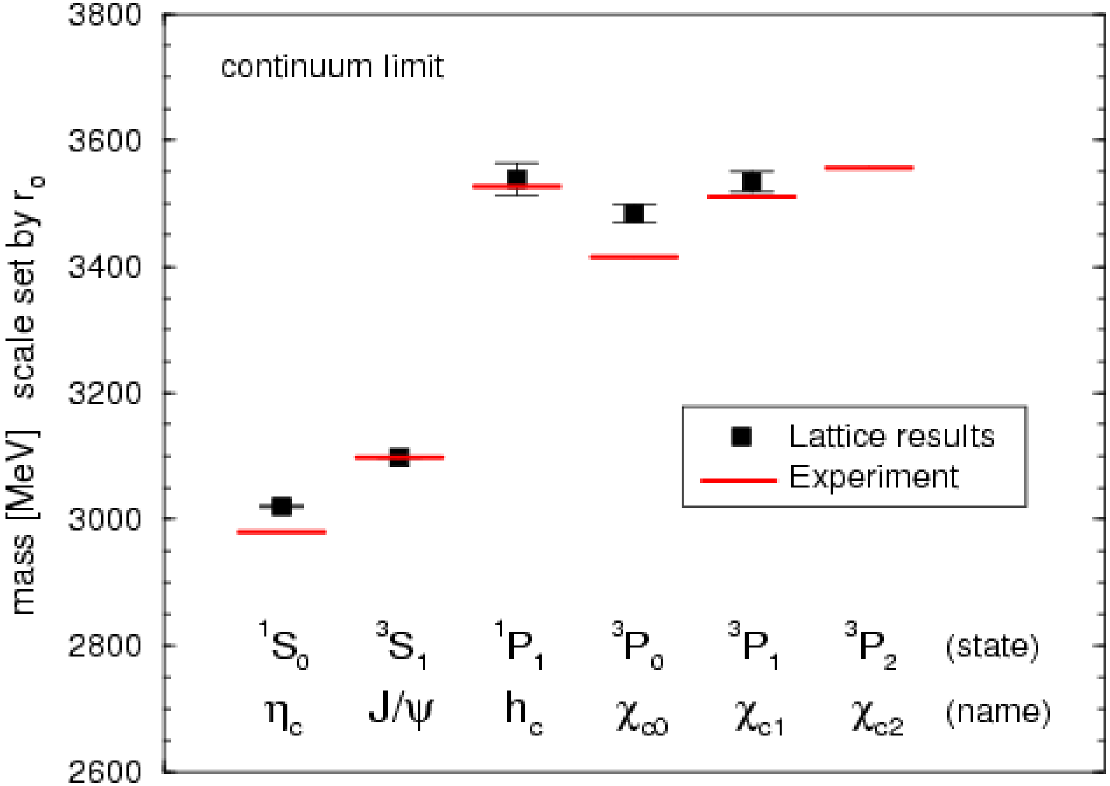
  matching with a physical charmonium mass

  m(J/ψ) = 3.097 GeV

  m(η_c) = 2.980 GeV

  } ~ 6 % uncertainty

Charmonium spectrum



QCD–TARO Collaboration. JHEP08 (2003) 022

- nonperturbative clover
- continuum limit

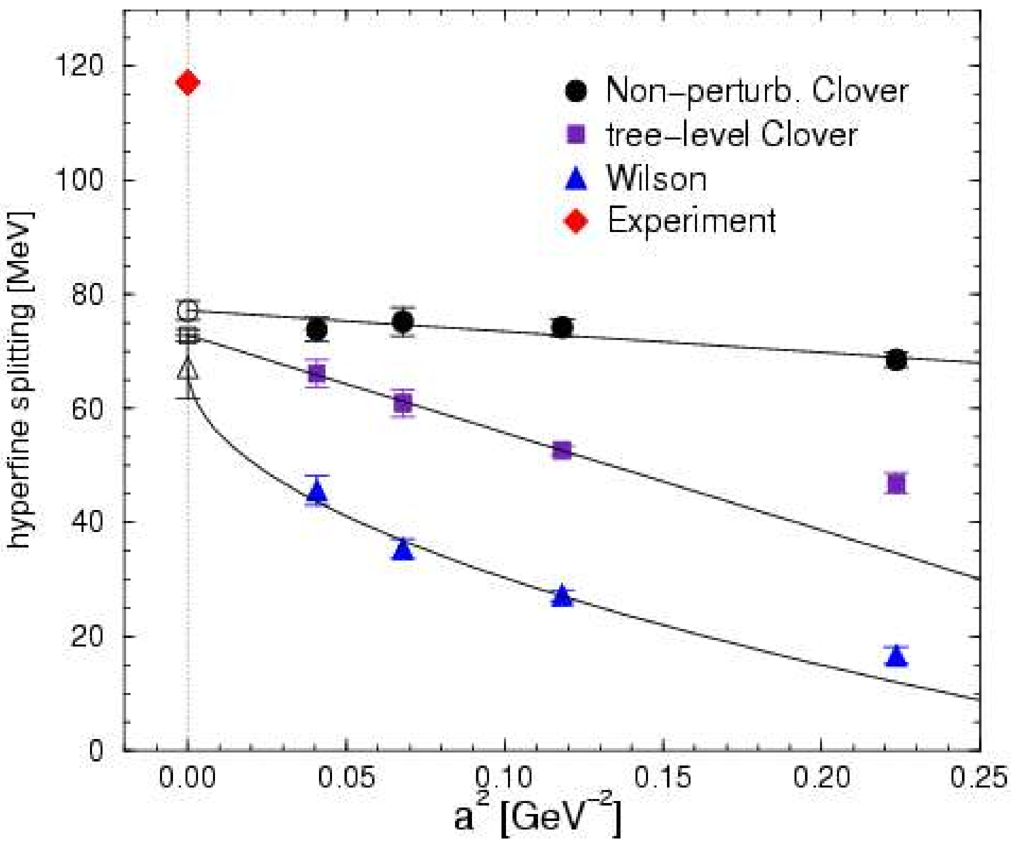
quark mass set by $m(J/\psi)$

scale set by r_0

The results are consistent with the previous works

hyperfine splitting is smaller than the Exp. value

Hyperfine splitting in the a=0 limit



- nonperturbative clover $O(a^2)$
- tree-level clover $O(g^2 a), O(a^2)$
- standard Wilson $O(a)$

- nonperturbative clover shows small scale violation even for $m(J/\psi)\,a \sim 1.4$
- other Dirac op. consistent with each other in the a=0 limit for $m(J/\psi)\,a < 1$

$m(J/\psi) - m(\eta_c)$ = 77 MeV in the continuum limit

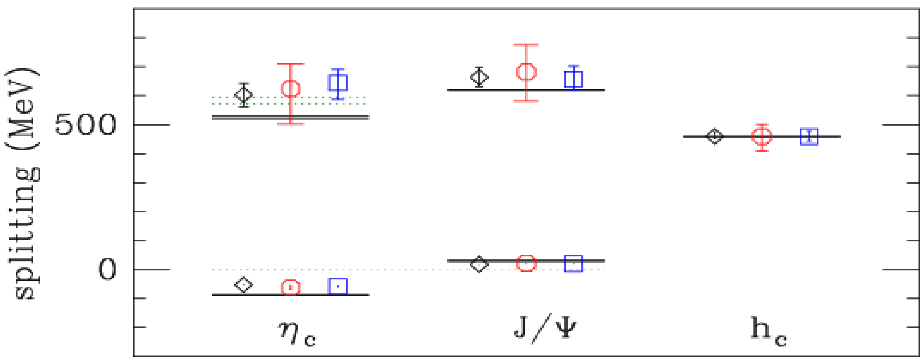
30–40% smaller than the experimental value = 117 MeV

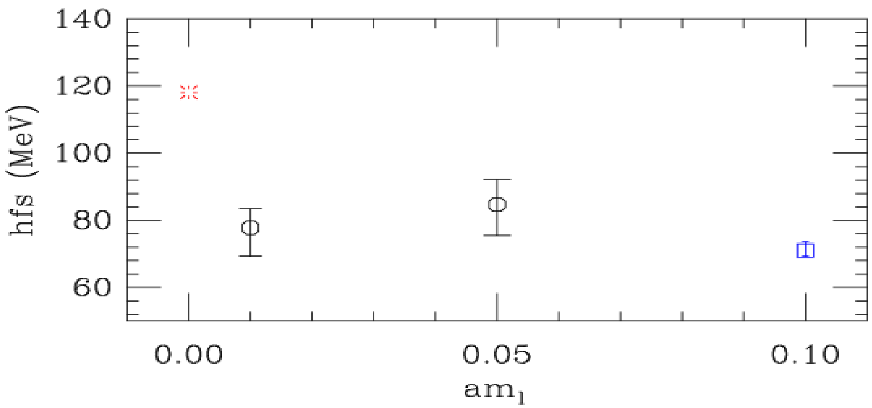
Contained uncertainties

$m(J/\psi) - m(\eta_c) = 77(2)(6)$ MeV (117MeV in Experiment)

- statistical error ~ 2 MeV
- quark mass determination (m_{charm}) ~ 5 MeV
- continuum extrapolation ~ 3 MeV
- finite volume effects ~ checked & negligible
- scale determination (1/a) ~ dynamical quark effects
- choice of Dirac operators ~ well controlled
- dynamical quark effects
- disconnected diagram contributions

Dynamical quark effects





Nucl. Phys. B(PS)119(2003)586

M.di Pierro et al.

- valence quark : Fermilab action
- sea quark : KS quark

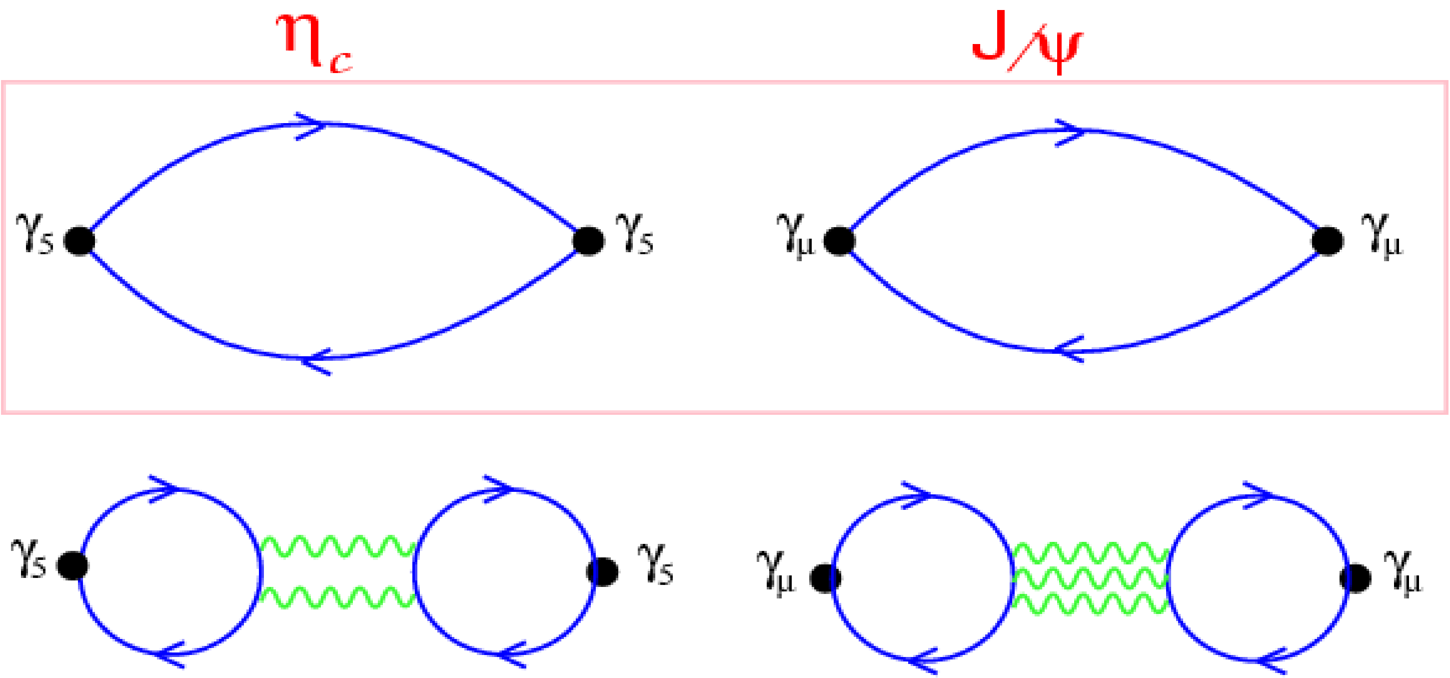
  Nf = 0, 3, 2+1
- no continuum limit a = 0.13 fm

Other full QCD studies → Dynamical quark effects ~ 10%

T.Manke et al., Phys. Rev. D62 (2000) 114508.

C.Stewart and R.Koniuk, Phys. Rev. D63(2001) 054503.

OZI forbidden “disconnected” diagram



Disconnected diagrams are neglected
because high cost & very small contribution

however, it may contribute to HFS ~ O(10) MeV ?

Charmonium correlators

$$C(t)=\sum_{\vec{x}}\langle Tr[\Gamma D^{-1}(0,\vec{0};t,\vec{x})\Gamma D^{-1}(t,\vec{x};0,\vec{0})]\rangle$$

$$D(t)=\sum_{\vec{x}}\langle Tr[\Gamma D^{-1}(0,\vec{0};0,\vec{0})]Tr[\Gamma D^{-1}(t,\vec{x};t,\vec{x})]\rangle$$

$D^{-1}(t,\vec{x};t',\vec{x}')$: quark propagator

- $\Gamma=\gamma_5,\gamma_\mu$ (Pseudoscalar, Vector)
- source & sink operators are extended with $\phi(\vec{x})\propto\exp(a|\vec{x}|^p)$
- disconnected diagrams are evaluated with the Z2-noise method

Lattice setup

- Sea quark : Nf=2 KS quark : $a m_q = 0.1$

  plaquette gauge : $\beta = 5.50$

  lattice size : $12^3 \times 24$

  lattice spacing : a=0.16fm (1/a=1.2GeV) set by r_0

  16,000 traj. (measurement at every 5 traj.)

☆ cutoff is not so sufficient for m_{charm} ,this is an exploratory study

- Valence quark : Fermilab action

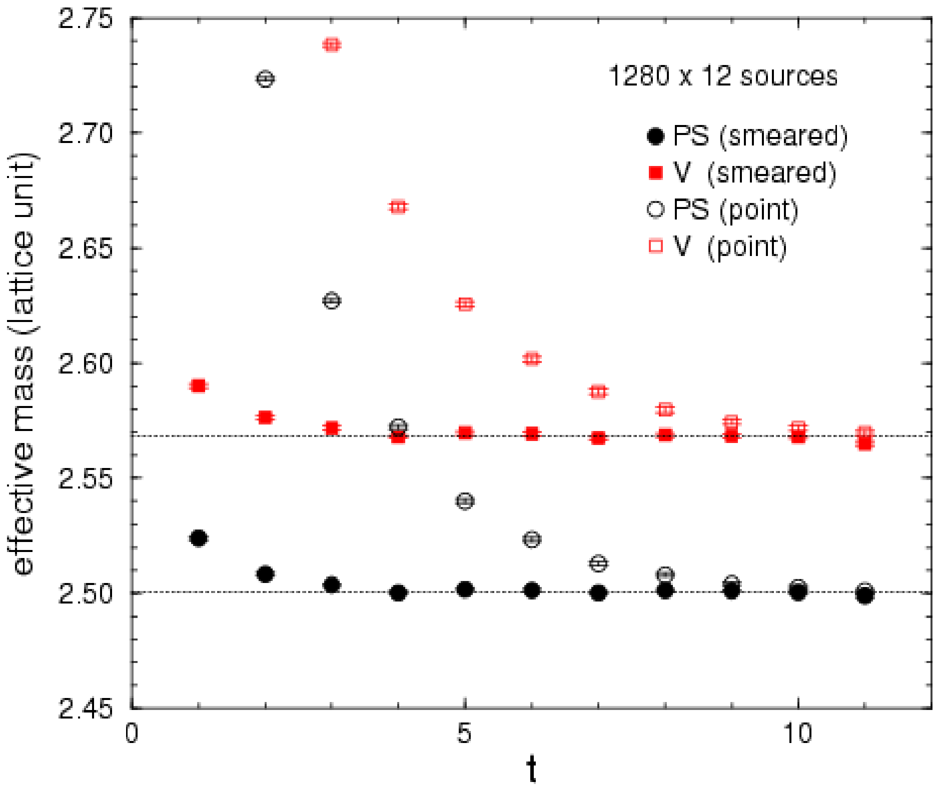
  Csw : tadpole improved tree-level (u_0 in Landau gauge)

  quark mass set by m(J/ψ)

- Z2-noise method

  N_{noise}=600

Smeared operators



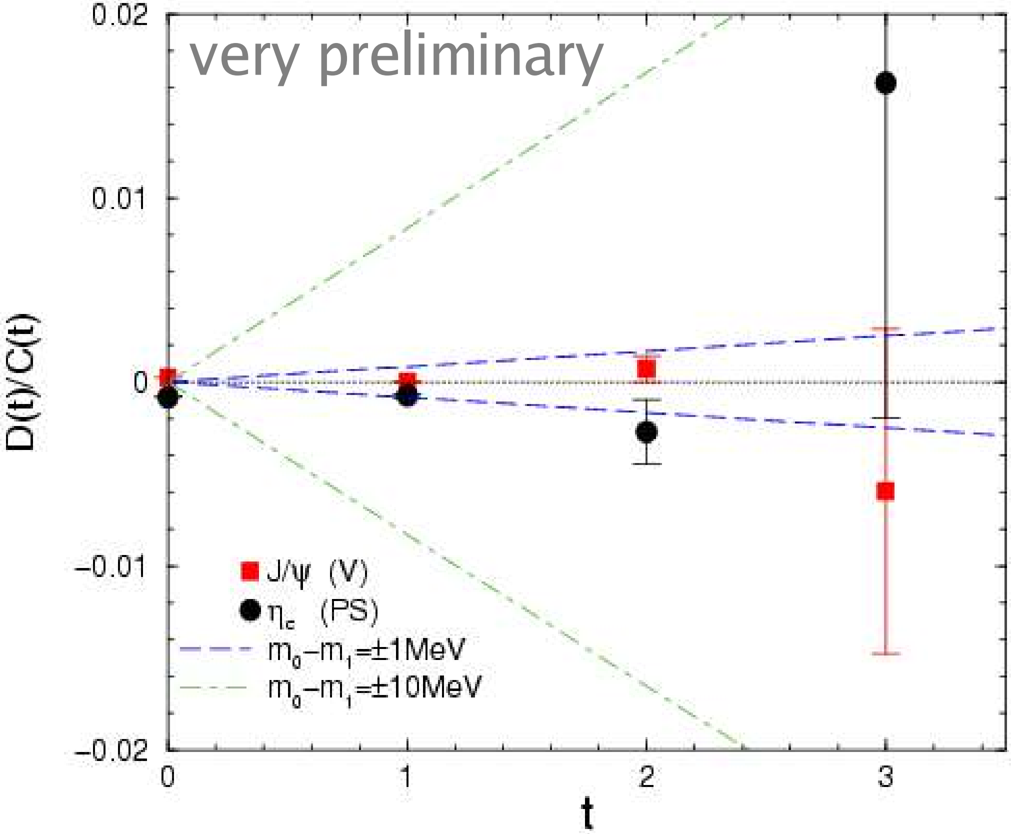
Effective mass plot
of connected diagram C(t)

Smearing functions $\phi(\vec{x})$ are
determined from wavefunction

Ground states dominate at t~2

$$a(\, m(\, J/\psi\,) - m(\, \eta_c\,)\,) = 0.0676 \,(\sim 81 \text{ MeV})$$

Disconnected diagram contributions



When ground state dominates,
We have

$$C(t)=c\exp(-m_1 t)$$

$$C(t)+D(t)=d\exp(-m_0 t)$$

$$\Downarrow$$

$$\frac{D(t)}{C(t)}=\frac{d}{c}\exp[(m_1-m_0)t]-1$$

- Too noisy at large t
- Contributions of the disconnected diagram may be very small ?

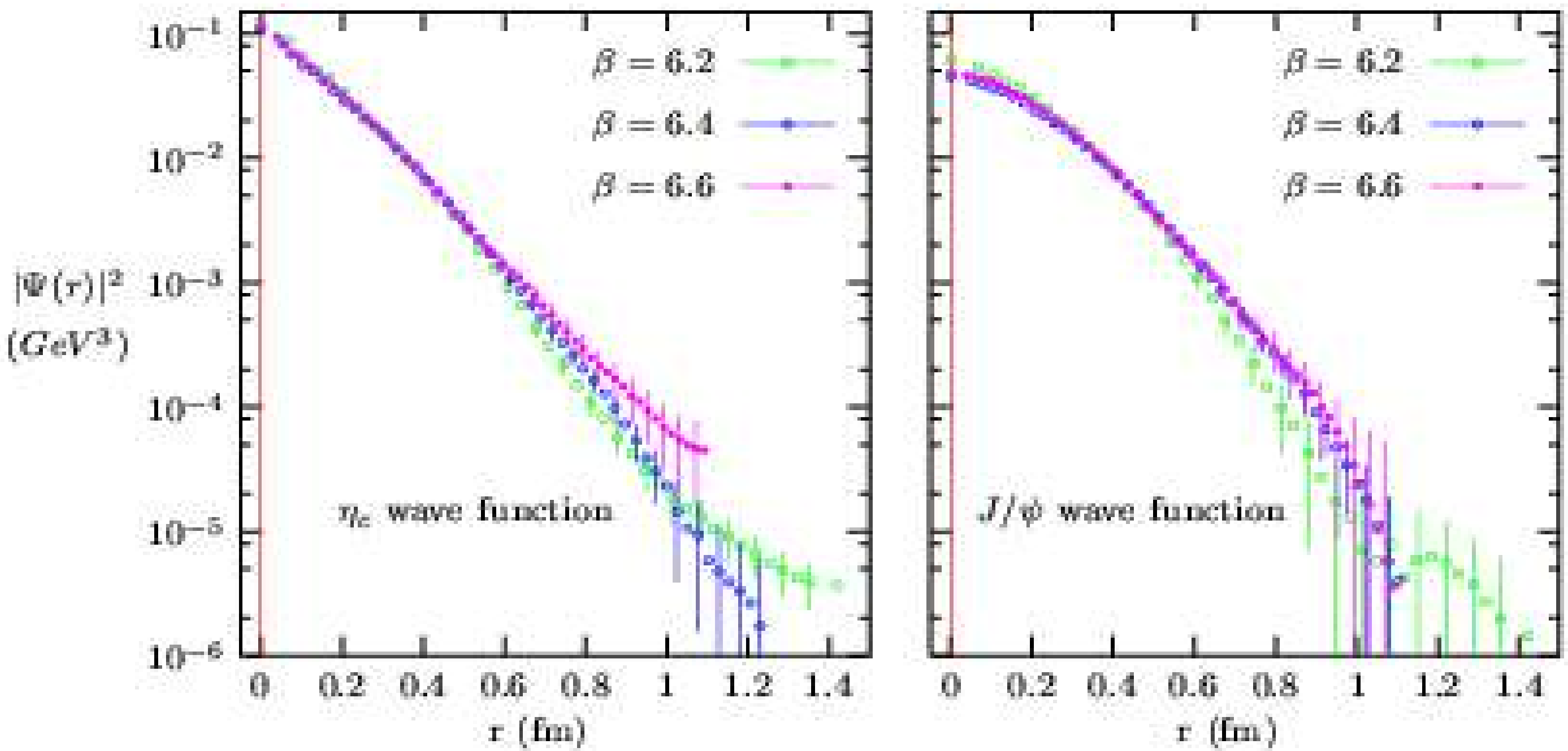
Summary & Outlook

We study the problem of charmonium HFS
and consider a possibility of disconnected diagram contributions

- Isotropic lattice + Nonperturbative improved clover quark
  is good choice for the charmonium spectrum
  In quenched QCD, $m(J/\psi) - m(\eta_c)$ is
  30 – 40% smaller than the experimental value

- Disconnected diagram contributions are very small
  or hidden by large error
  - Same calculations with smaller valence quark mass
    ➡ extrapolation from light quark mass region

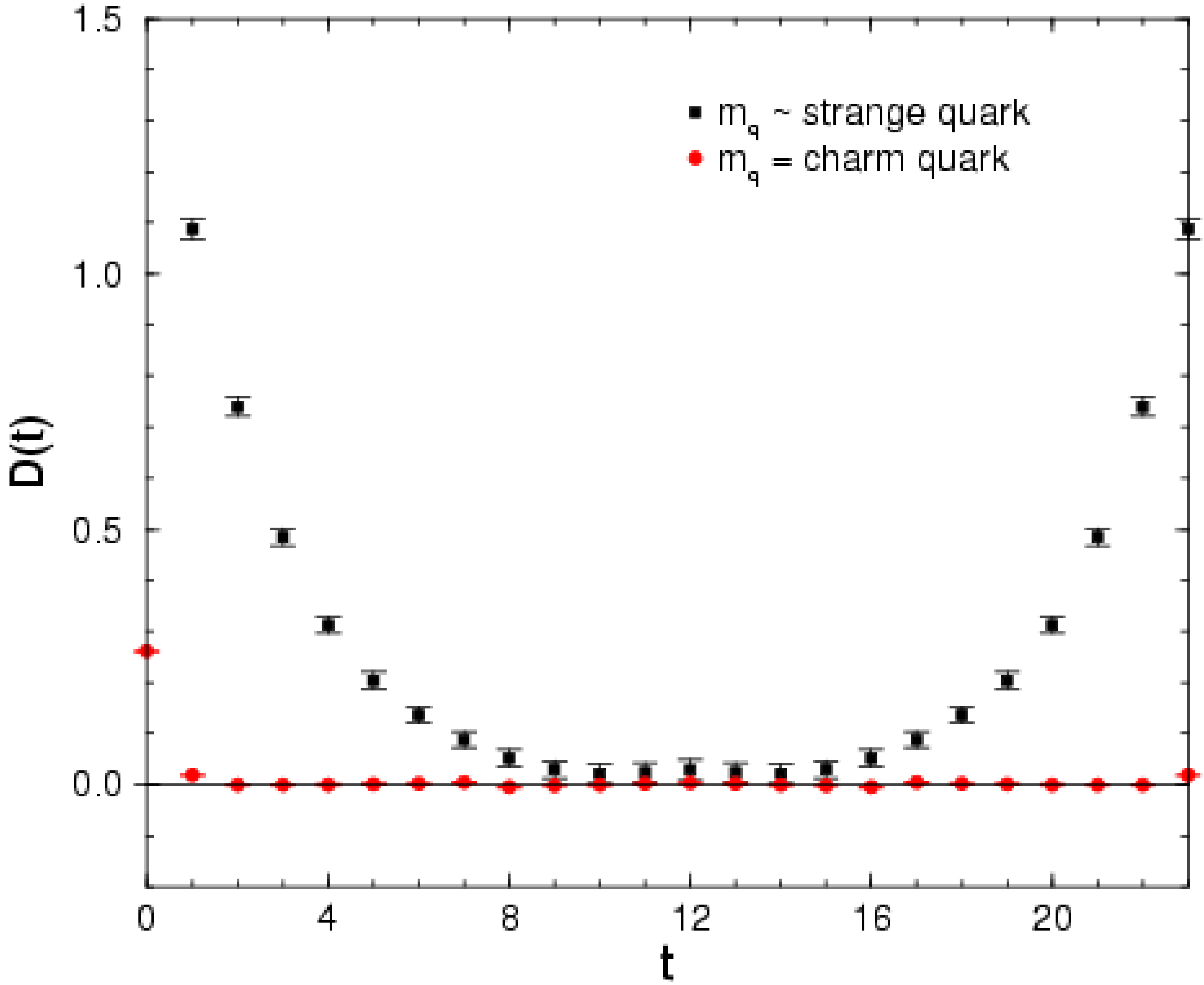
Appendix: Wave function



- scaling violations are very small
- $\Psi(\vec{0})$'s agree with phenomenological expectations qualitatively

In a heavy quark model : $\Psi_{\eta_c}(\vec{0}) > \Psi_{J/\psi}(\vec{0})$

Appendix: lighter valence quarks



Appendix: Z2-noise method