

# Quarkonium correlators at finite temperature



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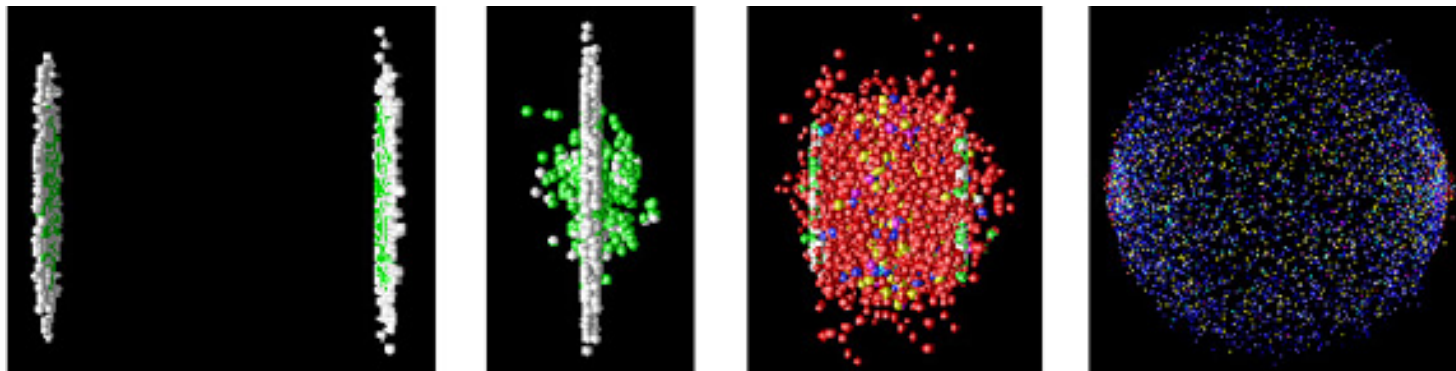
(WHOT-QCD Collaboration)

*Nara Women's University, Nara, Japan , December 2nd 2008*

# Contents of this talk

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from the Phenix group web-site



- Introduction

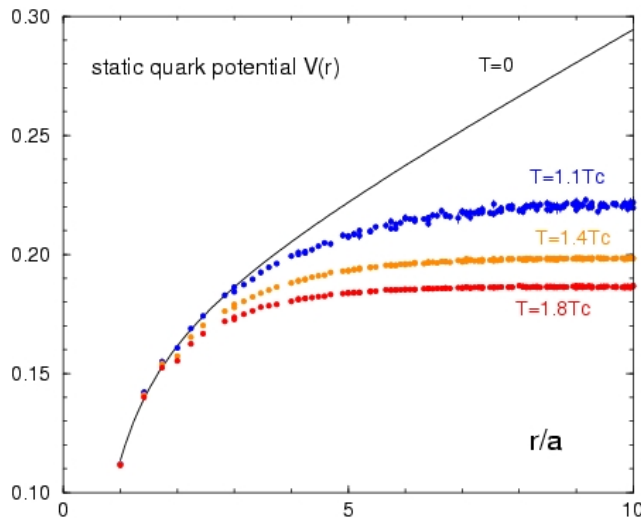
- Quark Gluon Plasma &  $J/\psi$  suppression
- Lattice studies on  $J/\psi$  suppression

- Our approach to study charmonium dissociation

- Charmonium wave functions at  $T > 0$

- Summary & future plan

# $J/\psi$ suppression as a signal of QGP



**Confined phase:**  
linear raising potential  
→ bound state of  $c - \bar{c}$

**De-confined phase:**  
Debye screening  
→ scattering state of  $c - \bar{c}$

T.Hashimoto et al.('86), Matsui&Satz('86)

## Lattice QCD calculations:

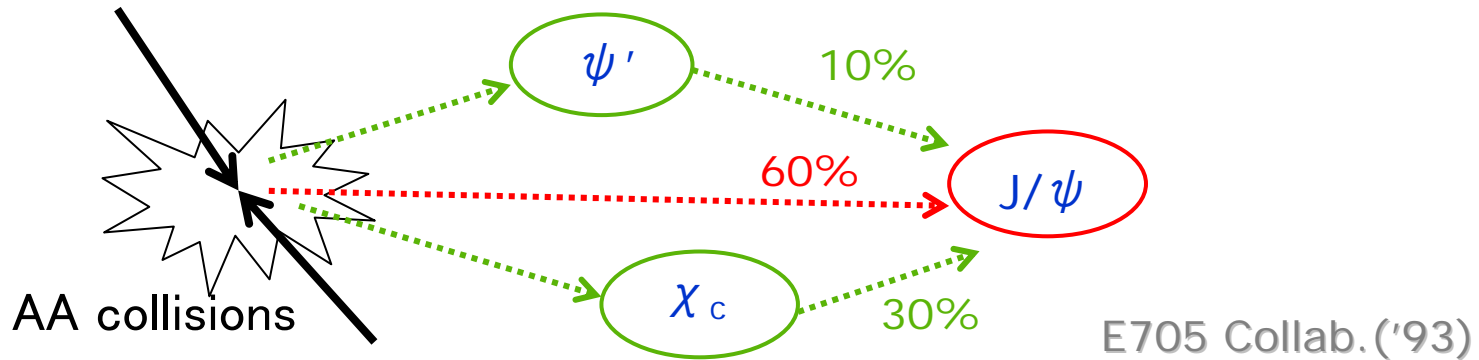
**Spectral function by MEM:** T.Umeda et al.('02), S.Datta et al.('04),  
Asakawa&Hatsuda('04), A.Jakovac et al.('07), G.Aatz et al.('06)

**Wave func.:** T.Umeda et al.('00)

**B. C. dep.:** H.Iida et al. ('06)

→ all calculations indicate that  $J/\psi$  survives till  $1.5T_c$  or higher

# Sequential $J/\psi$ suppression scenario



$J/\psi$ (1S) :	$J^{PC} = 1^{--}$	M=3097MeV	(Vector)
$\psi$ (2S) :	$J^{PC} = 1^{--}$	M=3686MeV	(Vector)
$\chi_{c0}$ (1P) :	$J^{PC} = 0^{++}$	M=3415MeV	(Scalar)
$\chi_{c1}$ (1P) :	$J^{PC} = 1^{++}$	M=3511MeV	(AxialVector)

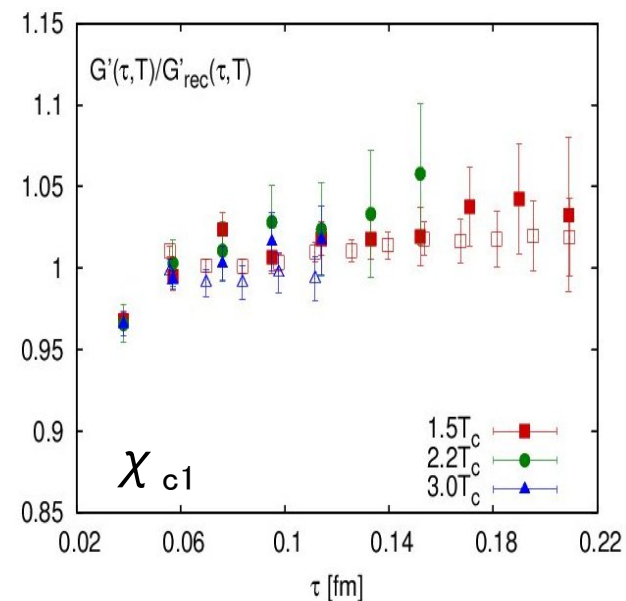
PDG('06)

It is important to study dissociation temperatures for not only  $J/\psi$  but also  $\psi$  (2S),  $\chi_c$ 's

# Current status on charmonium $T_{\text{dis}}$

- Lattice QCD studies ( by MEM analysis ) indicate
  - $J/\psi$  may survive up to  $T=1.5T_c$  or higher
  - $\chi_c$  may dissolve just above  $T_c$   
*e.g. A.Jakovac et al. (2007)*
  - no results on excited states,  $\psi'$
- The 2nd statement may be misleading (!)  
small change even in P-wave correlators  
up to  $3T_c$  w/o the constant mode
- On the other hand,  
potential model studies suggest  
charmonium dissociation may also  
provide small change in the correlators  
*e.g. A.Mocsy et al. (2007)*

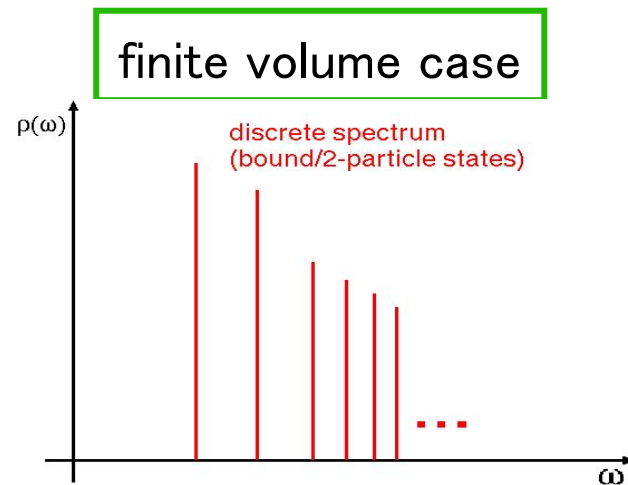
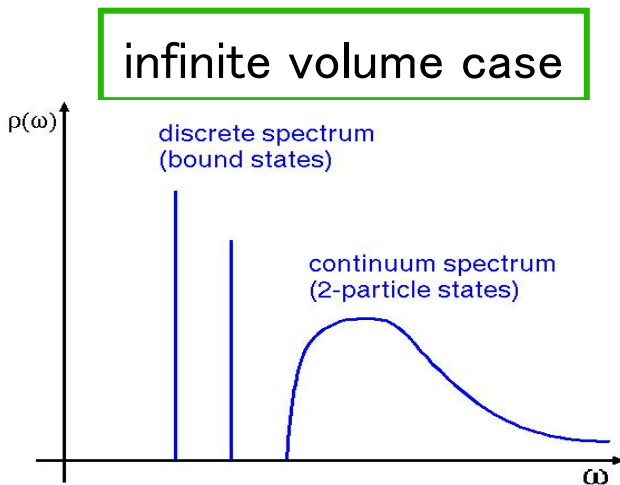
Therefore we would like to investigate  $T_{\text{dis}}$   
using new approaches in Lattice QCD  
without Bayesian analyses



**Fig. 3.** The ratio of the derivatives  $G'(\tau, T)/G'_{\text{rec}}(\tau, T)$  in the scalar channel (top) and axial-vector channel (bottom) calculated at  $\beta = 7.192$ . The results from anisotropic lattice calculations at  $\beta = 6.5$  [26] are also shown (open symbols).

*P.Petreczky (2008)*

# Spectral functions in a finite volume box



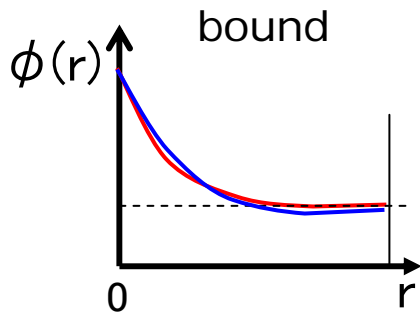
In a finite volume,  
discrete spectra does not always indicate bound states !

In order to study a few lowest states,  
the variational analysis is one of the most reliable approaches !

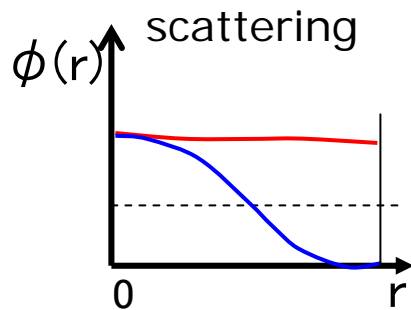
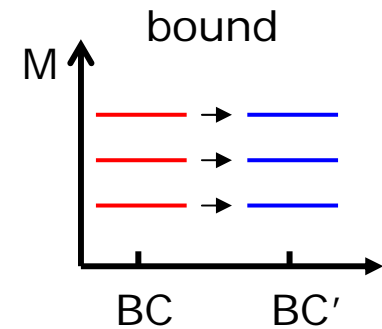
$N \times N$  correlation matrix :  $C(t)$

$$C(t)\psi = \lambda(t, t_0)C(t_0)\psi \quad \lambda_i(t, t_0) = e^{-E_i(t-t_0)}$$

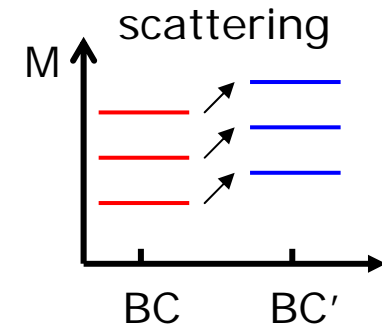
# Boundary condition dependence



The wave functions are localized,  
their energies are insensitive to B.C.



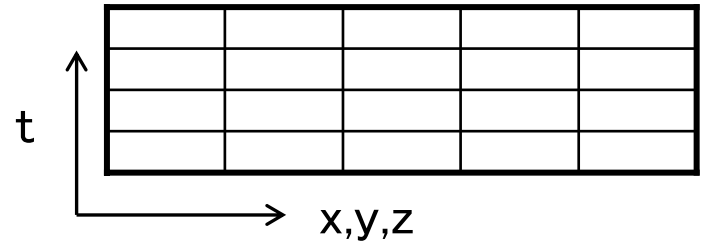
The momenta depends on B.C.,  
the scattering state energies  
are sensitive to B.C.



*The idea has been already applied to the charmonium study  
in H. Iida et al., PRD74, 074502 (2006).*

# Lattice setup

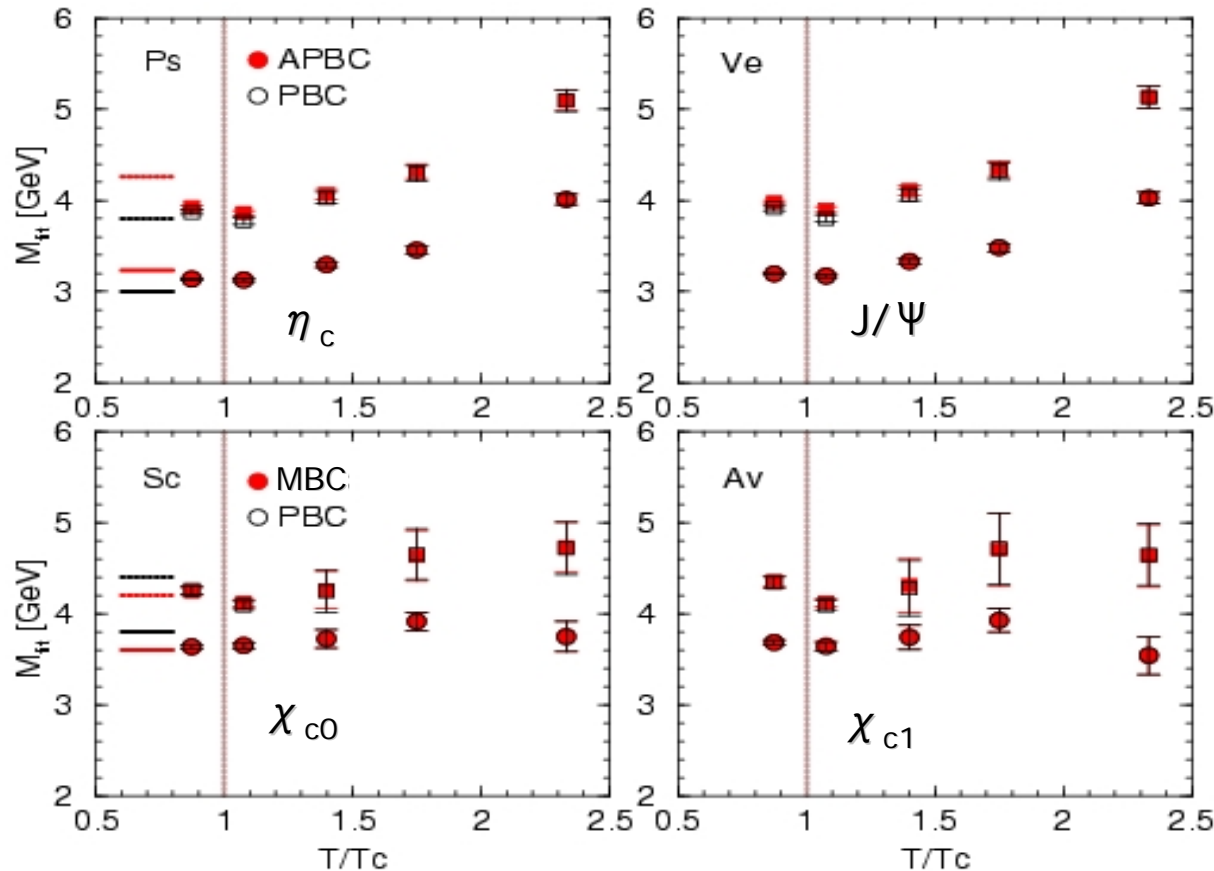
- Quenched approximation ( no dynamical quark effect )
- Anisotropic lattices
  - lattice spacing :  $a_s = 0.0970(5)$  fm
  - anisotropy :  $a_s/a_t = 4$
- $r_s=1$  to suppress doubler effects
- Variational analysis with 4 x 4 correlation matrix



$N_t$	32	26	20	16	12
$T/T_c$	0.88	1.08	1.40	1.75	2.33
# of conf.					
$V=16^3$	300	300	300	300	300
$V=20^3$	300	300	300	300	300
$V=32^3$	—	—	—	—	100



# Temperature dependence of charmonium spectra



$$q(x_i + L_i) = b_i q(x_i)$$

$b_i = 1$  : periodic  
 $b_i = -1$  : anti-periodic

PBC :  $b=(1, 1, 1)$   
 APBC :  $b=(-1, -1, -1)$   
 MBC :  $b=(-1, 1, 1)$

an expected shift  
 in  $V=(2\text{fm})^3$   
 (free quark case)  
 $\sim 200\text{MeV}$

- No significant mass shift in the different B.C.s
- only statistical error (large systematic error at higher temp.)

# Wave functions at finite temperature

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Temp. dependence of ( Bethe-Salpeter ) "Wave function"

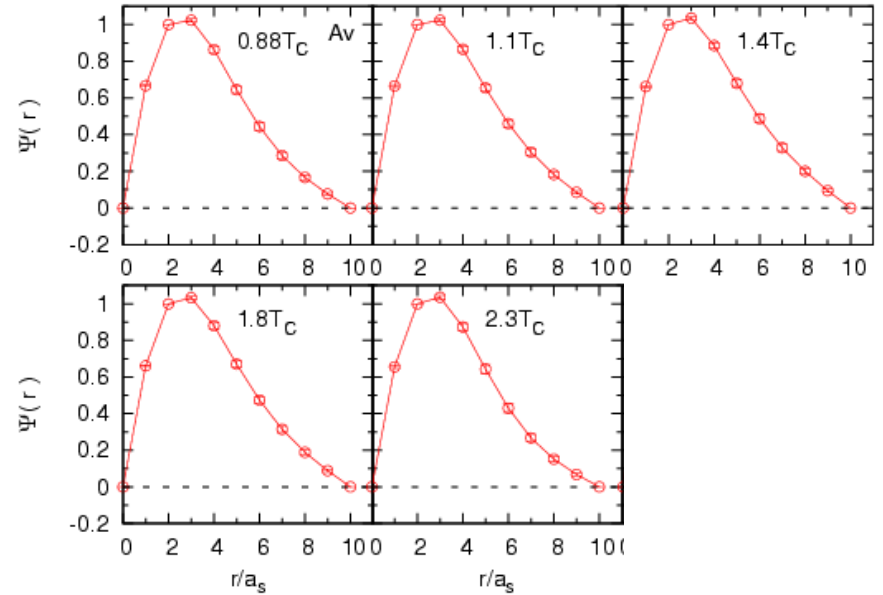
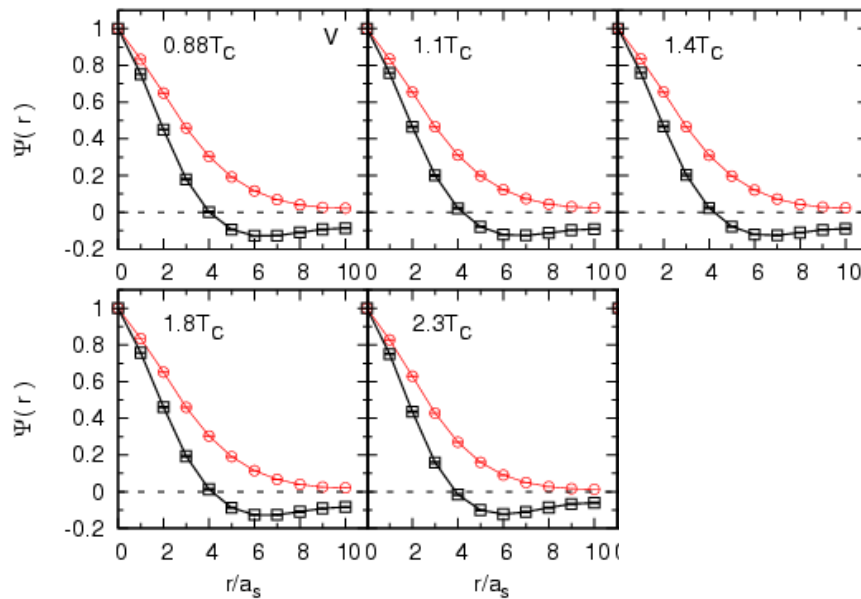
$$BS(\vec{r}, t) = \sum_{\vec{x}} \langle \bar{q}(\vec{x} + \vec{r}, t) \Gamma q(\vec{x}, t) \bar{q}(\vec{0}, 0) \Gamma q(\vec{0}, 0) \rangle$$

$$\Psi(|\vec{r}|, t) = BS(\vec{r}, t) / BS(\vec{r}_0, t)$$

$$\Gamma = \begin{cases} \gamma_5 & \text{(Ps)} \\ \gamma_i & \text{(Ve)} \quad (i = 1, 2, 3) \\ \sum_j \left( \vec{\partial}_j \gamma_j - \overleftarrow{\partial}_j \gamma_j \right) & \text{(Sc)} \\ \sum_{j,k} \epsilon_{ijk} \left( \vec{\partial}_j \gamma_k - \overleftarrow{\partial}_j \gamma_k \right) & \text{(Av)} \quad (i = 1, 2, 3) \end{cases}$$

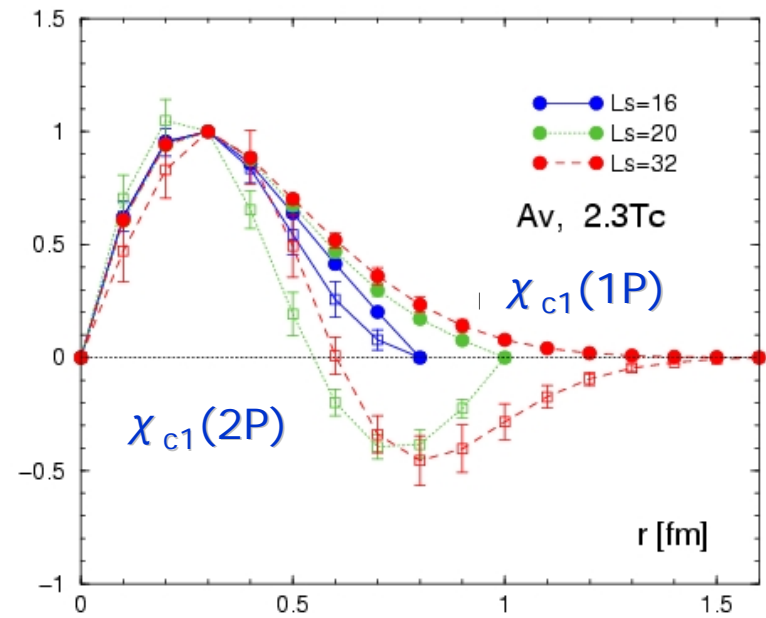
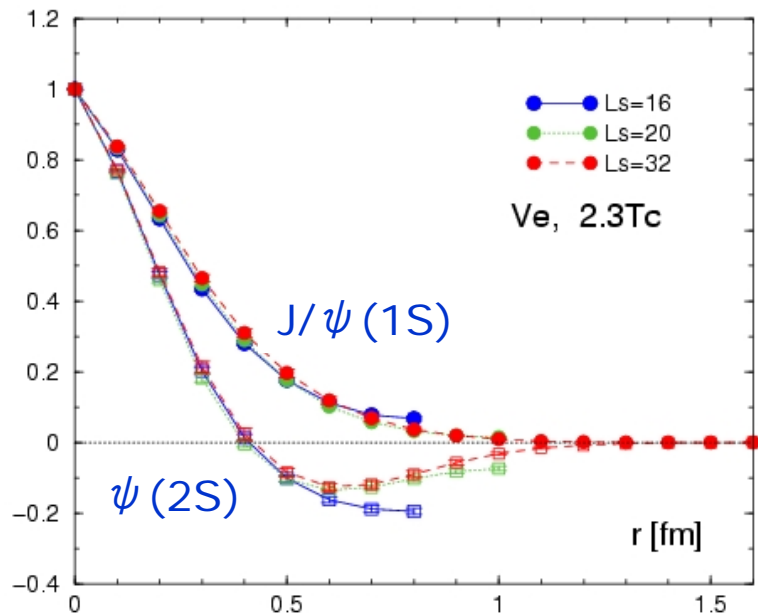
using the eigen functions of the variational analysis  
→ we can extract the wave functions of each states

# Charmonium wave functions at finite temperatures



- Small temperature dependence in S- and P-wave states
- No signal of scattering states even at  $T=2.3T_c$
- Large volume is necessary for P-wave states.

# Volume dependence at $T=2.3T_c$



- No signal of scattering states even at  $T=2.3T_c$
- P-wave state W.F. depends on volume, but these are well localized.

# Summary and future plan

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We investigated  $T_{\text{dis}}$  of charmonia from Lattice QCD using another approach to study charmonium at  $T > 0$  without Bayesian analysis

- boundary condition dependence
- Wave function (Volume dependence)

No hint for unbound  $c\bar{c}$  quarks up to  $T = 2.3 T_c$

→ The result may affect the scenario of  $J/\psi$  suppression.

## Future plan

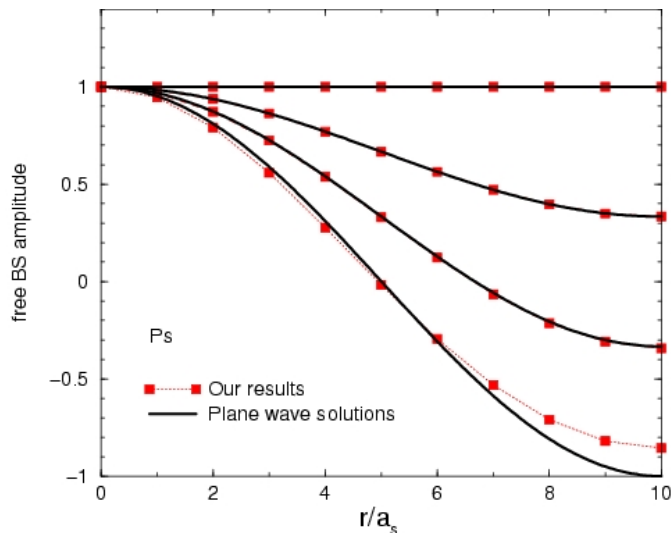
- Interpretations of the experimental results on  $J/\psi$  suppression
- Full QCD calculations (  $N_f=2+1$  Wilson is now in progress )

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Thank you very much !!

# Wave functions in free quark case

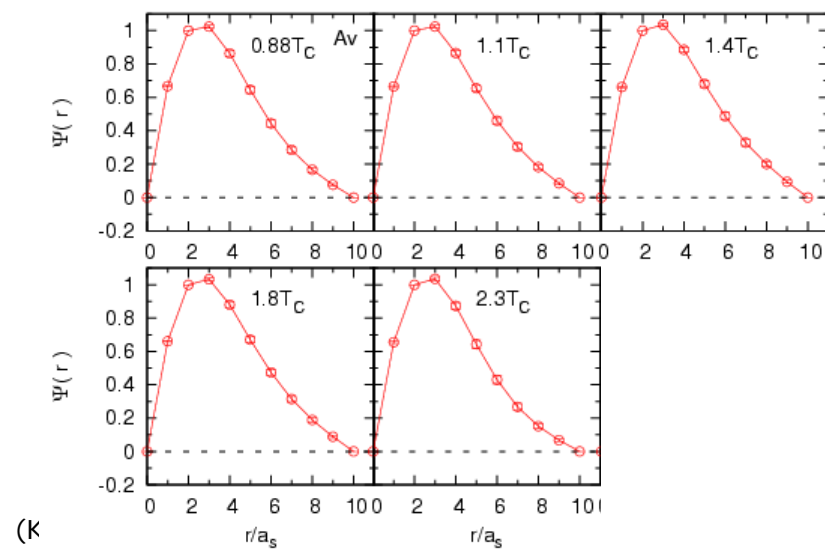
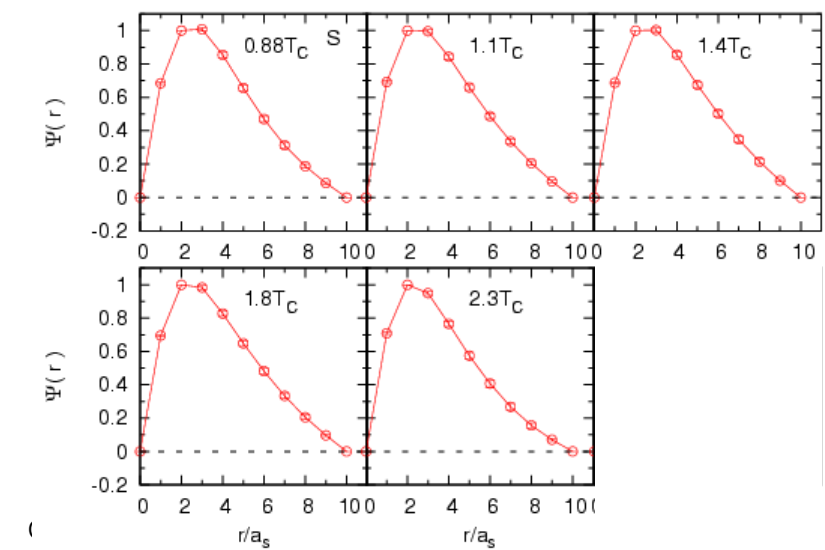
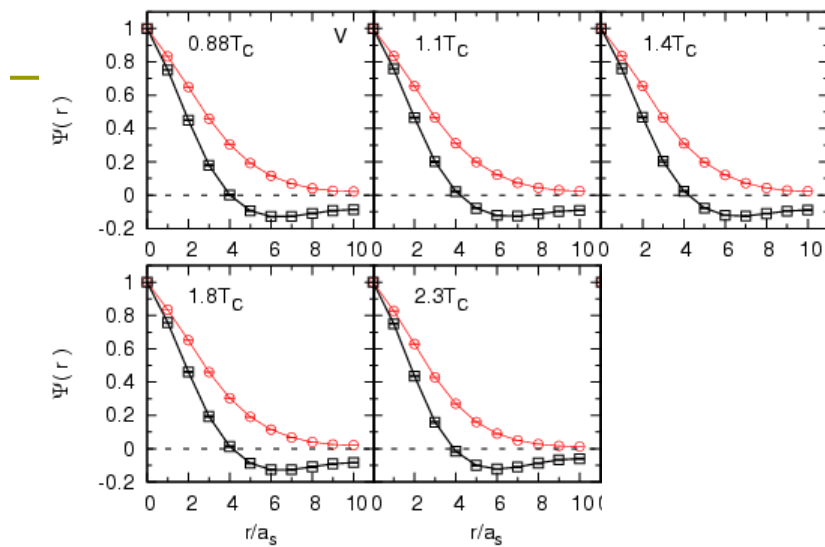
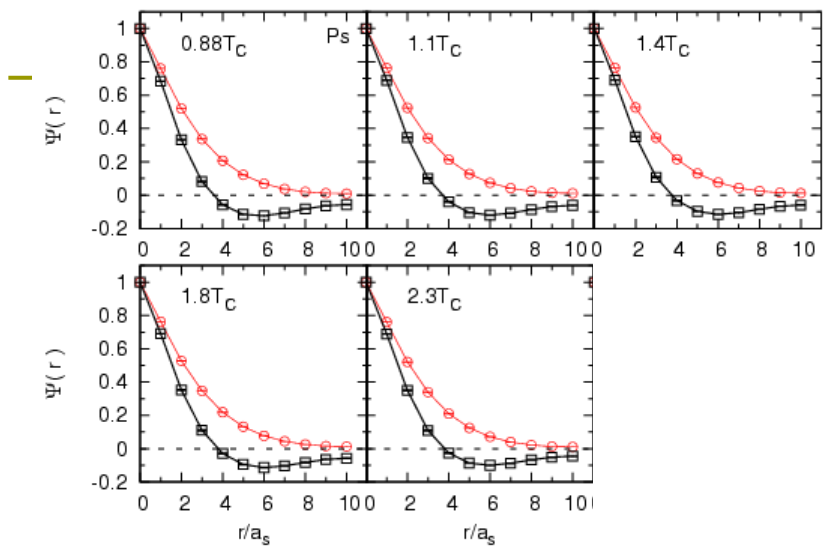
Test with free quarks (  $L_s/a=20$ ,  $ma=0.17$  )  
in case of S-wave channels



- Free quarks make trivial waves with an allowed momentum in a box

$$\Psi_k(|\vec{r}|, t) = \frac{\sum_{\vec{p}=\vec{k}} \cos(p_1 r_1) \cos(p_2 r_2) \cos(p_3 r_3)}{\sum_{\vec{p}=\vec{k}} 1}$$

- The wave function is constructed with eigen functions of 4 x 4 correlators
- Our method well reproduces the analytic solutions ( ! )



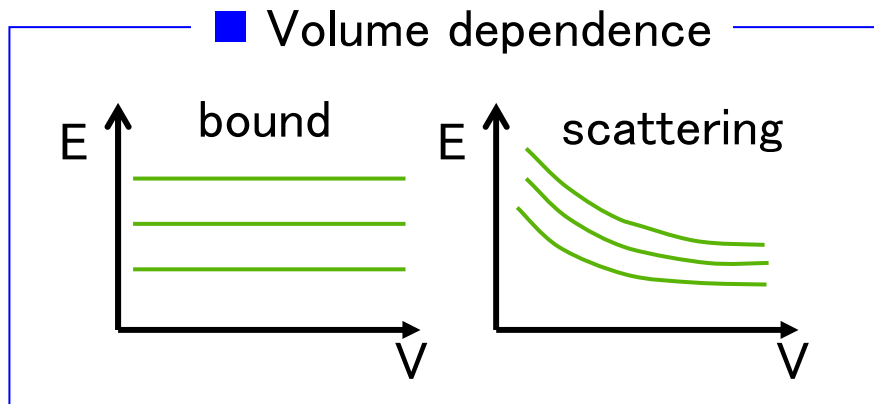
(J)

(K)



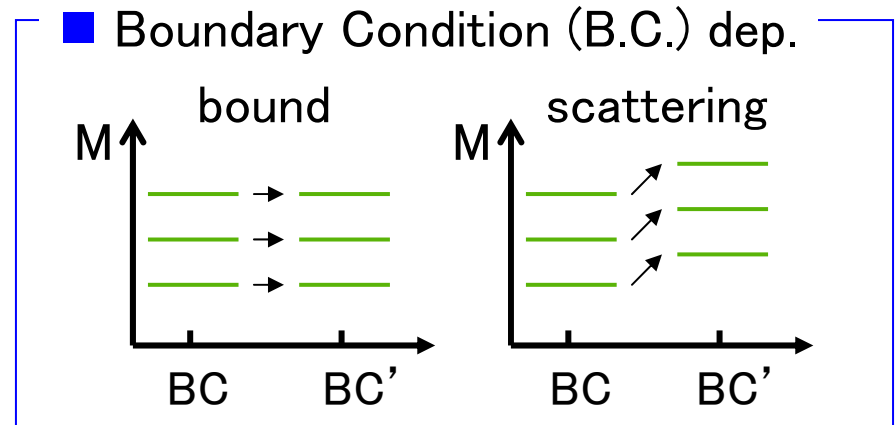
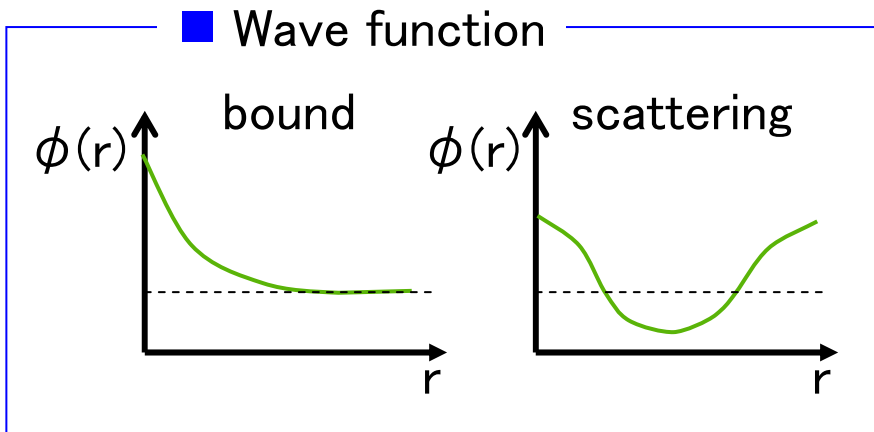
# Bound state or scattering state ?

We know three ways to identify the state in a finite volume



$E$  : energy  
 $V$  : volume

$\Phi(r)$  : wave function  
 $r$  :  $c - \bar{c}$  distance



H.Iida et al.('06), N.Ishii et al.('05)

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- Lattice QCD studies ( by MEM analysis ) indicate
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*e.g. A.Jakovac et al. (2007)*
  - no results on excited states,  $\psi'$

- The 2nd statement may be misleading (!)  
small change even in P-wave state  
up to  $1.4T_c$  w/o the constant mode

- On the other hand,  
the potential model studies suggest  
charmonium dissociation may also  
provide small change in the correlators  
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Therefore we would like to investigate  $T_{\text{dis}}$   
using new approaches in Lattice QCD  
without Bayesian analyses

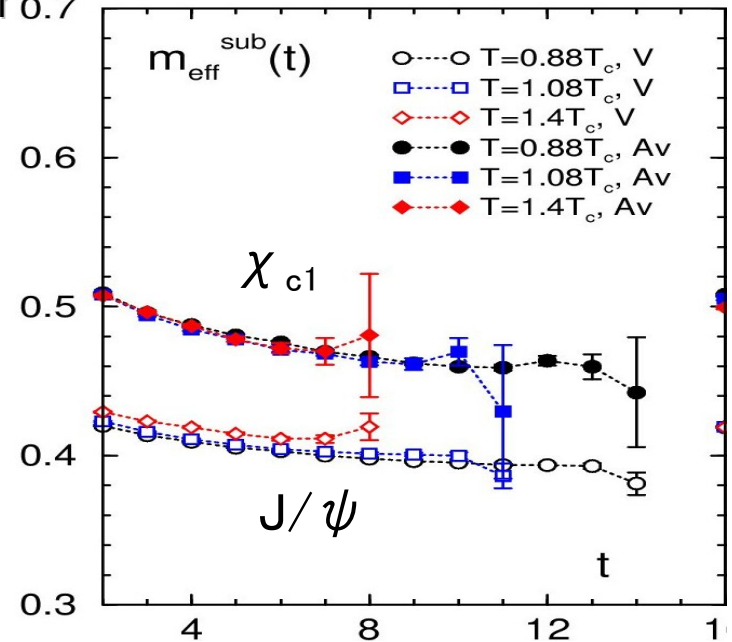


Fig: Temp. dependence of  $M_{\text{eff}}(t)$   
for  $J/\psi$ ,  $\chi_{c1}$  w/o constant mode.  
*T.Umeda (2007)*