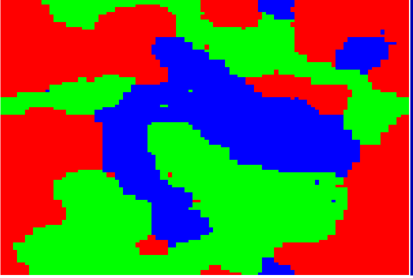
QCD thermodynamics on isotropic lattices

Frithjof Karsch
Brookhaven National Laboratory

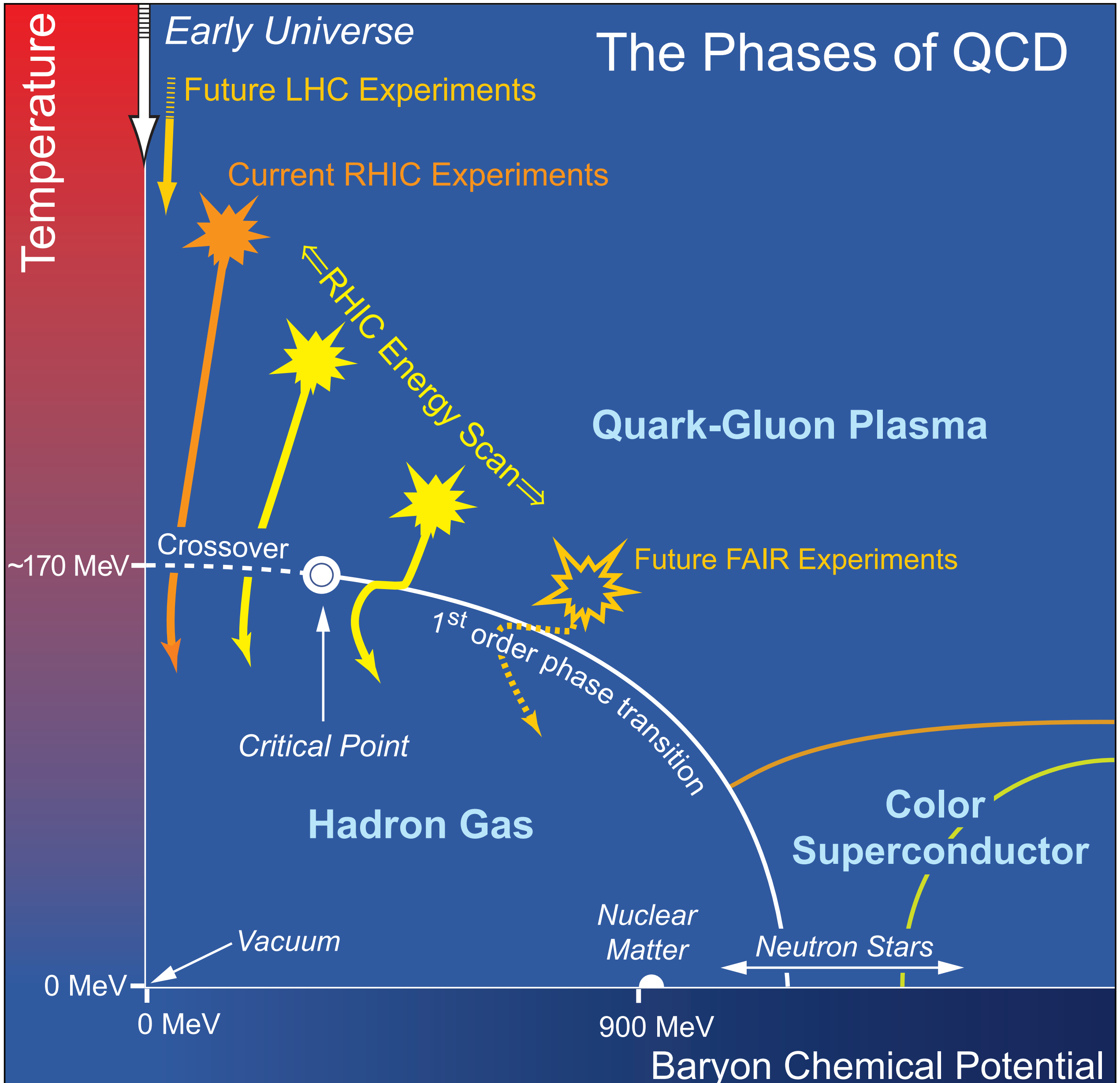
1987

1987

G. Burgers, F. Karsch, A. Nakamura and I. O. Stamatescu, QCD On Anisotropic Lattices, Nucl. Phys. B304, 587 (1988)



High Temperature/Density QCD



physics of the early universe

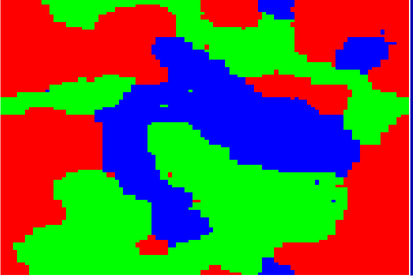
hot: $T \sim 10^{12} K$

our common interest

- Equation of State, T_c
- Transport Coefficients
- hadrons in medium
- finite density QCD
- Critical Point

properties of compact stars

dense: $n_B \sim 10 n_{NM}$



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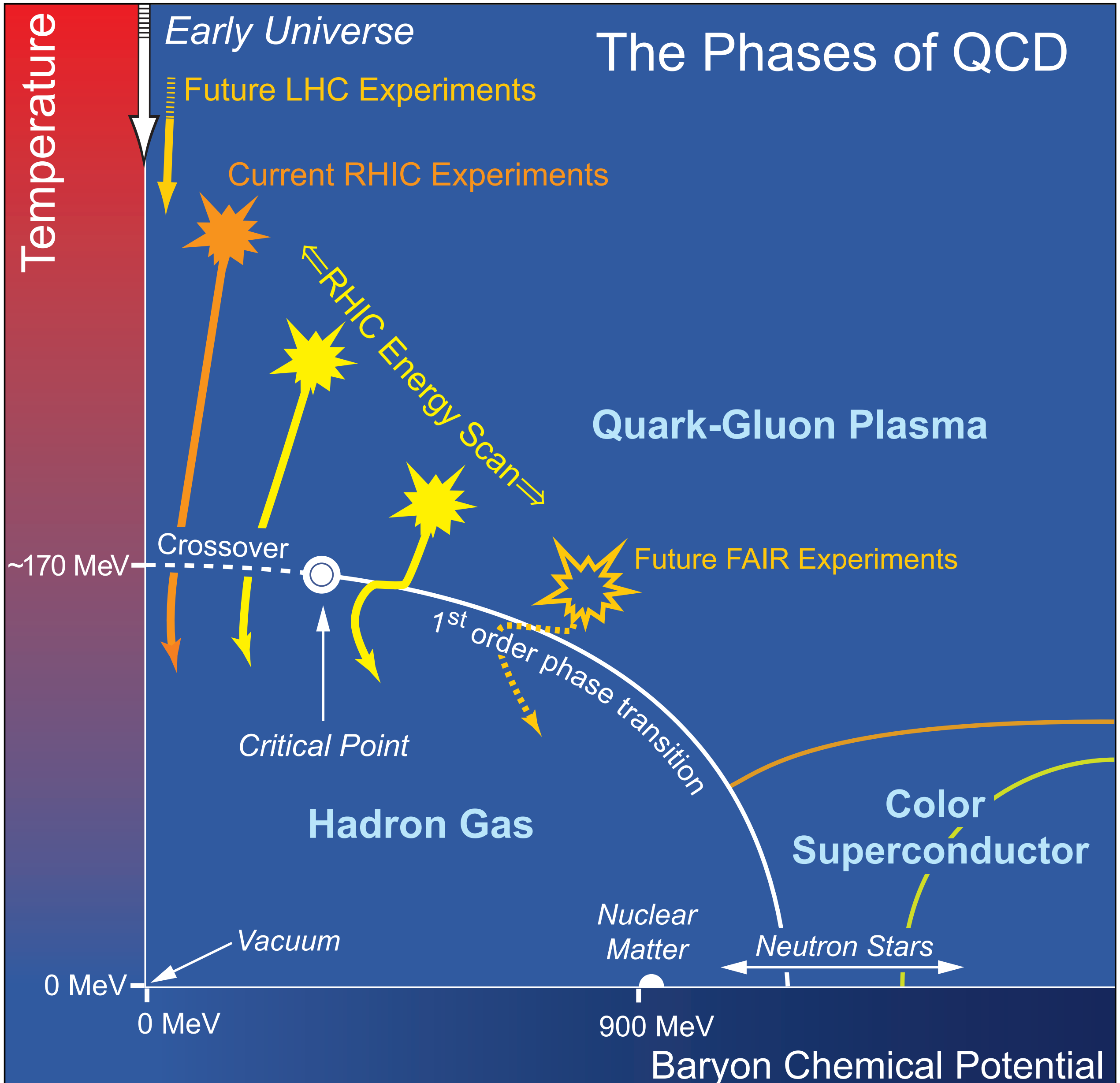

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Bulk thermodynamics

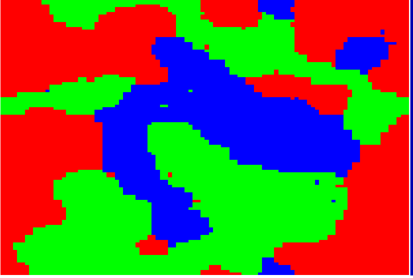
Goal: QCD thermodynamics with realistic quark masses in (2+1)-f QCD and controlled extrapolation to the continuum limit;

$\Rightarrow$ T_c, EoS,.. for $\mu_q \geq 0$

RBC-Bielefeld collaboration

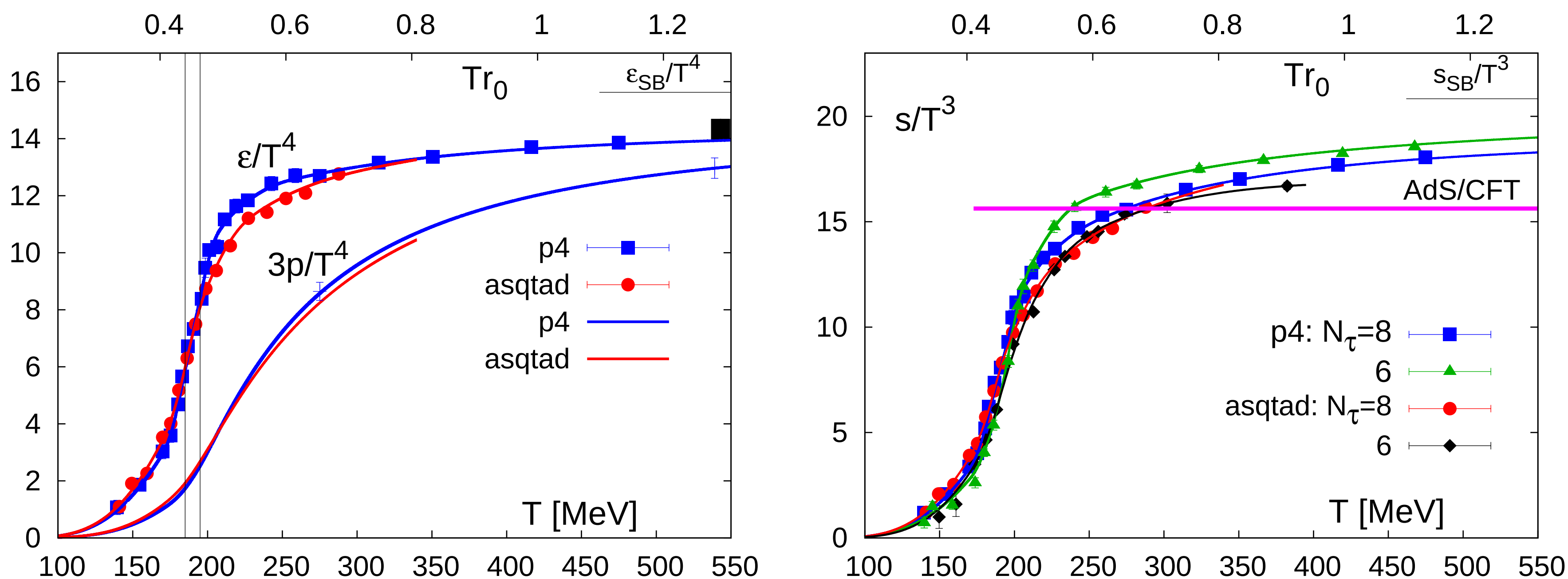
PRD77, 014511 (2008)

- $N_\tau = 4,\ 6$: bulk thermodynamics on a line of constant physics (LCP):
  (i) use $m_l = 0.1 m_s$, corresponding to $m_\pi \simeq 220$ MeV;
  (ii) tune m_s to physical strange quark mass using $m_K,\ m_{\bar{s}s}$
  at all values of the cut-off
- analyze EoS in a wide T-range: $140\ \mathrm{MeV} \lesssim T \lesssim 800\ \mathrm{MeV}$
- extend analysis to $N_\tau = 8$;
  compare p4 and asqtad results:
  joint project of RBC, Bielefeld, MILC, LANL and LLNL
  $\Rightarrow$ hotQCD collaboration, arXiv:0903.4379



Pressure, Energy and Entropy

- p/T^4 from integration over $(\epsilon - 3p)/T^5$;
  starting integration at $T = 0$ MeV with $p(0) = 0$;
  use hadron resonance gas at $T_0 = 100$ MeV to estimate systematic error:
  $[p(T_0)/T_0^4]_{HRG} \simeq 0.265$
- high-T region is well under control; significant deviations from conformal limit AND AdS/CFT



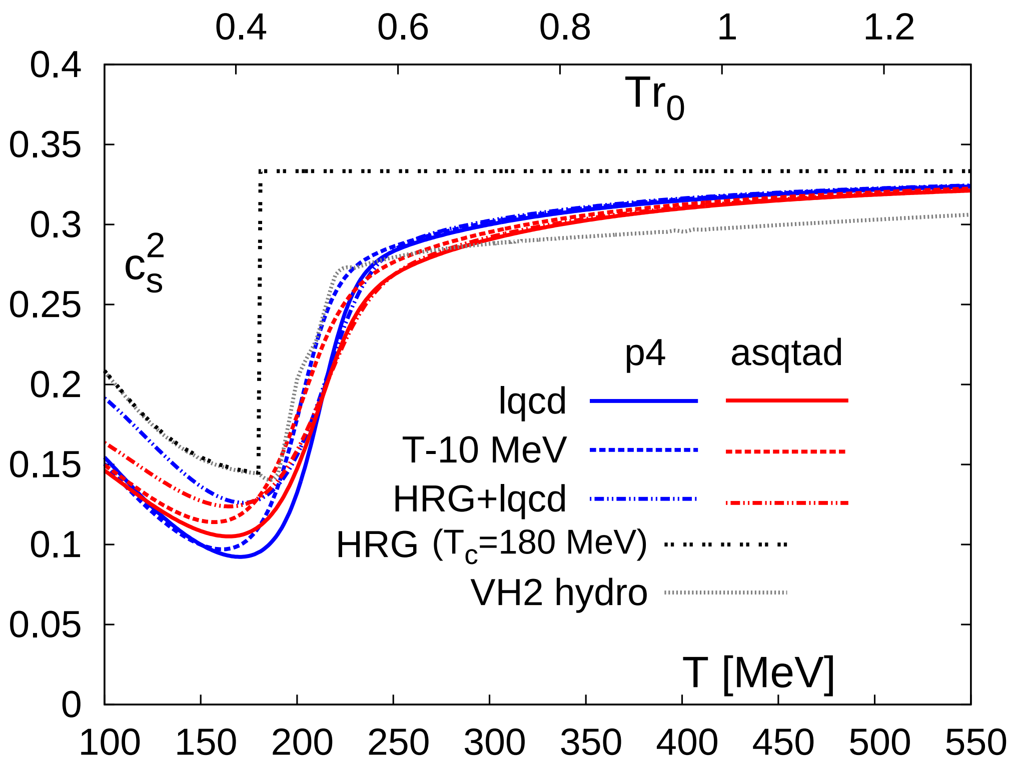
band:
$185\ \mathrm{MeV} \leq T \leq 195\ \mathrm{MeV}$

hotQCD: p4 vs. asqtad (arXiv:0903.4379)

EoS and velocity of sound

- $p/\epsilon \Rightarrow$ velocity of sound:

$$c_s^2 = \frac{\mathrm{d}p}{\mathrm{d}\epsilon} = \epsilon \frac{\mathrm{d}(p/\epsilon)}{\mathrm{d}\epsilon} + \frac{p}{\epsilon} \equiv \frac{s}{c_V}$$



hydro-expansion:

$p/\epsilon < 1/3$

$\Rightarrow$ slows down expansion;

$\Rightarrow$ increases plasma lifetime

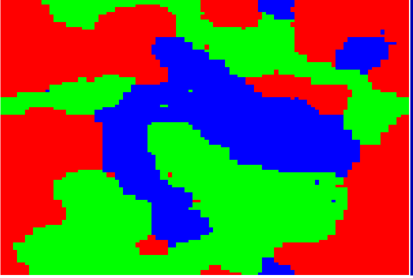
e.g.

$1 \leq \epsilon$ [GeV/fm^3]≤ 10

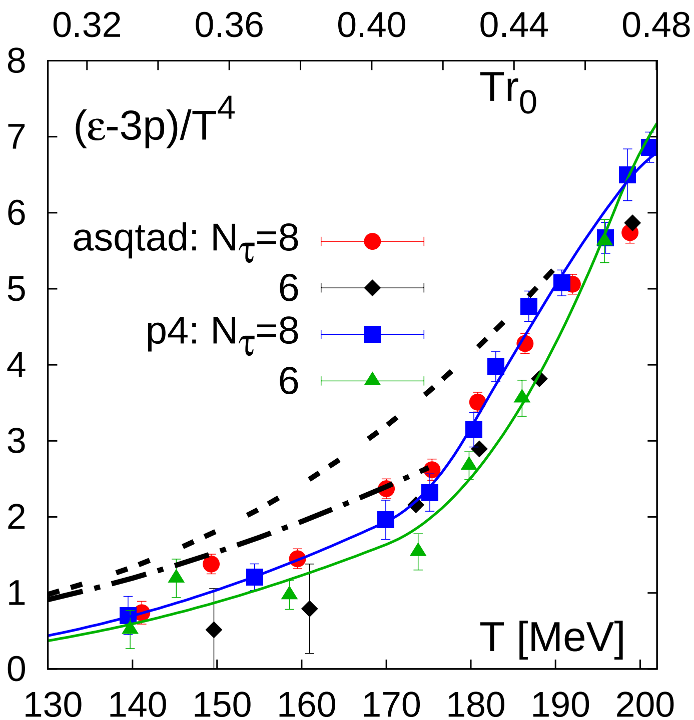
$\Rightarrow \Delta\tau \simeq$ 5.5 fm (ideal gas)

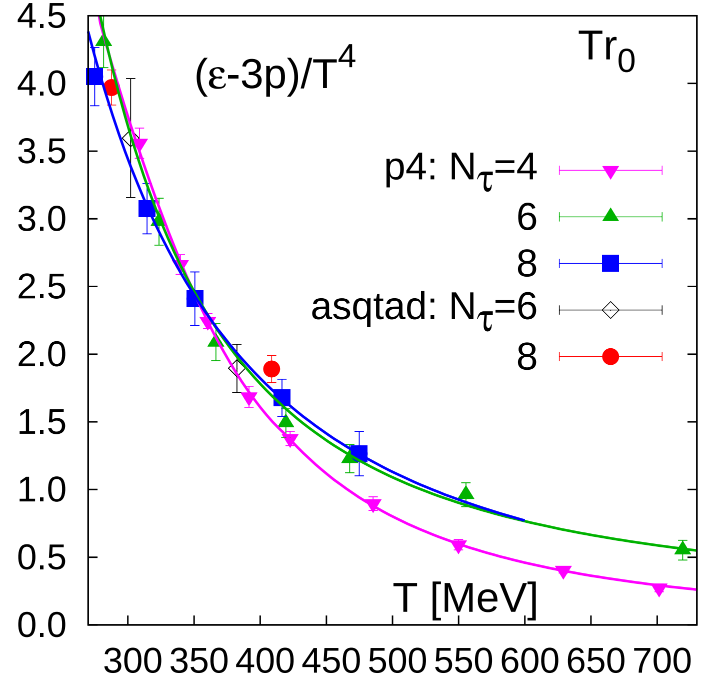
$\Rightarrow \Delta\tau \simeq$ 7 fm (LGT EoS)

hotQCD



Open Issues: Low and High-T asymptotics



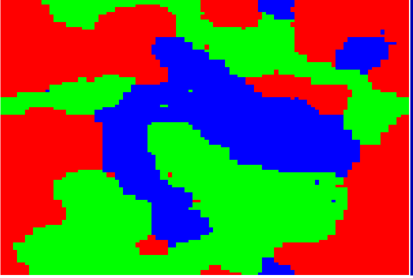


LGT vs resonance gas

- approach to continuum limit
  $\longrightarrow$ $N_\tau = 6,\ 8$
  $\longrightarrow$ $\mathcal{O}(5MeV)$ shift of T-scale
- LGT below HRG for $T \lesssim 180$ MeV
  coarser lattice, larger cut-off effects
  but: Which HRG?
  M_{max} =1.5 GeV, 2.5 GeV,..

LGT vs. pert. theory

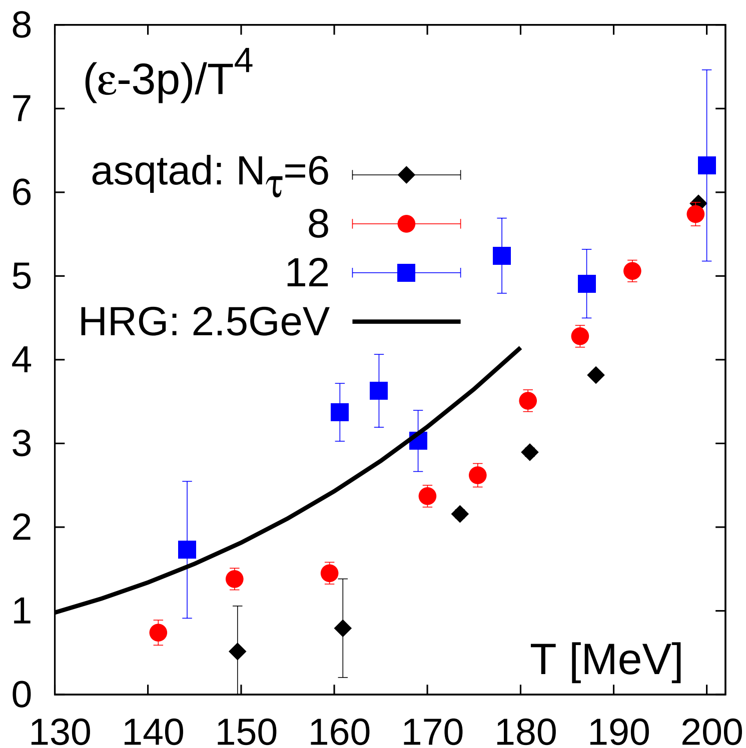
- approach to continuum limit
  $\longrightarrow$ $N_\tau = 6,\ 8$: no significant cut-off dependence for $T \gtrsim 300$ MeV
- strong deviations from conformal limit:
  find $(\epsilon - 3p)/T^4 \sim a/T^2 + b/T^4$ for $300\text{MeV} \lesssim T \lesssim 700\text{MeV}$



Latest News on EoS calculations

- hotQCD started to do calculations on $48^3 \times 12$ lattices at $m_\pi \simeq 150$ MeV and a physical kaon mass

  start observing agreement with a resonance gas model?



hotQCD preliminary

HRG:

$m_{max} = 2.5$ GeV

$\Rightarrow$ observe shift in transition region;

$\Rightarrow T_c \lesssim 170$ MeV

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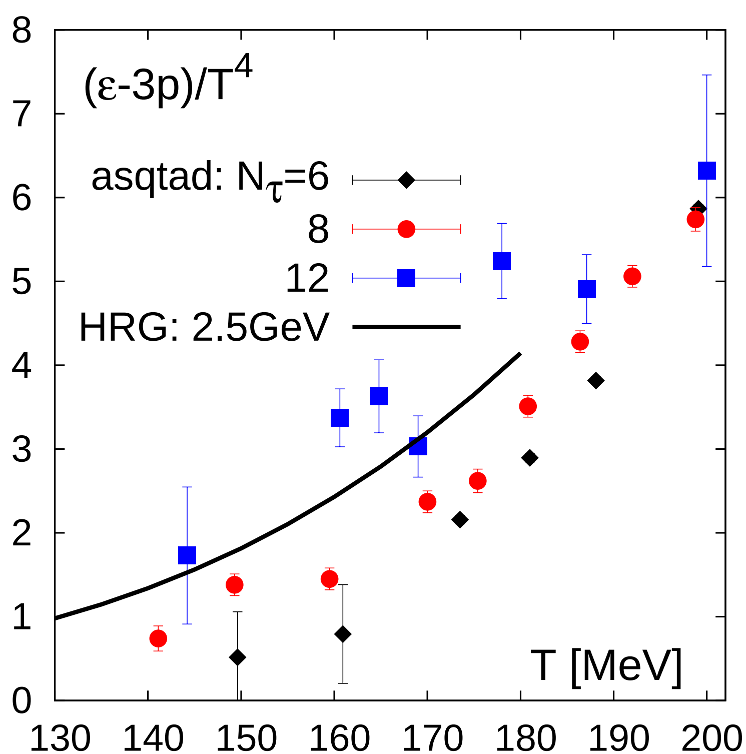

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Yamagata 1995

start of the German-Japanese workshop series

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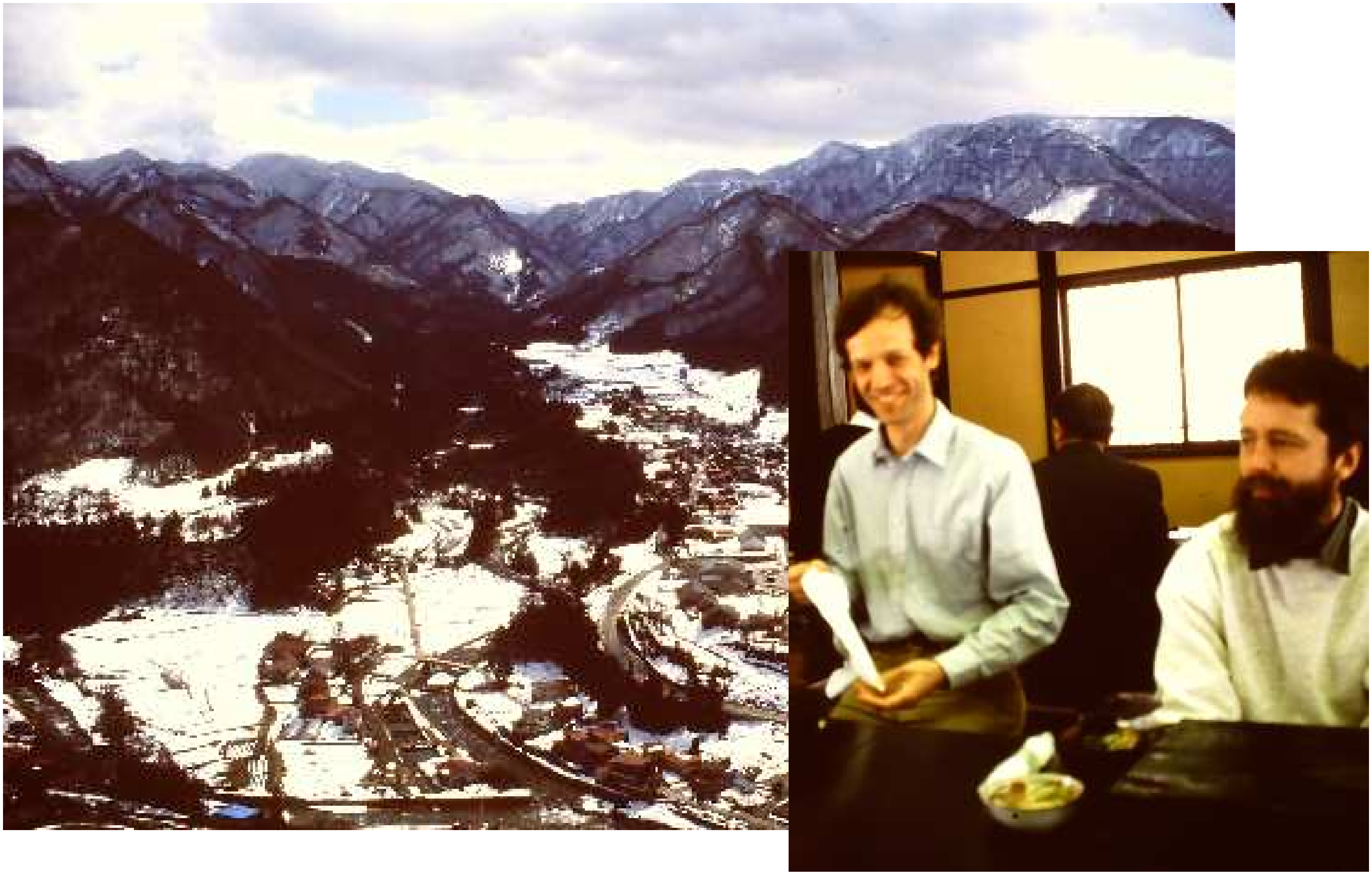


QCDTARO

An international computer experiment, Prog. Theor. Phys. Suppl. 122 (1996) 41

Prepared for Japan-Germany Seminar on QCD on Massively Parallel Computers, Yamagata, Japan, 16-18 Mar 1995

ABSTRACT:

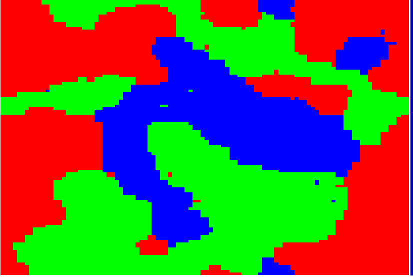
We present the Japanese-European QCD-TARO collaboration. We discuss the hardware basis and the concept of analyses and present selected results. We comment on the chances and bottlenecks connected to the non-local, international character of the collaboration.

QCDTARO

An international computer experiment, Prog. Theor. Phys. Suppl. 122 (1996) 41

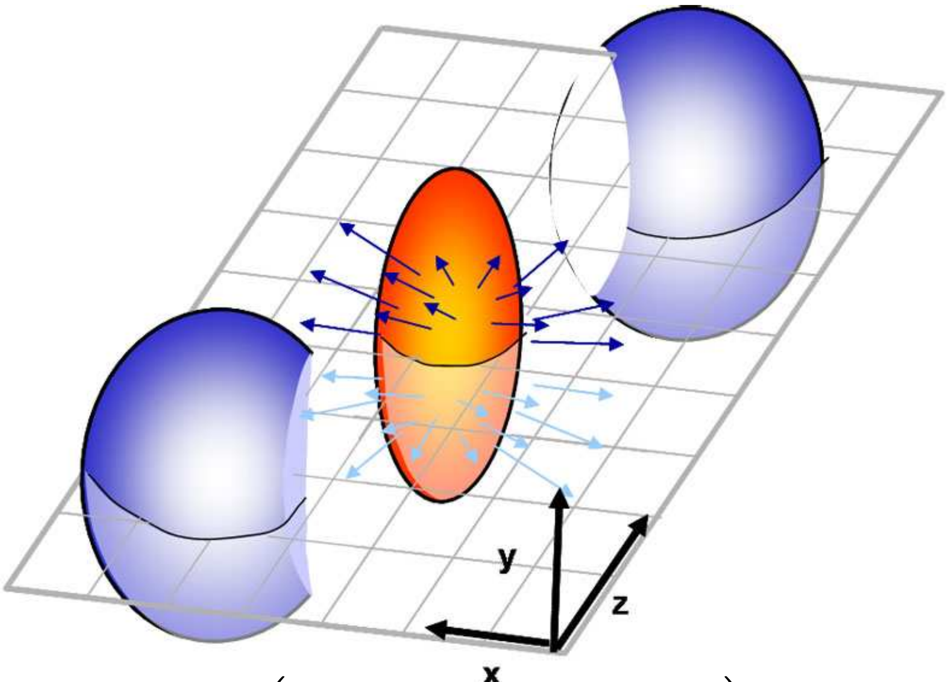
Prepared for Japan-Germany Seminar on QCD on Massively Parallel Computers, Yamagata, Japan, 16-18 Mar 1995

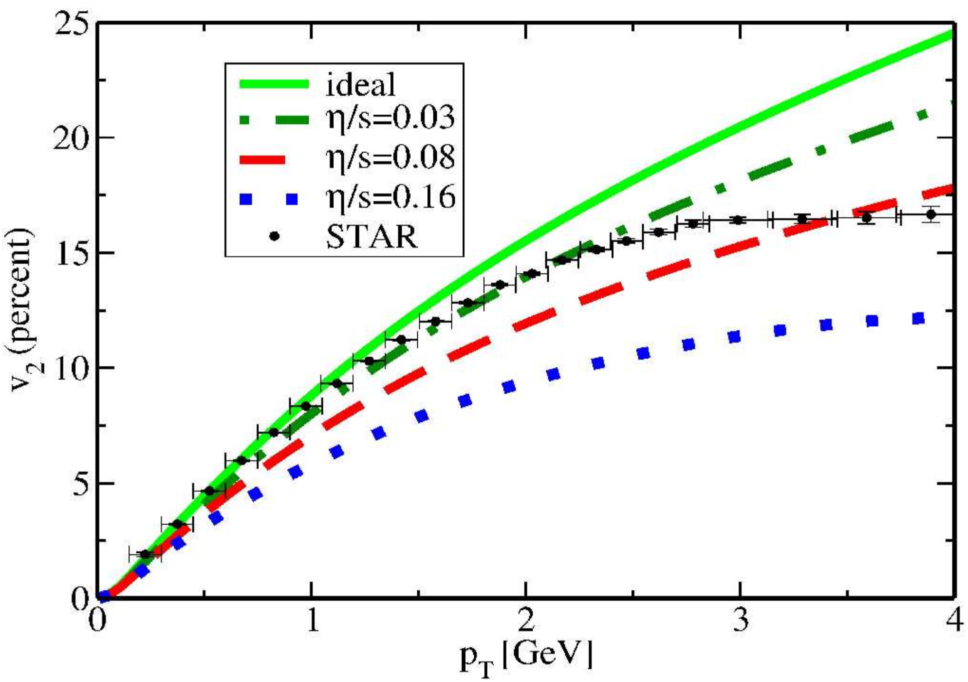
a unique international collaboration



Viscosity and elliptic flow at RHIC

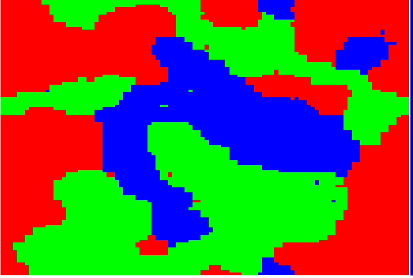
- asymmetric interaction region $\Rightarrow$ asymmetric pressure gradients $\Rightarrow$ asymmetric flow velocities: elliptic flow
- hydrodynamic modeling of flow pattern requires small shear viscosity over entropy: η/s
- lattice calculations give small η/s: $\eta/s \simeq 0.1$ for $T/T_c \sim (1.2 - 1.7)$





$$\frac{dN}{d\phi} = \frac{1}{2\pi}\left(1 + 2\sum_{n=1}^{\infty} v_n \cos(\phi)\right)$$

P. Romatschke, U. Romatschke, PRL 99, 172301 (2007)



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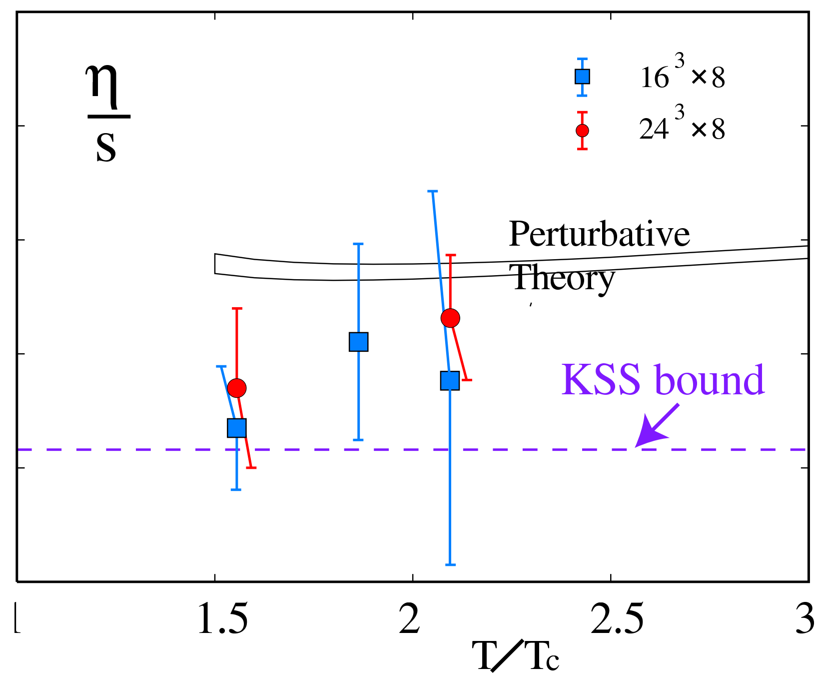
spectral representation of energy-momentum correlator

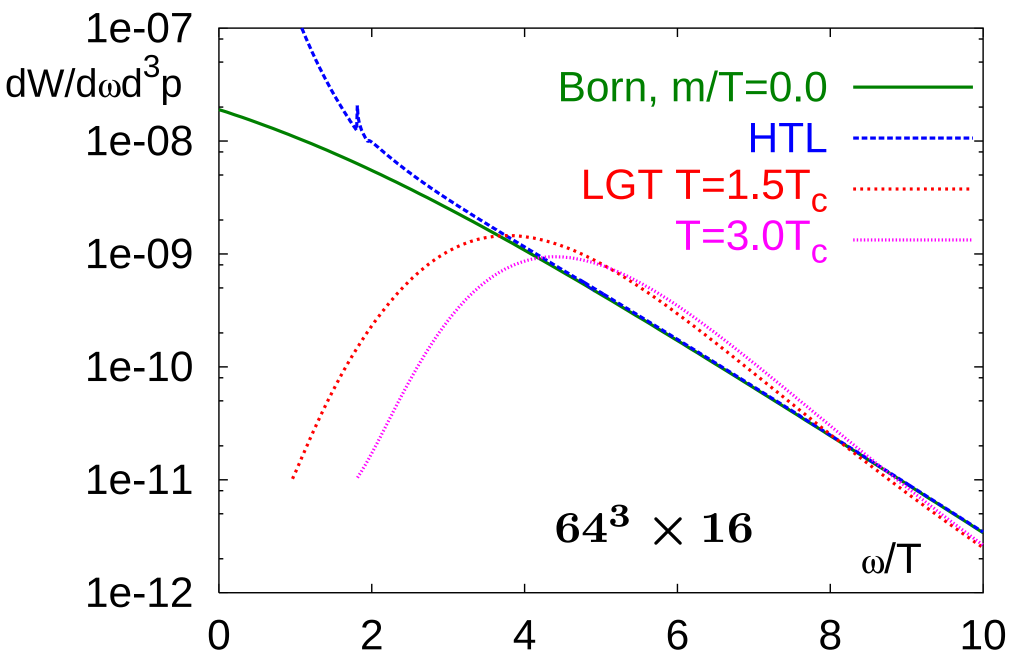
$$\langle T_{12}(0) T_{12}(t) \rangle = \int_0^\infty d\omega \, \rho(\omega) \, \frac{\cosh(\omega(t - 1/2T))}{\sinh(\omega/2T)}$$

$\Rightarrow$ shear viscosity $\eta = \lim_{\omega \to 0} \frac{\rho(\omega)}{\omega}$



A. Nakamura and S. Sakai, PRL 94 (2005) 072305

Thermal dilepton rates: HTL and lattice calculations



thermal dilepton rate

$$\frac{\mathrm{d}W}{\mathrm{d}\omega \mathrm{d}^3p} = \frac{5\alpha^2}{27\pi^2} \frac{\rho_V(\omega,\vec{p},T)}{\omega^2(\mathrm{e}^{\omega/T}-1)}$$

HTL and lattice disagree for

$\omega/T \lesssim (3-4)$

FK et al, PLB530 (2002) 147

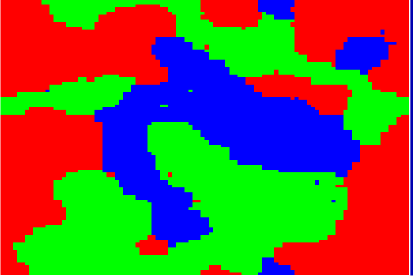
- infra-red sensitivity of HTL-calculations ⇔ ”massless gluon” cut in HTL-propagator
- infra-red sensitivity of lattice calculations ⇔ thermodynamic limit, $V \to \infty$

- $VT^3 = (N_\sigma/N_\tau)^3 < \infty \quad \Rightarrow$ momentum cut-off: $p/T > 2\pi N_\tau/N_\sigma$

⇒ need large lattices to analyze infra-red regim

need $N_\tau \sim \mathcal{O}(30)$ AND $N_\sigma \sim 6\ N_\tau$

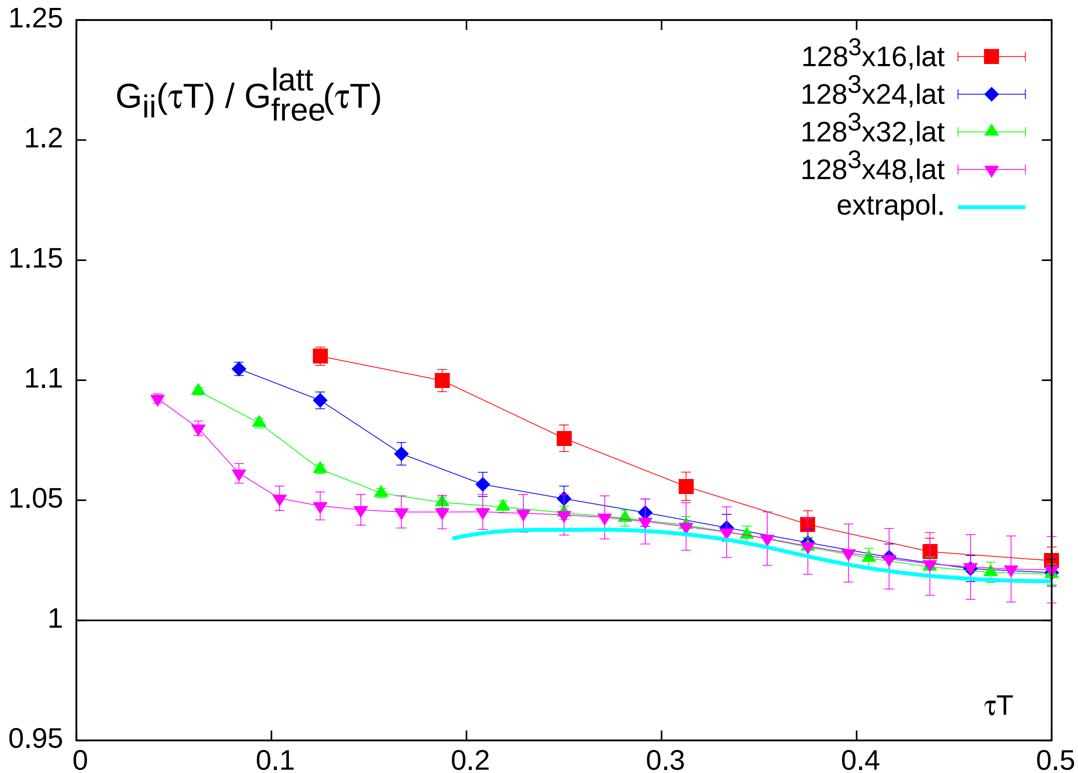
⇒ in future also thermal photon rates



Latest News on dilepton rates

- Bielefeld-BNL-GSI started to calculate dilepton rates (vector spectral functions) on lattices up to $128^3 \times 48$ at $T = 1.5T_c$

start observing agreement with hard thermal loop perturbation theory

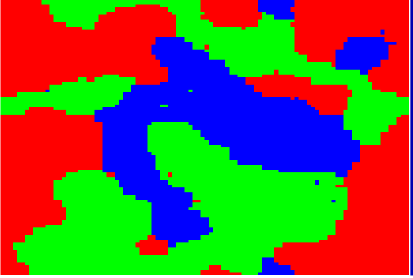


Bielefeld-BNL-GSI preliminary

extrapolation:

$\mathcal{O}(a^2)$ extrap. of spline interploated correlation functions

$\Rightarrow$ $N_\tau = 48$ close to continuum result for $\tau T \gtrsim 0.2$



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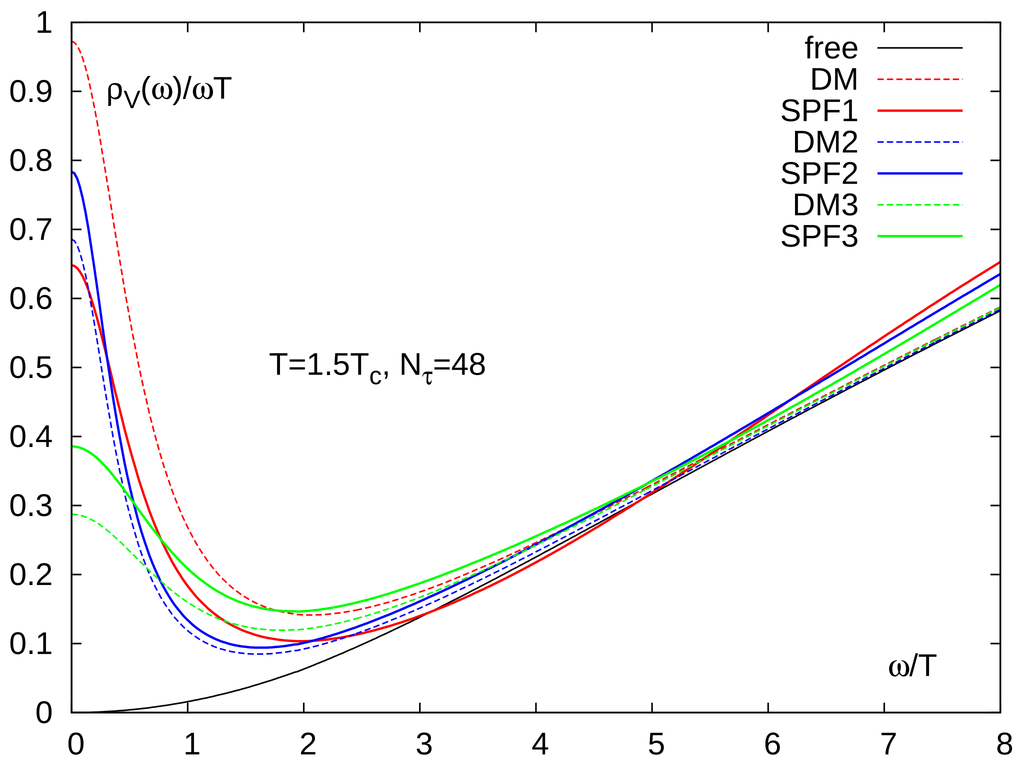

Bielefeld-BNL-GSI preliminary

check sensitivity of low frequncy regime to choice of default model:

$\Rightarrow$ evidence for change from $\rho_V(\omega) \sim \omega^2$ to $\rho_V(\omega) \sim \omega$ at $\omega/T \simeq 2$

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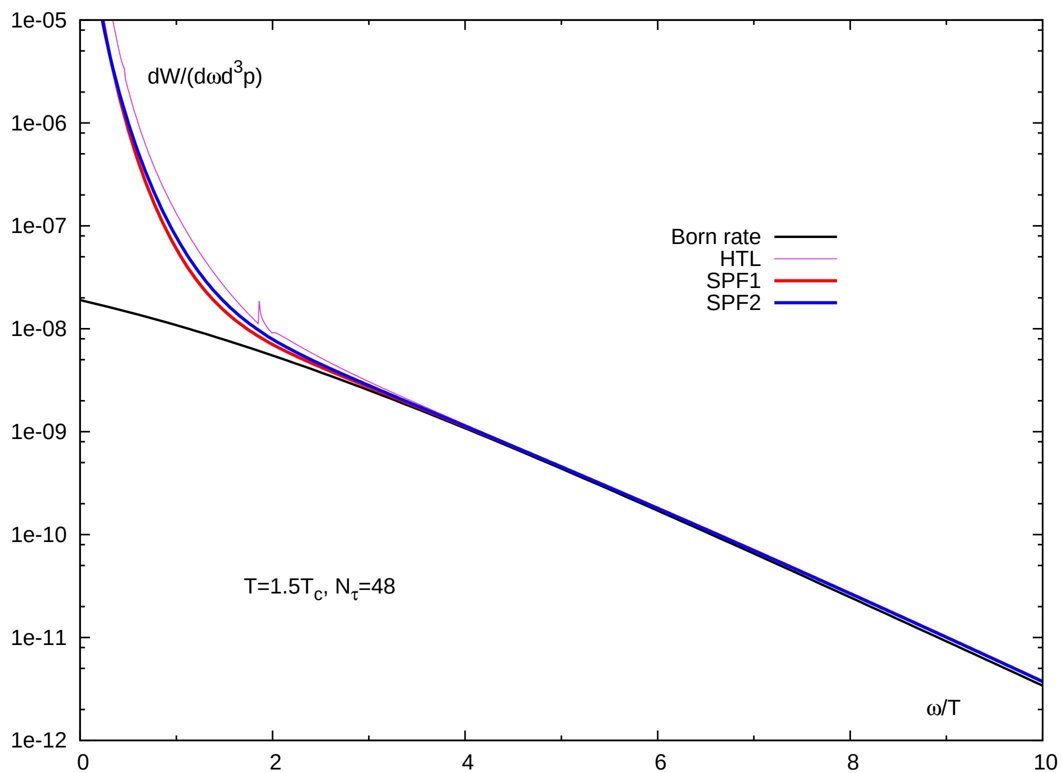

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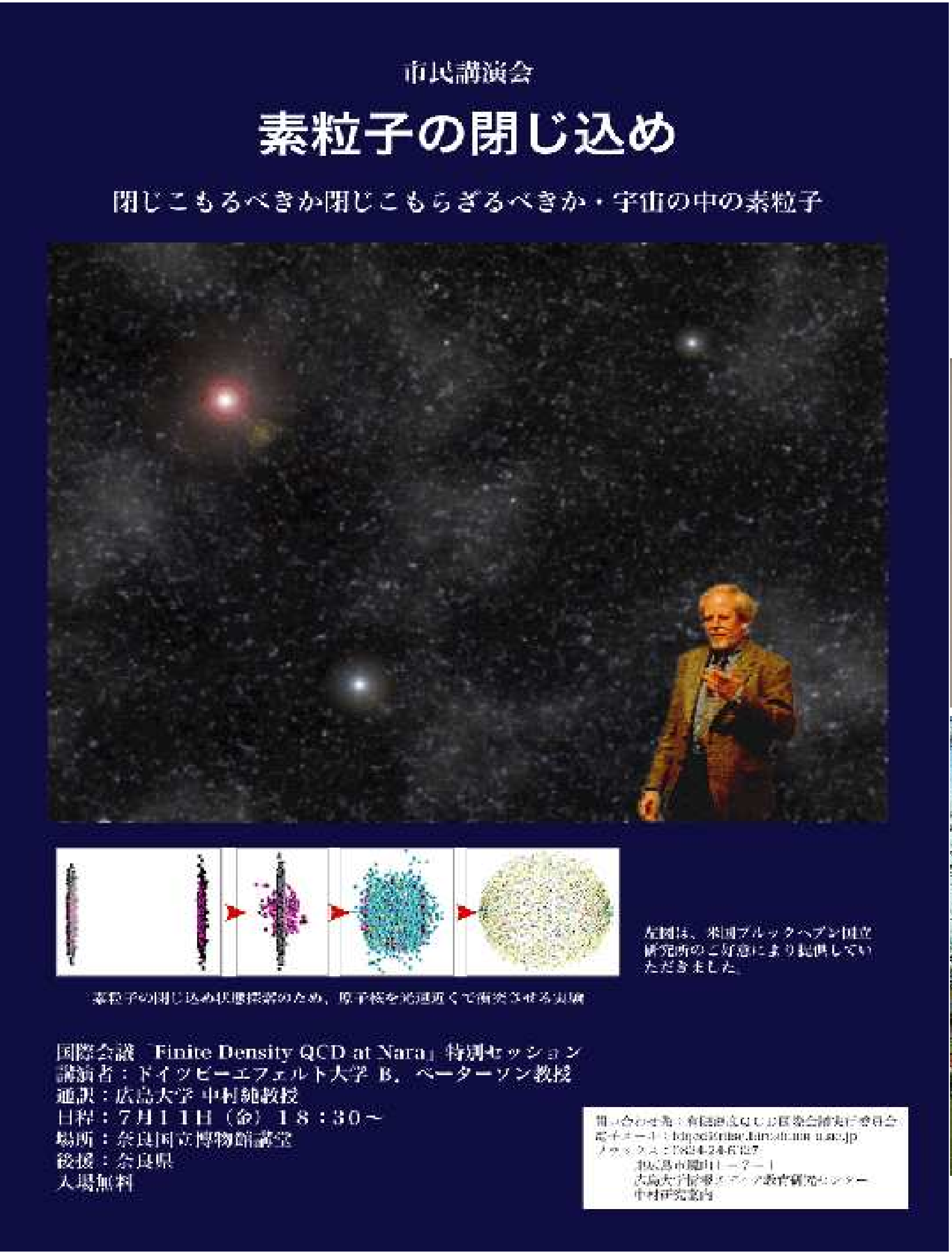
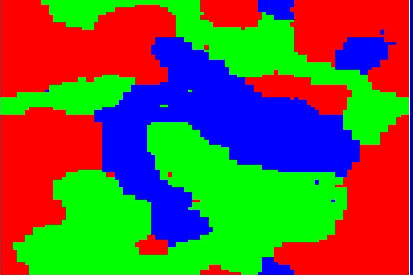


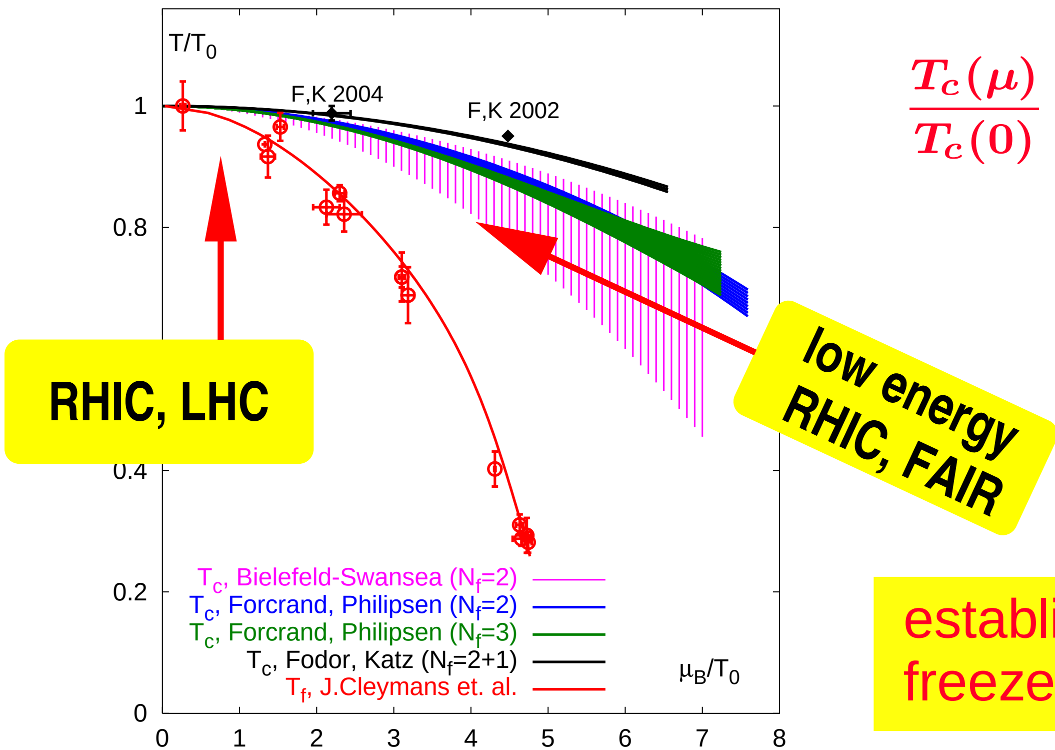
Extreme QCD
Nara, 2003



Extending the phase diagram to non-vanishing chemical potential

non-zero baryon number density: $\mu > 0$

$$Z(V,T,\mu) = \int \mathcal{D}\mathcal{A}\mathcal{D}\psi\mathcal{D}\bar{\psi}\ \mathrm{e}^{-S_E(V,T,\mu)}$$
$$= \int \mathcal{D}\mathcal{A}\mathcal{D}\ det\ M(\mu)\ \mathrm{e}^{-S_E(V,T)}$$



$\frac{T_c(\mu)}{T_c(0)}$: $1 - 0.0056(4)(\mu_B/T)^2$

deForcrand, Philipsen (imag. μ)

$1 - 0.0078(38)(\mu_B/T)^2$

Bielefeld-Swansea

($\mathcal{O}(\mu^2)$ reweighting)

search for critical point

establish relation between
freeze-out and critical behavior

O(N) scaling analysis

$$M \equiv h^{1/\delta} f_G(z) \qquad ; \qquad z = t/h^{1/\beta\delta}$$

- 3 parameter fit: t_0, h_0, T_c $\qquad t = \frac{1}{t_0}\frac{T - T_c}{T_c}\ ,\ h = \frac{1}{h_0}\frac{m_l}{m_s}$
- use only data for $m_l/m_s \le 1/20$, $\beta \in [3.285, 3.31]$

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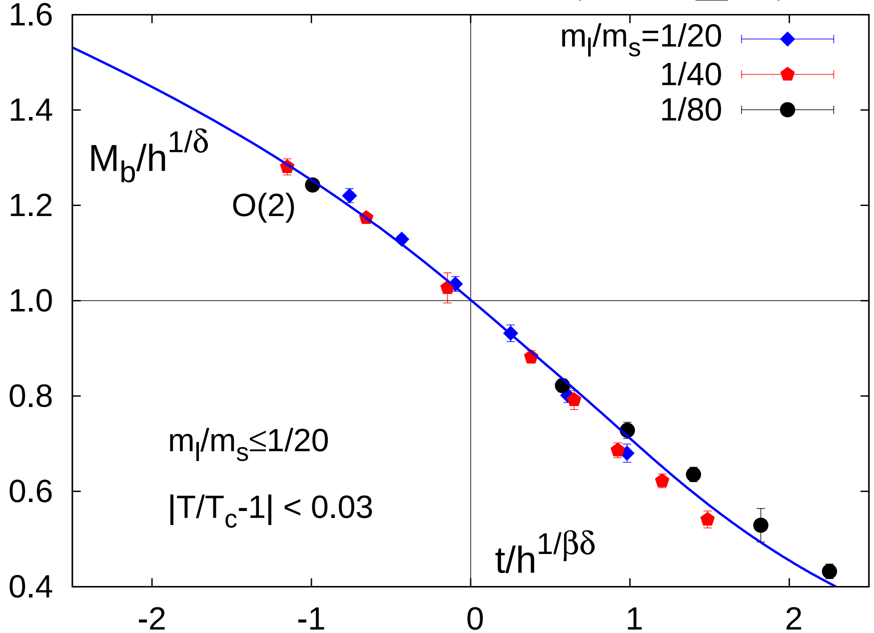
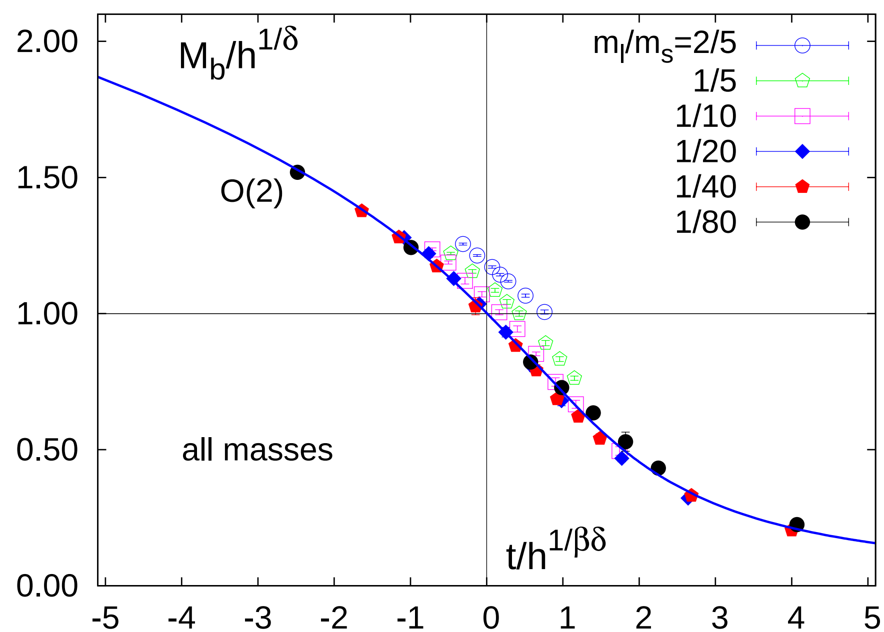
order parameter: $m_l/m_s \leq 1/20$



order parameteri: all masses



$\Rightarrow t_0, h_0, T_c$ provide input to scaling analysis at $\mu > 0$

Thermal fluctuations of the order parameter

$$\chi_t \equiv \frac{\partial M}{\partial T} = \frac{1}{t_0 T_c}\frac{\partial M}{\partial t} = \frac{1}{t_0 T_c} h^{(\beta-1)/\beta\delta} f'_G(z)$$

$$t = \frac{1}{t_0}\frac{T - T_c}{T_c} \; , \; h = \frac{1}{h_0}\frac{m_l}{m_s}$$

- non-zero chemical potential: reduced temperature depends (in leading order) on all couplings that do not break chiral symmetry:

$$t = \frac{1}{t_0}\left(\frac{T - T_c}{T_c} + \kappa_\mu \left(\frac{\mu_l}{T}\right)^2\right)$$

$$\Rightarrow \qquad c_2^{\bar\psi\psi} \equiv \left.\frac{\partial^2 M}{\partial(\mu_l/T)^2}\right|_{\mu_l=0} = \frac{2\kappa_\mu}{t_0 T_c}\frac{\partial M}{\partial t} = \frac{2\kappa_\mu}{t_0 T_c} h^{(\beta-1)/\beta\delta} f'_G(z)$$

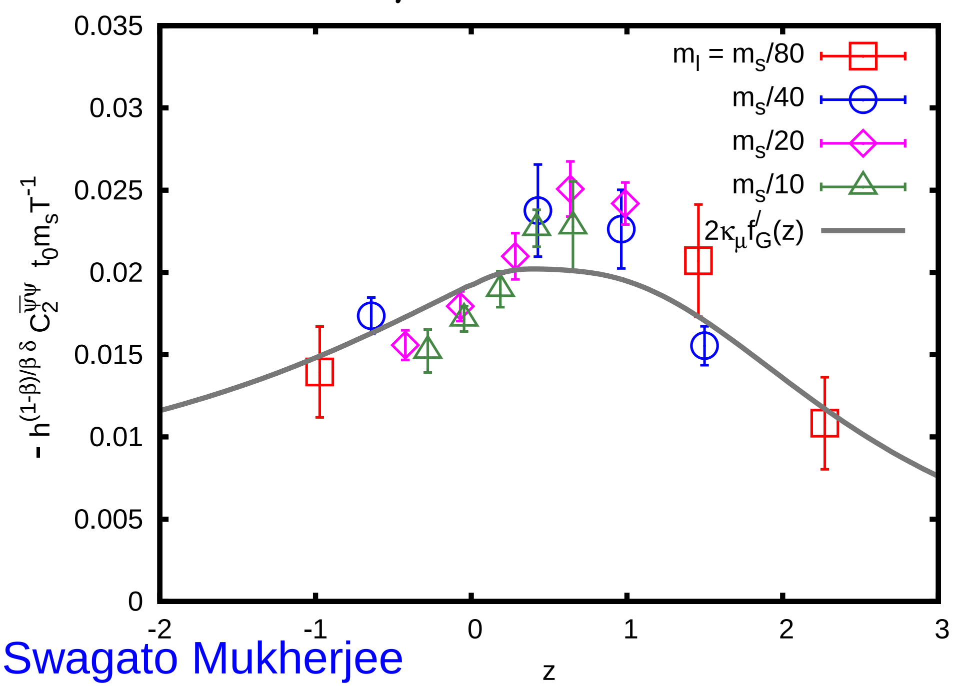
Thermal fluctuations of the order parameter

- curvature of the critical line in the chiral limit:

$$t = 0 \quad \Leftrightarrow \quad \frac{T}{T_c} = 1 - \kappa_\mu \left(\frac{\mu_l}{T} \right)^2$$

- t_0, T_c are known from scaling analysis of $\langle \bar{\psi}\psi \rangle$

  $\Rightarrow \kappa_\mu$ from fit with scaling curve to data for χ_t



- Preliminary:

  fit to $O(2)$ scaling curve

  $\kappa_\mu = 0.033(1)$

  $(\chi^2/dof = 1.9)$

Swagato Mukherjee

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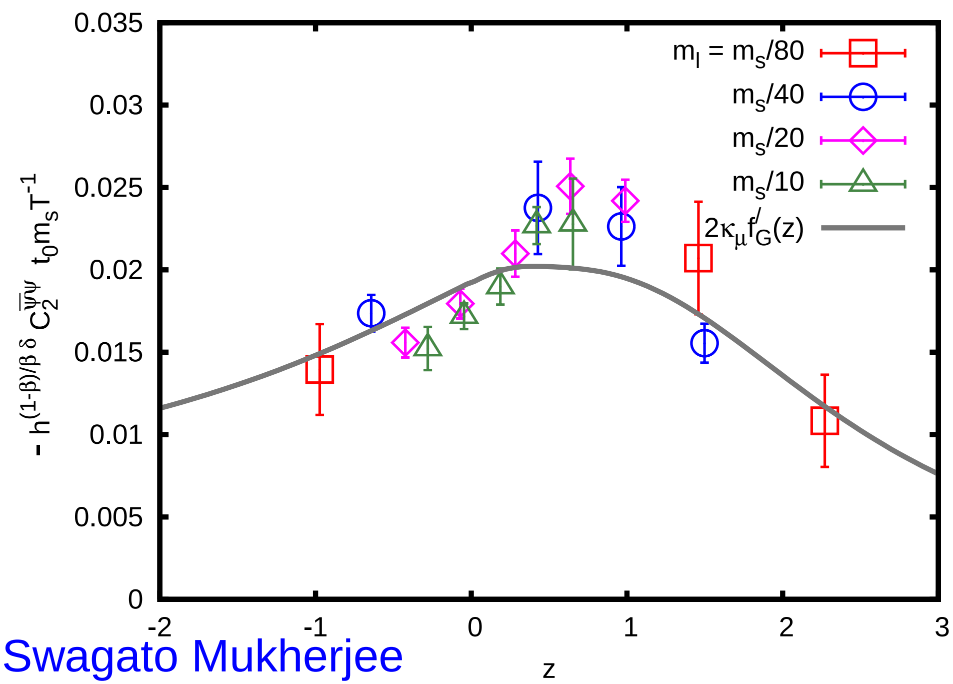
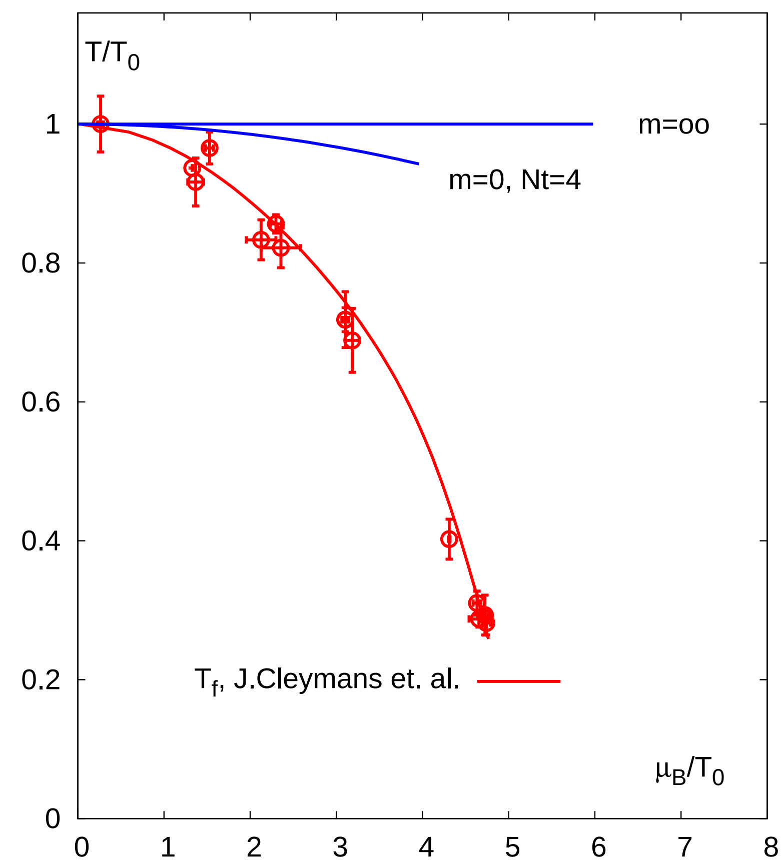
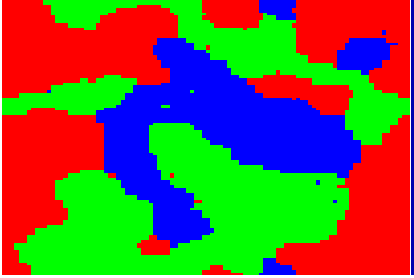
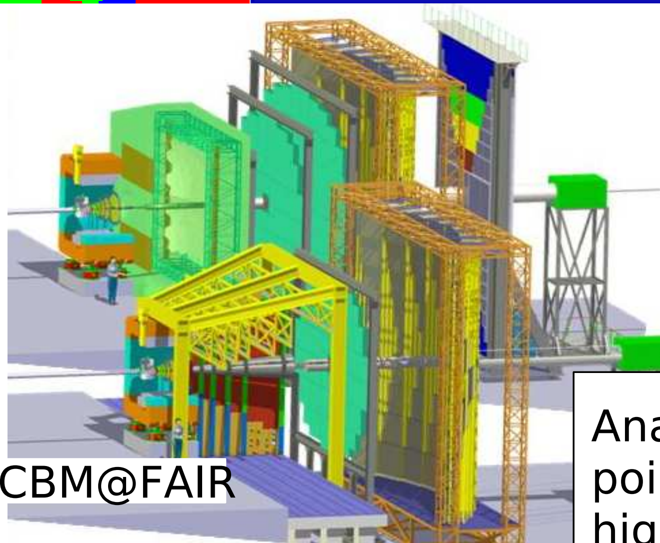




Swagato Mukherjee



Future finite density calculations

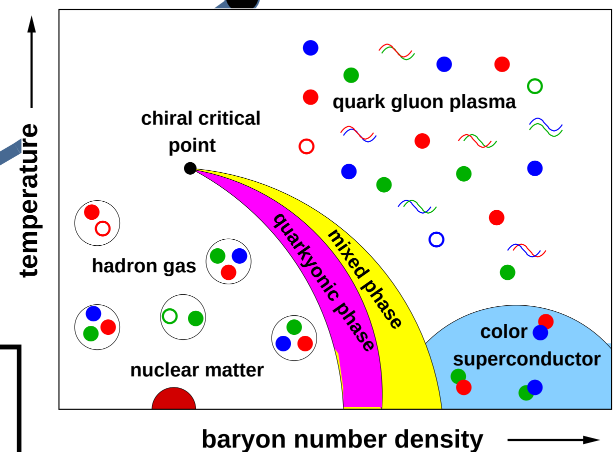


- Results on the existence/non-existence of a critical point in the QCD phase diagram
- Analysis of first order transition line using canonical simulations

Critical surface from calculations with unimproved staggered fermions

Analysis of critical point/surface with highly improved staggered fermions

Critical point estimates from Taylor expansions on coarse lattices



10x tera — 100x tera — peta — 10x peta — 100x peta — exa

-flop year sustained

Extreme Scale Computing Workshop
Washington, Feb. 2009

Happy Birthday

It seems pretty but, its rather hard to understand. Somehow it fills my head with ideas - only I don’t know exactly what they are!

Alice’s Adventure in Wonderland

Heidelberg 1991

http://www.thphys.uni-heidelberg.de/ stamates/NAKA2010 All the best from Nucu