The hyperfine splitting of charmonium on the lattice

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Preliminary

Verification of

Quantum Chromodynamics (QCD)

Lattice regularization

- Hadron spectroscopy
  - Light hadron spectrum (with/without dynamical quarks)

    some projects in progress

    by CP-PACS, JLQCD, MILC, UKQCD, ···
  - Charmonium spectrum

    Charmonia properties are well established

    by experiments

Contents

Charmonium hyperfine splitting (HFS) on the lattice

1) Introduction
   problems of the charmonium HFS
2) Quenched charmonium spectrum
   systematic study for the charmonium HFS
3) Discussion
   remaining uncertainties of HFS
4) Contribution of a disconnected diagram
5) Conclusion

Status of experiments

$\eta_c(1S)$

$J^{PC} = 0^{-+}$

mass = 2979.7 ± 1.5 MeV

width = 16.0 +3.6 − 3.2 MeV

$J/\psi(1S)$

$J^{PC} = 1^{--}$

mass = 3096.87 ± 0.04 MeV

width = 87 ±5 KeV

$$m(J/\psi) - m(\eta_c) = 117.2 \pm 1.5 \text{ MeV}$$

Part-I Quenched charmonium spectrum

Problems for heavy quarks on a lattice

Charmonium spectrum on lattice QCD

lattice cutoff $1/a \gg m_{charm}$ is necessary

→ large computational cost !

- Effective theories

  NRQCD, etc.

  e.g. *Trottier, Phys. Rev. D55 (1997) 6844*

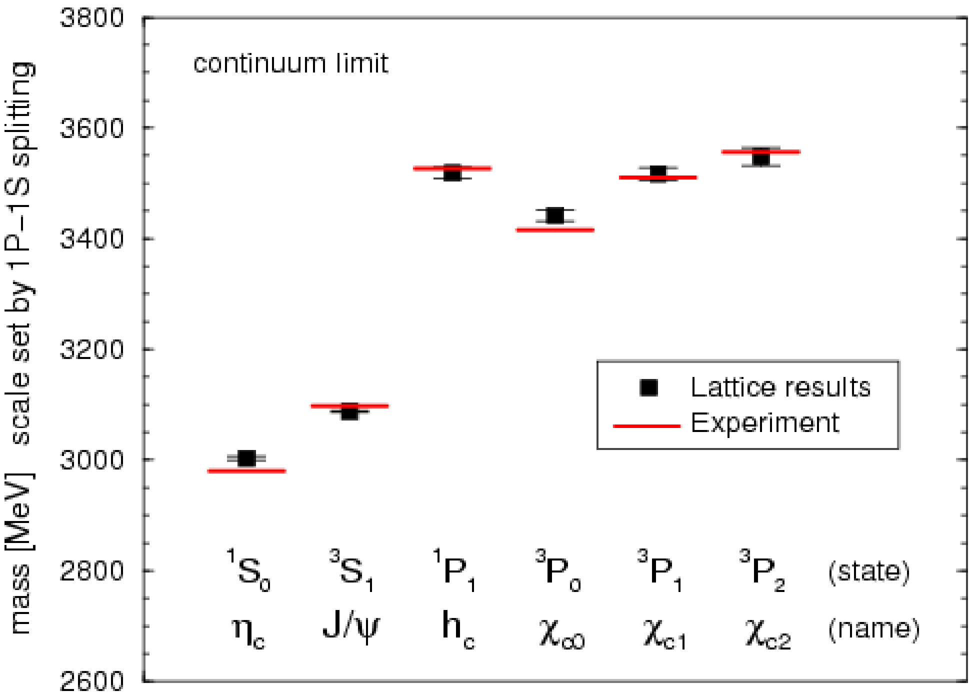
- Relativistic framework

  Fermilab action, anisotropic lattice

  e.g. *CP-PACS, Phys. Rev. D65 (2002) 094508*

Status of lattice results

M.Okamoto et al. (CP-PACS), Phys. Rev. D65 (2002)094508



M.Okamoto et al. Phys. Rev. D65 (2002) 094508

- quenched anisotropic lattice
- tree-level clover quark
- continuum extrapolation

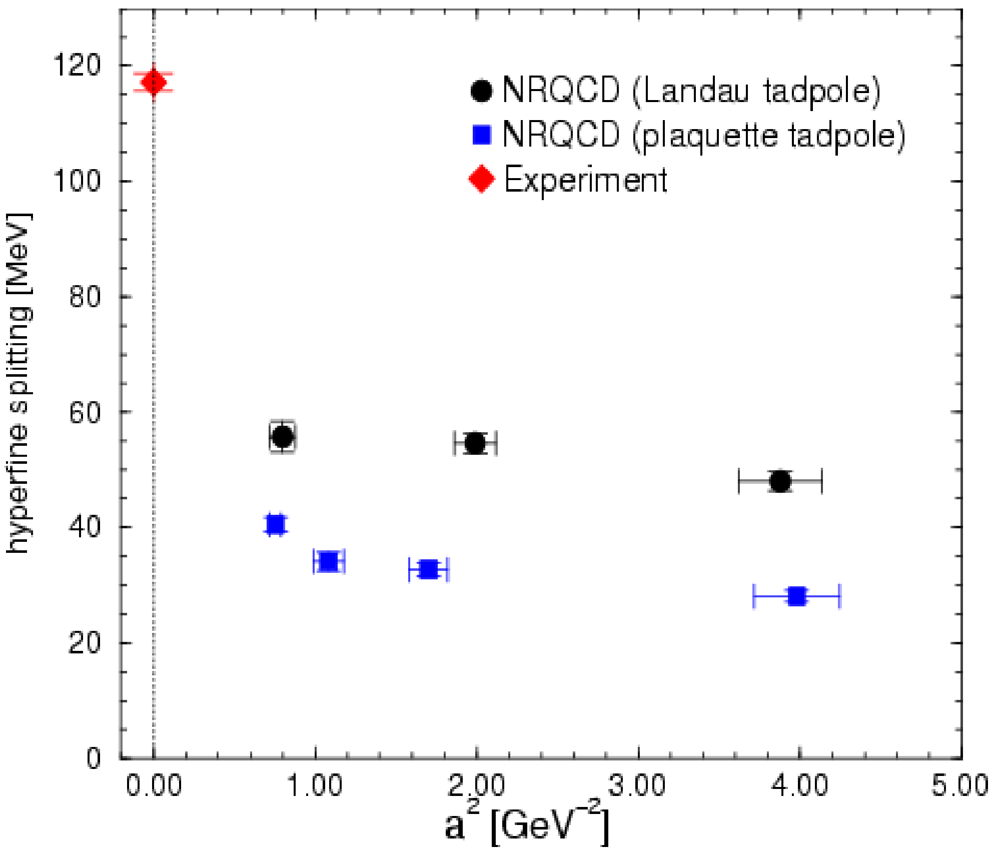
  with $1/a_s = 1.0 \sim 2.9$ GeV

Good agreement with the experimental values

except for the hyperfine splitting

HFS in NRQCD

H. Trottier, Phys. Rev. D55 (1997) 6844

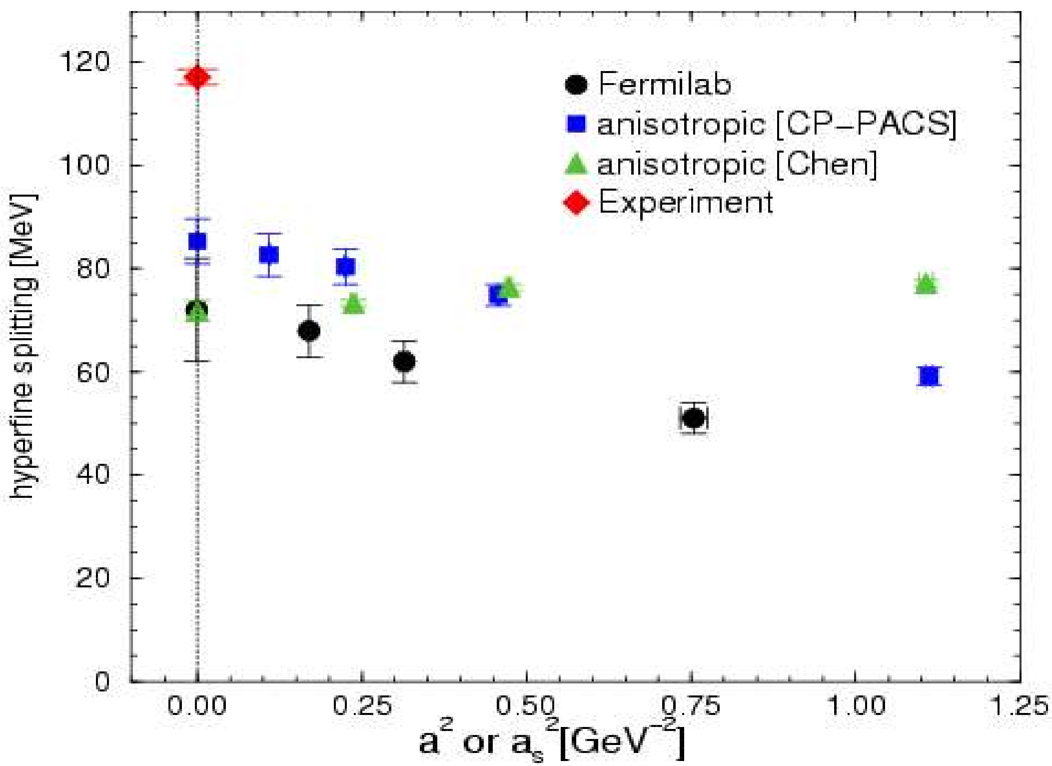


- quenched simulation
- next-to-leading order correction
- lattice cutoff in the range of $1/a$ = 0.5 ~ 1.2 GeV

X m_{charm} is not so heavy

X no continuum limit

HFS in relativistic frameworks



quenched calculations

Fermilab action

A.El-Khadra et al., [Lattice'91]

Anisotropic lattice action

CP-PACS, PRD65('02)094508

P.Chen, PRD64('01)034509

- anisotropic lattice also needs $1/a_s \gg m_{charm}$ for reliable continuum extrapolations
- large dependence on Dirac operators

A study by QCD-TARO Collaboration

QCD-TARO Collabration, JHEP08 (2003) 022

- Quenched QCD (without dynamical quarks)
- Isotropic lattices with large cutoff $1/a \gg m_{charm}$
- Nonperturbatively improved clover quark

  M.Lüscher et al., Nucl.Phys.B491 (1997) 323
- Continuum extrapolation

Charmonium correlation functions

$$C(t)=\sum_{\vec{x}} \langle \mathrm{Tr}[\Gamma D^{-1}(0,\vec{0};t,\vec{x})\Gamma \gamma_5 D^{-1}(0,\vec{0};t,\vec{x})^{\dagger} \gamma_5] \rangle$$

- point source & point sink correlators
- zero momentum projection

Name	$^{(2s+1)}L_J$	J^{PC}	Γ	mass (GeV)
η_c	$^1S_0$	0^{-+}	γ_5	2.980(2)
J/ψ	$^3S_1$	1^{--}	γ_i	3.097
h_c	$^1P_1$	1^{+-}	σ_{ij}	3.526
χ_{c0}	$^3P_0$	0^{++}	1	3.415
χ_{c1}	$^3P_1$	1^{++}	$\gamma_i\gamma_5$	3.511

Lattice setup

Gauge field : plaquette gauge

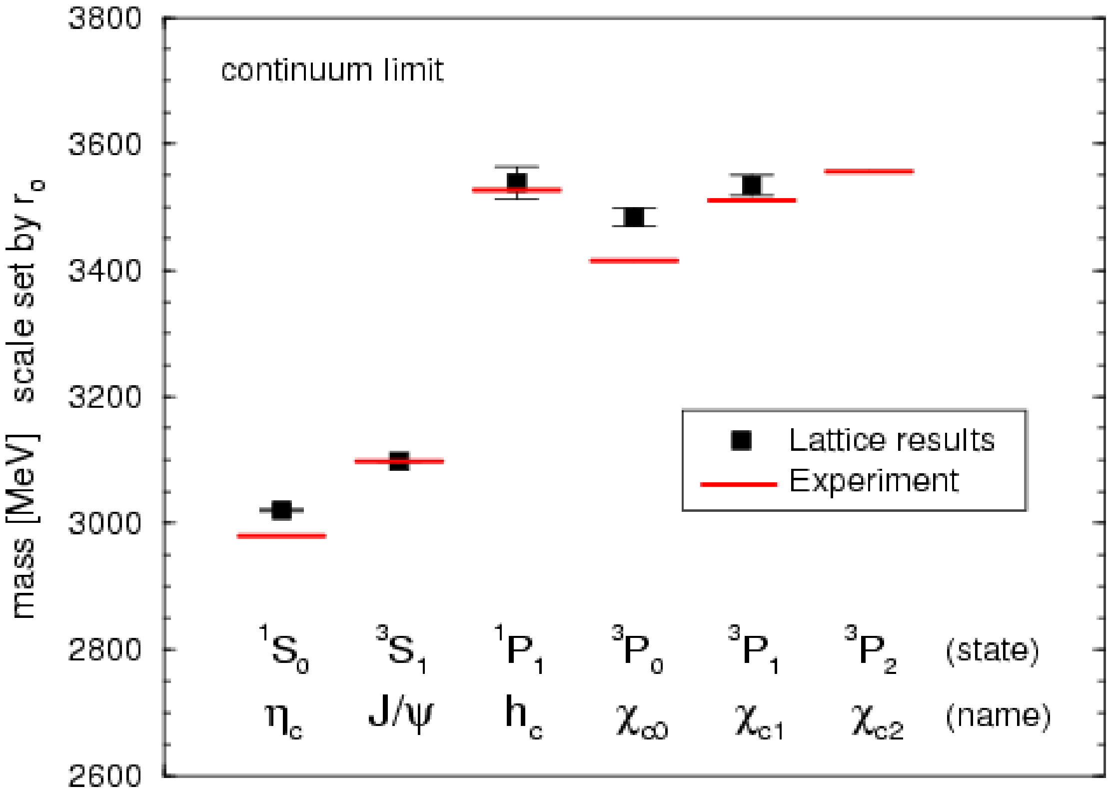
Quark field : nonperturbatively improved clover quark

(tree level clover quark, standard Wilson)

β	$L^3 \times T$	a(fm)	La(fm)	Csw	#conf
6.0	$18^3 \times 48$	0.0931	1.68	1.769	190
6.2	$24^3 \times 72$	0.0677	1.62	1.614	90
6.4	$32^3 \times 96$	0.0513	1.64	1.526	60
6.6	$32^3 \times 96$	0.0397	1.27	1.467	130

scale set by r_0

Charmonium spectrum



QCD–TARO Collaboration. JHEP08 (2003) 022

- nonperturbative clover
- continuum limit

quark mass set by m(J/ψ)

scale set by r_0

The results are consistent with the previous works

hyperfine splitting is smaller than the Exp. value

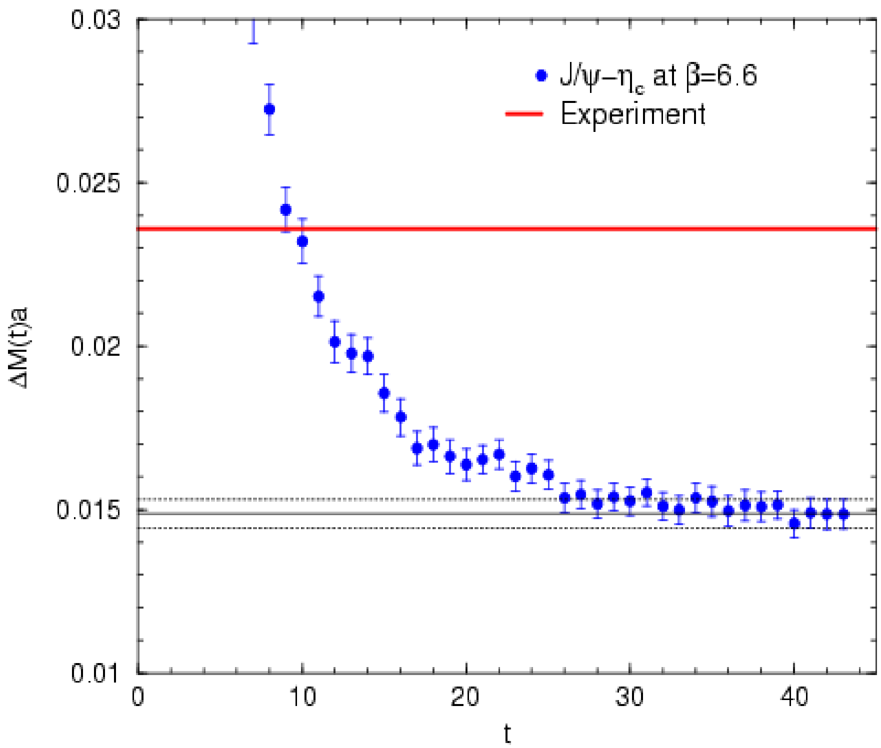
Contained uncertainties

- statistical error
- quark mass determination (m_{charm})
- continuum extrapolation
- finite volume effects
- scale determination (1/a)
- choice of Dirac operators
- dynamical quark effects
- disconnected diagram contributions

Effective mass plot of HFS

$$R(t) \equiv \frac{\langle C(t) \rangle}{\langle C(t+1) \rangle} = \frac{\cosh[(T/2-t)M(t)]}{\cosh[(T/2-t-1)M(t)]}$$

1: $\Delta M(t) \equiv M_B(t) - M_A(t)$ 2: $\frac{R_A(t)}{R_B(t)} \sim e^{-\Delta M(t)}$



- 1: and 2: are consistent each other within error
- 2: is less noisy than 1:

we use the definition 2:

Scale determination

- Lattice cutoff
  - Sommer scale $r_0 \simeq 0.5\ \mathrm{fm}$ *[R.Sommer,NPB411('94)839]*

$$r_0 \left. \frac{dV(r)}{dr} \right|_{r=r_0} = 1.65$$

$V(r)$: Static quark potential

using the nucleon mass amounts to $r_0 \simeq 0.55\ \mathrm{fm}$

r_0 ~ 10 % uncertainty (due to dynamical quark effects)

  - Hadron mass (e.g. m(1P)−m(1S))

for example CP-PACS results ~ 16 % uncertainty

This discrepancy caused by

scaling violation & dynamical quark effects

Charm quark mass determination

matching with a physical charmonium mass

$m(J/\psi) = 3.097$ GeV
$m(\eta_c) = 2.980$ GeV

} ~ 6% uncertainty

only mass splittings of chamonia can be predicted

alternative possibilities

D, D_s meson mass or decay constant

free of OZI ambiguities

Finite volume effect

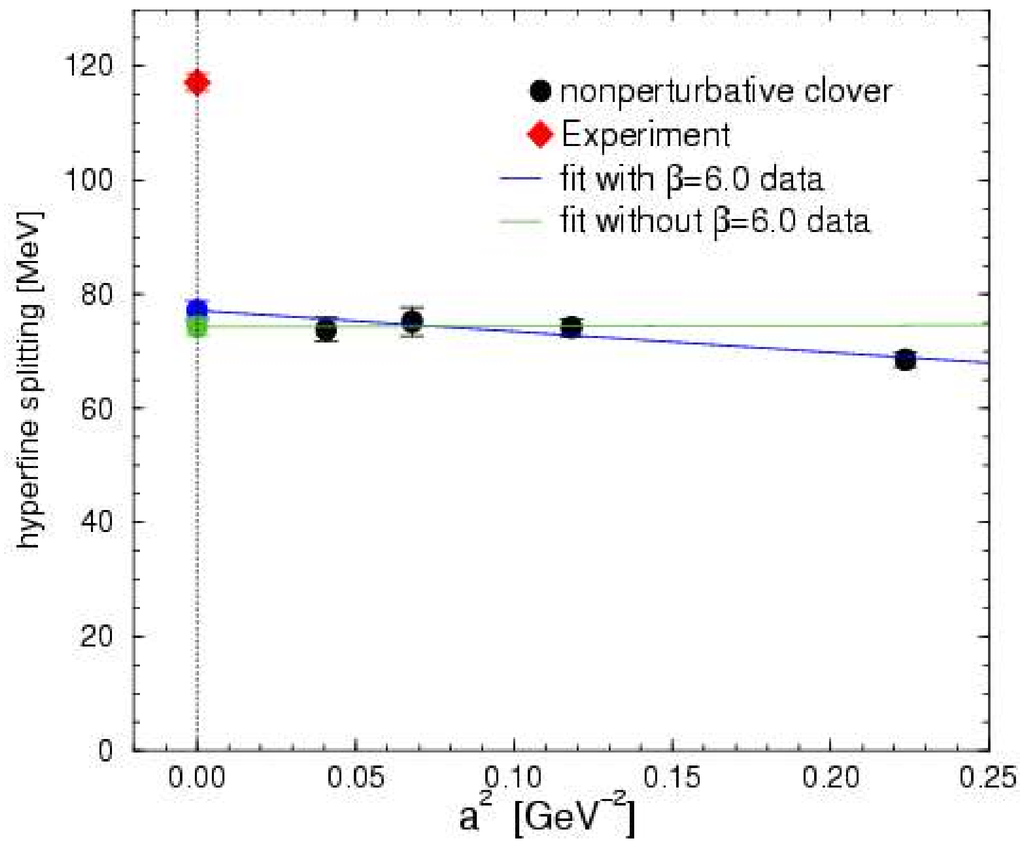
L	La (fm)	$^1S_0$ (MeV)	$^3S_1$ (MeV)	$^3S_1 - {}^1S_0$ (MeV)
8	0.75	2958(10)	3019(12)	61.4(4.4)
10	0.93	2953(5)	3023(6)	70.6(2.5)
12	1.12	2957(4)	3032(5)	75.4(2.7)
14	1.30	2947(3)	3020(4)	72.6(1.9)
16	1.49	2952(3)	3025(4)	74.9(2.1)
18	1.68	2949(2)	3021(3)	72.5(1.5)

- Lattice spacing is fixed to 0.093fm (β =6.0)
- nonperturbatively improved clover Dirac op. with κ =0.11925
- averaged over 100conf. (190 for L=18)

finite volume effect is negligible at La > 1.1fm

this effect is discussed again using wave functions

Continuum extrapolation



Nonperturbative Clover

lattice artifact ~ $O(a^2)$

Continuum extrapolation

linearly in a^2

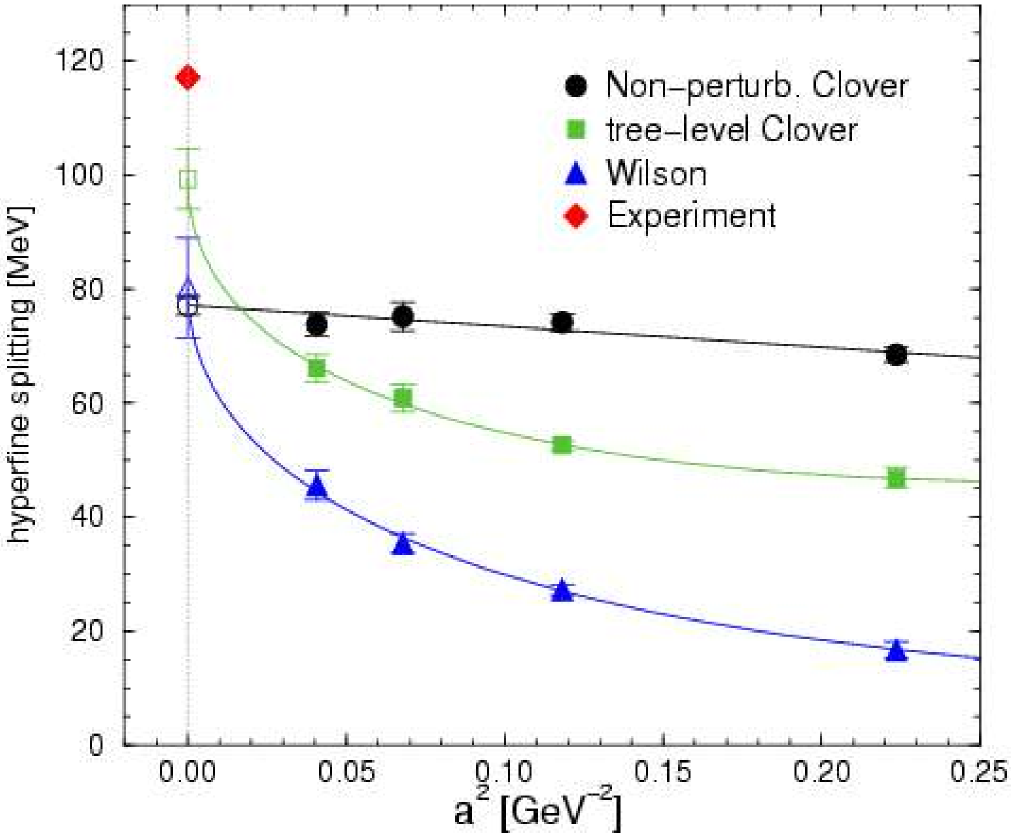
$\Delta M = 77(2)\,\mathrm{MeV}$

at the continuum limit

different extrapolations including or not β = 6.0

➡ systematic uncertainty of extrapolation ~ 3 MeV

Choice of Dirac operators : I



(1) using the all four β

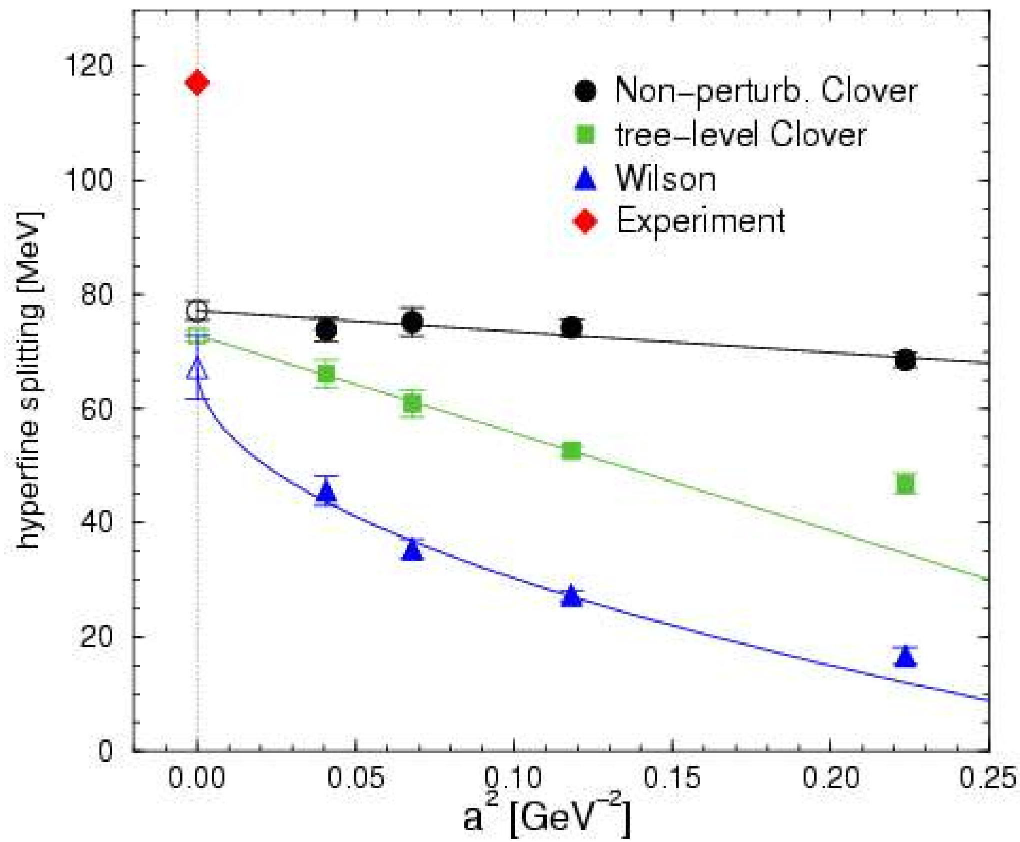
- nonperturbative clover $\mathrm{O}(a^2)$
  linearly in a^2
- for tree-level clover
  including a and a^2 terms
- for Wilson
  including a and a^2 terms

scale violations to be:

$\mathrm{O}(a)$ for the Wilson Dirac operator

$\mathrm{O}(g^2 a), \mathrm{O}(a^2)$ for the tree-level clover Dirac operator

Choice of Dirac operators : II



(1) using the three largest β

- nonperturbative clover $O(a^2)$
  linearly in a^2
- for tree-level clover
  linearly in a^2
- for Wilson
  linearly in a

Wilson & tree-level Clover need very large cutoff
for a reliable continuum extrapolation

→ Dirac operator dependence is well controlled

Contained uncertainties

$m(J/\psi) - m(\eta_c) = 77(2)(6)$ MeV (117MeV in Experiment)

- statistical error ~ 2 MeV
- quark mass determination (m_{charm}) ~ 5 MeV
- continuum extrapolation ~ 3 MeV
- finite volume effects ~ negligible
- scale determination (1/a) ~ dynamical quark effects
- choice of Dirac operators ~ well controlled
- dynamical quark effects
- disconnected diagram contributions

Wave function

HFS in non-relativistic approximation

⟶ solving the Schredinger eq. with non-rela. Coulomb pot.

0-th order : HFS = 0

degeneracy is removed by spin-spin interaction

1-st order : HFS = $\frac{32\pi\alpha_s(m_q)}{9m_q^2}|\Psi_{NR}(0)^2|$

where non-rela. wave function $\Psi_{NR}(r)=\frac{1}{\sqrt{8\pi\rho^3}}e^{\frac{-r}{2\rho}}$, $\rho=\frac{3}{4\alpha_s m_q}$

$$\Psi_{\eta_c}(0)=\left(1+\delta_{NP}+\left(\frac{1}{2}-\nu\right)8\alpha_s^2(\mu)9\right)\Psi_{NR}(0)$$

$$\Psi_{J/\psi}(0)=\left(1+\delta_{NP}-\left(\frac{1}{6}+\nu\right)8\alpha_s^2(\mu)9\right)\Psi_{NR}(0)$$

δ_{NP} : non-perturb. correction

$\alpha_s(\mu)$: strong coupling at scale μ

$\nu\approx 7.241\times 10^{-2}$

S.Titard et al, Phys. Rev. D51 (1995) 6348.

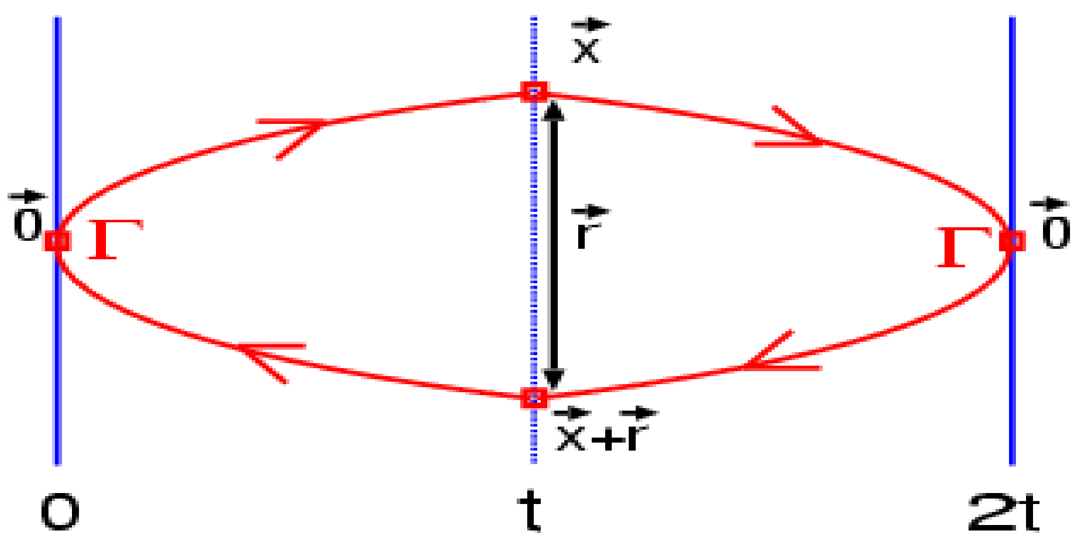
Gauge invariant wave function

$$\left|\Psi(\vec{r})^2\right| = \langle c\bar{c}|(\psi_c\psi_c)(0)(\psi_{\bar{c}}\psi_{\bar{c}})(\vec{r})|c\bar{c}\rangle$$

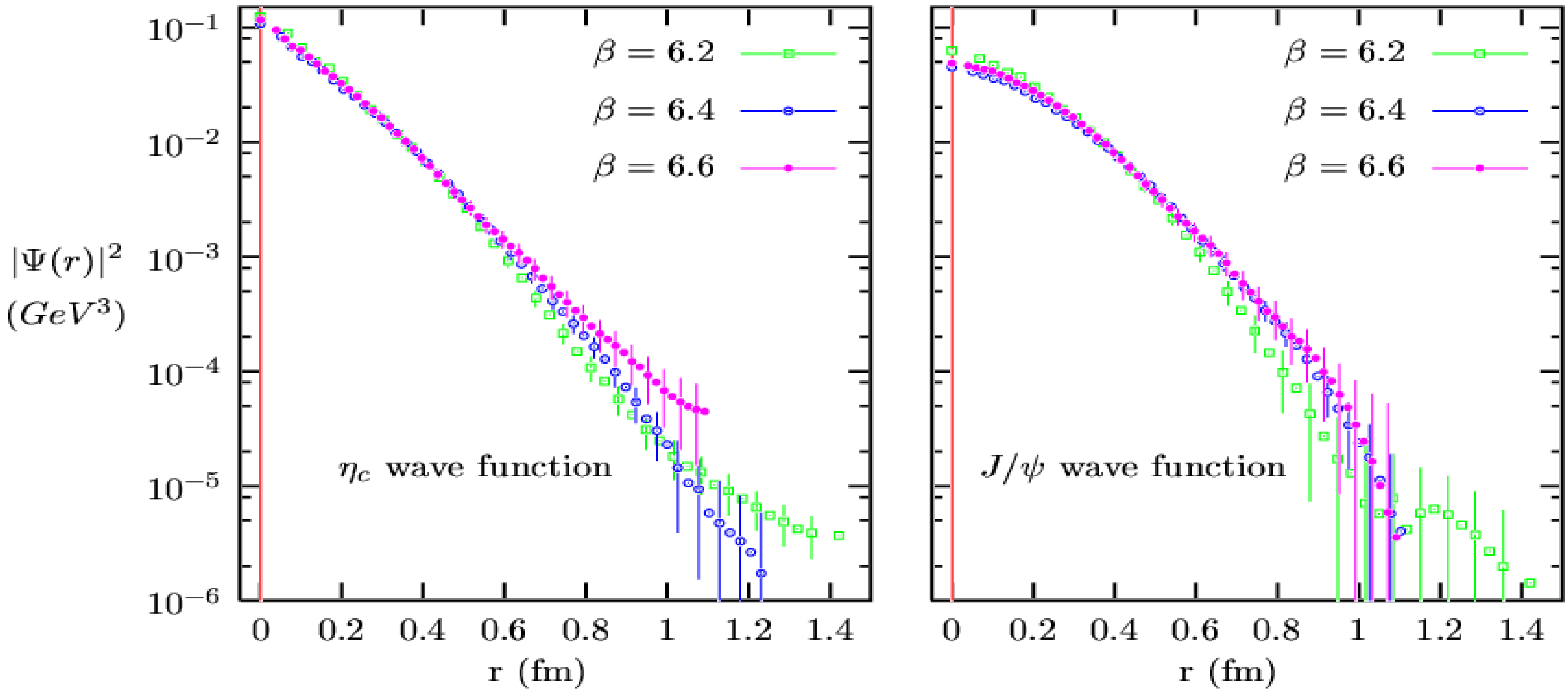
$$= \lim_{t\to\infty} N \sum_{\vec{x}} \langle \mathrm{Tr}[S(\vec{0},0;\vec{x},t)S(\vec{0},2t;\vec{x},t)\Gamma\gamma_5$$

$$S(\vec{0},2t;\vec{x}+\vec{r},t)\,S(\vec{0},0;\vec{x}+\vec{r},t)\gamma_5\Gamma]\rangle$$

C.Alexandrou et al., Phys. Rev. D66 (2002) 094503



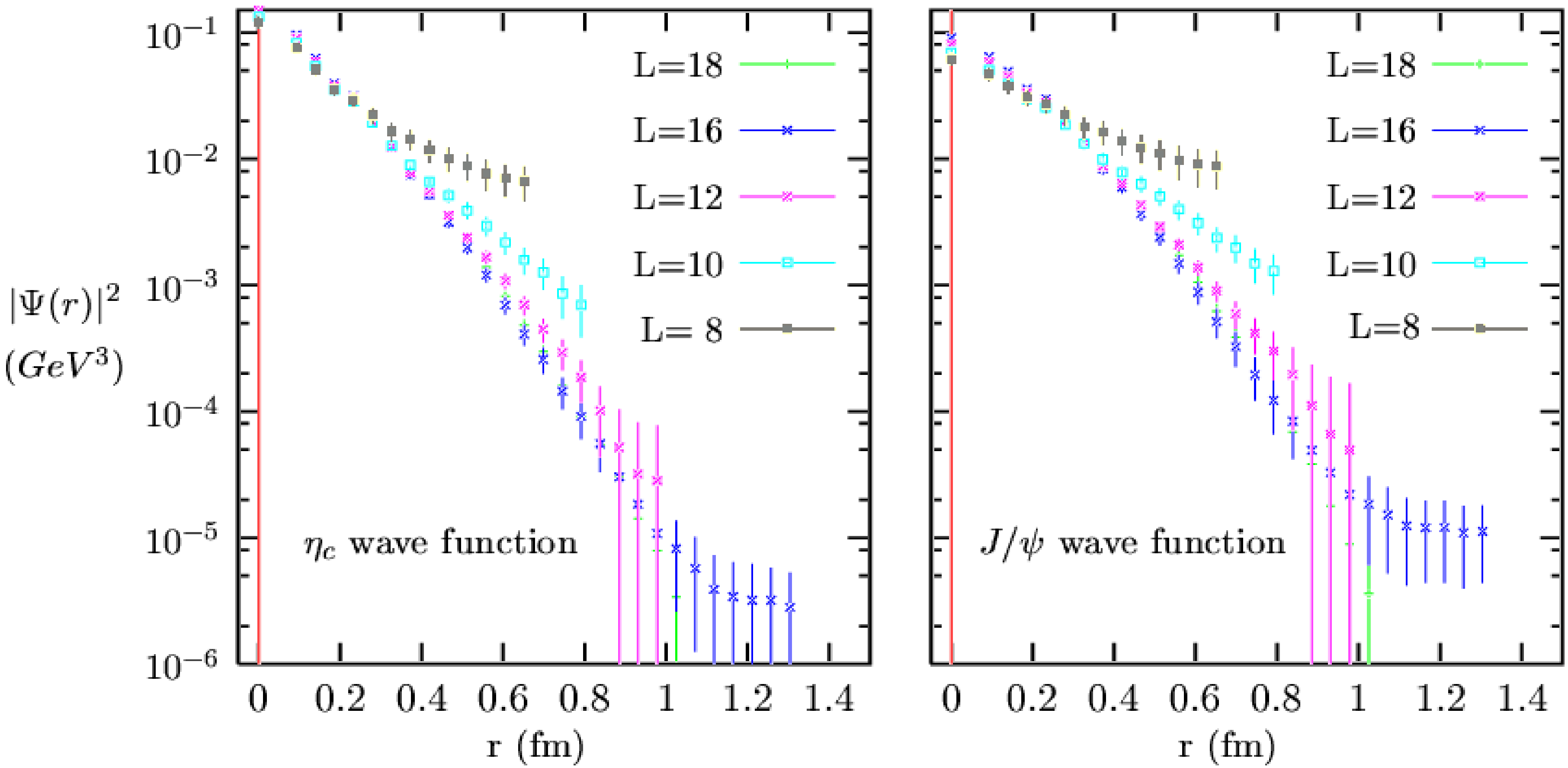
Charmonium wavefunctions



- scaling violations are very small
- $\Psi(\vec{0})$'s agree with phenomenological expectations qualitatively

In a heavy quark model : $\Psi_{\eta_c}(\vec{0}) > \Psi_{J/\psi}(\vec{0})$

Finite volume effect : 2



Discussion

Possible uncertainties of HFS

Charmonium hyperfine splitting is 30~−40% smaller than Exp.

- dynamical quark effects
- disconnected diagram contributions

→ 2-nd part of this talk

Dynamical quark effects

There is no systematic study including continuum extrapolation

Improved NRQCD on Nf=2 staggered full QCD config.

C.Stewart and R.Koniuk, Phys. Rev. D63 (2001) 054503.

NRQCD on Nf=2 Wilson full QCD config.

CP-PACS, Phys. Rev. D62 (2000) 114508.

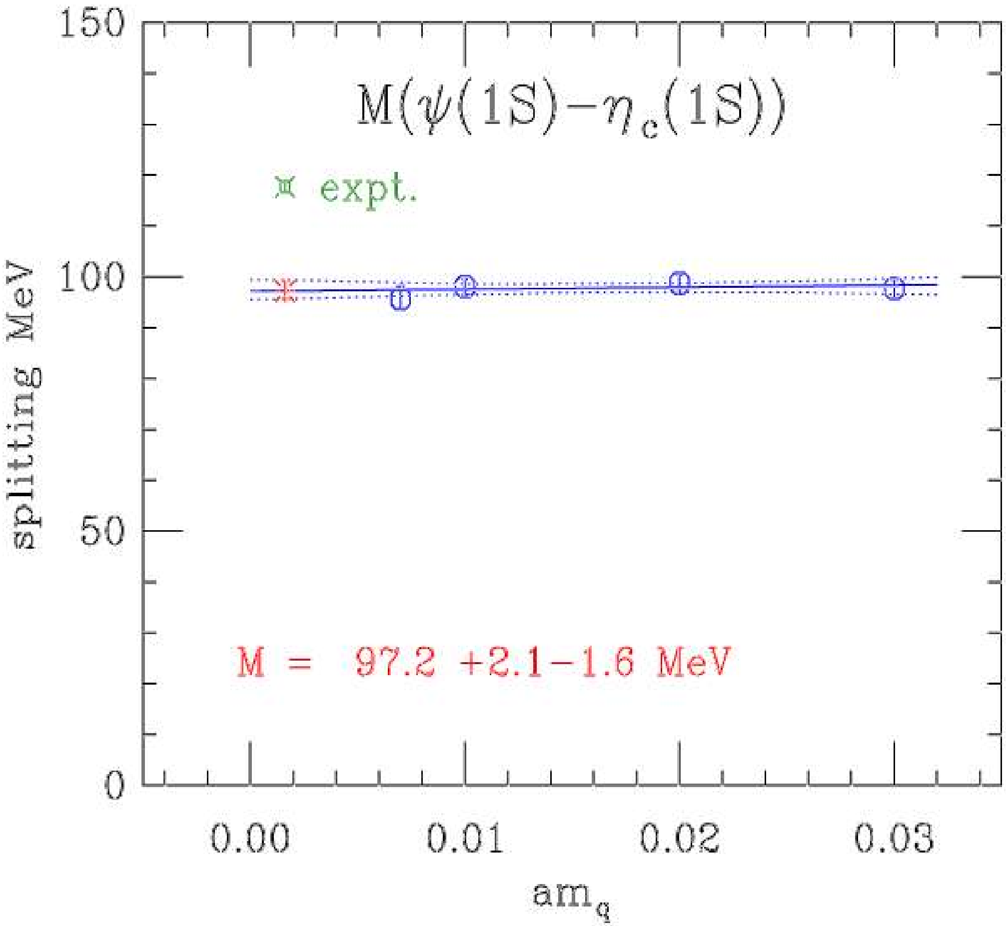
Fermilab action on Nf=2+1 staggered full QCD config.

M.di Pierro et al., Nucl. Phys. B(PS)119 (2003) 586.

↓

Dynamical quark effects ~ at most 10%

Dynamical quark effects



M.di Pierro et al., hep-lat/0310042

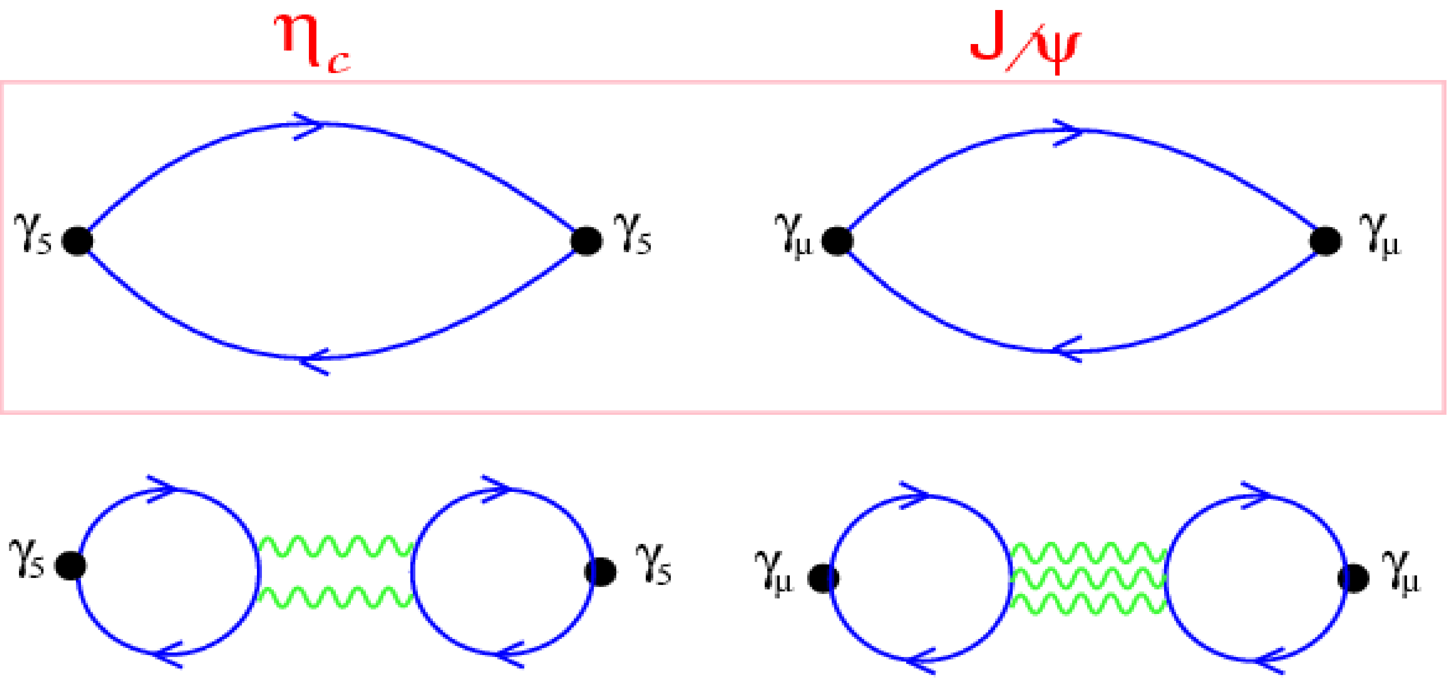
- valence quark :
  Fermilab action
  tadpole improved tree-level
- sea quark :
  Improved staggered quarks
  Nf = 2+1 (am_s = 0.05)
- fixed lattice cutoffs 1/a=1.55GeV

theory/exp.=0.82(2) (=0.6 quenched result at 1/a=1.5GeV)

clover coefficient : (tadpole improved) tree-level

Part-II Contribution of disconnected diagram

OZI forbidden “disconnected” diagram



Disconnected diagrams are neglected
because high cost & very small contribution

however, it may contribute to HFS ~ O(10) MeV ?

Charmonium correlators

$$C_{con}(t)=\sum_{\vec{x}}\langle \mathrm{Tr}[\Gamma D^{-1}(0,\vec{0};t,\vec{x})\Gamma D^{-1}(t,\vec{x};0,\vec{0})]\rangle$$

$$C_{dis}(t)=\sum_{\vec{x}}\langle \mathrm{Tr}[\Gamma D^{-1}(0,\vec{0};0,\vec{0})]\mathrm{Tr}[\Gamma D^{-1}(t,\vec{x};t,\vec{x})]\rangle$$

$D^{-1}(t,\vec{x};t',\vec{x}')$: quark propagator

- $\Gamma=\gamma_5,\gamma_\mu$ (Pseudoscalar, Vector)
- source & sink operators are extended with $\phi(\vec{x})\propto\exp(a|\vec{x}|^p)$
- disconnected diagrams are evaluated with the Z2-noise method

Z2 noise method

Stochastic estimation of $\mathrm{Tr}[D^{-1}(t,\vec{x};t,\vec{x})\Gamma]$

using noise vectors $R_i(x)$

$$\frac{1}{N_{NV}}\sum_{i=1}^{N_{NV}} R_i(x) \to 0, \qquad \frac{1}{N_{NV}}\sum_{i=1}^{N_{NV}} (R_i^{\dagger}(x)R_i(y)) \to \delta_{x,y}$$

we use complex Z2 noise

$$\frac{1}{N_{NV}}\sum_{i=1}^{N_{NV}} R_i^{\dagger}(x)D^{-1}(t,\vec{x};t,\vec{y})\Gamma R_i(y) \to \mathrm{Tr}[D^{-1}(t,\vec{x};t,\vec{x})\Gamma]$$

- This method can treat smeared operators
- Error of the method can be controlled by N_{NV}

Lattice setup

- Sea quark : Nf=2 KS quark : $a\,m_q$ = 0.1

  plaquette gauge : β = 5.50

  lattice size : $12^3 \times 24$

  lattice spacing : a=0.16fm (1/a=1.2GeV) set by r_0

  16,000 traj. (measurement at every 5 traj.)

  ☆ cutoff is not so sufficient for m_{charm} ,this is an exploratory study

- Valence quark : Fermilab action

  Csw : tadpole improved tree-level (u_0 in Landau gauge)

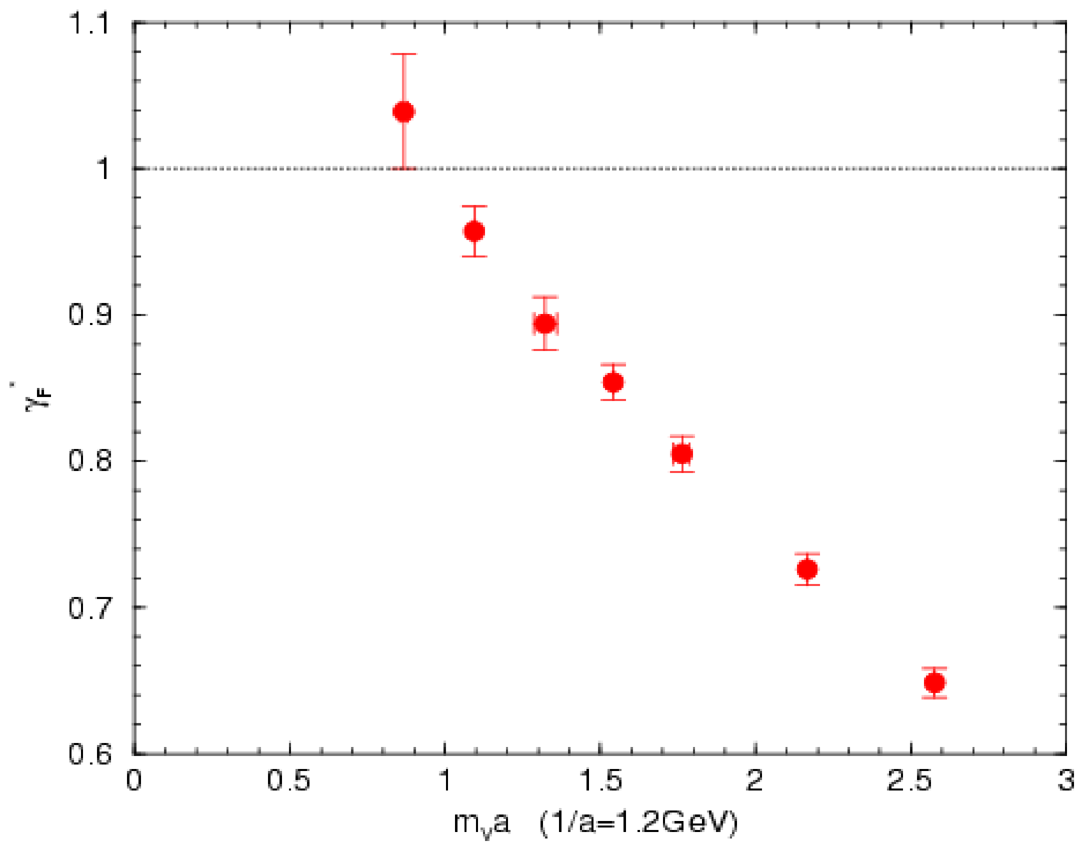
  quark mass set by m(J/ψ)

- Z2-noise method

Parameter tuning

On shell matching with $M_{rest} = M_{kinetic}$

$\gamma_F \equiv \kappa_s / \kappa_t$ is tuned by the dispersion relation of mesons

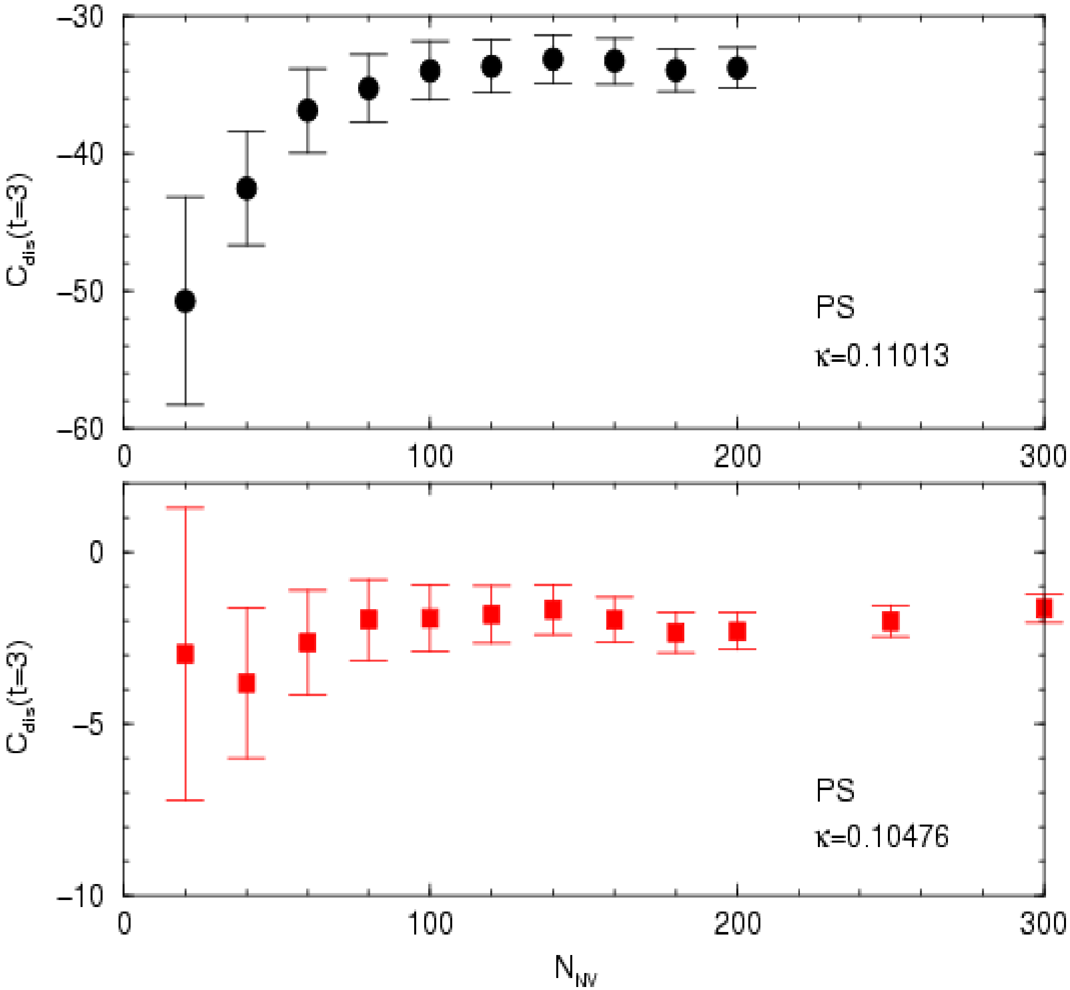


Fermilab action become standard clover action in light quark region

$\gamma_F^* \to 1$ at $m_q \to 0$

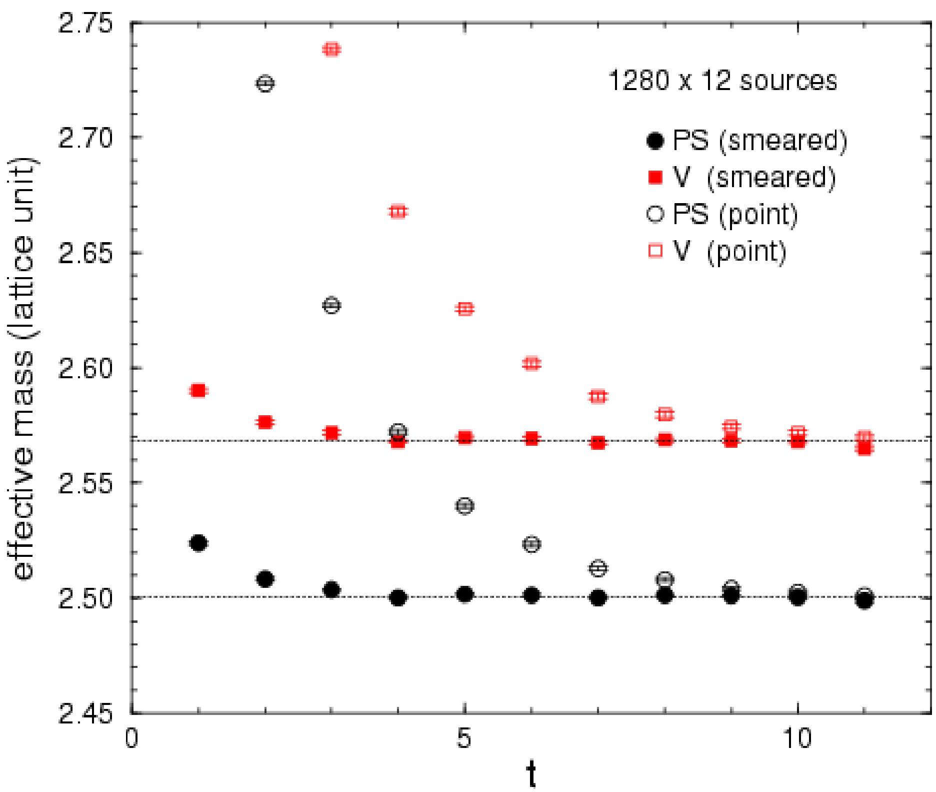
Z2-noise estimation

number of noise vector N_{NV} dependence



κ	m_V	N_{NV}
0.11294	0.867	100
0.11013	1.094	200
0.10732	1.323	200
0.10476	1.542	300
0.09342	2.578	600

Smeared operators



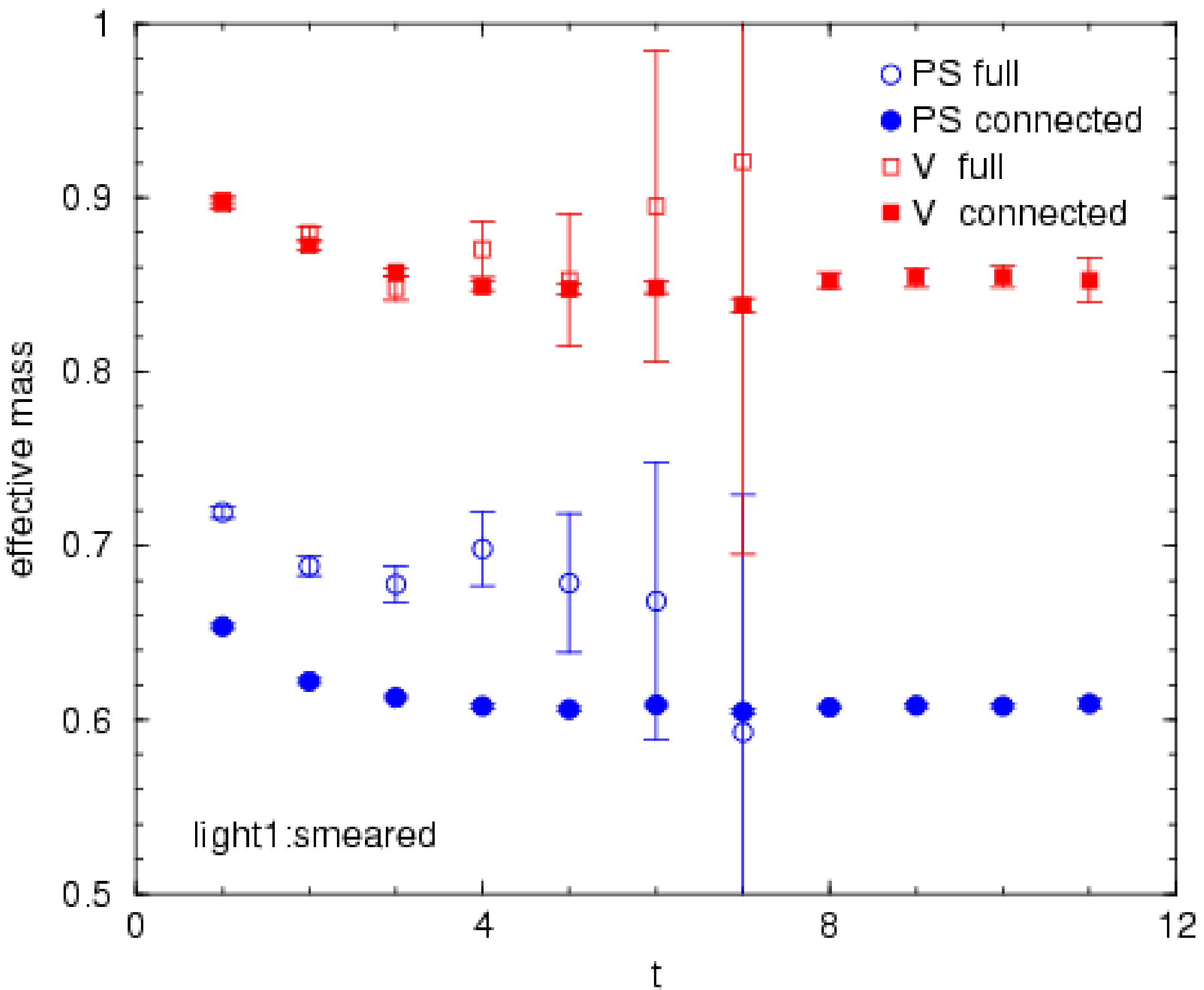
Effective mass plot
of connected diagram C(t)

Smearing functions $\phi(\vec{x})$ are
determined from wavefunction

Ground states dominate at t~2

$a(\, m(J/\psi) - m(\eta_c)\,) = 0.0676$ (~ 81 MeV)

Effective masses of full correlator (1/5)



$$\frac{m_q}{m_{charm}} \sim 0.34$$

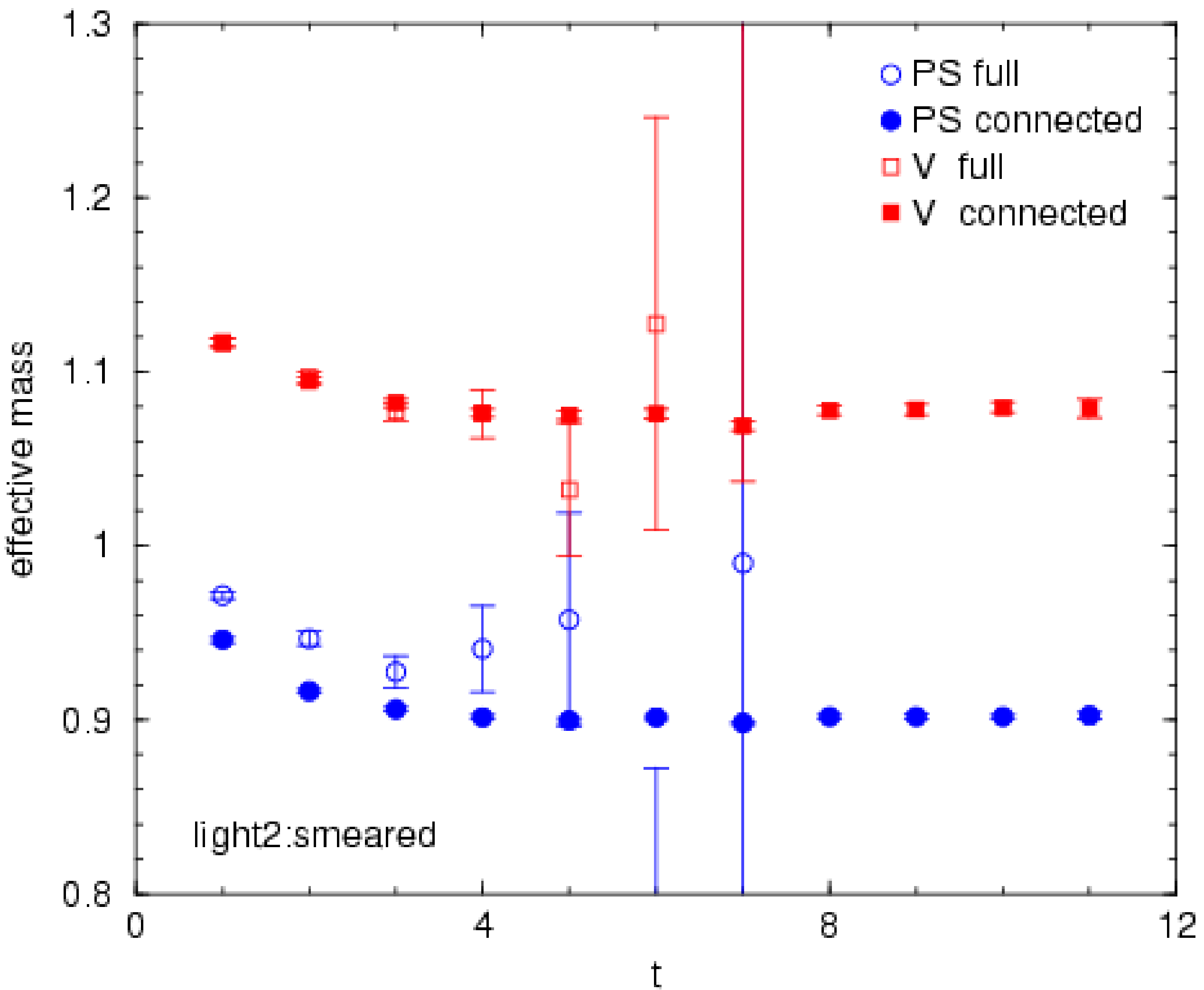
κ = 0.11294

1920 conf.

N_{NV} = 100

smeared operators

Effective masses of full correlator (2/5)



$\frac{m_q}{m_{charm}} \sim 0.42$

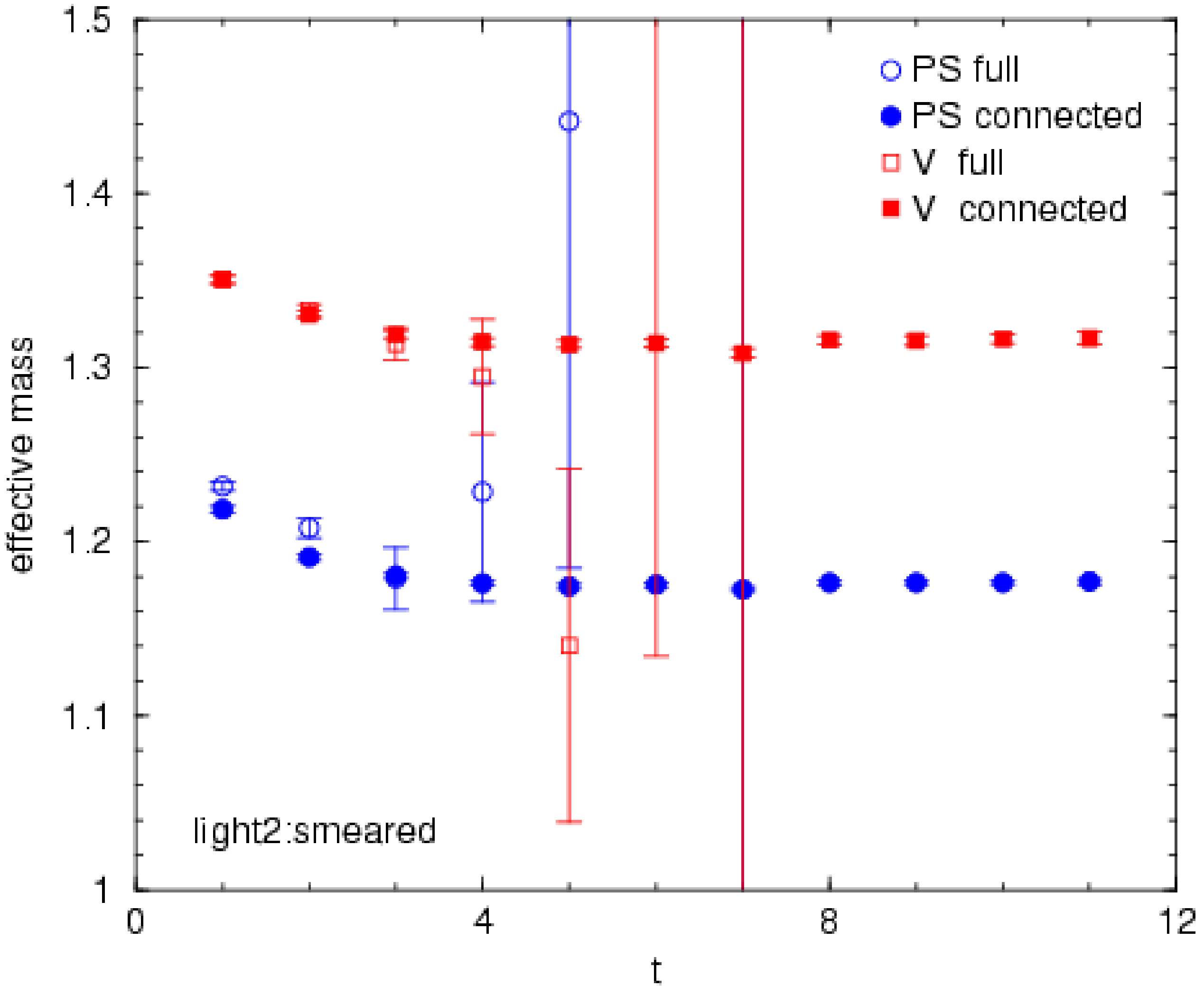
$\kappa = 0.11013$

1920 conf.

$N_{NV} = 200$

smeared operators

Effective masses of full correlator (3/5)



$$\frac{m_q}{m_{charm}} \sim 0.51$$

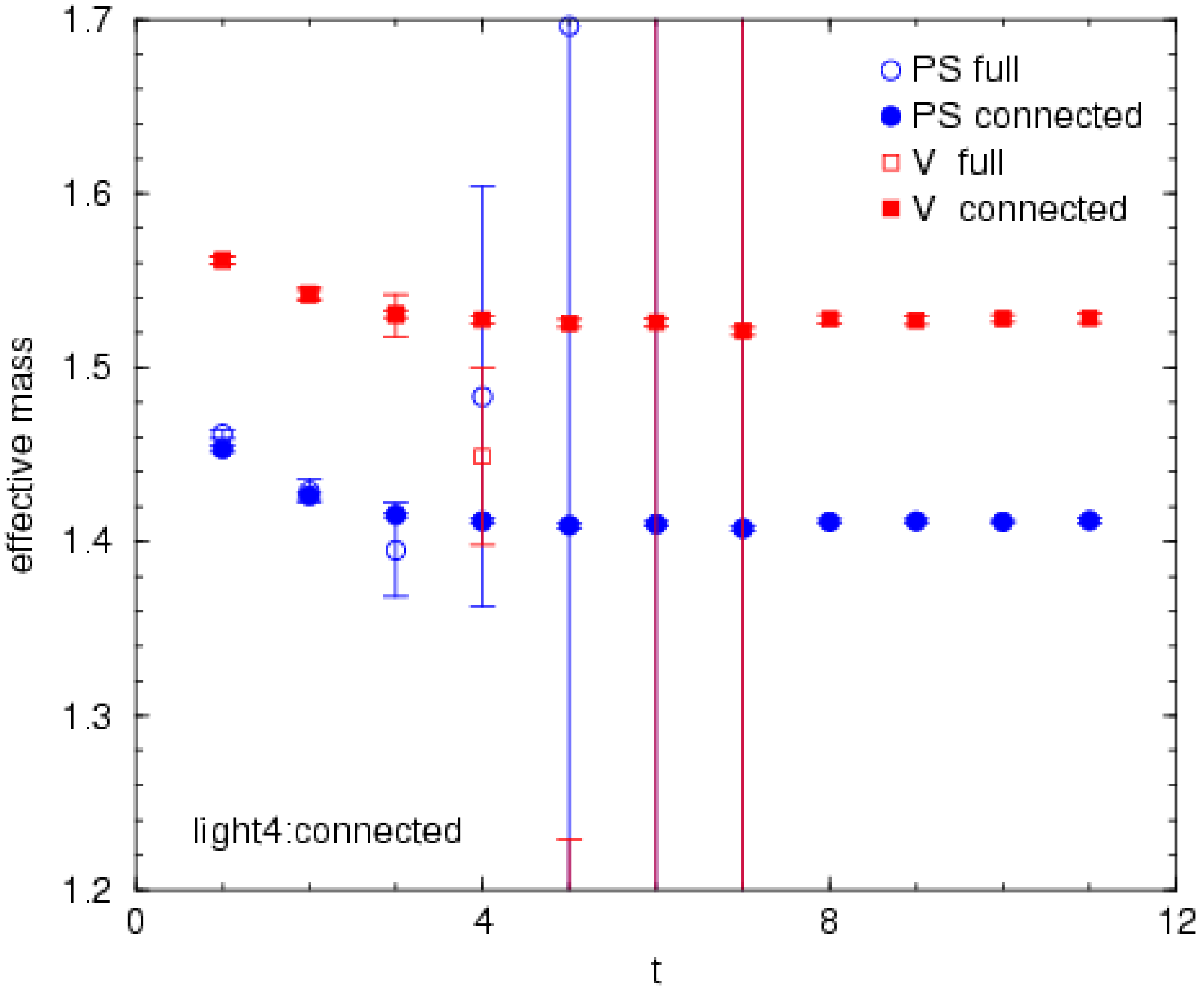
κ = 0.10732

1920 conf.

N_{NV} = 200

smeared operators

Effective masses of full correlator (4/5)



$\frac{m_q}{m_{charm}} \sim 0.60$

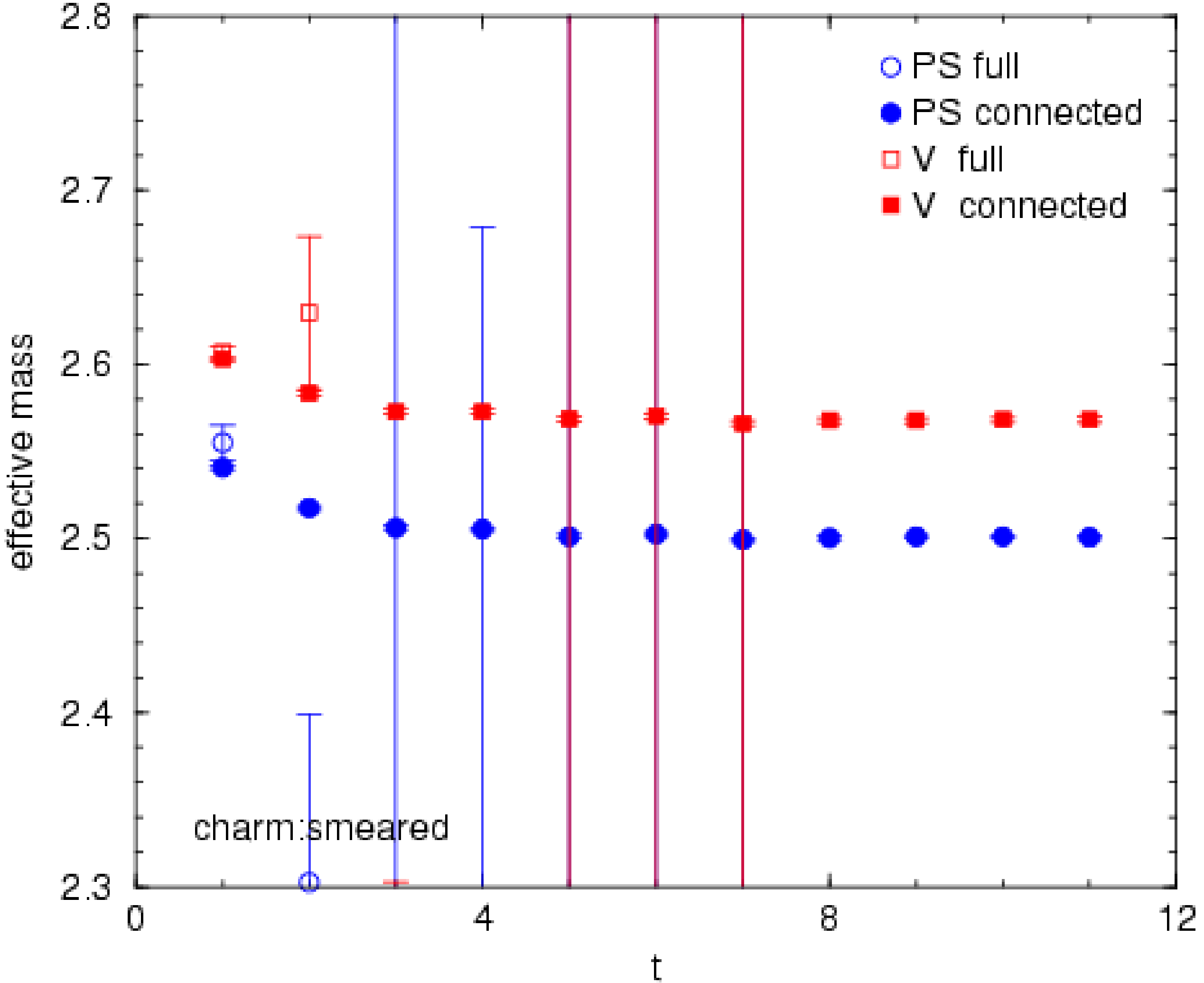
κ = 0.10476

1920 conf.

N_{NV} = 300

smeared operators

Effective masses of full correlator (5/5)



$$\frac{m_q}{m_{charm}} \sim 1.00$$

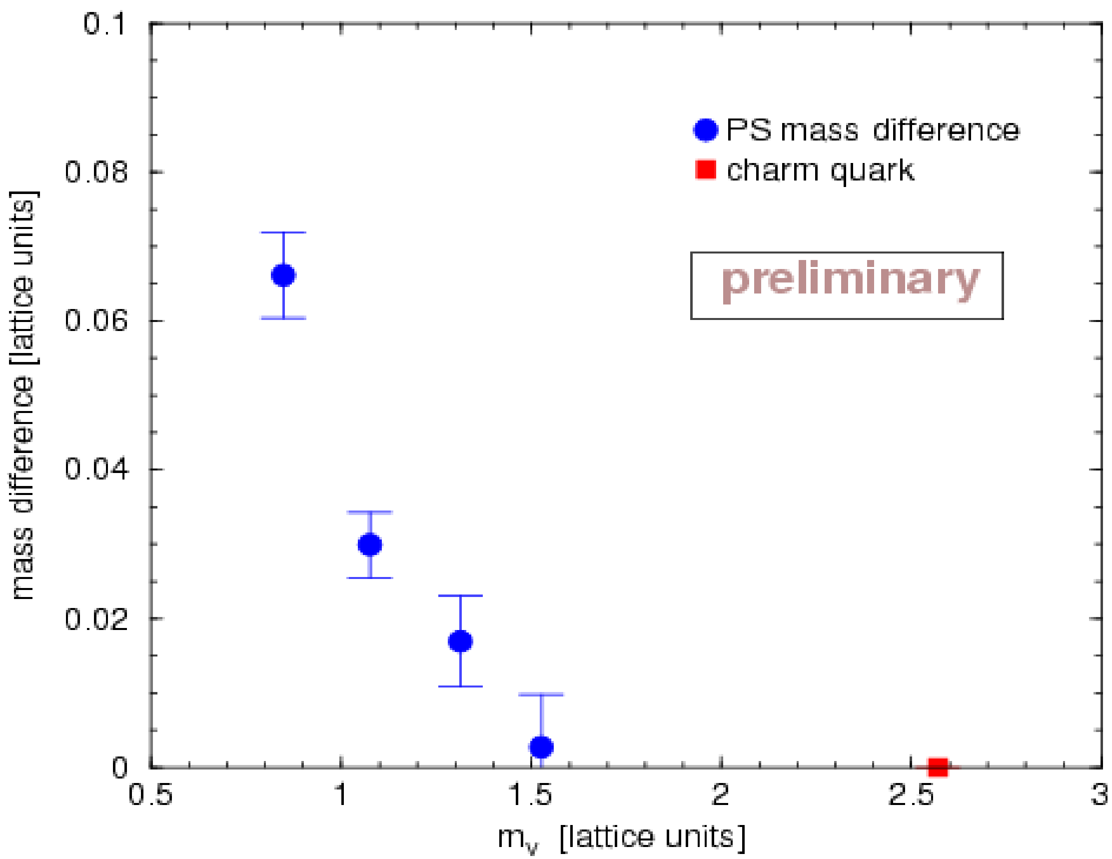
κ = 0.09342

3200 conf.

N_{NV} = 600

smeared operators

Contribution of disconnected diagram



- very small contribution for vector channel
- contribution of pseudoscalar channel

← $m_{full}(t=2) - m_{con}(t=2)$

The contribution of disconnected diagram is quickly suppressed as quark mass increase

Summary & Outlook

We study the problem of charmonium HFS
and consider a possibility of disconnected diagram contributions

- Isotropic lattice + Nonperturbative improved clover quark
is good choice for the charmonium spectrum
In quenched QCD, $m(J/\psi) - m(\eta_c)$ is
30 – 40% smaller than the experimental value

- Disconnected diagram contributions are very small
or hidden by large error

▶ calculations with small sea quark mass & large lattice cutoff

Full QCD simulations including the continuum limit