A new approach to QCD thermodynamics on the lattice

Takashi Umeda (YITP, Kyoto Univ.)
for WHOT-QCD Collaboration

This talk is (partly) based on arXiv:0809.2842 [hep-lat]

T.U, S. Ejiri, S. Aoki, T. Hatsuda, K. Kanaya,
Y. Maezawa, and H. Ohno
(WHOT-QCD Collaboration)

YITP seminar, Kyoto, Japan, 17 Dec. 2008

Contents of this talk

Our aim is to investigate
QCD Thermodynamics with Wilson-type quarks

- Brief review on Lattice QCD at finite T (zero μ)
- Why do we need "Hot QCD with Wilson-type quarks" ?
- Why is "Hot QCD with Wilson-type quarks" difficult ?
- How do we overcome the difficulties ?
  - We propose "T-integration method"
  - Test with the SU(3) gauge theory
- Summary and Outlook

Introduction

Physics in Lattice QCD at finite temperature

- Phase diagram in (T, μ, m_{ud}, m_s)
- Transition temperature
- Equation of state (e, p, s,...)
- Heavy quarkonium
- Transport coefficients (shear/bulk viscosity)
- Finite chemical potential
- etc...

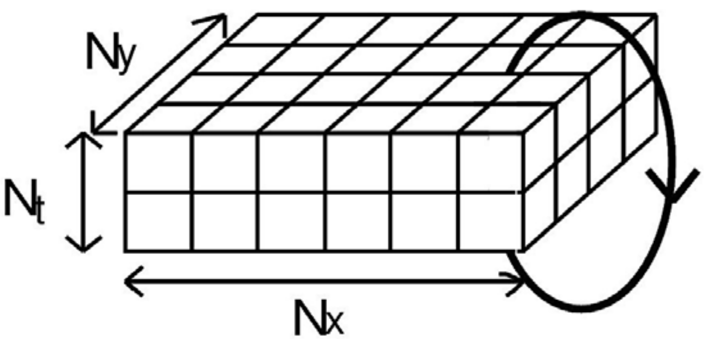
(Phase diagram, transition temperature, equation of state: quantitative studies)

(Heavy quarkonium, transport coefficients, finite chemical potential, etc.: qualitative studies)

These are important to study

- Quark Gluon Plasma in Heavy Ion Collision exp.
- Early universe
- Neutron star
- etc...

Hot QCD on the lattice



Finite T Field Theory on the lattice

- 4dim. Euclidean lattice
- gauge field $U_\mu(x)$ → periodic B.C.
- quark field q(x) → anti-periodic B.C.
- Temperature $T=1/(N_t a)$

Input parameters :	β (=6/g^2)	(lattice gauge coupling)
(Nf=2+1 QCD)	am_{ud}	(light (up & down) quark masses)
	am_s	(strange quark mass)
	N_t	(temperature)

(*) lattice spacing "a" is not an input parameter

$a=a(\beta, am_{ud}, am_s)$

Temperature $T=1/(N_t a)$ is varied by a at fixed N_t

Fermions on the lattice

Lattice QCD

Path integral is carried out by Monte Carlo Integration

QCD action is defined on the lattice

Fermion doubling problem

- naive discretization causes 2^4 doublers
- Nielsen-Ninomiya's No-go theorem

  → Doublers appear unless chiral symmetry is broken

- Staggered (KS) fermion
  - 16 doublers = 4 spinors x 4 flavors ("tastes")
  - Remnant U(1) symmetry
  - Fourth root trick : still debated
  - Numerical cost is low

and …

Fermions on the lattice

- Wilson fermion
  - adds the Wilson term to kill extra 2^4-1 doublers
  - breaks chiral symmetry explicitly → additive mass renorm.
  - Improved version (Clover fermion) is widely used.
  - Numerical cost is moderate
- Domain Wall fermion
  - 5dim. formulation
  - Symmetry breaking effect $m_{res} \to 0$ as $N_5 \to \infty$
  - Numerical cost is high
- Overlap fermion
  - Exact chiral symmetry
  - Numerical cost is very high

} Wilson-type fermions

Recent lattice calculations of EOS

Hot-QCD Collab. (2007～)

- RBC-Bielefeld: N_t=4,6,8 Staggered (p4) quark
  pion mass ~ 220MeV, N_f=2+1
  Phys. Rev. D77 (2008) 014511
- MILC: N_t=4,6,8 Staggered (Asqtad) quark
  pion mass ~ 220MeV, N_f=2+1
  Phys. Rev. D75 (2007) 094505

Wuppertal: N_t=4,6 Staggered (stout) quark
pion mass ~ 140MeV, N_f=2+1
JHEP 0601 (2006) 089

CP-PACS: N_t=4,6 Wilson (MFI Clover) quark
pion mass ~ 500MeV, N_f=2
Phys. Rev. D64 (2001) 074510

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Problems in QCD Thermo. with KS fermions

Many QCD thermo. calc. were done with KS fermions.

- Phase diagram
  N_f=2 massless QCD → O(4) critical exponets
  KS fermion does not exhibit expected O(4) scaling
  (Wilson fermion shows O(4), but at rather heavy masses)

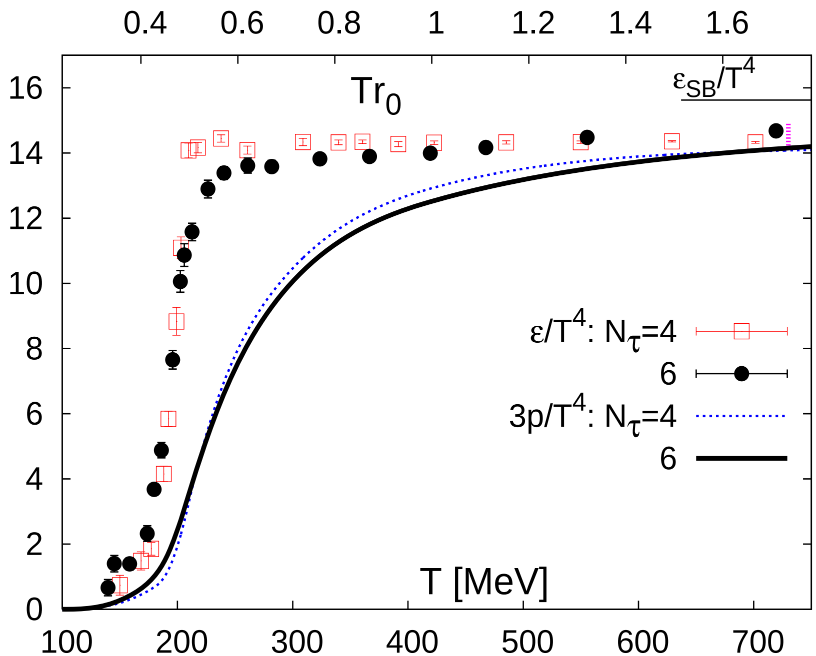
- Transition temperature (crossover transition in KS studies)
  KS results are not consistent with each other

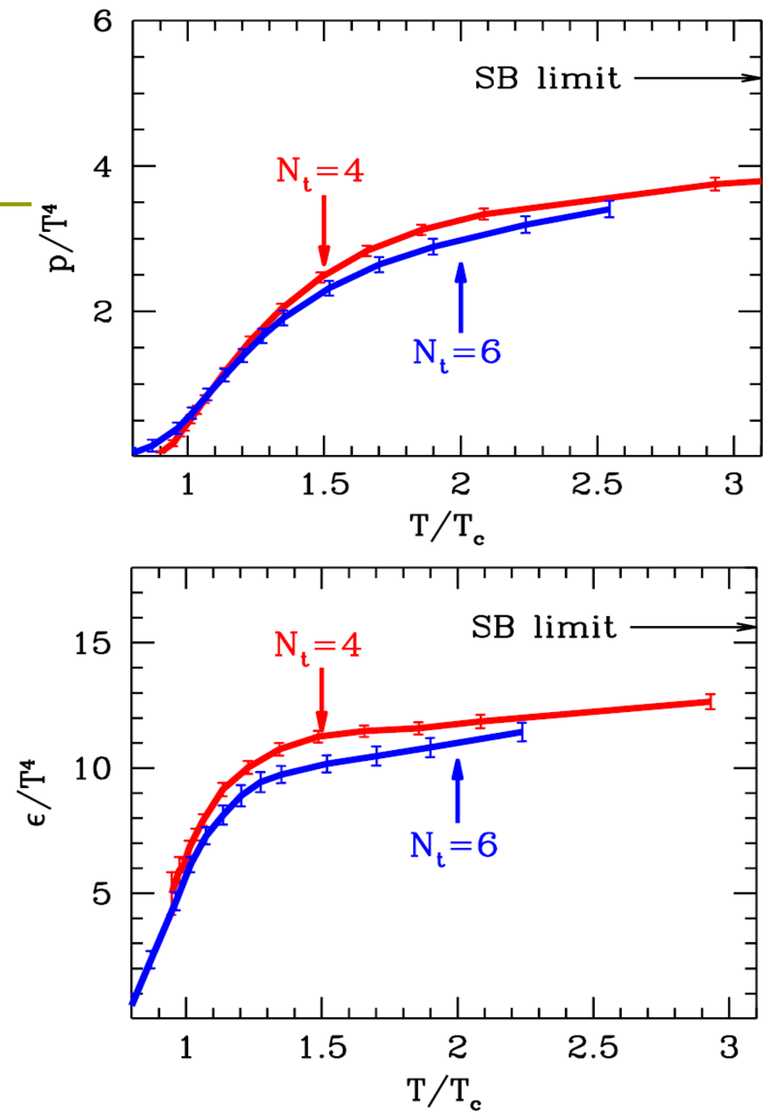
MILC :	169(12)(4)MeV(*)	*Phys. Rev. D71 (2005) 034504*
RBC-Bi :	192(7)(4)MeV	*Phys. Rev. D74 (2006) 054507*
Wuppertal :	151(3)(3)MeV	*Phys. Lett. B643 (2006) 46*

(*)T_c at m_q=0

- EOS
  KS results are not consistent with each other
  MILC & RBC-Bi are consistent (N_t=4,6,8)
  Wuppertal (N_t=4,6)

EOS with KS fermions



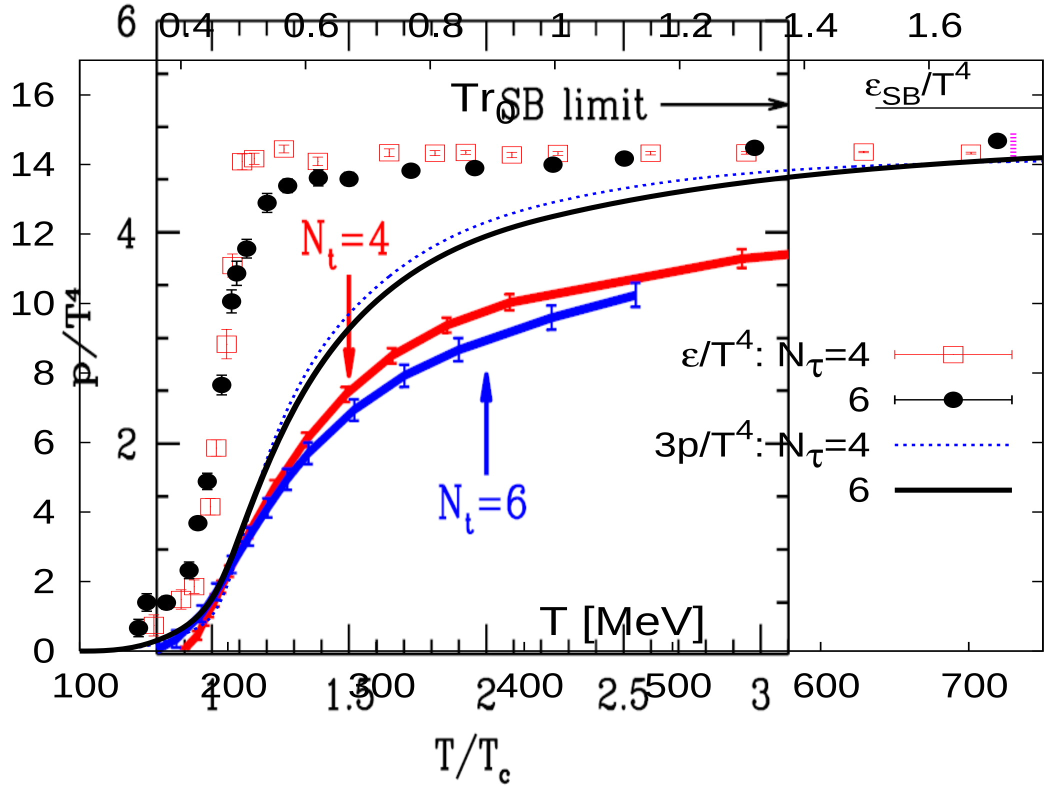


M.Chen et al. (RBC-Bielefeld) Phys. Rev. D77 (2008) 014511.

Y.Aoki et al. (Wuppertal) JHEP 0601 (2006) 089.

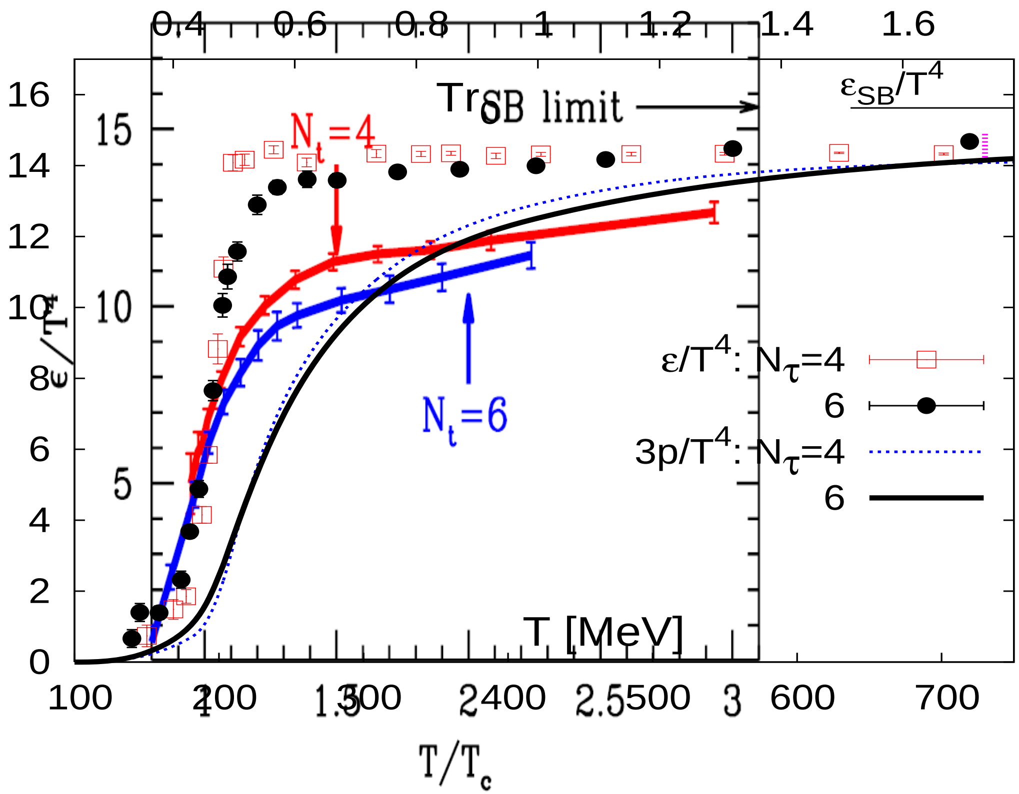
EOS with KS fermions

RBC-Bi vs Wuppertal for pressure p/T^4



EOS with KS fermions

RBC-Bi vs Wuppertal for energy density e/T^4



- ✓ N_t is small yet ?
- ✓ rooted trick ?
- ✓ flavor symmetry violation ?
- ✓ other systematic errors ?

We have to study the QCD-EOS with Wilson-type fermions !!

Contents of this talk

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QCD Thermodynamics with Wi

- Hot QCD requires huge computational cost
→ e.g. EOS calculation
- Wilson-type quarks requires large lattice cutoff simulations

- Brief review on Lattice QCD at finite T (
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Integral method to calculate pressure p/T^4

$$p = \frac{T}{V} \ln Z$$ for large volume system

Lattice QCD can not directly calculate the partition function $\ln Z$

however its derivative is possible $$\frac{\partial}{\partial \beta} \ln Z = - \left\langle \frac{\partial S_{QCD}}{\partial \beta} \right\rangle$$

One can obtain p as the integral of derivative of p

high temp.

low temp.
with p≃0

$$\left. \frac{p}{T^4} \right|_{\beta_0}^{\beta} = \frac{1}{VT^3} \int_{\beta_0}^{\beta} d\beta' \frac{\partial}{\partial \beta'} \ln Z$$

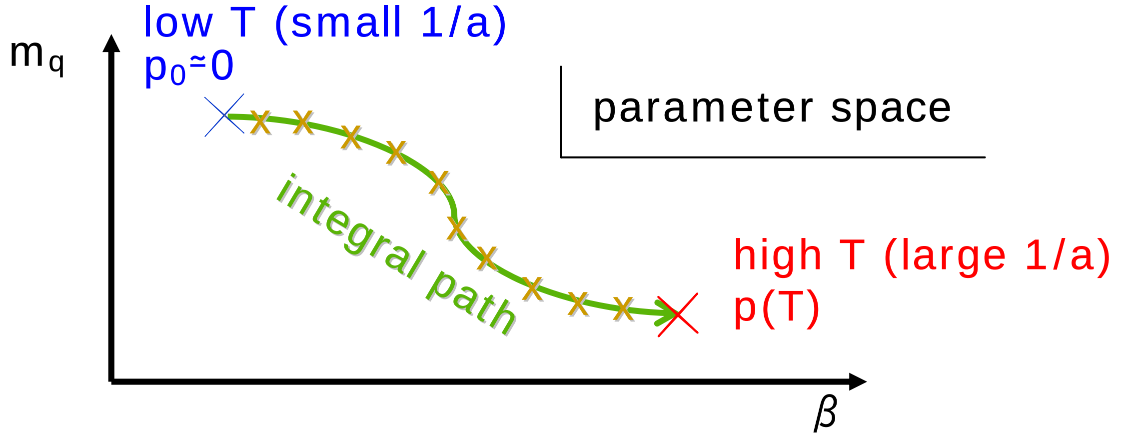
$$= -N_t^4 \int_{\beta_0}^{\beta} d\beta' \frac{1}{N_s^3 N_t} \left(\langle \frac{\partial S_{QCD}}{\partial \beta} \rangle_{T>0} - \langle \frac{\partial S_{QCD}}{\partial \beta} \rangle_{T=0} \right)$$

T=0 subtraction

Line of constant physics (LCP)

In case of $N_f=2+1$ QCD

there are three (bare) parameters: β , (am_{ud}) and (am_s)



- The physics (observables) should be kept along the integral path.
  ➔ Line of Constant Physics (LCP) defined at T=0
  Inaccuracy of the LCP is a source of systematic error in EOS.
- Integral on the path is carried out numerically.
  T=0 subtractions are necessary at each point.

Numerical cost for EOS calculations

In the EOS calculation,

T=0 calculations dominate in spite of T>0 study.

- Search for a Line of Constant Physics (LCP)
- T=0 subtraction at each temperature

T=0 simulations are time consuming.

- N_t is sufficiently large (e.g. 24^3x24 at T=0, 24^3x6 at T>0)
- small Dirac eigenvalue (larger cost for $D^{-1}(x,y)$)

(cost at T=0) = (5~20) x (cost at T>0)

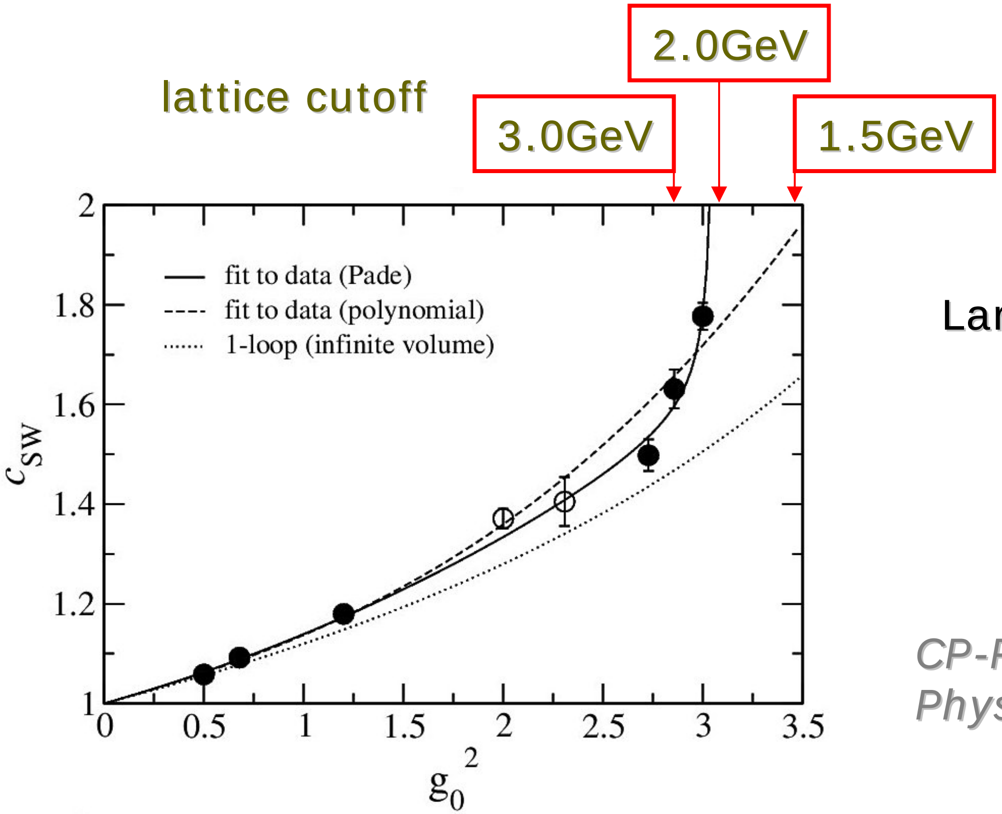
Even with the Staggered fermions,

EOS at N_t=8 is the best with current computer resources.

Further problems in Wilson-type quarks

Nonperturbative improvement of Wilson fermions :
clover coefficient c_{sw} by the Schrodinger functional method

lattice cutoff

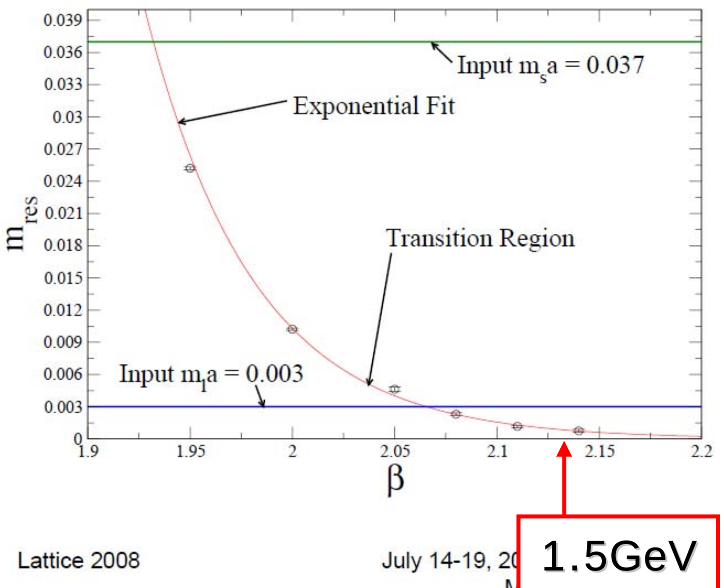


Large uncertainty of c_{sw}
at $1/a < 2$GeV

CP-PACS,
Phys. Rev. D73 (2006) 034501

Further problems in Wilson-type quarks

Residual quark mass m_{res} in Domain Wall fermion



- $\mathbf{m_{res}}$ increases exponentially with decreasing β for L_s=32
- Total quark mass is not fixed with β → increases as we go to coarser lattices
- $\mathbf{m_{res}}$ dominated by non-perturbative lattice dislocations, suppressed by only $1/L_s$

Lattice 2008 July 14-19, 20 Mary 7

Residual quark mass is not well controlled at 1/a < 2GeV (at typical L_s)

RBC & Hot-QCD, Lattice 2008

RBC & HOT-QCD Collab. gave up N_t=8, L_s=32 Domain Wall project.

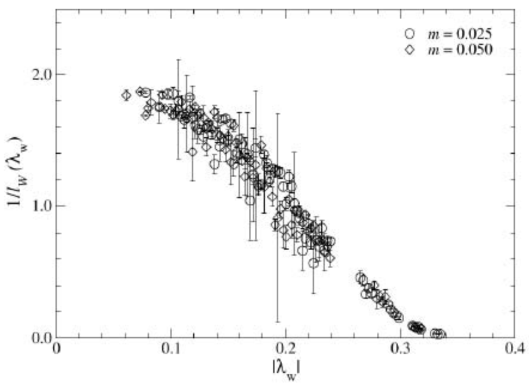
→ N_t=8, L_s=96 project on progress

Further problems in Wilson-type quarks

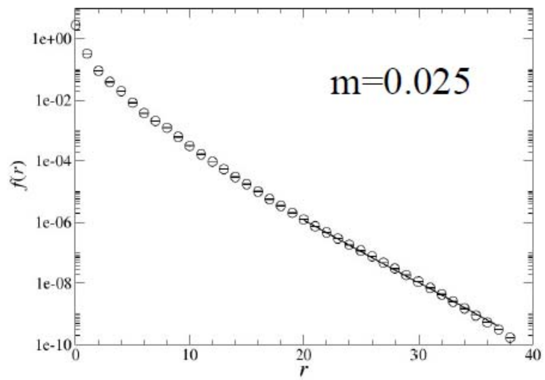
Locality

JLQCD, 2008; JLQCD (Yamada et al.), Proc. of Lattice 2006

- At beta=2.3 (Nf=2) overlap fermion



Localization length of low eigenmodes of H_W



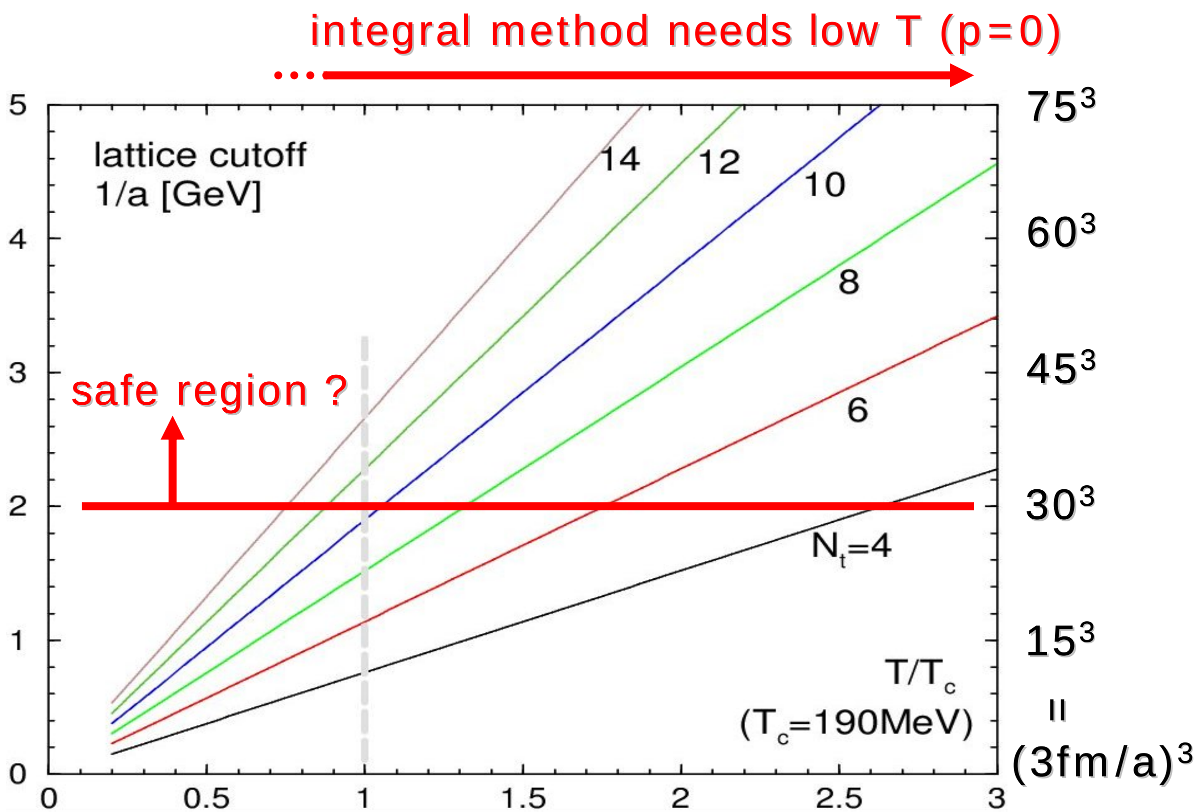
Localization of overlap operator
$l = 0.25\text{fm} (\sim 2a)$

JLQCD, TQFT(YITP) 2008

Coarse lattice generally causes various problems.
→ 1/a > 2GeV is safe to calculate physics at T=0 & T>0.

How large Nt is safe ?

T vs 1/a at various Nt



- KS fermion results are not sufficient to finalize QCD-EOS in lattice QCD
- EOS calc. is very costly many T=0 simulations
- Wilson-type fermions needs larger 1/a
- Situation for T_c calc. is similar to the EOS
- Phase diagram study needs more cost !!

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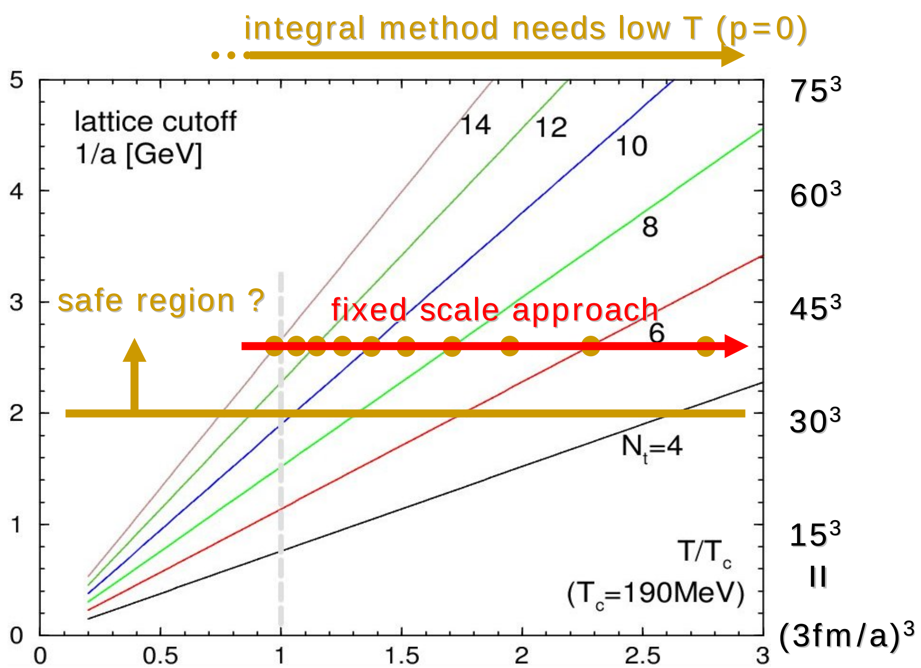
Fixed scale approach to study QCD thermodynamics

Temperature $T=1/(N_t a)$ is varied by N_t at fixed $a(\beta, m_{ud}, m_s)$



- Advantages
  - LCP is trivially exact
  - T=0 subtraction is done with a common T=0 sim. (T=0 high. stat. spectrum)
  - easy to keep large 1/a at whole T region
  - easy to study T effect without V, 1/a effects
- Disadvantages
  - T resolution by integer N_t
  - program for odd N_t
  - (1/a)/T = const. is not suited for high T limit study

T-integration method to calculate the EOS

We propose a new method ("T-integration method")
to calculate the EOS at fixed scales

T.Umeda et al. (WHOT-QCD) arXiv:0809.2842 [hep-lat]

Our method is based on the trace anomaly (interaction measure),

$$\frac{\epsilon - 3p}{T^4} = \left(\frac{N_t^3}{N_s^3}\right) a \frac{d\beta}{da} \left\langle \frac{dS}{d\beta} \right\rangle_{sub}$$

and the thermodynamic relation.

$$\frac{\epsilon - 3p}{T^4} = T \frac{\partial (p/T^4)}{\partial T}$$

$$\Rightarrow \quad \frac{p}{T^4} = \int_0^T dT' \, \frac{\epsilon - 3p}{T'^5}$$

Simulation parameters (isotropic lattices)

We present results from SU(3) gauge theory as a test of our method

- plaquette gauge action on N_s^3 x N_t lattices
- Jackknife analysis with appropriate bin-size

To study scale- & volume-dependence,
we prepare 3-type of lattices.

(1) β =6.0, V=(16a)^3
1/a=2.1GeV

β	N_s	N_t	T[MeV]	conf.
6.0	16	16	~ 0	350k
6.0	16	10	210	350k
6.0	16	9	230	250k
6.0	16	8	260	200k
6.0	16	7	300	100k
6.0	16	6	350	50k
6.0	16	5	420	50k
6.0	16	4	530	50k
6.0	16	3	700	50k

(2) β =6.0, V=(24a)^3
1/a=2.1GeV

β	N_s	N_t	T[MeV]	conf.
6.0	24	16	~ 0	150k
6.0	24	10	210	250k
6.0	24	9	230	200k
6.0	24	8	260	150k
6.0	24	7	300	100k
6.0	24	6	350	50k
6.0	24	5	420	50k
6.0	24	4	530	50k
6.0	24	3	700	50k

(3) β =6.2, V=(22a)^3
1/a=2.5GeV

β	N_s	N_t	T[MeV]	conf.
6.2	22	22	~ 0	250k
6.2	22	13	220	350k
6.2	22	12	240	350k
6.2	22	11	270	350k
6.2	22	10	290	250k
6.2	22	9	320	200k
6.2	22	8	360	200k
6.2	22	7	420	100k
6.2	22	6	490	100k
6.2	22	5	580	50k
6.2	22	4	730	50k

Simulation parameters (anisotropic lattice)

Anisotropic lattice is useful to increase Temp. resolution,
we also test our method on an anisotropic lattice $a_s \neq a_t$

- plaquette gauge action on N_s^3 x N_t lattices
  with anisotropy $\xi = a_s/a_t = 4$

β	N_s	N_t	T[MeV]	conf.
6.1	20	80	~ 0	220k
6.1	20	32	250	520k
6.1	20	30	270	220k
6.1	20	29	280	220k
6.1	20	28	290	220k
6.1	20	27	300	220k
6.1	20	26	310	220k
6.1	20	24	340	220k
6.1	20	22	370	220k
6.1	20	20	410	220k
6.1	20	18	450	220k
6.1	20	16	510	220k
6.1	20	14	580	220k
6.1	20	12	680	220k
6.1	20	10	810	220k
6.1	20	8	1010	220k

$\beta = 6.1$, $\xi = 4$
$V = (20a_s)^3$
$= (1.95\text{fm})^3$
$1/a_s = 2.0\text{GeV}$
$1/a_t = 8.1\text{GeV}$

- EOS calculation
- static quark free energy

β	N_s	N_t	T[MeV]	conf.
6.1	20	30	270	220k
6.1	20	29	280	220k
6.1	20	28	290	220k
6.1	20	27	300	220k
6.1	20	26	310	220k
6.1	30	30	270	80k
6.1	30	29	280	100k
6.1	30	28	290	180k
6.1	30	27	300	100k
6.1	30	26	310	80k
6.1	40	30	270	70k
6.1	40	29	280	130k
6.1	40	28	290	300k
6.1	40	27	300	140k
6.1	40	26	310	70k

$V = (20a_s)^3$
$= (1.95\text{fm})^3$

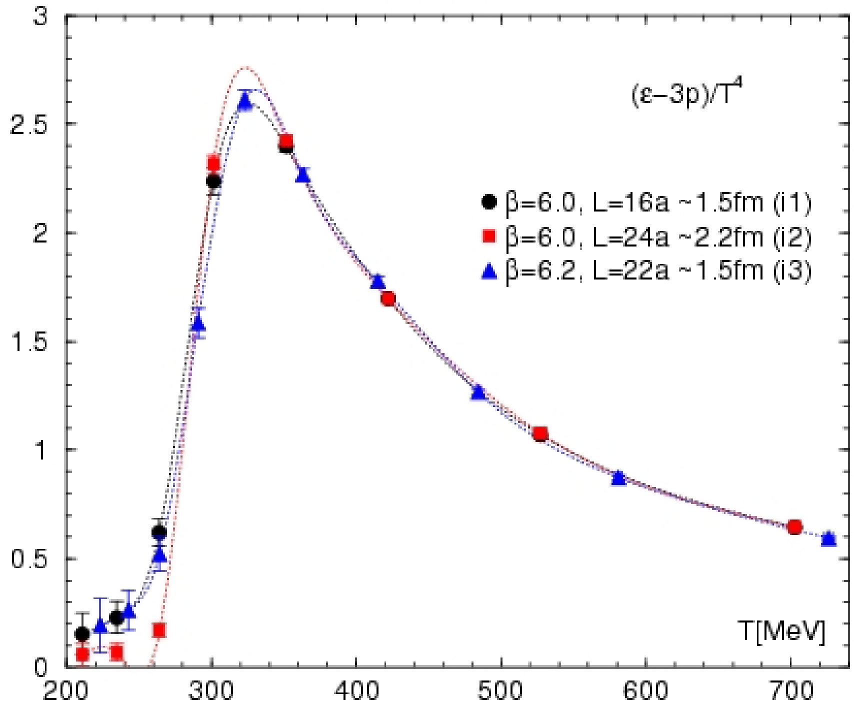
$V = (30a_s)^3$
$= (2.92\text{fm})^3$

$V = (40a_s)^3$
$= (3.89\text{fm})^3$

- critical temp.

Trace anomaly $(e - 3p)/T^4$ on isotropic lattices

$$\frac{\epsilon - 3p}{T^4} = \left(\frac{N_t^3}{N_s^3}\right) a\frac{\partial\beta}{\partial a}\left\langle\frac{\partial S_g}{\partial\beta}\right\rangle_{sub}$$



dotted lines : cubic spline

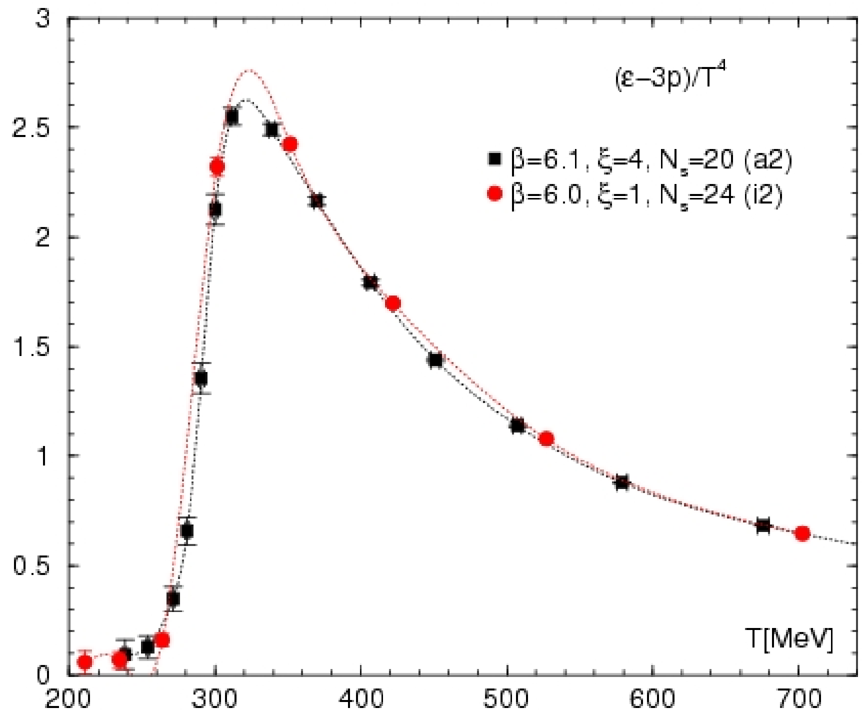
(1) β =6.0, 1/a=2.1GeV, V=$(1.5\text{fm})^3$
(2) β =6.0, 1/a=2.1GeV, V=$(2.2\text{fm})^3$
(3) β =6.2, 1/a=2.5GeV, V=$(1.5\text{fm})^3$

beta function : G.Boyd et al. ('96)
lattice scale r_0 : R.Edwards et al. ('98)

- Excellent agreement between (1) and (3)
  → scale violation is small
  1/a=2GeV is good
- Finite volume effect appears below & near T_c
  → volume size is important
  V=$(2\text{fm})^3$ is necessary.

Trace anomaly $(e - 3p)/T^4$ on aniso. lattice

$$\frac{\epsilon - 3p}{T^4} = \left(\frac{N_t^3}{N_s^3 \xi^3}\right) a_s \frac{\partial\beta}{\partial a_s}\bigg|_\xi \left\langle \frac{\partial S_g}{\partial\beta}\bigg|_\xi \right\rangle$$



dotted lines : cubic spline

(1) ξ =4, $1/a_s$=2.0GeV, V=(2.0fm)^3

(2) ξ =1, 1/a=2.1GeV, V=(2.2fm)^3

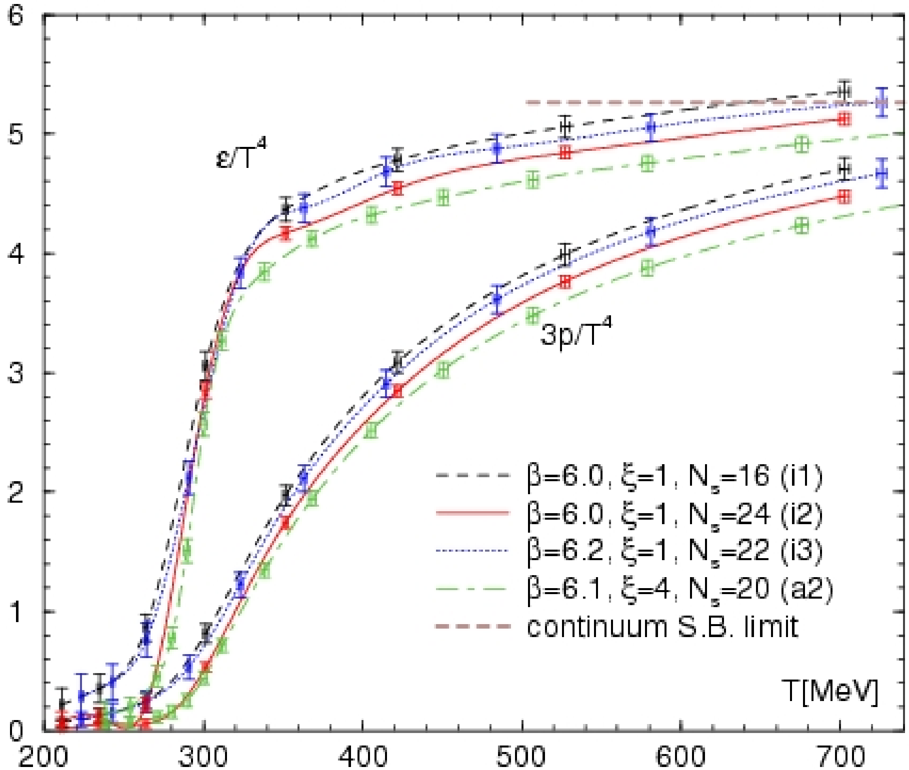
beta function : obtained by r_0/a_s fit
r_0/a_sdata H.Matsufuru et al. ('01)

$\frac{\partial\xi_0}{\partial\beta}\Big|_\xi$ is required in SU(3) gauge theory.

T.R.Klassen ('98)

- Anisotropic lattice is useful to increase Temp. resolution.

Pressure & Energy density



- Integration $\left(\frac{p}{T^4} = \int_0^T dT' \frac{\epsilon - 3p}{T'^5}\right)$

  is performed with the cubic spline of (e-3p)/T^4

- Cubic spline vs trapezoidal inte. yields small difference ~ 1σ

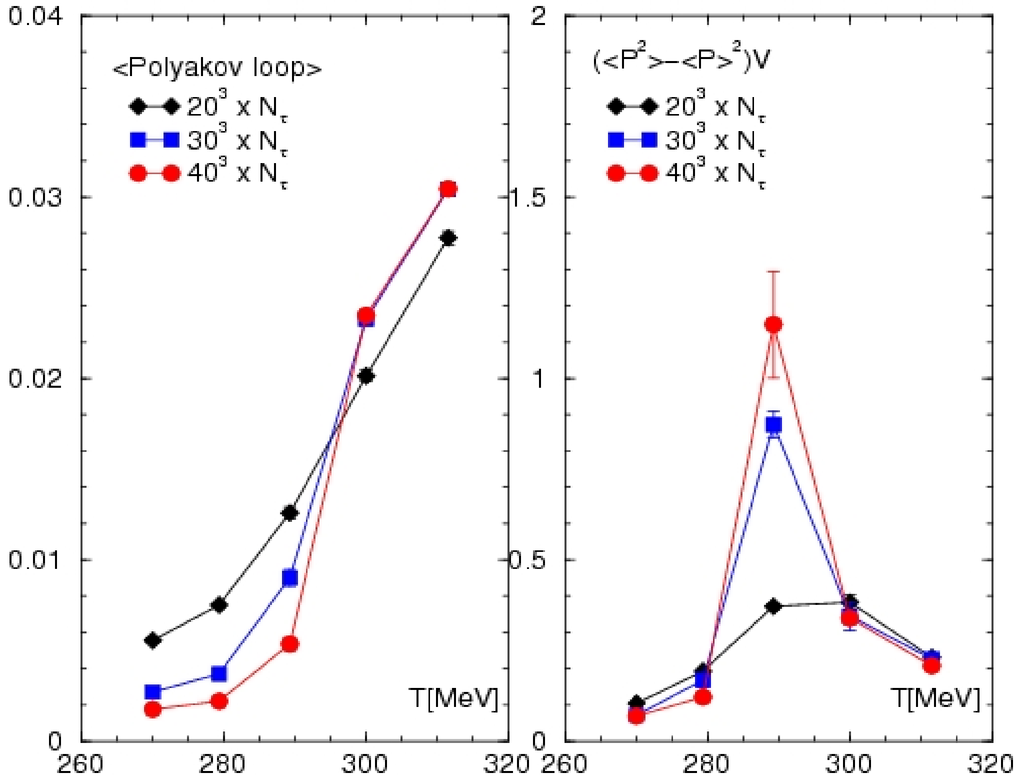
- Our results are roughly consistent with previous results.

- Unlike the fixed N_t approach, scale/temp. is not constant.

  → Lattice artifacts increase as temperature increases.

Our fixed scale approach with "T-integration method" works well !!

Transition temperature at fixed scale

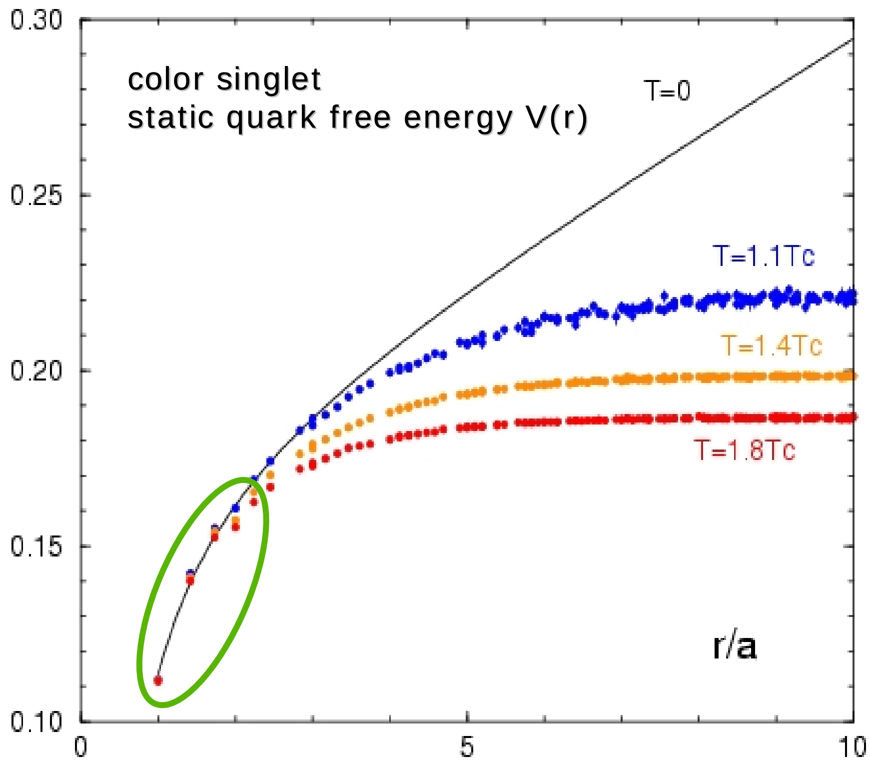


T-dependence of the (rotated) Polyakov loop and its susceptibility

- No renormalization is required upto overall factor due to the fixed scale.
- Rough estimation of critical temperature is possible.

T_c = 280~300 MeV at β =6.1, ξ =4 (SU(3) gauge theory)

Static quark free energy at fixed scale



Static quark free energies at fixed scale

- Due to the fixed scale, no renomalization constant is required.

→ small thermal effects in V(r) at short distance (without any matching)

- Easy to study temperature effect of V(r) without scale & volume effects

Toward full QCD calculations and new ideas at T>0 & μ >0

- There are many projects on high statistics full QCD at T=0.
  PACS-CS, JLQCD, MILC, RBRC, etc...
  - some basic quantities at T=0 are studied
  - T=0 config. are open to the public (by ILDG)

  our method requires no additional T=0 simulation !!

- We have already generated T>0 configurations
  using CP-PACS/JLQCD parameter
  (N_f=2+1 Clover+RG, 1/a=3GeV, pion mass ~ 500MeV)

- Our final goal is to study thermodynamics on
  the physical point (pion mass ~ 140MeV)
  with N_f=2+1 Wilson quarks (PACS-CS)
  or exact chiral symmetry with N_f=2+1 Overlap quarks (JLQCD)

- We are looking for new ideas to study other physics on our config.
  (density correlations, J/psi suppression, finite density...)

Backup slides

(e-3p)/T^4

our (a2),(i2) results vs Nt=4,6,8 in Ref[9]

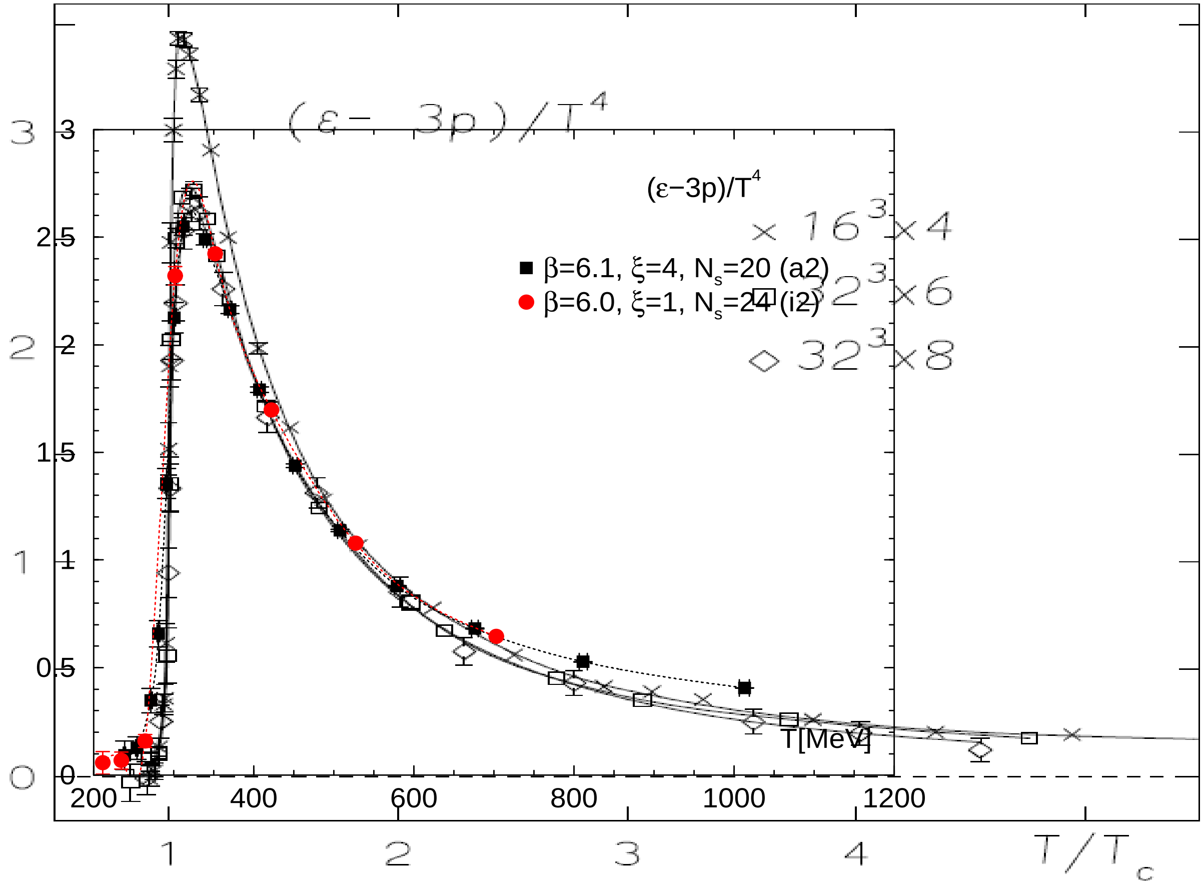
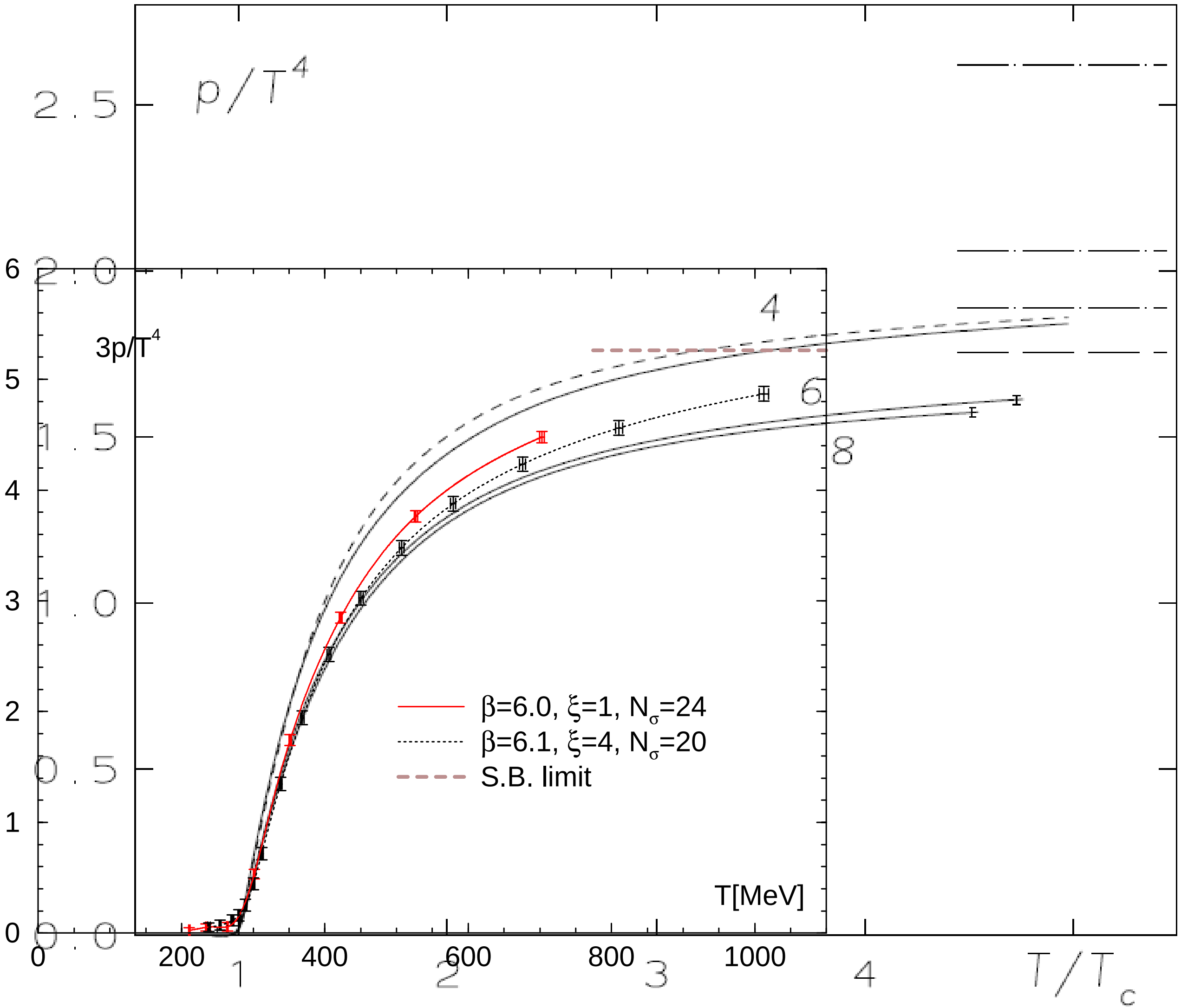


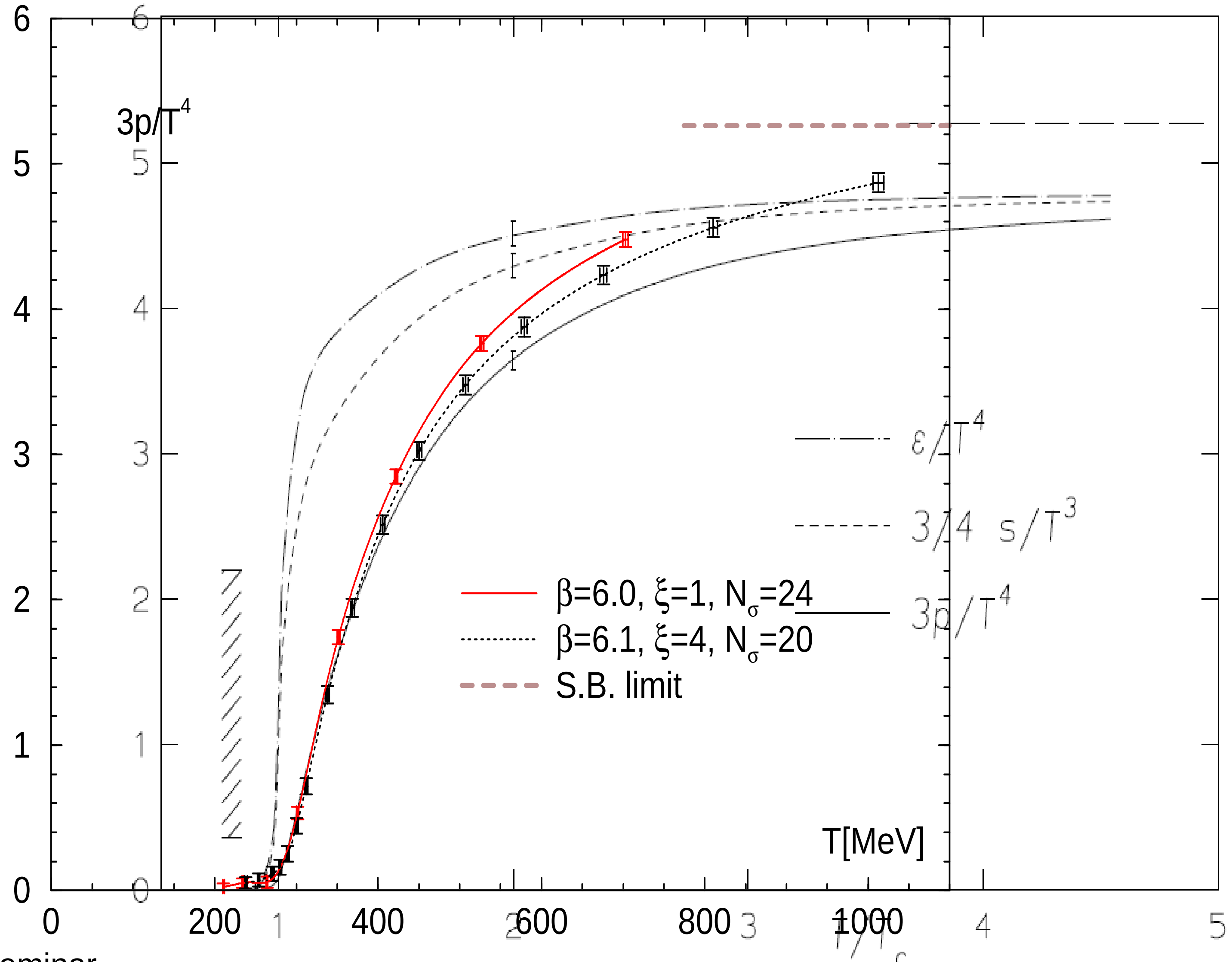
Figure 5: The difference $(\epsilon - 3p)/T^4$.

pressure

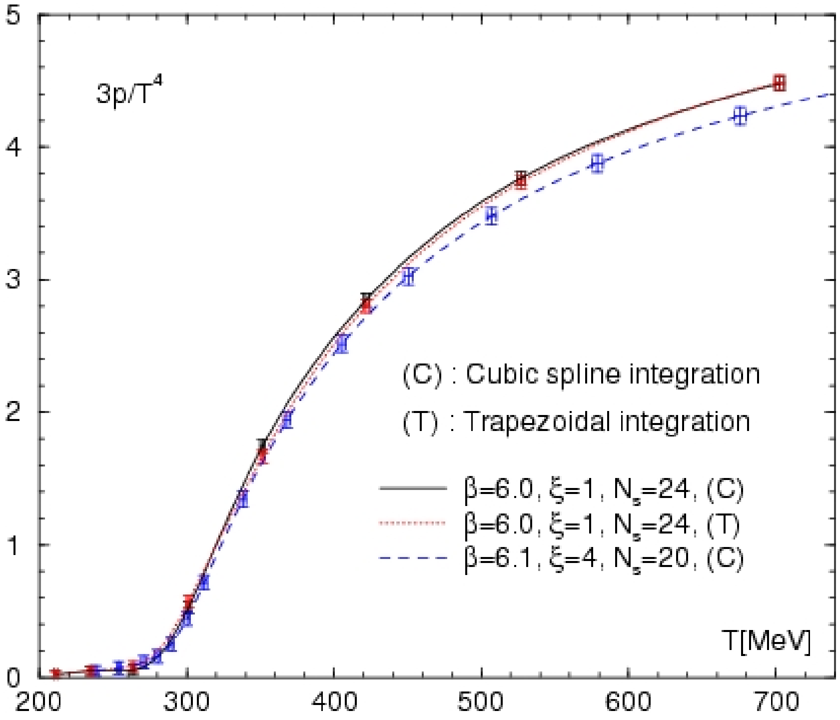
our (a2),(i2) results vs Nt=4,6,8 in Ref[9]

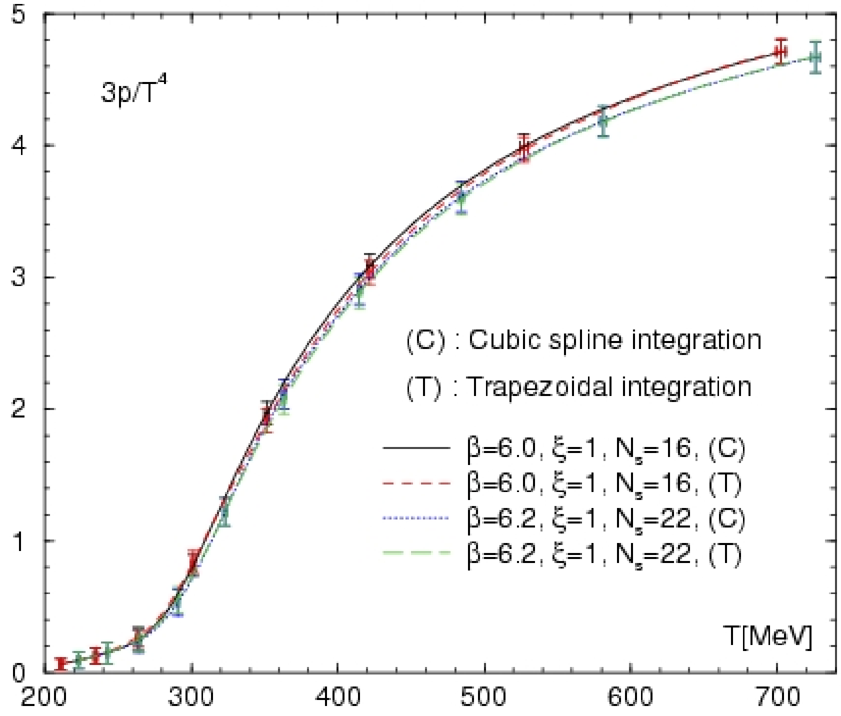


pressure
our (a2),(i2) results vs continuum limit in Ref[9]

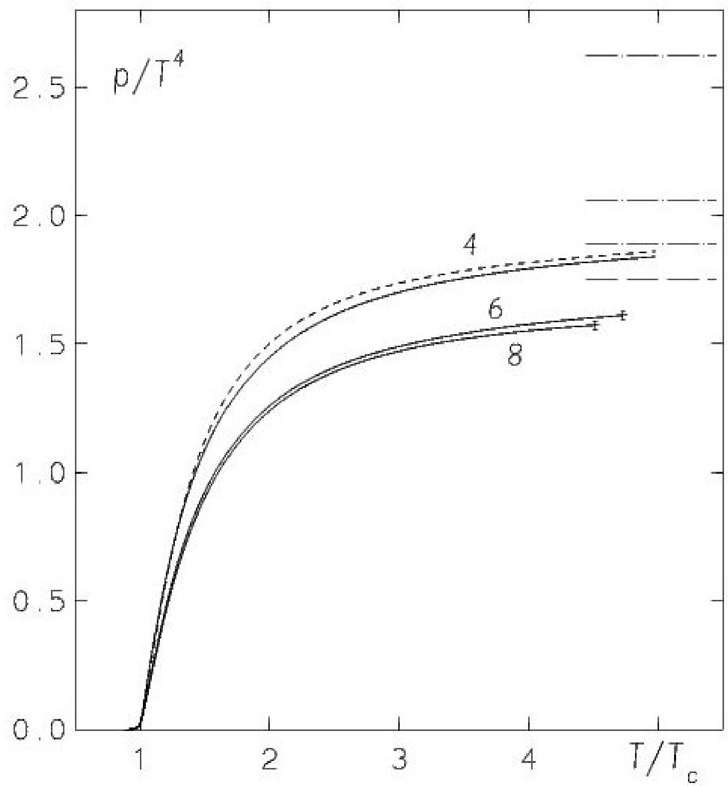


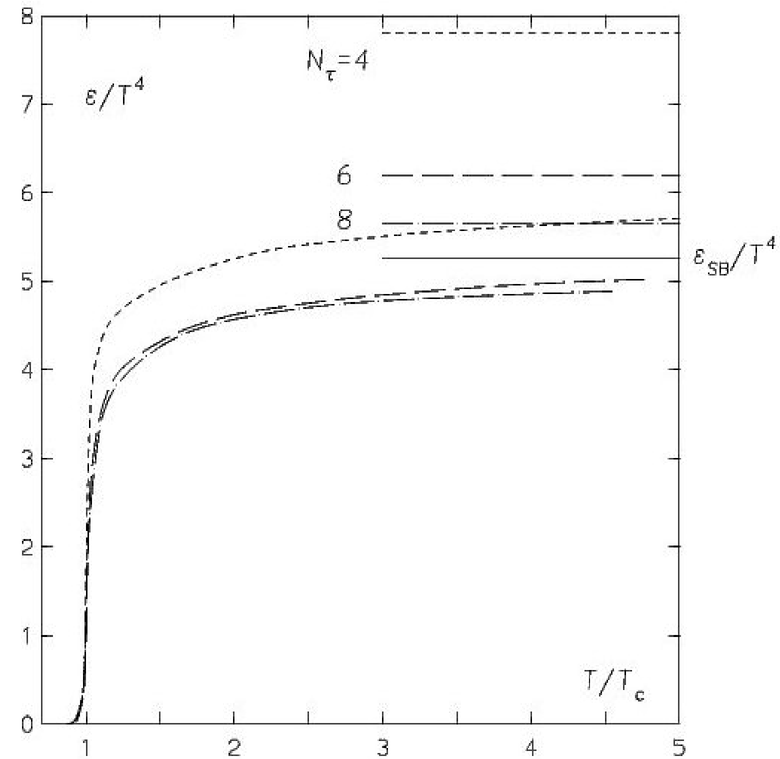
Pressure & Energy density





Pressure & Energy density



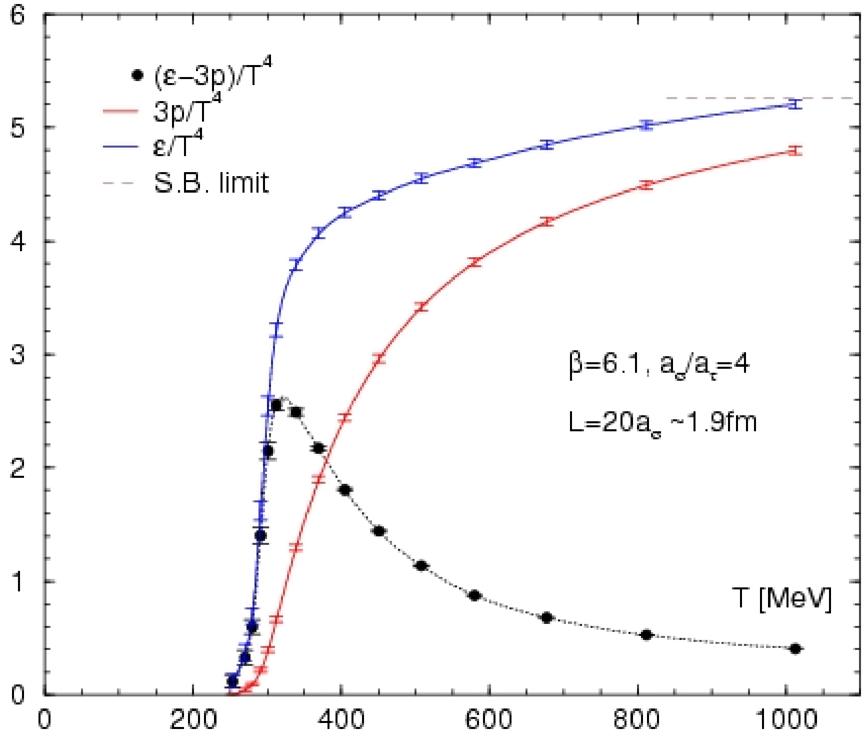


G.Boyd et al. ('96)

EOS on an anisotropic lattice

$$\frac{\epsilon - 3p}{T^4} = \left(\frac{N_\tau^3}{N_\sigma^3 \xi^3}\right) a_\sigma \frac{\partial \beta}{\partial a_\sigma}\bigg|_\xi \left\langle \frac{\partial S_g}{\partial \beta}\bigg|_\xi \right\rangle$$

beta function : obtained by r_0/a_σ fit
r_0/a_σ data H.Matsufuru et al. ('01)



- Anisotropic lattice is useful to increase Temp. resolution.
- Results are roughly consistent with previous & isotropic results
- Additional coefficients are required to calculate $(e-3p)/T^4$

$\frac{\partial \xi_0}{\partial \beta}\Big|_\xi$ is required in SU(3) gauge theory.

→ T.R.Klassen ('98)

Recent lattice calculations for Tc

RBC-Bielefeld: Nt=4,6,8 Staggered (p4) quark
pion mass ≥ 140MeV, Nf=2+1

MILC: Nt=4,6,8 Staggered (Asqtad) quark
pion mass ≥ 220MeV, Nf=2+1

Wuppertal: Nt=4,6,8,10 Staggered (stout) quark
pion mass ~ 140MeV, Nf=2+1

DIK: Nt=8,10,12 Wilson (Clover) quark
pion mass ≥ 500MeV, Nf=2

WHOT-QCD: Nt=4,6 Wilson (MFI Clover) quark
pion mass ≥ 500MeV, Nf=2

RBC-HOT: Nt=8 Domain Wall quarks
pion mass ~ 250MeV?, Nf=2+1