# Hot wave function from lattice QCD

T. Umeda (Hiroshima Univ.)
H. Ohno, K. Kanaya (Univ. of Tsukuba)

for WHOT-QCD Collaboration

広島大学

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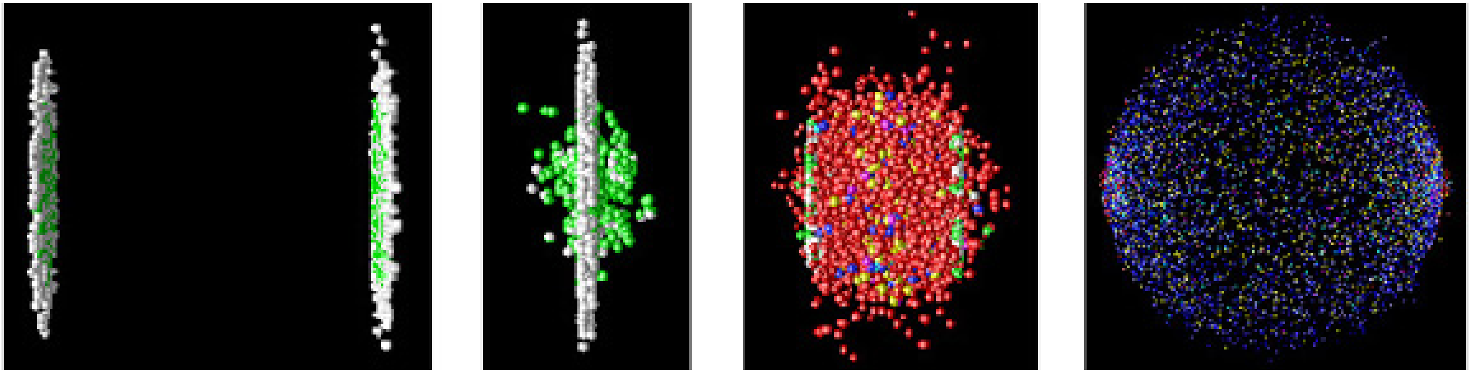
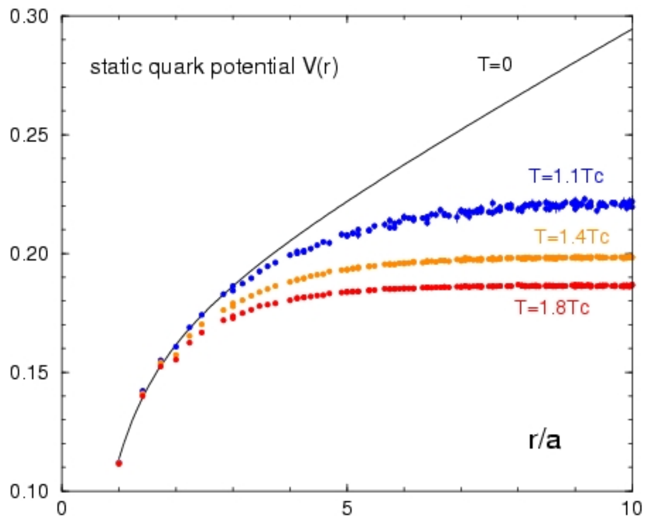
# Contents of this talk

from the Phenix group web-site



- Introduction
  - -- Quark Gluon Plasma & J/$\psi$ suppression
  - -- Lattice studies on J/$\psi$ suppression
- Our approach to study charmonium dissociation
- Charmonium wave functions at T>0
- Discussion & Summary

# $J/\psi$ suppression as a signal of QGP




Confinement phase:
 linear raising potential
 → bound state of c - $\bar{c}$

De-confinement phase:
 Debye screening
 → scattering state of c - $\bar{c}$

T.Hashimoto et al.('86), Matsui&Satz('86)

<u>Lattice QCD calculations:</u>

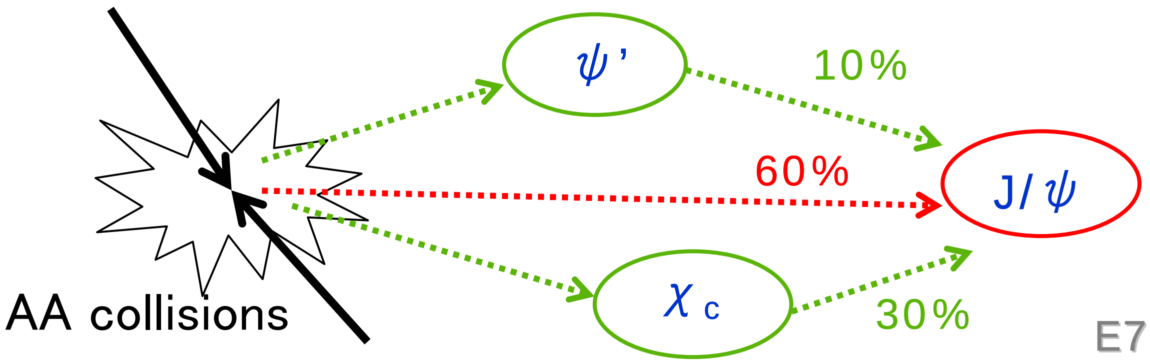
Spectral function by MEM: T.Umeda et al.('02), S.Datta et al.('04), Asakawa&Hatsuda('04), A.Jakovac et al.('07), G.Aatz et al.('06)

Wave func.: T.Umeda et al.('00)

B. C. dep.: H.Iida et al. ('06)

→ all calculations conclude that $J/\psi$ survives till $1.5T_c$ or higher

# Sequential J/ψ suppression scenario




| | | | | |
|---|---|---|---|---|
| $J/\psi$ (1S) | : | $J^{PC} = 1^{--}$ | M=3097MeV | (Vector) |
| $\psi$ (2S) | : | $J^{PC} = 1^{--}$ | M=3686MeV | (Vector) |
| $\chi_{c0}$ (1P) | : | $J^{PC} = 0^{++}$ | M=3415MeV | (Scalar) |
| $\chi_{c1}$ (1P) | : | $J^{PC} = 1^{++}$ | M=3511MeV | (AxialVector) |

PDG('06)

It is important to study dissociation temperatures for not only $J/\psi$ but also $\psi$(2S), $\chi_c$'s

# Hot QCD on the lattice

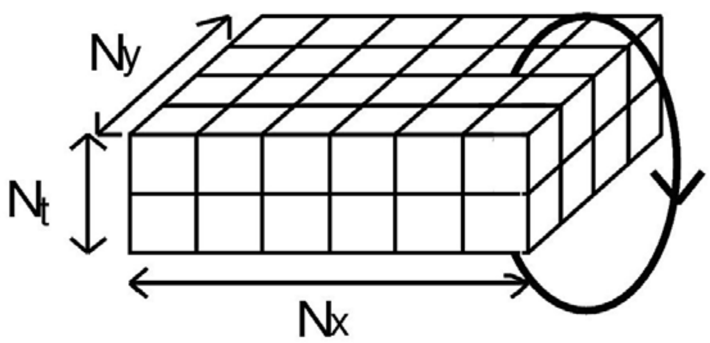
Lattice QCD enables us to perform

nonperturbative calculations of QCD

$$\langle X \rangle = \frac{1}{Z_{QCD}} \int Dq(x) D\bar{q}(x) DA_\mu(x) X(q, \bar{q}, A_\mu) e^{-S_{QCD}}$$

Path integral by Monte Carlo integration

QCD action on a lattice




Finite T Field Theory on the lattice

- 4dim. Euclidean lattice
- gauge field $U_\mu(x)$ → periodic B.C.
- quark field q(x) → anti-periodic B.C.
- Temperature $T=1/(N_t a)$

# Spectral function on the lattice

Thermal hadron (charmonium) correlation functions $C_H(\tau, T)$

$$
\begin{aligned}
C_H(\tau, T) &= \sum_{\vec{r}} \langle J_H(\tau, \vec{r}) J_H^{\dagger}(0, \vec{0}) \rangle \\
&= \int_0^{\infty} d\omega \sigma_H(\omega, T) \frac{\cosh(\omega(\tau - 1/2T))}{\sinh(\omega/2T)}
\end{aligned}
$$

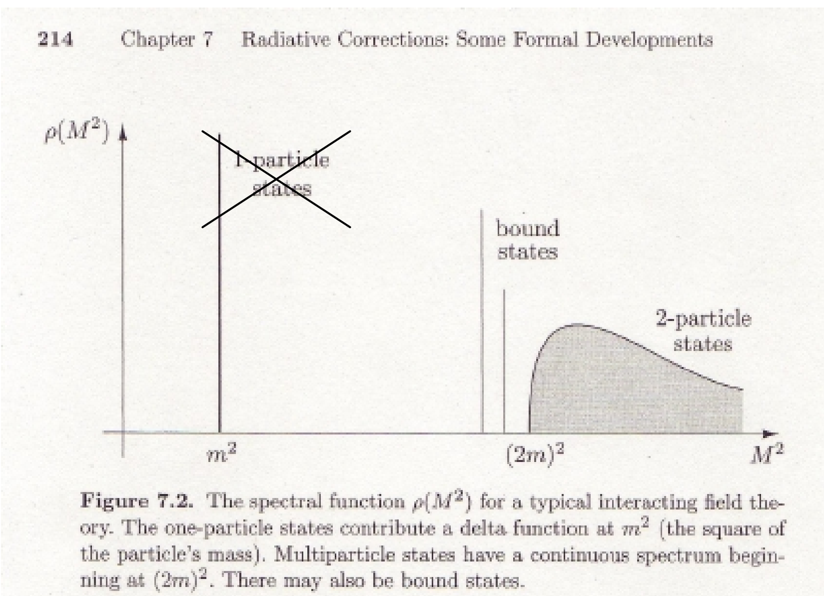

214 Chapter 7 Radiative Corrections: Some Formal Developments



**Figure 7.2.** The spectral function $\rho(M^2)$ for a typical interacting field theory. The one-particle states contribute a delta function at $m^2$ (the square of the particle's mass). Multiparticle states have a continuous spectrum beginning at $(2m)^2$. There may also be bound states.

*Michael E. Peskin, Perseus books (1995)*

Spectral function $\sigma_H(\omega, T)$

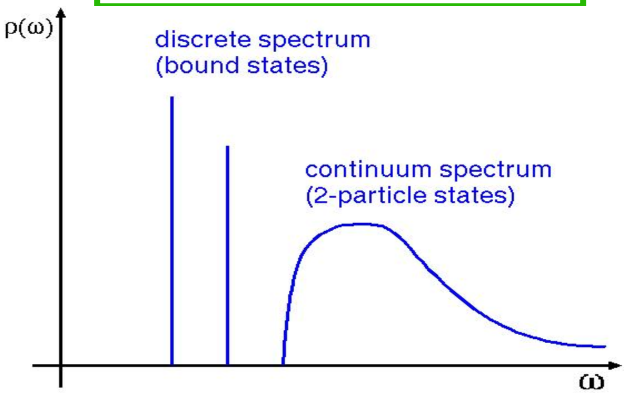
- discrete spectra
  → bound states
  charmonium states
- continuum spectra
  → 2-particle states
  $c\,\bar{c}$ scattering states
  melted charmonium

# Spectral functions in a finite volume

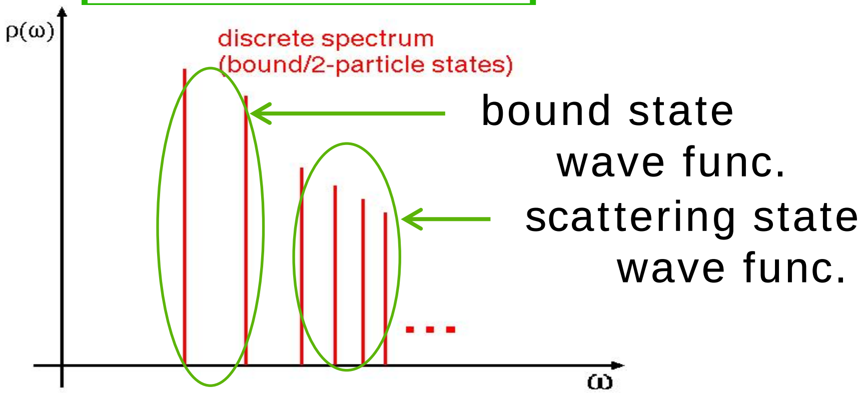
Momenta are discretized in finite ($V=L^3$) volume

$p_i/a = 2n_i\pi/L$ ($n_i=0,\pm1,\pm2,\ldots$) for Periodic boundary condition

infinite volume case



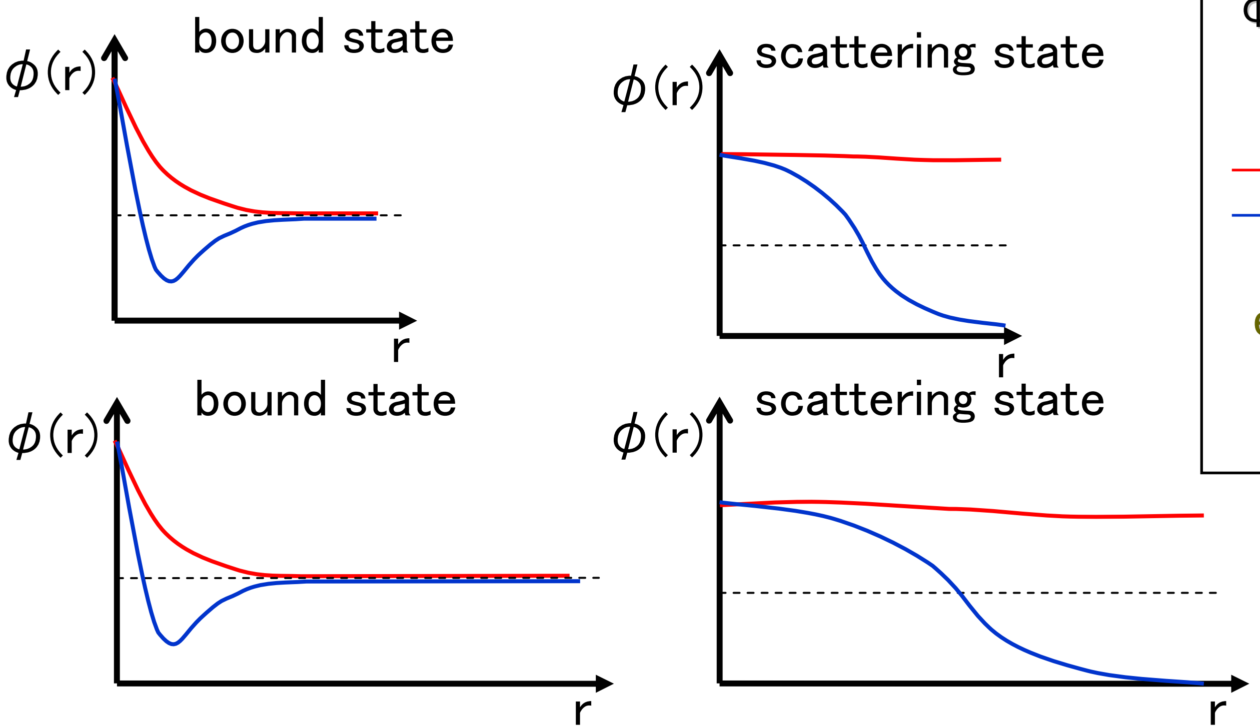

finite volume case




In a finite volume (e.g. Lattice simulations),
discrete spectra does not always indicate bound states !

Shape of wave functions may be good signature
to find out the charmonium melting.

# Bound state or scattering state ?




$\Phi(r)$: wave function
r : c - c distance

—— lowest state
—— next lowest state

examples for
· S-wave
· Periodic B.C

- local wave function
- small Vol. dependence

- non-local wave function
- Vol. dependence

# Wave functions at finite temperature

Temp. dependence of ( Bethe-Salpeter ) "Wave function"

$$BS(\vec{r},t) = \sum_{\vec{x}} \langle \bar{q}(\vec{x}+\vec{r},t)\Gamma q(\vec{x},t)\bar{q}(\vec{0},0)\Gamma q(\vec{0},0)\rangle$$

$$\Psi(|\vec{r}|,t) = BS(\vec{r},t)/BS(\vec{r}_0,t)$$

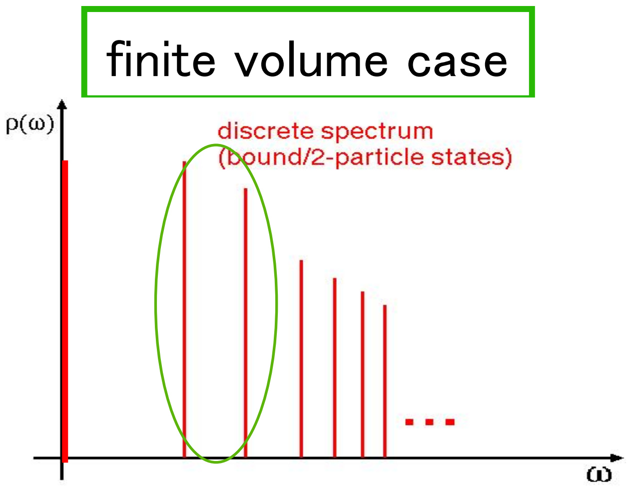
$$\Gamma = \begin{cases} \gamma_5 & \text{(Ps)} \\ \gamma_i & \text{(Ve)} \quad (i=1,2,3) \\ \sum_j \left(\overrightarrow{\partial}_j\gamma_j - \overleftarrow{\partial}_j\gamma_j\right) & \text{(Sc)} \\ \sum_{j,k} \epsilon_{ijk}\left(\overrightarrow{\partial}_j\gamma_k - \overleftarrow{\partial}_j\gamma_k\right) & \text{(Av)} \quad (i=1,2,3) \end{cases}$$

Remarks on wave function of quark-antiquark

- gauge variant → Coulomb gauge fixing
- large components of quark/antiqaurk

  → derivative operators for P-wave channels

# Technique to calculate wave function at T>0




It is difficult to extract higher states
from lattice correlators (at T>0)
even if we use MEM !!

It is important to investigate
a few lowest states (at T>0)

Constant mode can be separated
by the Midpoint subtraction

*T. Umeda (2007)*

In order to study a few lowest states,
the variational analysis is one of the most reliable methods !

N x N correlation matrix : C(t)

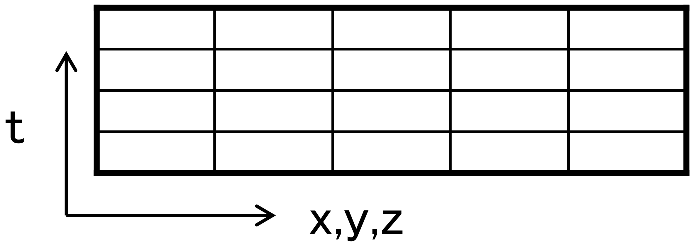
$$C(t)\psi = \lambda(t,t_0)C(t_0)\psi \qquad \lambda_i(t,t_0) = e^{-E_i(t-t_0)}$$

# Lattice setup

- Quenched approximation ( no dynamical quark effect )
- Anisotropic lattices

  lattice spacing : $a_s$ = 0.0970(5) fm

  anisotropy : $a_s/a_t = 4$

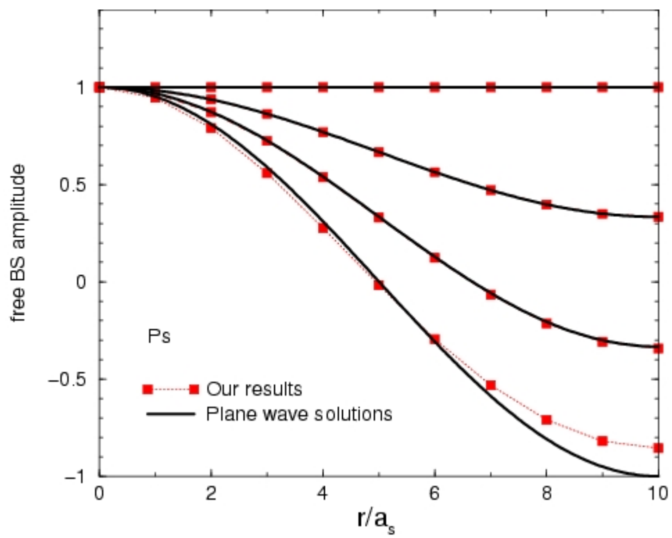



- $r_s$=1 to suppress doubler effects
- Variational analysis with 6 x 6 correlation matrix

| $N_t$ | 32 | 26 | 20 | 16 | 12 |
|---|---|---|---|---|---|
| $T/T_c$ | 0.88 | 1.08 | 1.40 | 1.75 | 2.33 |
| # of conf. | | | | | |
| $V=16^3$ | 300 | 300 | 300 | 300 | 300 |
| $V=20^3$ | 300 | 300 | 300 | 300 | 300 |
| $V=32^3$ | — | — | — | — | 200 |

# Wave functions in free quark case

Test with free quarks ( $L_s/a=20$, $ma=0.17$ )
in case of S-wave channels

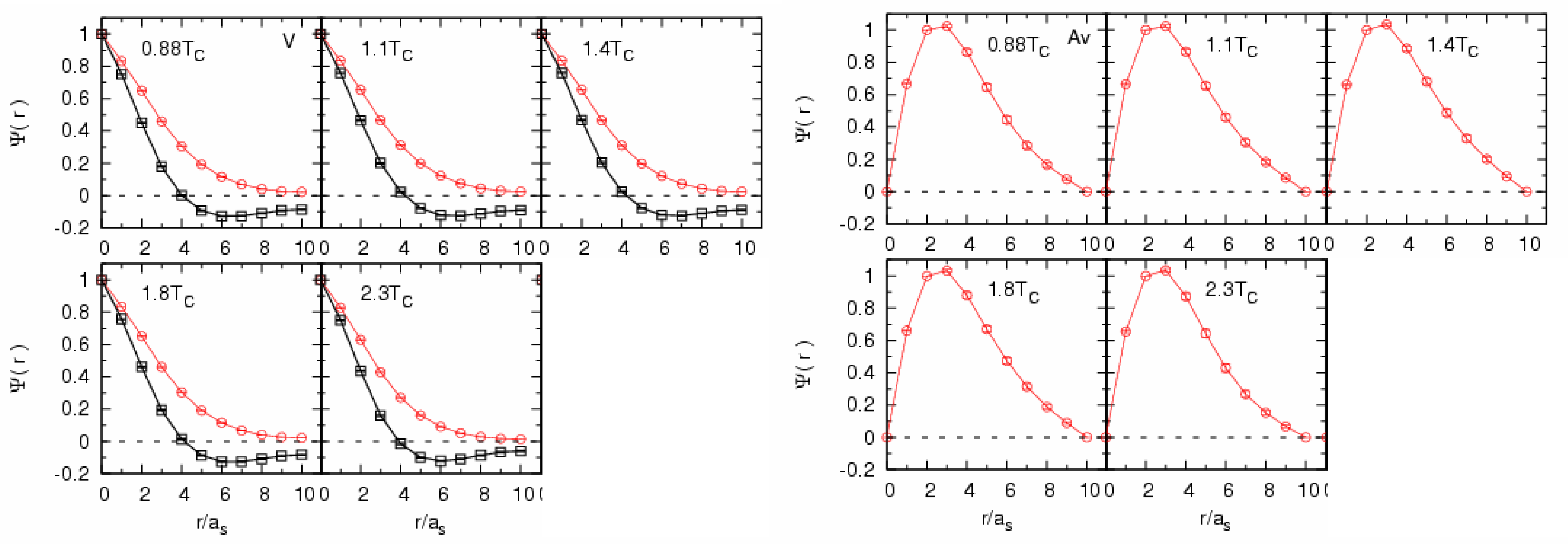



- Free quarks make trivial waves with an allowed momentum in a box

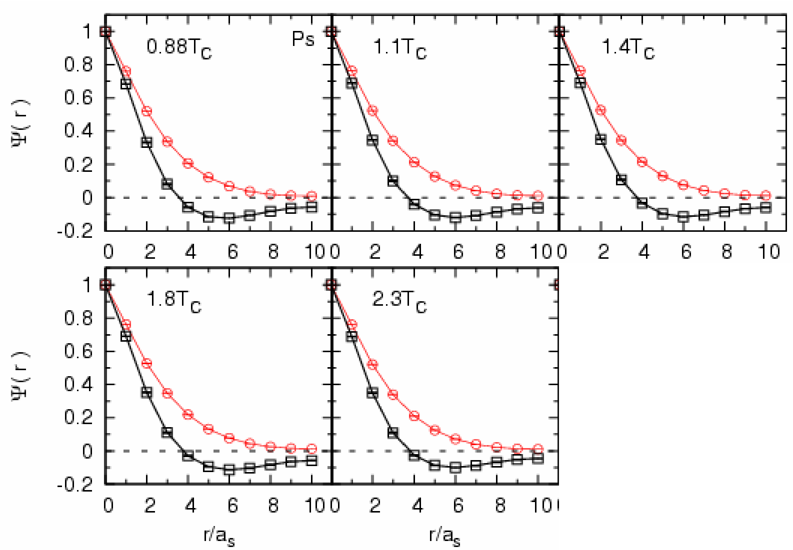
$$\Psi_k(|\vec{r}|,t) = \frac{\sum_{\vec{p}=\vec{k}} \cos(p_1 r_1)\cos(p_2 r_2)\cos(p_3 r_3)}{\sum_{\vec{p}=\vec{k}} 1}$$

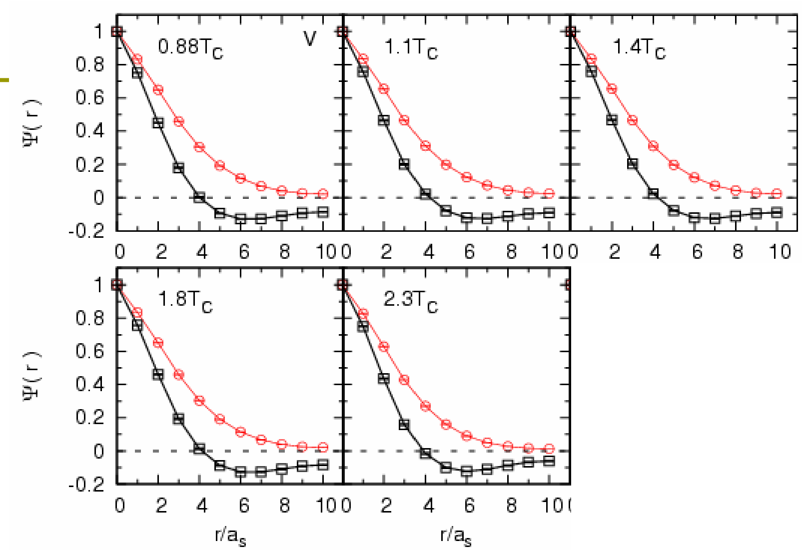
- The wave function is constructed with eigen functions of 6 x 6 correlators
- 6 types of Gaussian smeared operators
  $\phi(x) = \exp(-A|x|^2)$,
  A = 0.02, 0.05, 0.1, 0.15, 0.2, 0.25
- Our method well reproduces the known result ( ! )

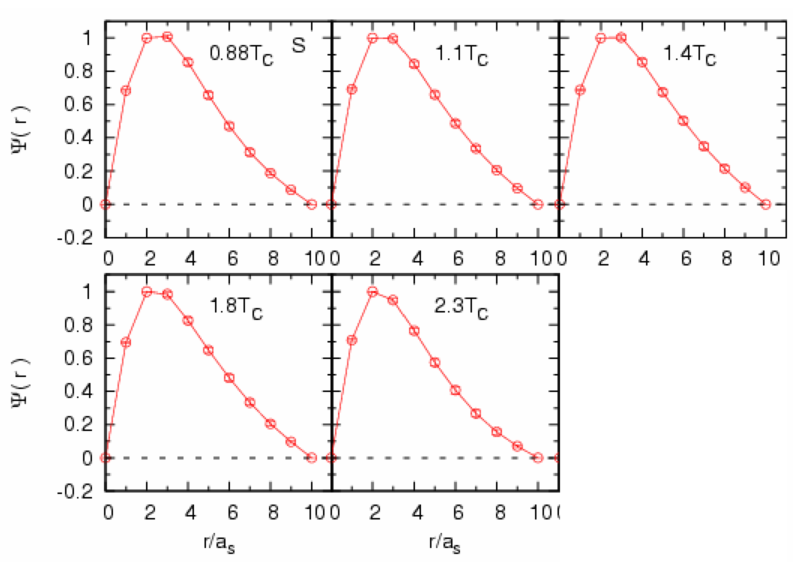
# Charmonium wave functions at finite temperatures

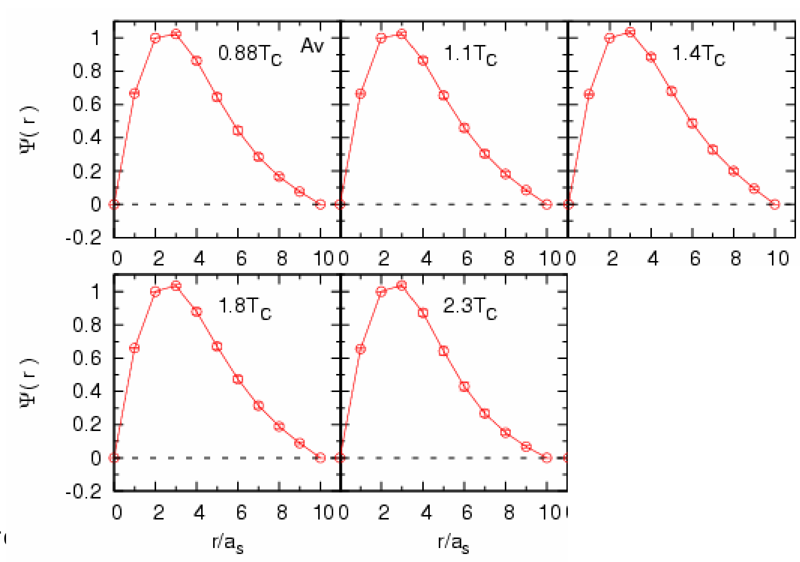



- Small temperature dependence in each channels
- Clear signals of bound states even at T=2.3Tc ( ! )
- $(2fm)^3$ may be small for P-wave states.

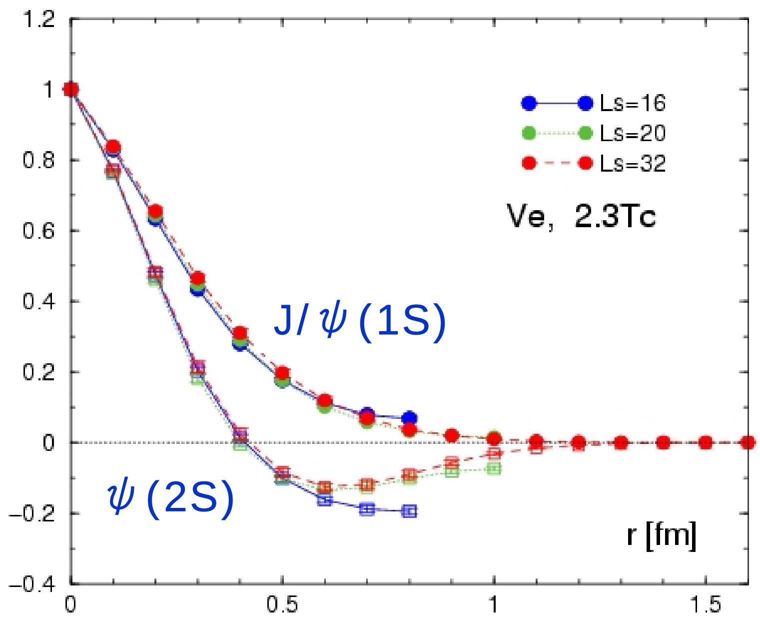

Ps
0.88T$_C$
1.1T$_C$
1.4T$_C$
1.8T$_C$
2.3T$_C$
Ψ( r )
r/a$_s$

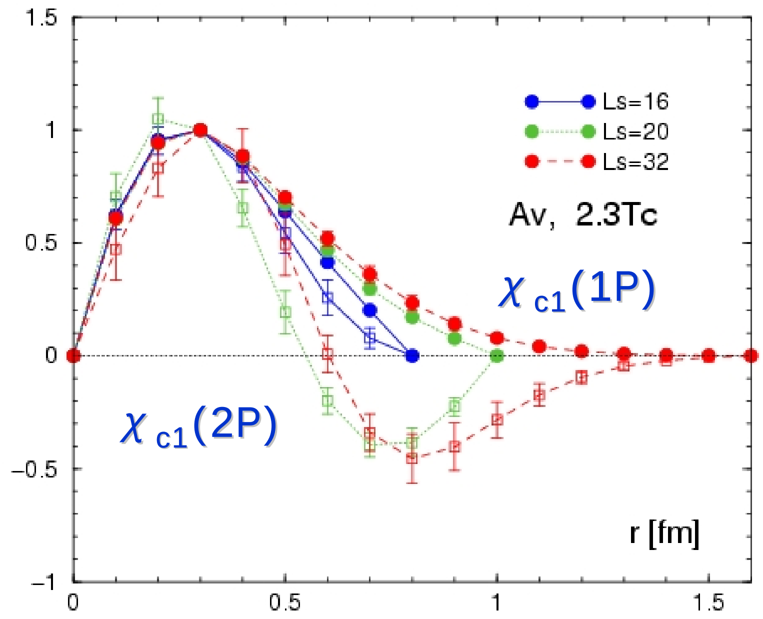

V
0.88T$_C$
1.1T$_C$
1.4T$_C$
1.8T$_C$
2.3T$_C$
Ψ( r )
r/a$_s$


S
0.88T$_C$
1.1T$_C$
1.4T$_C$
1.8T$_C$
2.3T$_C$
Ψ( r )
r/a$_s$


Av
0.88T$_C$
1.1T$_C$
1.4T$_C$
1.8T$_C$
2.3T$_C$
Ψ( r )
r/a$_s$

# Volume dependence at T=2.3Tc




- Clear signals of bound states even at T=2.3Tc ( ! )
- Large volume is necessary for P-wave states.

# Discussion

We found tight wave functions
up to 2.3Tc for S- & P- wave channels.

(1) Variational analysis works well ?

wave function for lowest/next-lowest state

+ contributions from higher states ← contaminations

our results suggest
there are no/small non-local wave functions
even in the higher states (!)

(2) Effects of interaction ?
It may be difficult to conclude whether bound or unbound.
in any case,
tight wave function is incompatible
with the J/$\psi$ suppression (!)

# Summary and future plan

We investigated $T_{dis}$ of charmonia from Lattice QCD without Bayesian (MEM) analysis using...

- Bethe-Salpeter "wave function"
- Volume dependence of the "wave function"

No evidence for unbound $c\text{-}\bar{c}$ quarks up to T = 2.3 Tc

→ The result may affect the scenario of J/$\psi$ suppression.

Future plan

- Possible scenarios for the experimental J/$\psi$ suppression
- Higher Temp. calculations ( T/Tc=3~5 )
- Full QCD calculations ( Nf=2+1 Wilson is now in progress )