Finite Temperature/Density Phase Diagram of Two-Color QCD

Seyong Kim

Sejong University

with S. Hands(Swansea) and J.-I. Skullerud(NUIM)

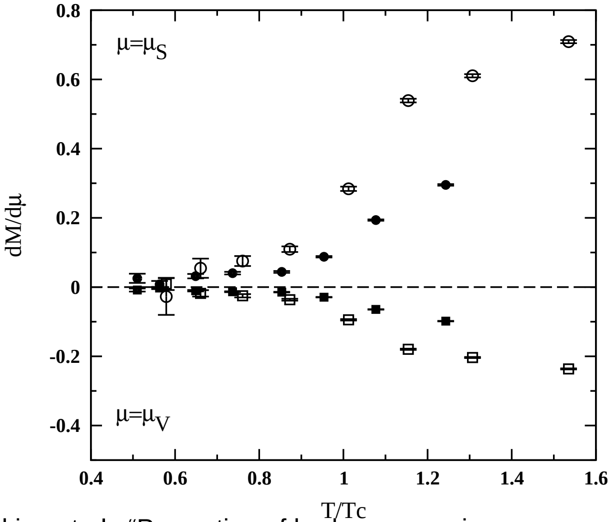
Reminiscence

祝生日

Prof. 中村純

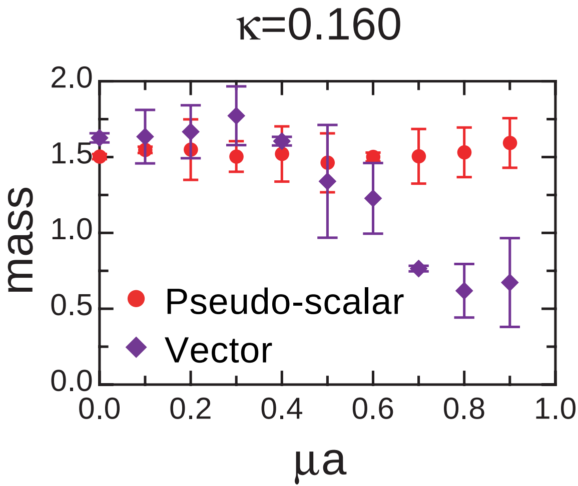
생일 축하합니다.
아쓰시 나카무라교수님

Reminiscence



- I. Pushkina et al., “Properties of hadron screening masses at finite baryonic density”, PLB609 (2005) 265

Reminiscence



•S. Muroya, A. Nakamura and C.Nonaka, “Behavior of hadrons at finite density: Lattice study of color SU(2) QCD”, PLB551 (2003) 305

2-Color QCD

- QCD-like theory: two-color QCD or QCD with adjoint quarks (Kogut et al, Nucl. Phys. B582 (2000) 477)

- Consider $N_f = 2$ Wilson fermion, two-color QCD

$$\mathcal{S} = \overline{\psi}_1 M(\mu)\psi_1 + \overline{\psi}_2 M(\mu)\psi_2 - J\bar{\psi}_1(C\gamma_5)\tau_2\bar{\psi}_2^{tr} + \bar{J}\psi_2^{tr}(C\gamma_5)\tau_2\psi_1, \quad (1)$$

where

$$M_{xy}(\mu) = \delta_{xy} - \kappa\sum_{\nu}\Big[(1-\gamma_\nu)e^{\mu\delta_{\nu 0}}U_\nu(x)\delta_{y,x+\hat{\nu}} + (1+\gamma_\nu)e^{-\mu\delta_{\nu 0}}U_\nu^\dagger(y)\delta_{y,x-\hat{\nu}}\Big]. \quad (2)$$

- $\det M(\mu)$ is real (but doesn't mean that it is positive)

$$\gamma_5 M(\mu)\gamma_5 = M^\dagger(-\mu) \quad (3)$$

$$(C\gamma_5)\tau_2 M(\mu)(C\gamma_5)^{-1}\tau_2 = M^*(\mu) \quad (4)$$

2-Color QCD

• Grassman variable integration turns the fermion part of the action into a determinant

$$\int d\overline{\psi} d\psi e^{-S_F} \rightarrow \mathrm{det} M(\mu) \qquad (1)$$

since $\mathrm{det} M(\mu) = \mathrm{det} M^*(\mu)$, $\mathrm{det} M(\mu)$ is real

• redefine $\overline{\phi} = -\psi^{\mathrm{tr}}{}_2 C\tau_2, \phi = C^{-1}\tau_2\psi^{\mathrm{tr}}{}_2$

$$\mathcal{S} = (\bar{\psi} \quad \bar{\phi}) \begin{pmatrix} M(\mu) & J\gamma_5 \\ -\bar{J}\gamma_5 & M(-\mu) \end{pmatrix} \begin{pmatrix} \psi \\ \phi \end{pmatrix} \equiv \bar{\Psi}\mathcal{M}\Psi. \qquad (2)$$

2-Color QCD

- With the redefinition

$$\mathcal{M}^\dagger \mathcal{M} = \begin{pmatrix} M^\dagger(\mu)M(\mu)+|\bar{J}|^2 & \\ & M^\dagger(-\mu)M(-\mu)+|J|^2 \end{pmatrix} \tag{1}$$

- Monte Carlo feasible despite the quark chemical potential

2-Color QCD

- QC_2D is asymptotically free
- There is spontaneous chiral symmetry breaking
  $\longrightarrow$ pion is light
- But qq is a color singlet $\longrightarrow$ diquark condensate does not break color symmetry

2-Color QCD

- For studying SU(2)

  finite chemical potential can be simulated

  IF N_c limit is smooth, QCD within $1/N_c$

  related to many condensed matter systems

- Maybe, against studying SU(2)

  mesons and baryons are different from SU(3)

  $q\bar{q}$ mesons, qq baryons

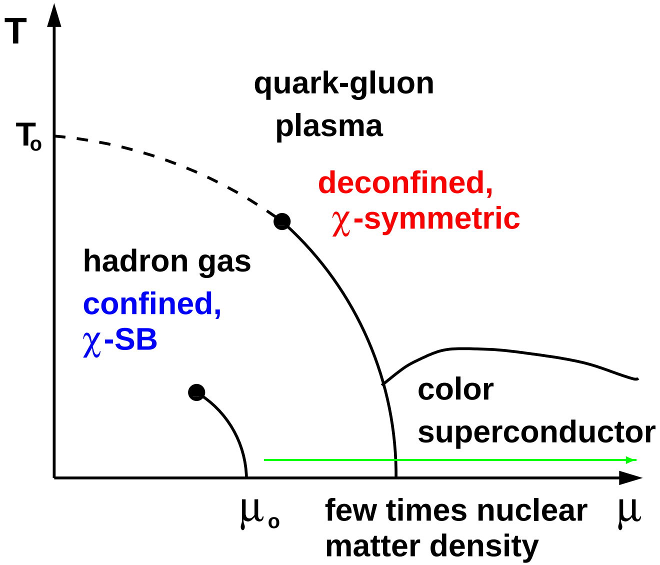
2-Color QCD

• two-color QCD with light staggered fundamental quarks has been studied and compared with chiral peturbation theory (J.B.Kogut,D.K.Sinclair,S.J.Hands and S.E. Morrison, PRD64 (2001) 094505, J.B.Kogut, D. Toublan, D.K. Sinclair, NPB642(2002) 181)

• two-color QCD with light staggered adjoint quarks (S.Hands, I. Montvay, S. Morrison, M. Oevers, L. Scorzato, J. Skullerud, Eur. Phys. J. C17 (2000) 285, S. Hands, I. Montvay, L. Scorzato, J. Skullerud, Eur. Phys. J. C22 (2001) 451)

• two-color QCD with light Wilson fundamental quarks (S. Muroya, A. Nakamura, C. Nonaka, PLB 551(2003) 205)

Low T, μ


T
quark-gluon
plasma
T_o
deconfined,
χ-symmetric
hadron gas
confined,
χ-SB
color
superconductor
μ_o
few times nuclear
matter density
μ

Low T, μ

- Two color QCD with heavy quark:

S. Hands, S. K., J.-I. Skullerud, Eur.Phys.J.C48:193,2006

$8^3 \times 16, \beta = 1.7, \kappa = 0.178,$ Wilson quark, Wilson gauge

$m_\pi a \sim 0.79(1), m_\pi/m_\rho = 0.779(4)$
$a = 0.230(5)(\text{fm}),\ T = 54(1)(\text{MeV})$

S. Hands, S. K., J.-I. Skullerud, arXiv:1001.1682

$12^3 \times 24, \beta = 1.9, \kappa = 0.168,$ Wilson quark, Wilson gauge

$m_\pi a \sim 0.68(1), m_\pi/m_\rho = 0.80(1)$
$a = 0.186(8)(\text{fm}),\ T = 44(2)(\text{MeV})$

Low T, μ

•χ-PT: Kogut et al, Nucl. Phys. B582 (2000) 477

$$n_{\chi PT} = \begin{cases} 0\,, & \mu < \mu_o\,, \\ 8N_f F^2 \mu \left(1 - \frac{\mu_o^4}{\mu^4}\right)\,, & \mu \geq \mu_o \end{cases} \tag{1}$$

$$p_{\chi PT} = 4N_f F^2 \left(\mu^2 + \frac{\mu_o^4}{\mu^2} - 2\mu_o^2\right) \tag{2}$$

$$\varepsilon_{\chi PT} = 4N_f F^2 \left(\mu^2 - 3\frac{\mu_o^4}{\mu^2} + 2\mu_o^2\right) \tag{3}$$

$$T_{\mu\mu} = \delta_{\chi PT} = 8N_f F^2 \left(-\mu^2 - 3\frac{\mu_o^4}{\mu^2} + 4\mu_o^2\right) \tag{4}$$

Low T, μ

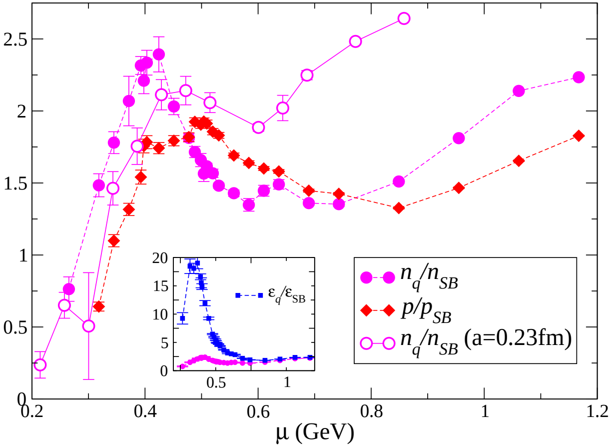
- BCS condensation

$$n_q \propto \mu^3 \tag{1}$$

$$\langle qq \rangle \propto \Delta\mu^2 \tag{2}$$

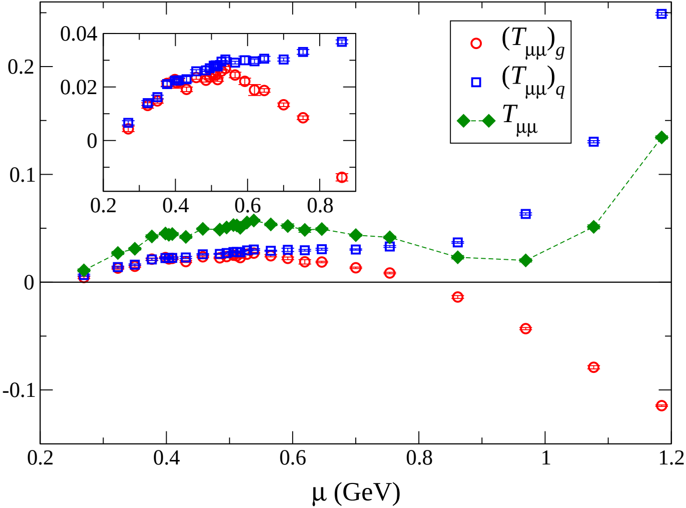
$$\varepsilon_q \propto \mu^4 \tag{3}$$

Low T, μ



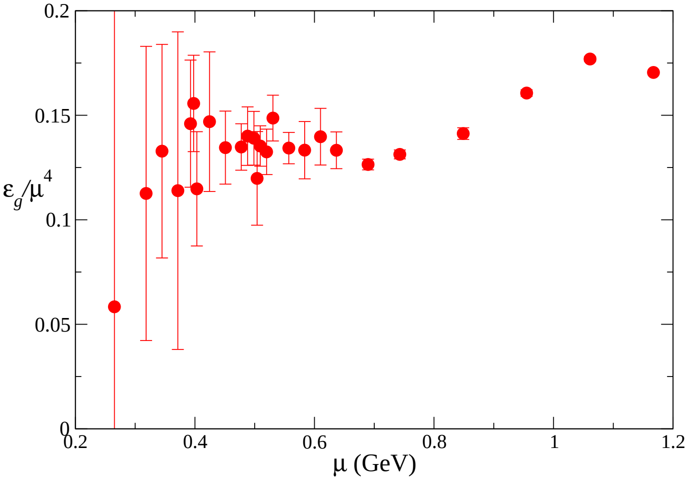
2.5
2
1.5
1
0.5
0
0.2
0.4
0.6
0.8
1
1.2
μ (GeV)
20
15
10
5
0
0.5
1
ε_q/ε_SB
n_q/n_SB
p/p_SB
n_q/n_SB (a=0.23fm)

Low T, μ

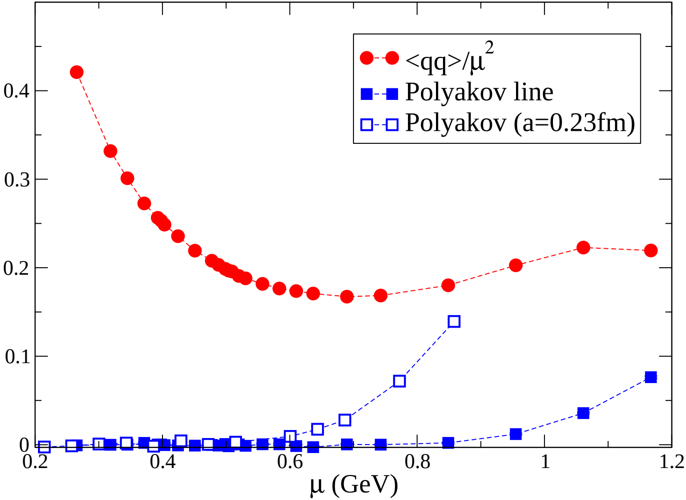


0.2
0.1
0
-0.1
0.2
0.4
0.6
0.8
1
1.2
μ (GeV)
0.04
0.02
0
0.2
0.4
0.6
0.8
$(T_{\mu\mu})_g$
$(T_{\mu\mu})_q$
$T_{\mu\mu}$

Low T, μ

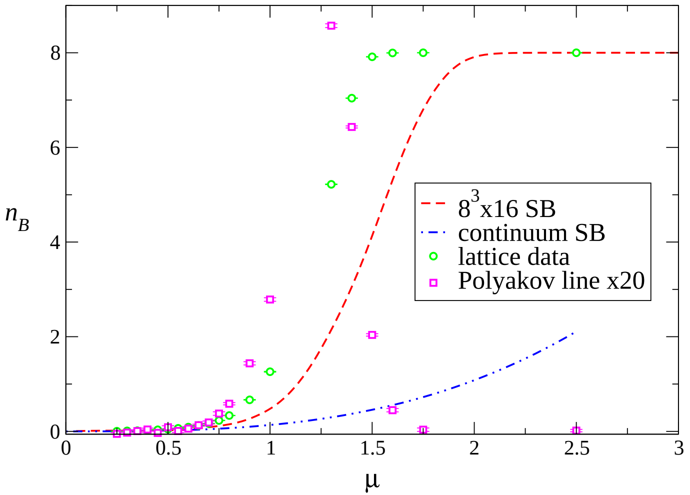


Low T, μ



<qq>/μ²
Polyakov line
Polyakov (a=0.23fm)
0
0.1
0.2
0.3
0.4
0.2
0.4
0.6
0.8
1
1.2
μ (GeV)

Low T, μ



8
6
4
2
0
n_B
0
0.5
1
1.5
2
2.5
3
μ
8^3x16 SB
continuum SB
lattice data
Polyakov line x20

Low T, μ

- Hint of three different phases

  Hadronic phase

  Bose-Einstein Condensed(BEC) phase

  Bardeen-Cooper-Schrieffer(BCS) phase

- BEC-BCS transition is not a sharp transition

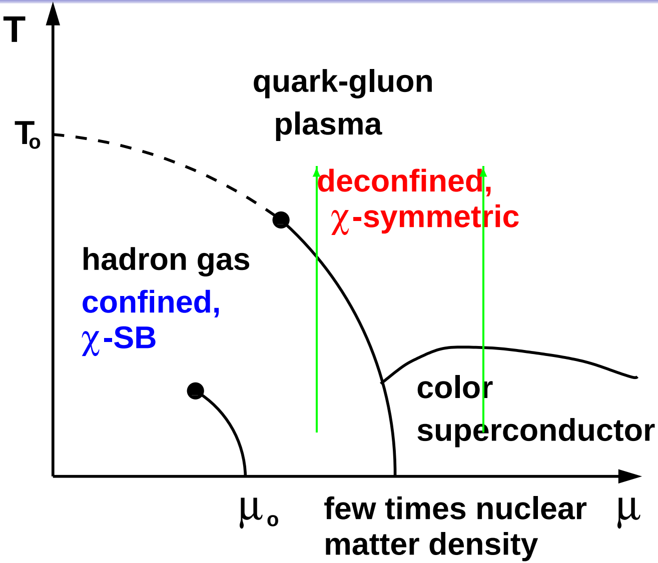
  strong attraction $\longrightarrow$ tightly bound boson

  $\longrightarrow$ boson condesation

  weaker attraction $\longrightarrow$ loosely bound Cooper pair

  $\longrightarrow$ superconducting phase

Finite T, μ


T
T_o
quark-gluon
plasma
deconfined,
χ-symmetric
hadron gas
confined,
χ-SB
color
superconductor
μ_o
few times nuclear
matter density
μ

Finite T, μ

- temperature is defined as $T = \frac{1}{N_\tau a}$

  $\leftarrow$ a is controlled by gauge coupling constant
  $\leftarrow$ different temperature means a different gauge coupling constant

- periodic boundary condition on bosonic field, anti-periodic boundary condition on fermionic field

- no. of spatial lattice sites (N_s) should be bigger than N_τ

- phases are distinguished by order parameters

Finite T, μ

- P. Nozieres, S.Schmitt-Rink, J. of Low Temp. Phys. 59, 195(1985)

finite temperature transition of BEC phase is different

$$T_c = (2\pi/M)(N_p/2.612)^{2/3}$$

from that of BCS phase

$$T_c = (e^{\gamma}/\pi)\Delta_F$$

Finite T, μ

- phase diagram of two-color QCD in strong coupling limit

  D.H. Adams, S. Chandrasekharan, NPB662 (2003) 220
  S. Chandrasekharan, F.-J. Jigan, PRD 74(2006) 014506
  Y. Nishida, K. Fukushima, T. Hatsuda, Phys. Rep. 398(2004) 281

Finite T, μ

- suitable for a grid application

S. Hwang, H. Kim (KISTI), S.K.,
"Large-scale deployment of Two-Color QCD simulations on the Grid using Ganga", submitted to Computer Physics Communications

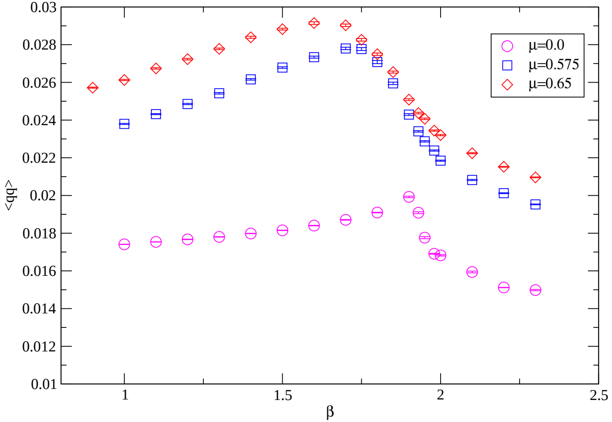
Finite T, μ

- Lattice simulation details

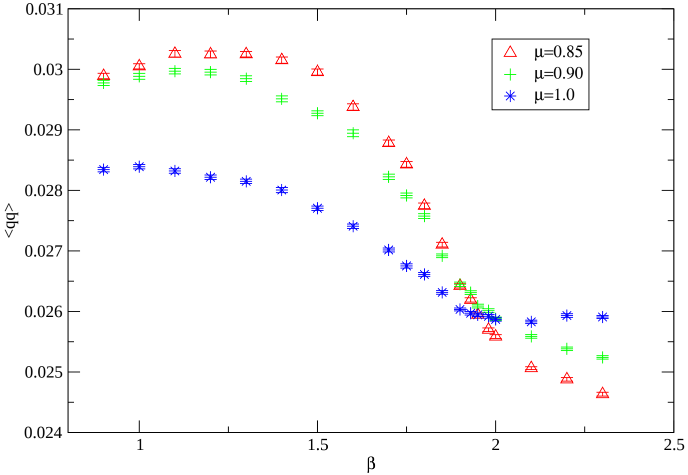
  $12^3 \times 6, \kappa = 0.168$

  $j = 0.04(0.05, 0.06), 0.9 \leq \beta \leq 2.4$

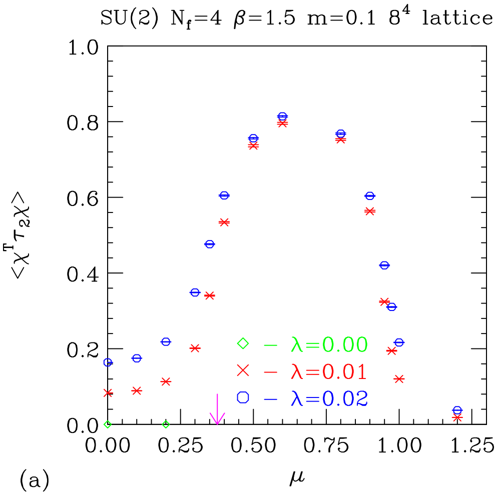
Finite T, μ


0.03
0.028
0.026
0.024
0.022
0.02
0.018
0.016
0.014
0.012
0.01
1
1.5
2
2.5
β
<qq>
μ=0.0
μ=0.575
μ=0.65

Finite T, μ

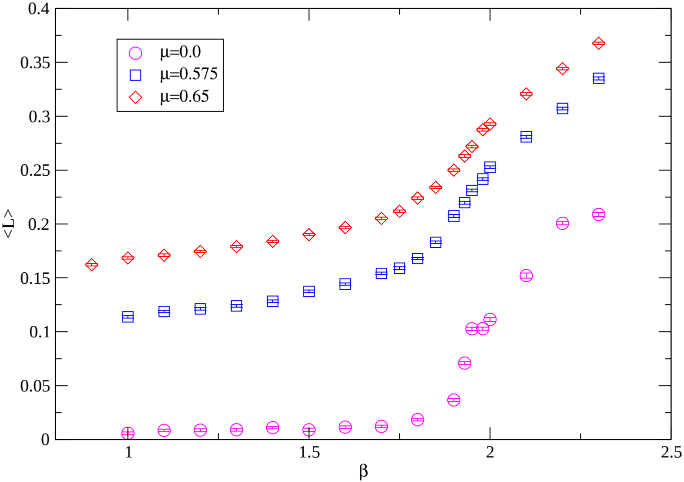

0.031
0.03
0.029
0.028
0.027
0.026
0.025
0.024
<qq>
1
1.5
2
2.5
β
μ=0.85
μ=0.90
μ=1.0

Finite T, μ

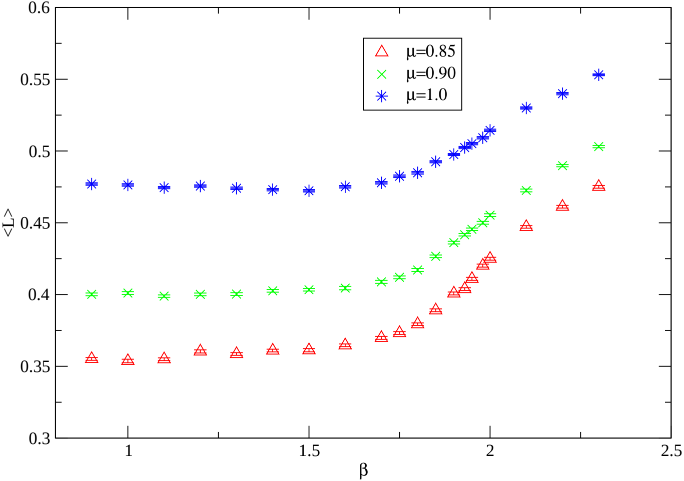


- J.B.Kogut,D.K.Sinclair,S.J.Hands and S.E. Morrison, PRD64 (2001) 094505

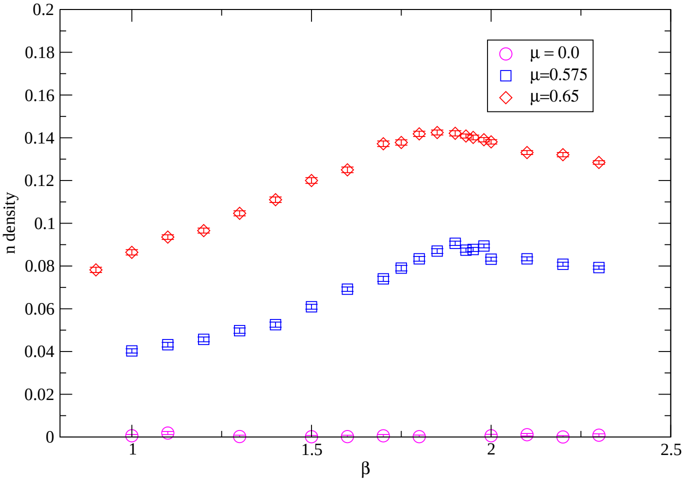
Finite T, μ



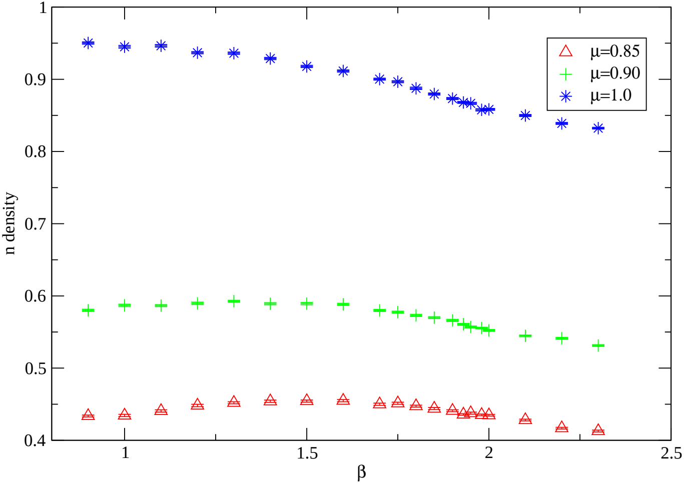
Finite T, μ



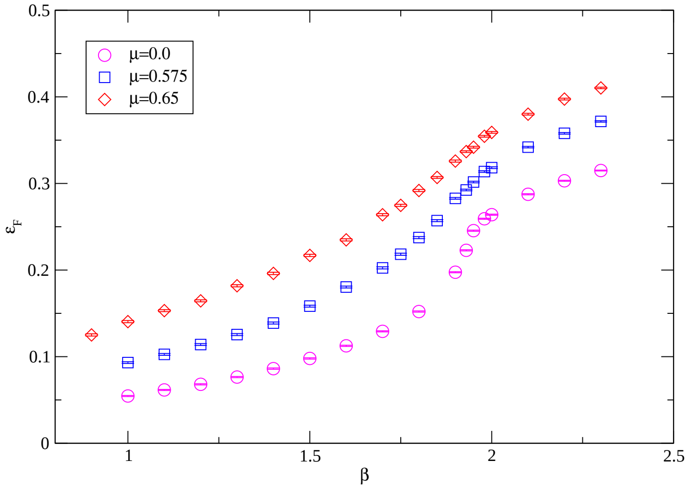
Finite T, μ


μ = 0.0
μ=0.575
μ=0.65
n density
β
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
1
1.5
2
2.5

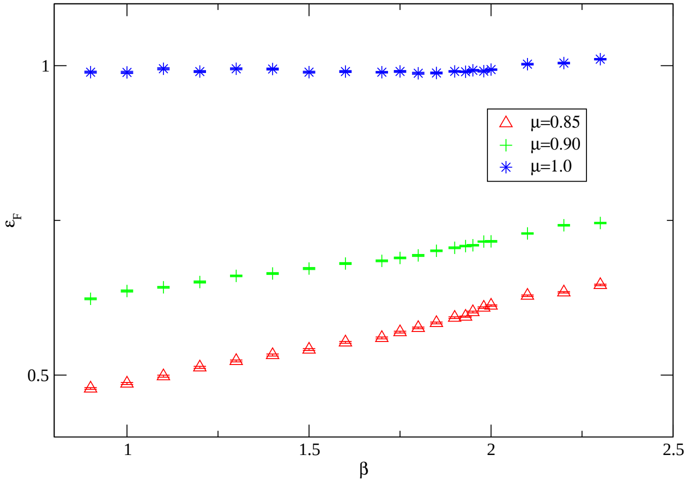
Finite T, μ



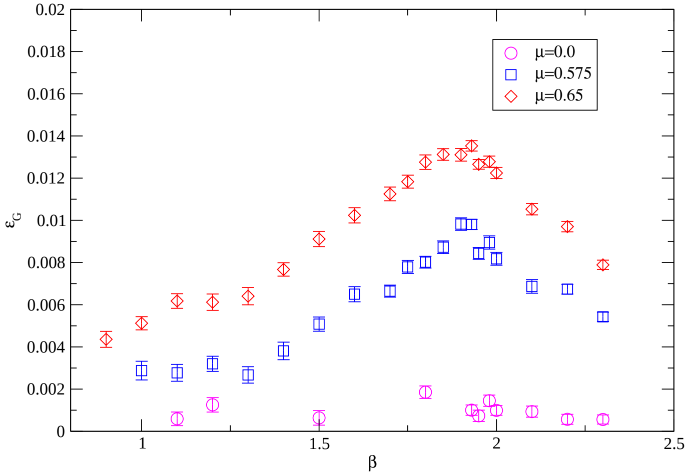
Finite T, μ



Finite T, μ

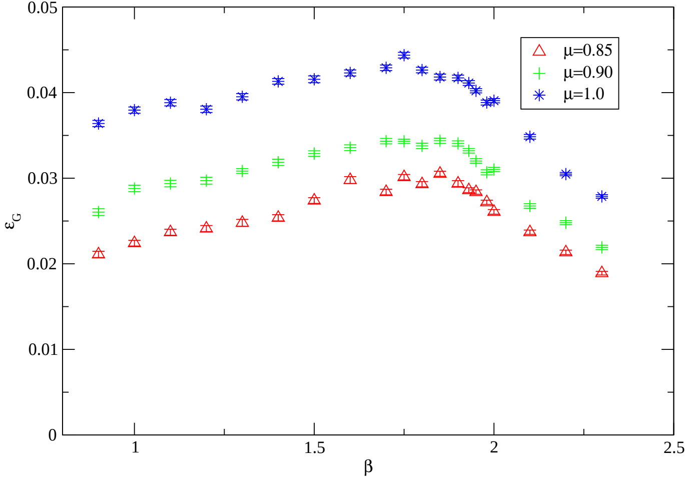


Finite T, μ



0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
1
1.5
2
2.5
β
εG
μ=0.0
μ=0.575
μ=0.65

Finite T, μ



Discussion

- diquark condensate behavior shows interesting behavior

  for $1.75 < \beta < 2.0$

  for $\mu = 0.575, 0.65$, the diquark condensate doesn't change much

- but for $\mu \geq 0.85$ the diquark condensate behaves differently from that for $\mu \leq 0.65$
- further study is needed
- many parameters to scan $\longrightarrow$ Grid computing