ⓒT.Hatsuda ?

HASUL Symposium

March 13, 2010

Hiroshima

HASUL ?

- Humilations and Surprises on the Lattice ?

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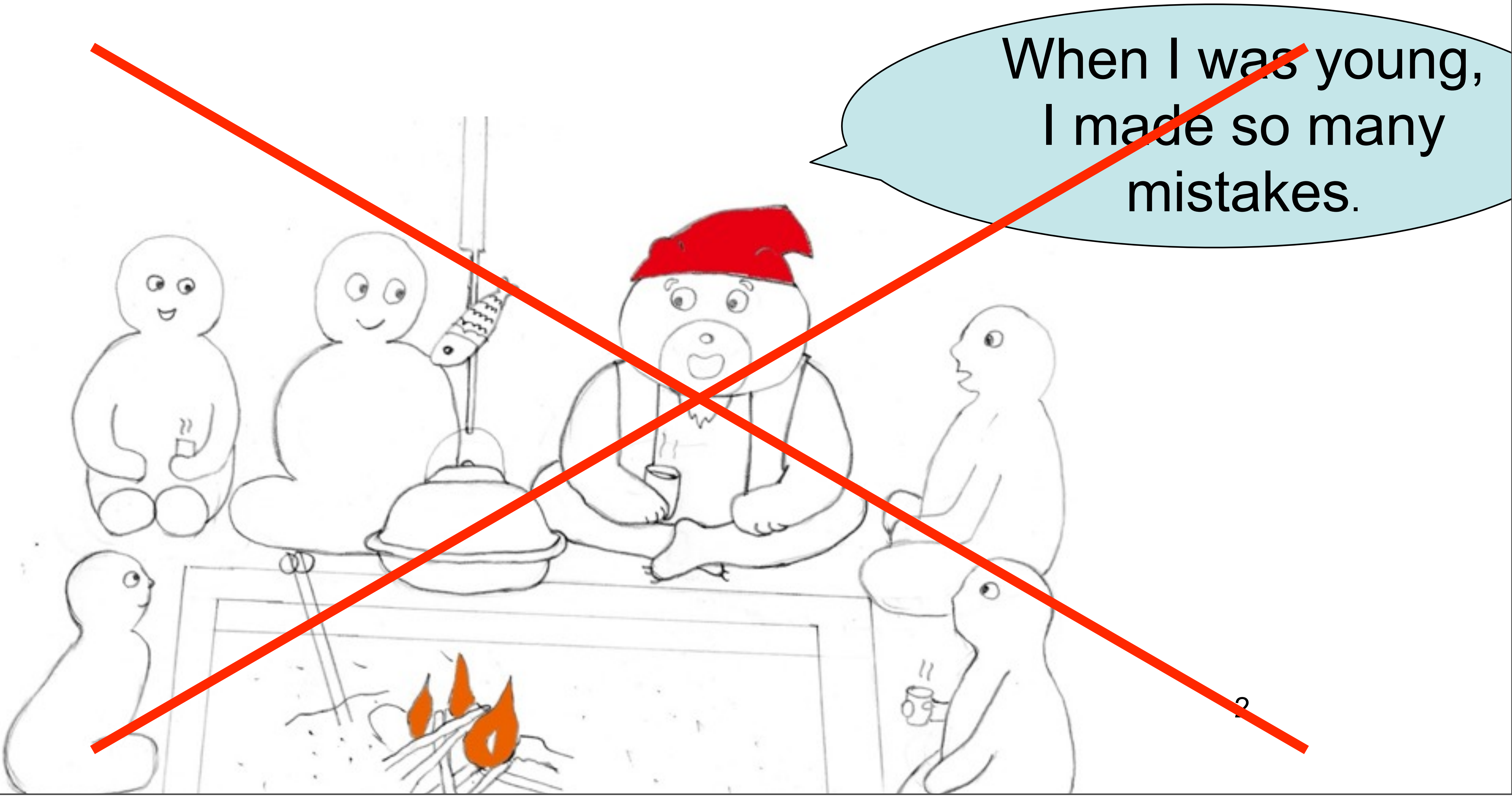


HASUL: Harmonies and Surprises

Status Report of X-Japan

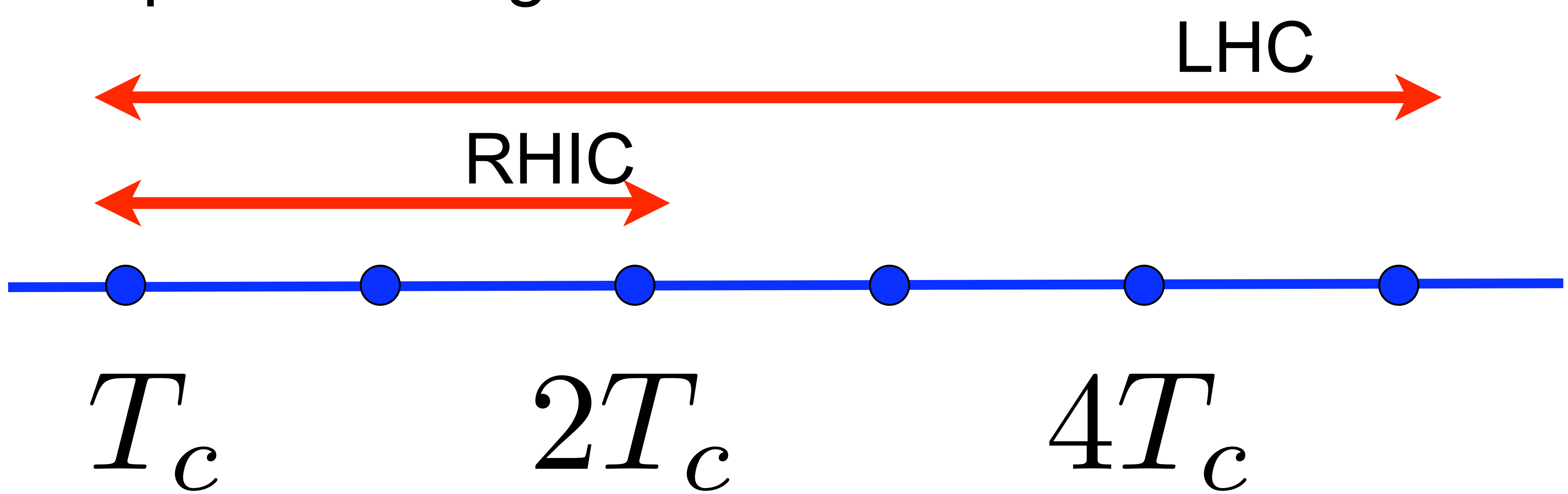
QCD

Japan

XQCD Japan:
Y.Nakagawa,
K.Nagata and
A.N

Targets of XQCD Japan

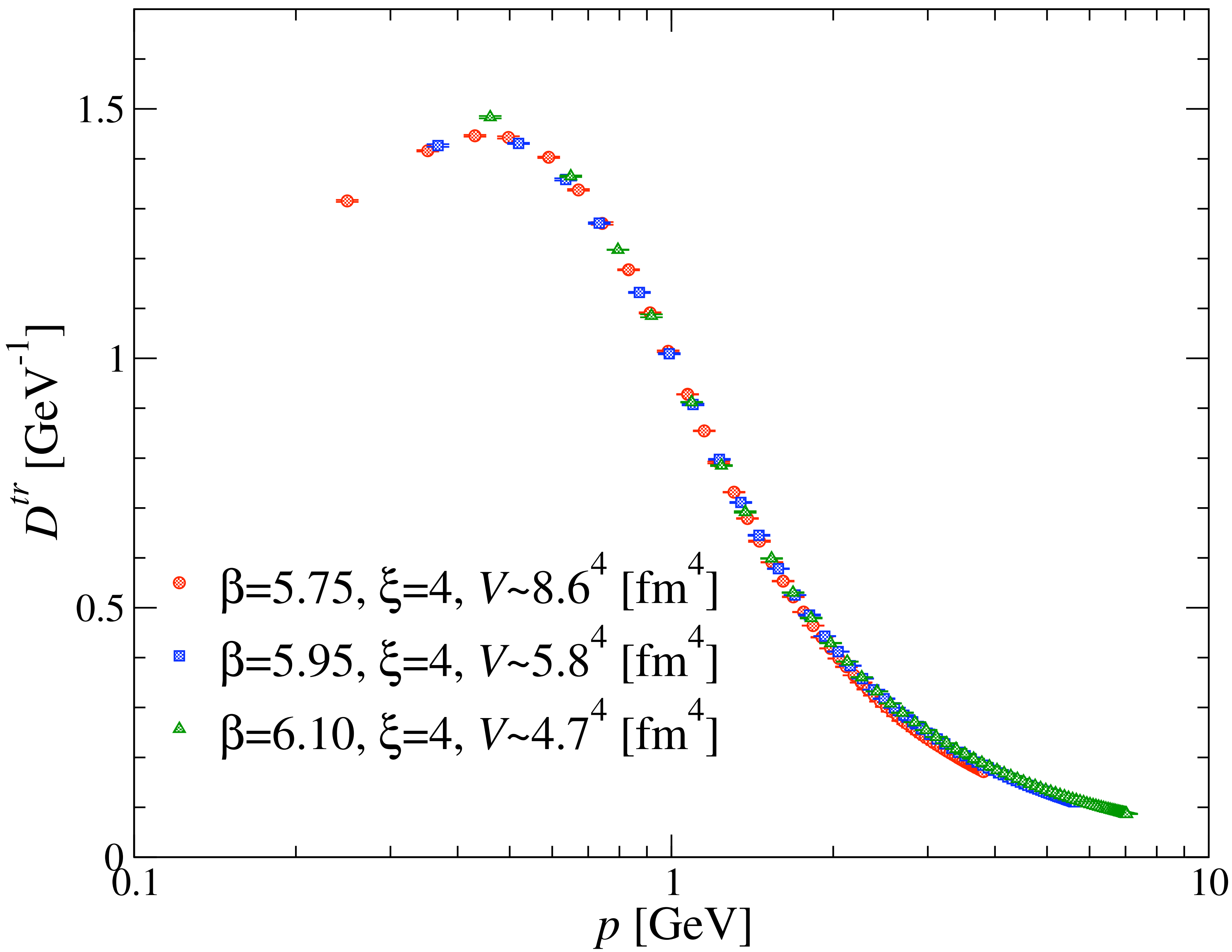
- Finite Temperature by Yoshiyuki Nakagawa
  - Mechanism of Confinement/Deconfinement
  - LHC Temperature regions is similar to RHIC Temperature regions or not



QGP arond Tc=4 is Perfect Liquid or Free Gas ?

Gluon Propagators at T=0 in Coulomb Gauge

$$\langle A_i(t,\vec{p})A_i(t,-\vec{p})\rangle$$



Suppresion at small P

$$\frac{Z}{p^2+M^2}$$

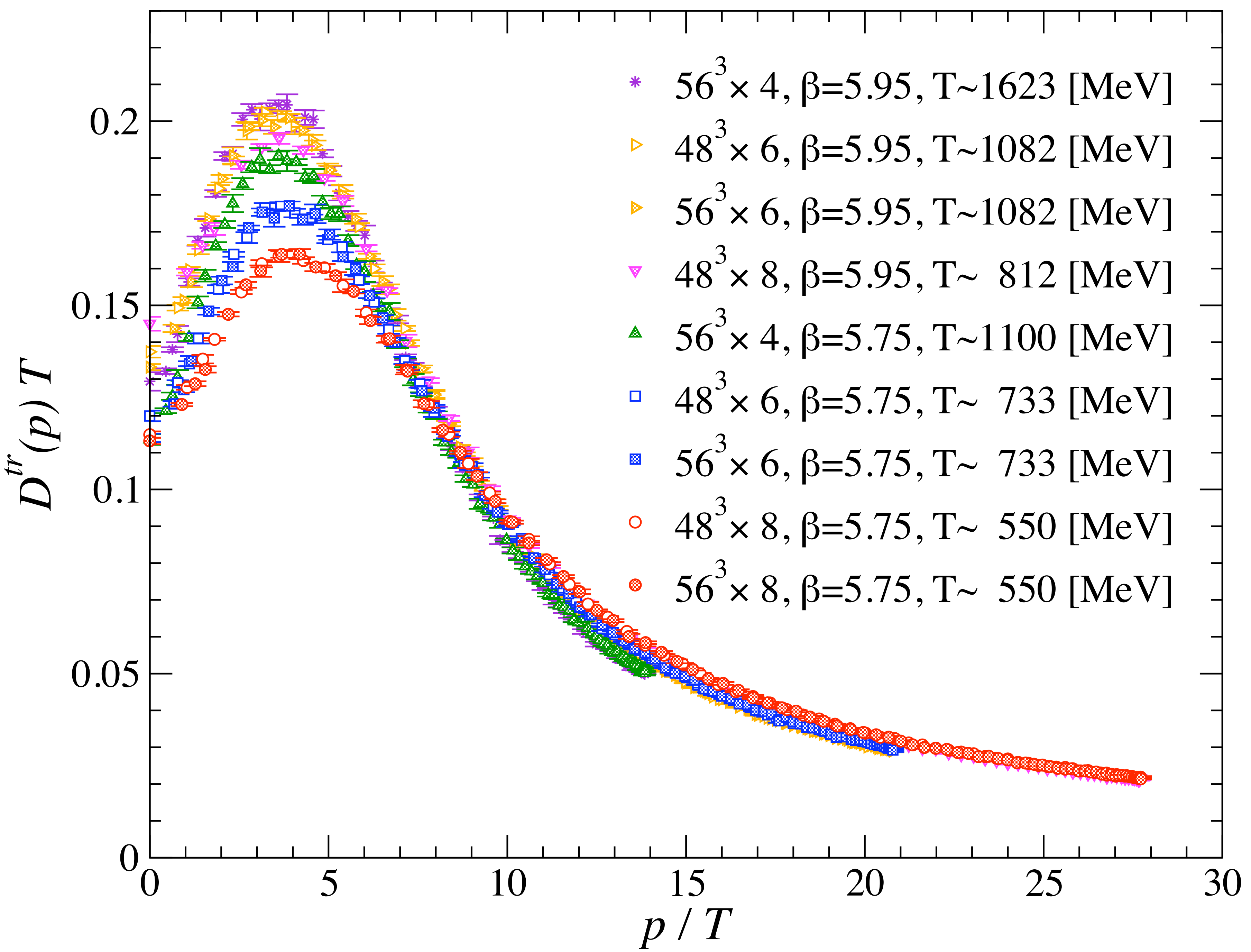
$$M(p) \rightarrow \mathrm{Large}$$

as $\quad p \rightarrow Large$

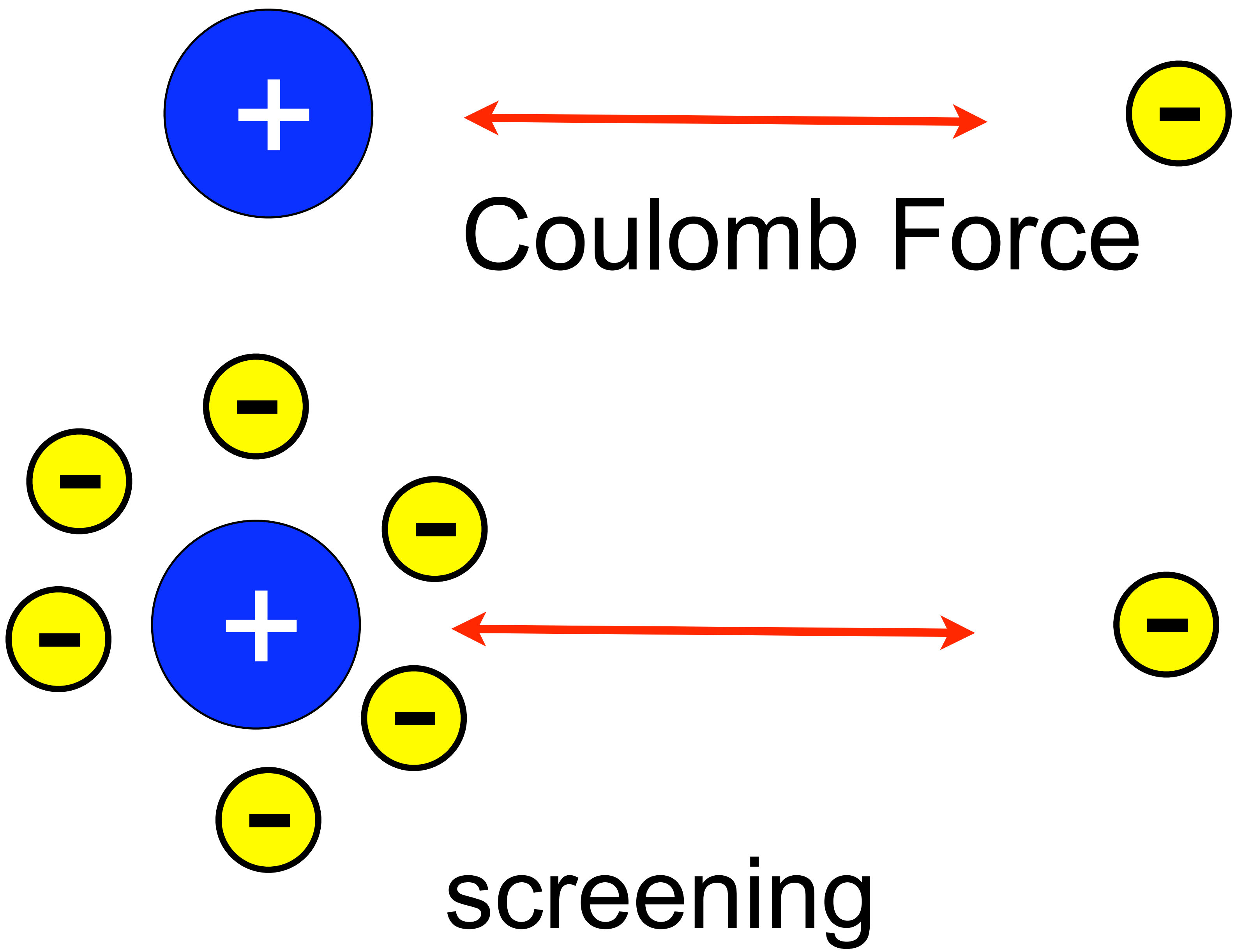
Gribov Ansatz

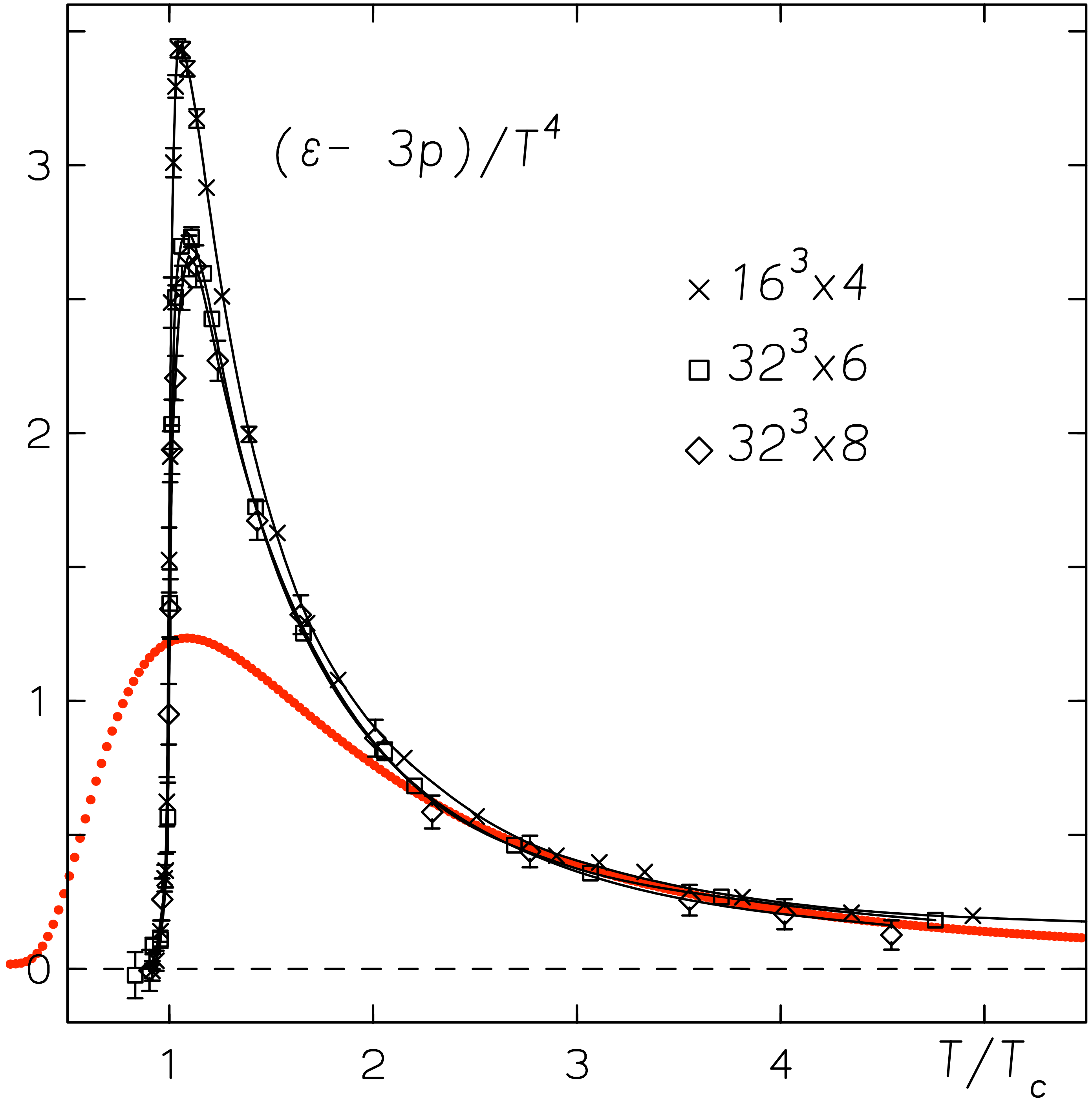
The suppresions are seen even at high T

Unrenormalized, 10000 sweeps, 100 confs, every 100 sweeps



In QED Plasma





D.Zwanziger
PhysRevLett.
94.182301
hep-lat/0610021

$$Z = \Pi \frac{1}{1 - \exp(-\beta E_n)}$$

$$E(k) = \sqrt{k^2 + M^2/k^2}$$

Gribov Ansatz

Target of XQCD Japan (2)

- Finite Density Lattice QCD with Wilson Fermions by Keitaro Nagata
  - Imaginary Chemical Potential
    - Canonical Approach
    - Information to PNJL
    - Analytical Continuation
      - Phase transition line
      - Gluon Propagators

- Competitors: Kentucky, Graz
-

Grand-Canonical

Z_{GC}

Canonical

Z_C

$$\int \mathcal{D}U\mathcal{D}\bar{\psi}\mathcal{D}\psi e^{-\beta S_G - \bar{\psi}\Delta\psi}$$

$$\hat{N} \equiv \int d^3x \bar{\psi}(x)\gamma_0\psi(x)$$

$$\color{red}{\delta(\hat{N} - Q)} = \int d\phi\, e^{i\phi(\hat{N}-Q)}$$

$$\phi = \frac{\mu_I}{T}$$

$$\frac{3}{2\pi}\int_{-\frac{\pi}{3}}^{+\frac{\pi}{3}} d\frac{\mu_I}{T}\, e^{-iQ\frac{\mu_I}{T}}\, Z_{GC}(i\mu_I)$$

μ_I has the period $\frac{2\pi T}{3}$

(Roberge-Weiss)

Grand-Canonical $\Leftrightarrow$ Canonical

$$Z_{GC} \qquad Z_C$$

$$\int \mathcal{D}U\mathcal{D}\bar{\psi}\mathcal{D}\psi e^{-\beta S_G - \bar{\psi}\Delta\psi} \delta(\hat{N}-Q)$$

$$\hat{N} \equiv \int d^3x \bar{\psi}(x)\gamma_0\psi(x)$$

$$\delta(\hat{N}-Q) = \int d\phi \, e^{i\phi(\hat{N}-Q)}$$

$$\phi = \frac{\mu_I}{T}$$

$$\frac{3}{2\pi}\int_{-\frac{\pi}{3}}^{+\frac{\pi}{3}} d\frac{\mu_I}{T} \, e^{-iQ\frac{\mu_I}{T}} \, Z_{GC}(i\mu_I)$$

μ_I has the period $\dfrac{2\pi T}{3}$

(Roberge-Weiss)

Grand-Canonical ⟺ Canonical

Z_{GC} $\qquad$ Z_C

$$Z_C = \int \mathcal{D}U\mathcal{D}\bar{\psi}\mathcal{D}\psi e^{-\beta S_G - \bar{\psi}\Delta\psi}\,\delta(\hat{N}-Q)$$

$$\hat{N} \equiv \int d^3x \bar{\psi}(x)\gamma_0\psi(x)$$

$$\delta(\hat{N}-Q) = \int d\phi\, e^{i\phi(\hat{N}-Q)}$$

$$\phi = \frac{\mu_I}{T}$$

$$\frac{3}{2\pi}\int_{-\frac{\pi}{3}}^{+\frac{\pi}{3}} d\frac{\mu_I}{T}\, e^{-iQ\frac{\mu_I}{T}}\, Z_{GC}(i\mu_I)$$

μ_I has the period $\frac{2\pi T}{3}$

(Roberge-Weiss)

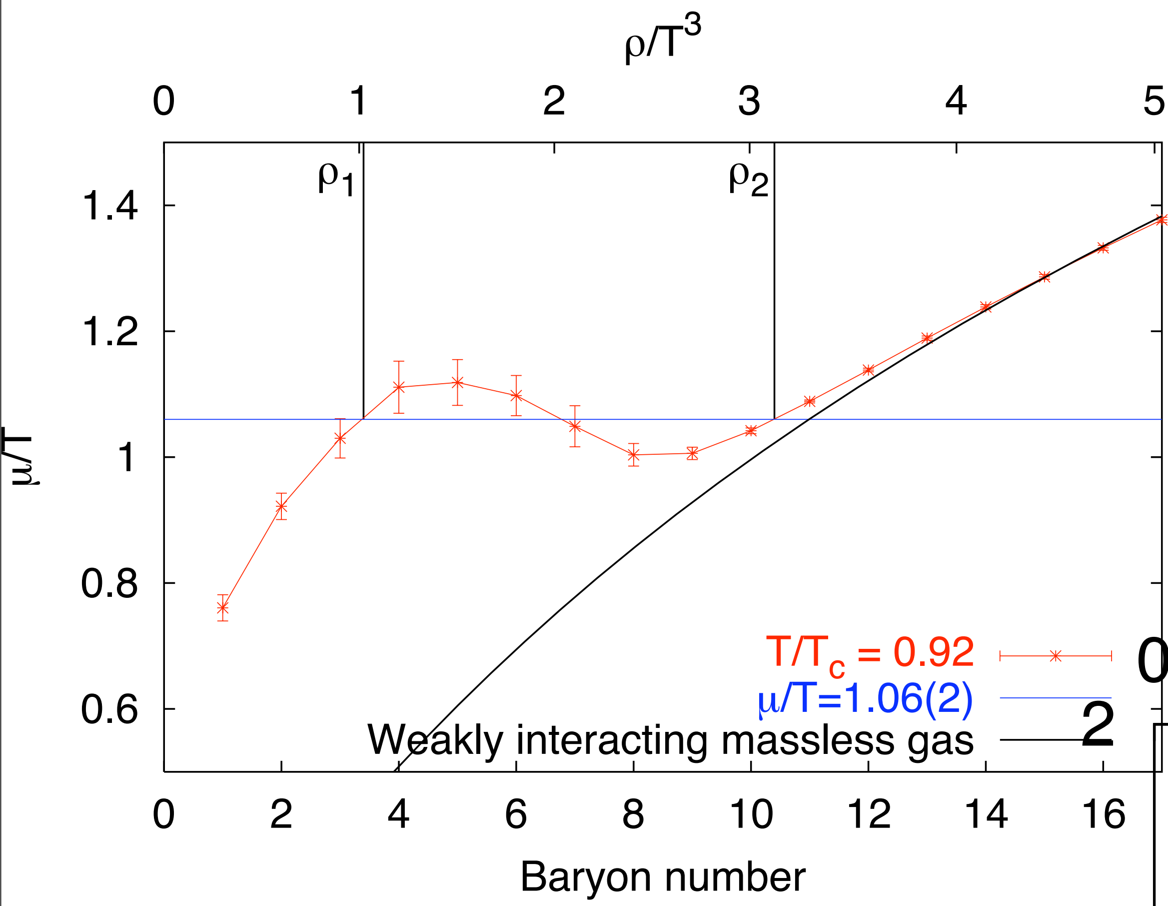
Canonical Approach

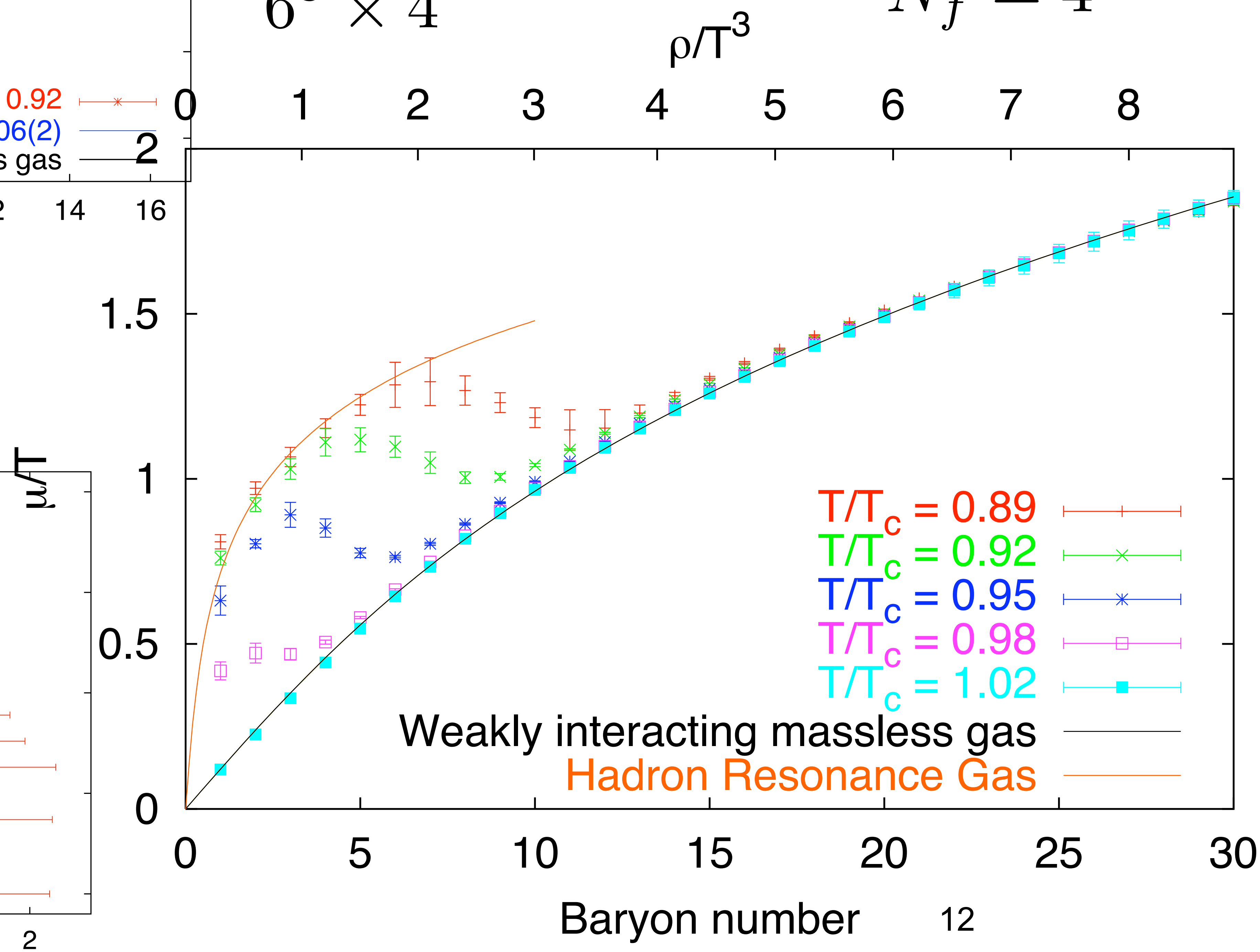
- Miller and Redlich
  - –Phys. Rev. D35 (1987) 2524
- A.Hasenfratz and Toussaint
  - – Nucl.Phys.B371 (1992) 539
- Engels, Kaczmarek, Karsch and Laermann
  - – Nucl.Phys. B558 (1999) 307 (hep-lat/9903030)
  - – hep-lat/9905022
- Forcrand and Kratochvila
  - – Nucl. Phys. B (P.S.) 153 (2006) 62 (hep-lat/0602024)
- A.Li, Meng, Alexandru, K-F. Liu
  - –PoS LAT2008:032 and 178 (arXiv:0810.2349, arXiv: 0811.2112)

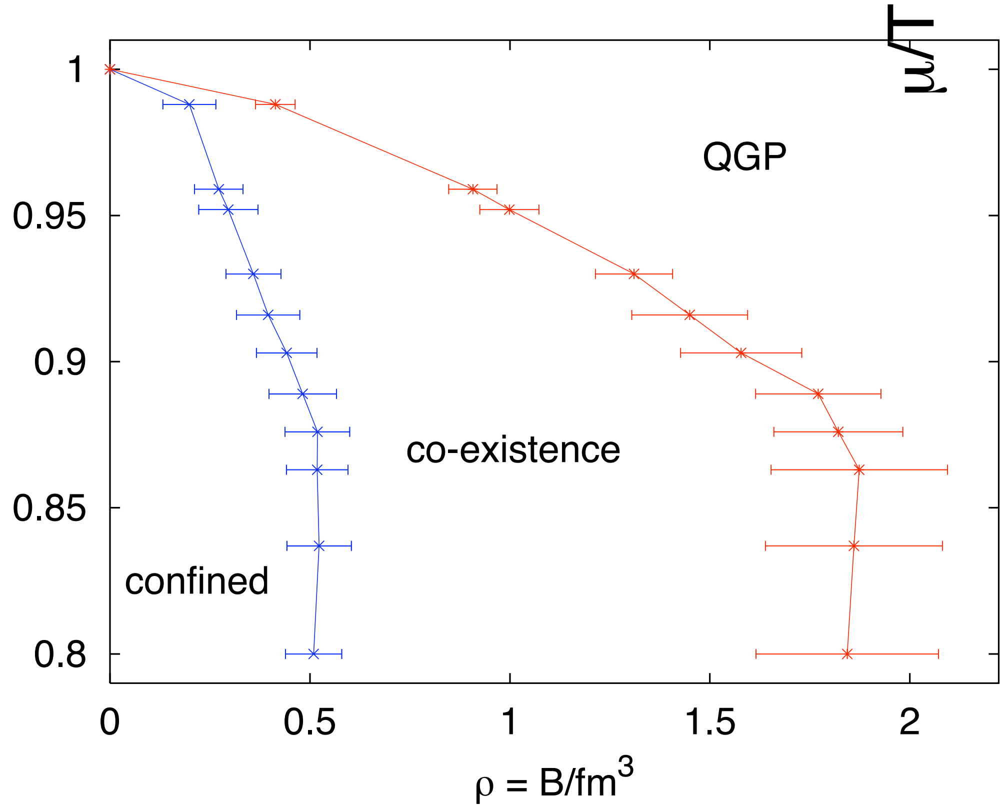
Forcrand- Kratochvila
Nucl. Phys. B (P.S.) 153 (2006) 62
hep-lat/0602024

$m_\pi = 300\mathrm{MeV}$ KS Fermions

$6^3 \times 4$ $N_f = 4$







Some Tricks behind

Gibbs Formula

- P.E.Gibbs, Phys.Lett. B172 (1986) 53-61

$$
\begin{aligned}
\Delta &= \frac{1}{z}\left(zB + z^2(-V^\dagger) + (-V)\right) \\
&= \frac{1}{z}\left(zB(-V) + z^2(-V^\dagger)(-V) + (-V)^2\right)(-V^{-1}) \\
&= \frac{1}{z}\left(zB(-V) + z^2 + (-V)^2\right)
\end{aligned}
$$

$$\boxed{z \equiv e^{-\mu}}$$

$$
V = \left(\begin{array}{c|c|c cc|c}
0 & U_4(t=1) & 0 & \cdots & & 0 \\
\hline
0 & 0 & U_4(t=2) & \cdots & & 0 \\
\hline
0 & 0 & 0 & \cdots & & \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
 & & & \cdots & U_4(t=N_t-2) & 0 \\
\hline
0 & 0 & & \cdots & 0 & U_4(t=N_t-1) \\
\hline
-U_4(t=N_t) & 0 & & \cdots & 0 & 0
\end{array}\right)
$$

$$\Delta = \frac{1}{z}\left(zB(-V) + z^2 + (-V)^2\right)$$

$$\begin{aligned}
\det \Delta &= z^{-N}\begin{vmatrix} -B(-V) - z & 1 \\ -V^2 & -z \end{vmatrix} \\
&= \left| \underbrace{\begin{pmatrix} BV & 1 \\ -V^2 & 0 \end{pmatrix}}_{P} - zI \right| \\
&= \det\left(P - zI\right) \\
&= \prod (\lambda_i - z)
\end{aligned}$$

- P is $(2 \times N_c \times N_x \times N_y \times N_z)^2$ (Matrix Reduction)
- Determinant for any value of μ

This formula is useful also in the Multi-Parameter Reweighting

Once the eigen-values are known, the fermion reweighting factors can be obtaied for any value of μ

$$\frac{\det \Delta(\mu)}{\det \Delta(0)} = \frac{\prod(\lambda_i - e^{-\mu})}{\prod(\lambda_i - 1)}$$

Kentucky Group's Trick

$$Z_C(Q) = \int \frac{d\phi}{2\pi} e^{-iQ\phi} Z_{GC}(\mu = i\pi T)$$

$$= \int \mathcal{D}U e^{-\beta S_G} \boxed{\int \frac{d\phi}{2\pi} e^{-iQ\phi} \det\Delta(\mu = i\phi T)}$$

How to calculate this part.

$$\det\Delta = e^{\mathrm{Tr}\log\Delta}$$

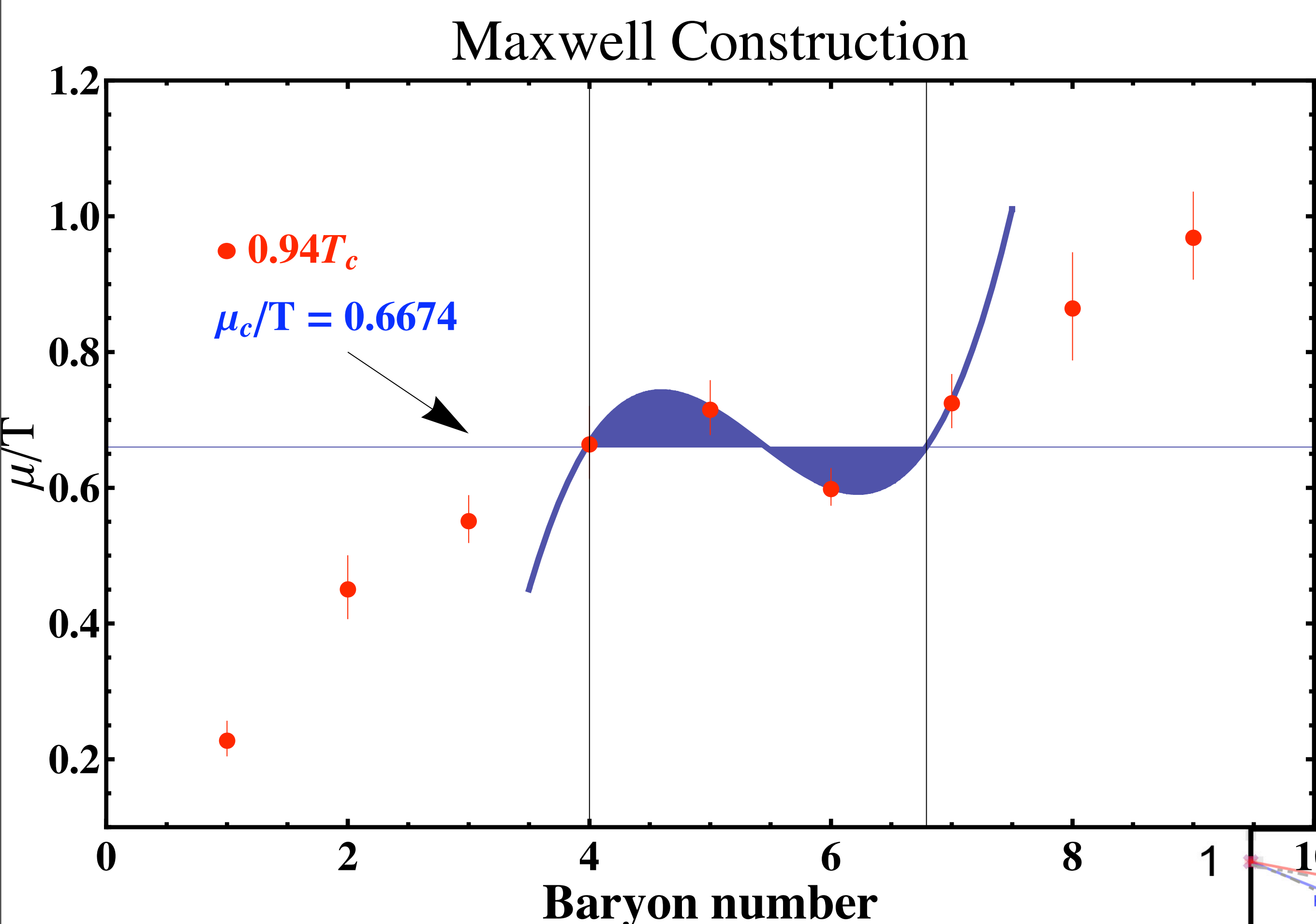
We expand log

$$\mathrm{Tr}\log\Delta = A_0 + \sum_n A_n \cos(n\phi + \delta_n)$$

n: winding number

$\boxed{}$ can be expressed by Bessel Functions

A.Li, Meng, Alexandru, K-F. Liu
PoS LAT2008:178 (arXiv:0810.2349)

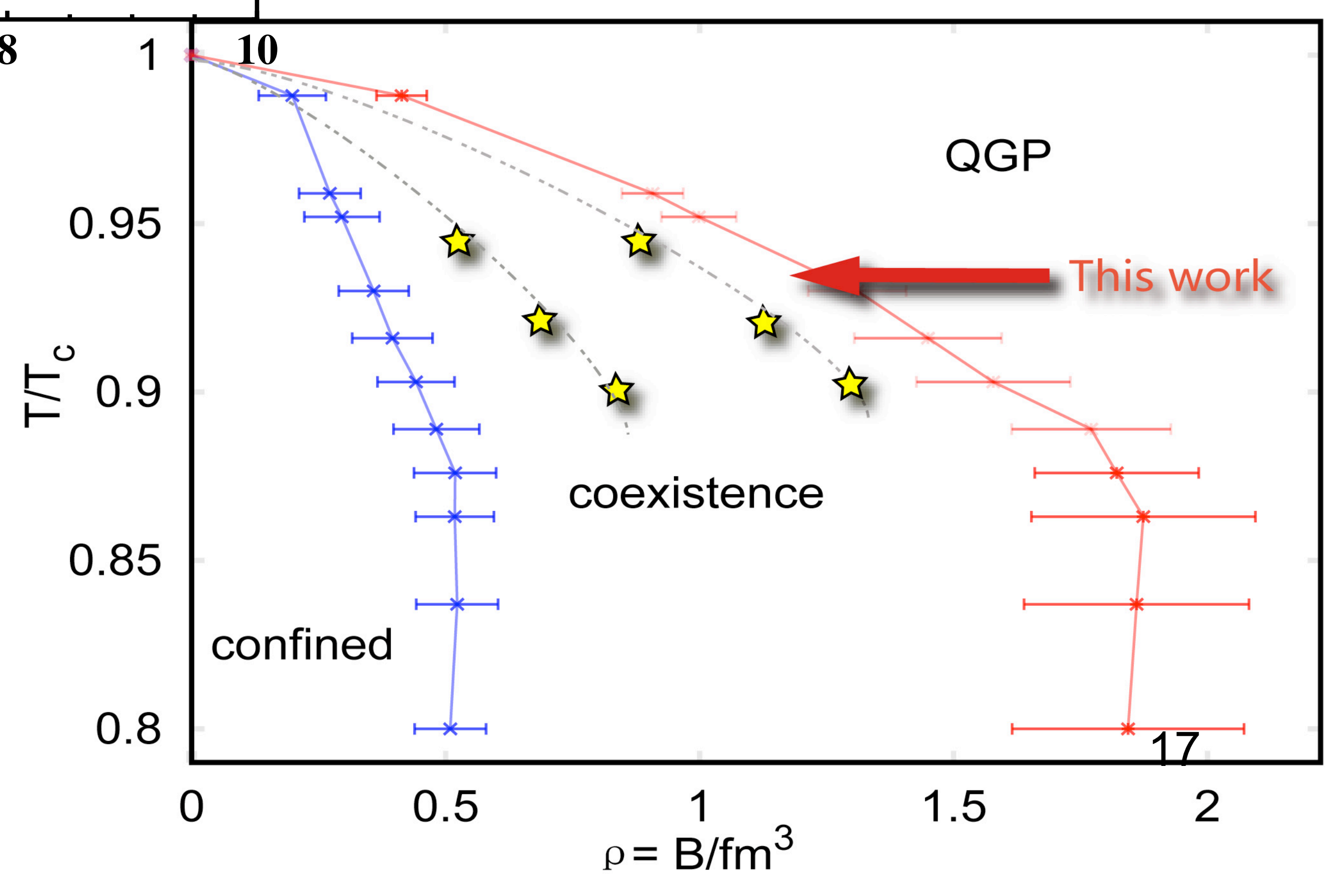


$N_f = 4$

$6^3 \times 4$

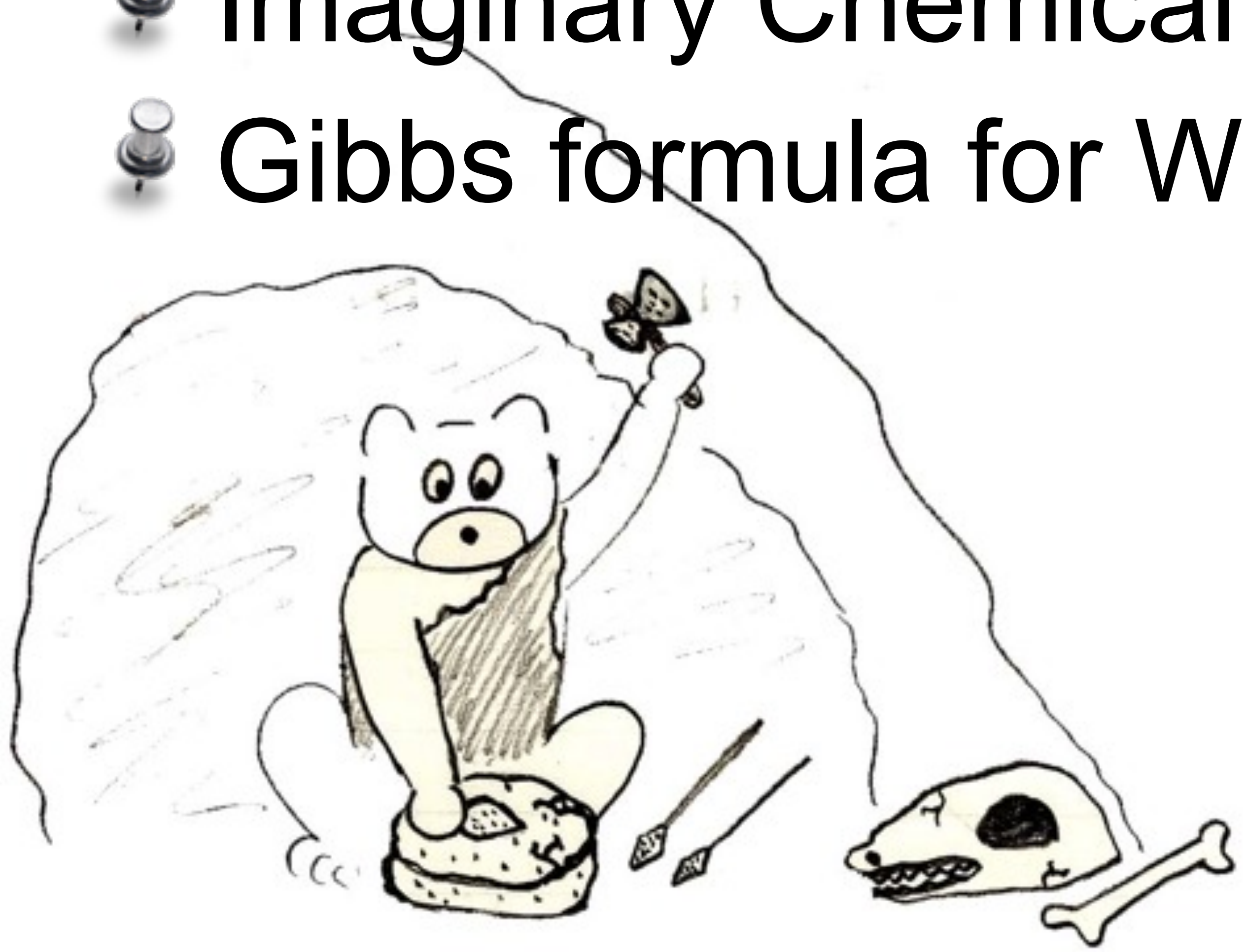
$m_\pi = 1\mathrm{GeV}$

Wilson Fermions



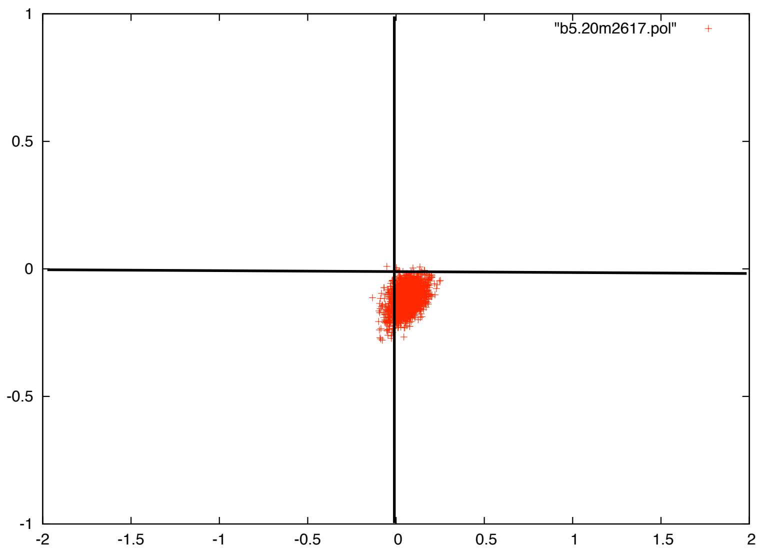
Prepare Tools

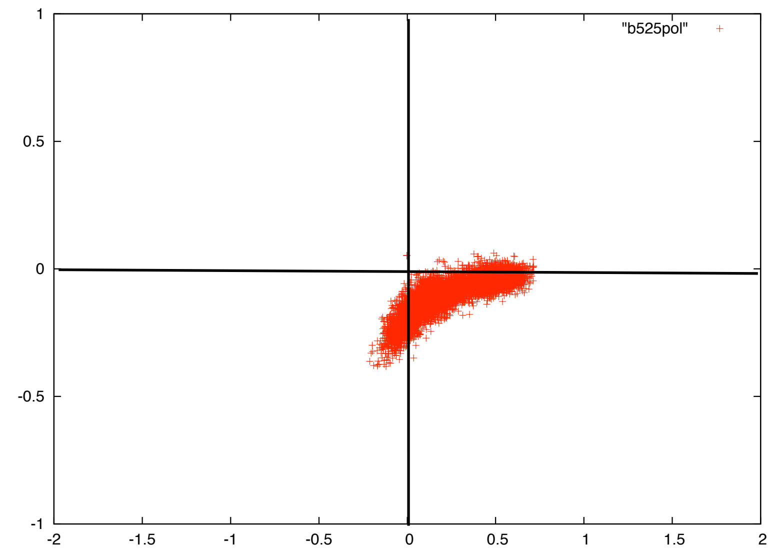
- Wilson Fermions with Clover Term
- Gauge Action with 1x1 + 1x2
- Taylor Expansion for small mu
- Imaginary Chemical Potential
- Gibbs formula for Wilson Fermions

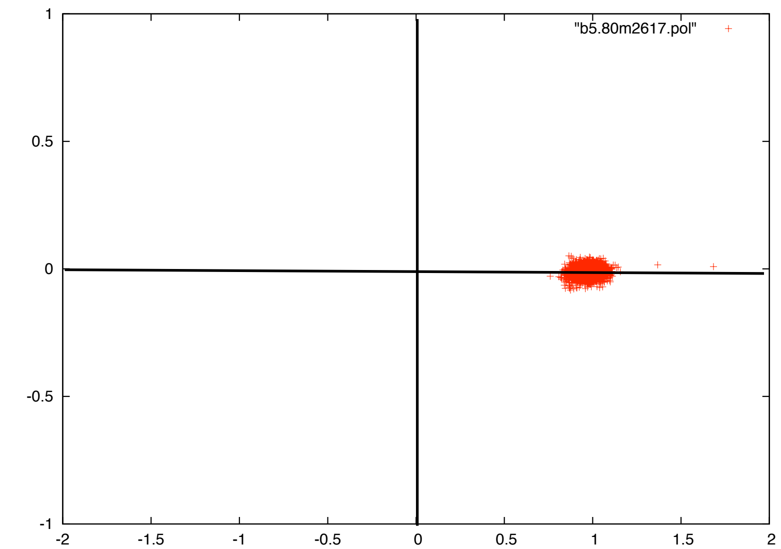


Polyakov Loops around $\frac{\mu_I}{T} \sim \frac{\pi}{3}$

Standard Wilson Actions



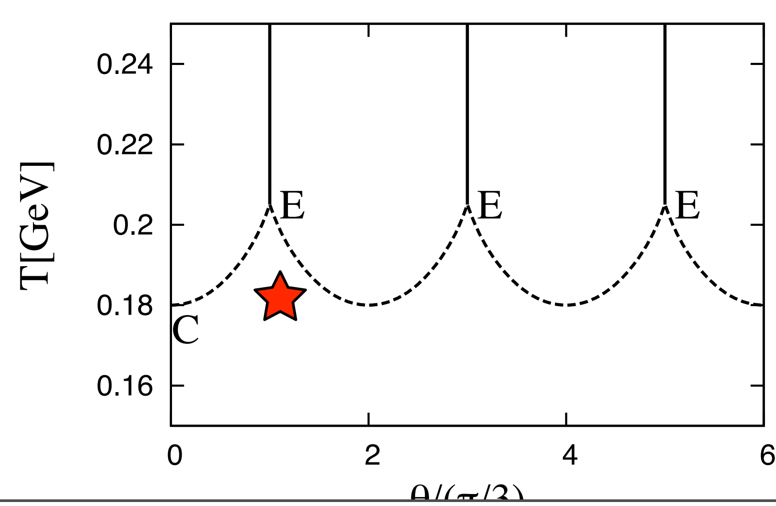


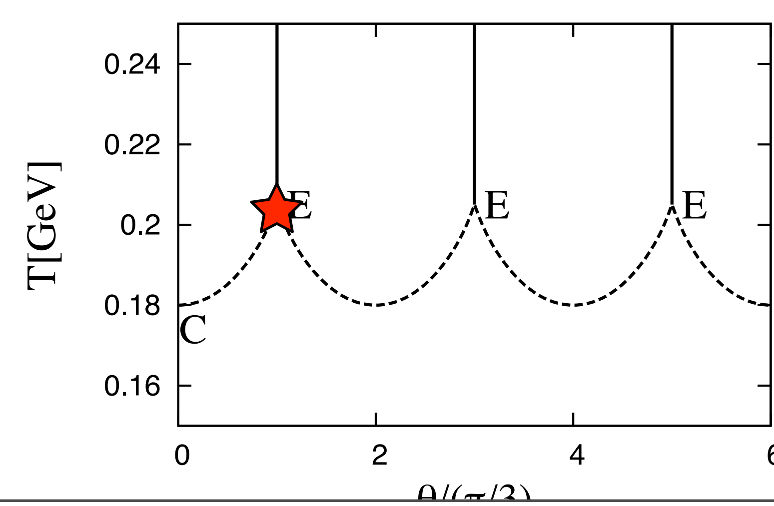


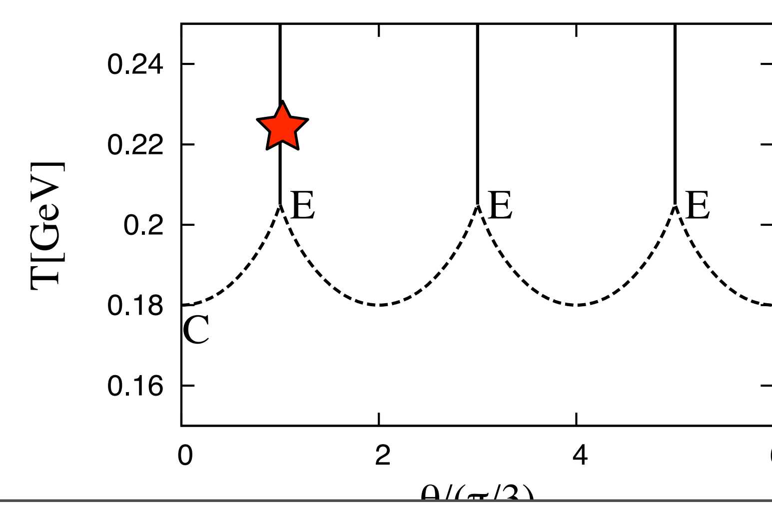
$T < T_c$
$(\beta = 5.20)$

$T \sim T_c$
$(\beta = 5.25)$

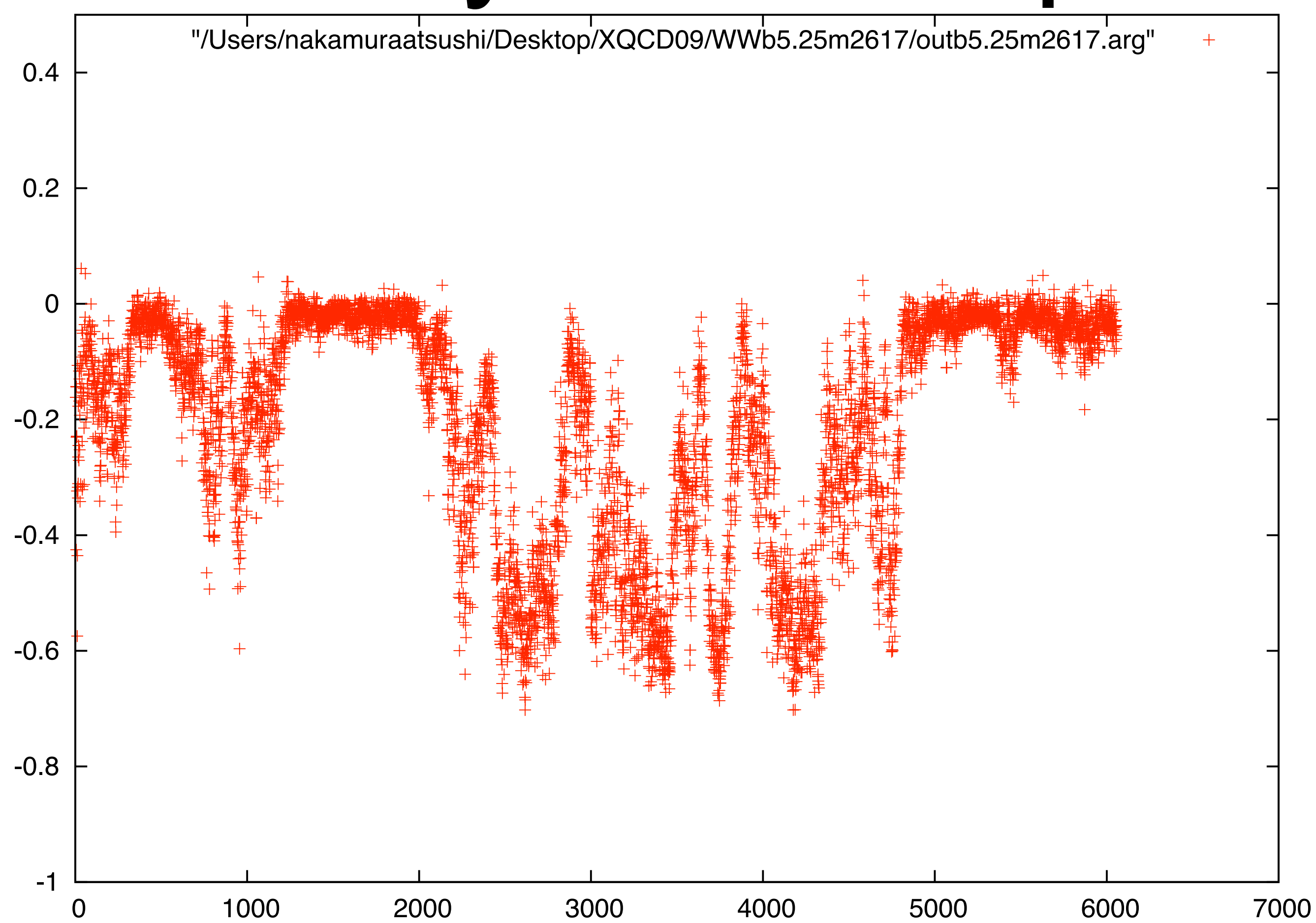
$T > T_c$
$(\beta = 5.80)$





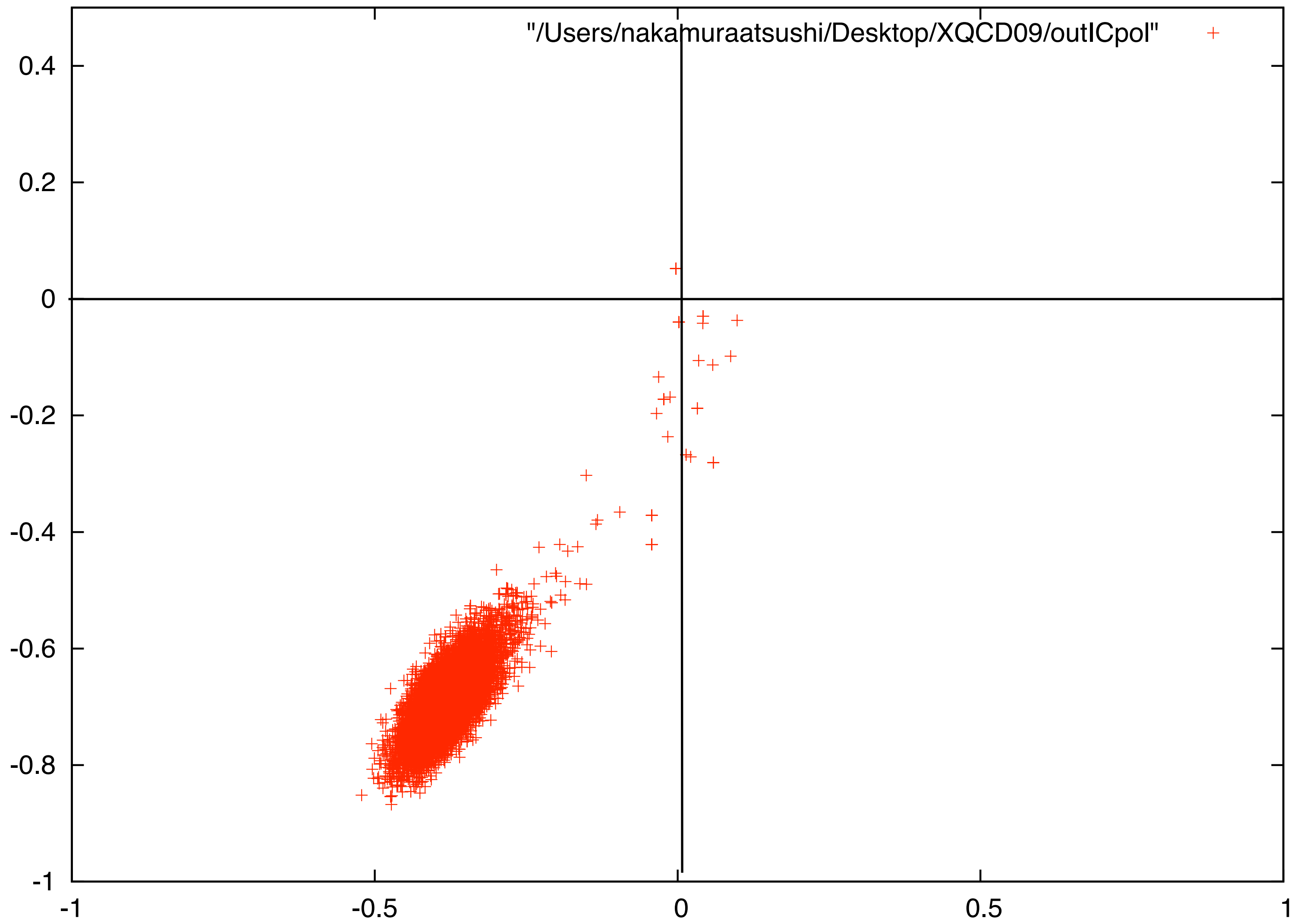


Argument of Polyakov Loop

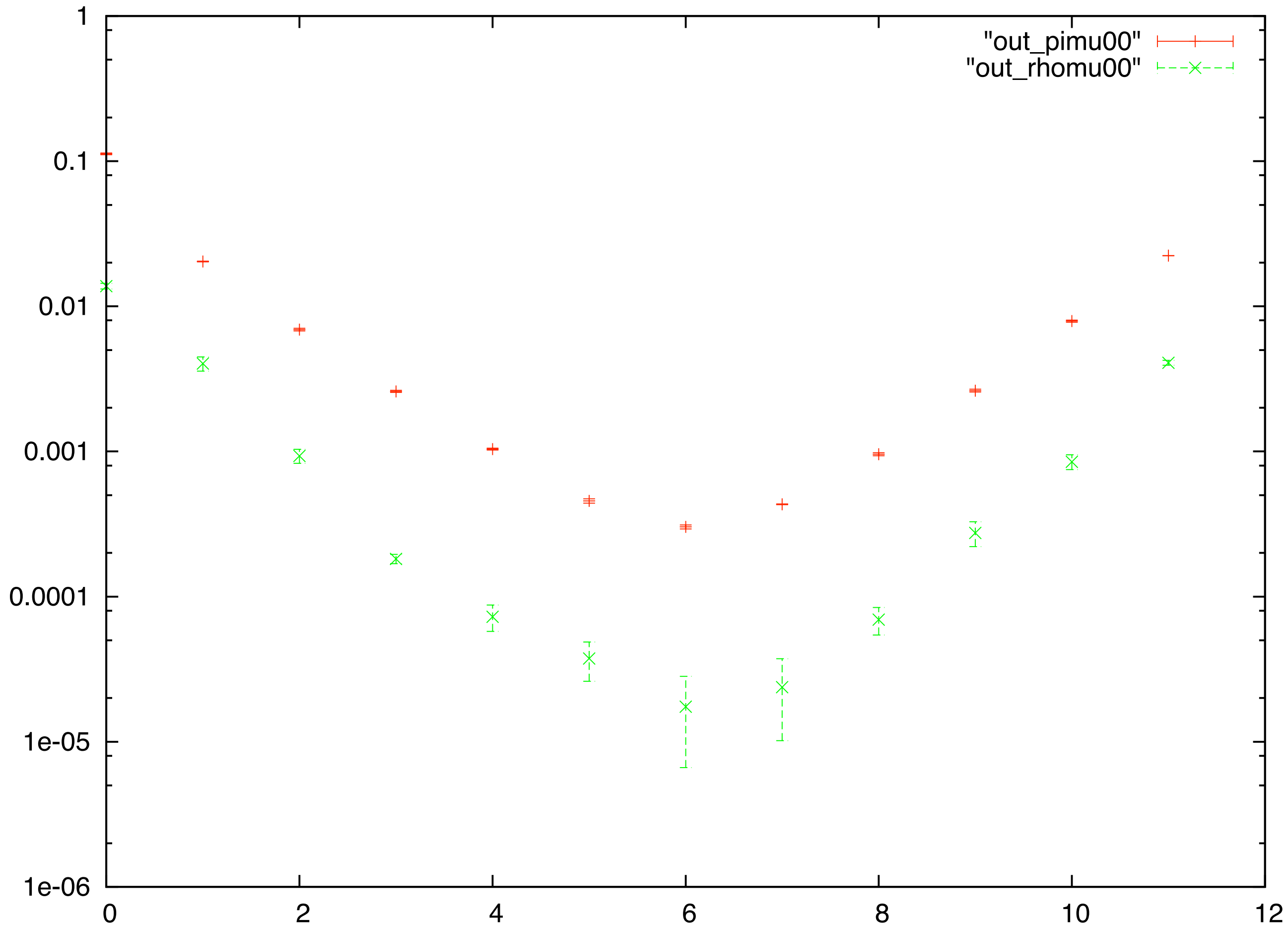


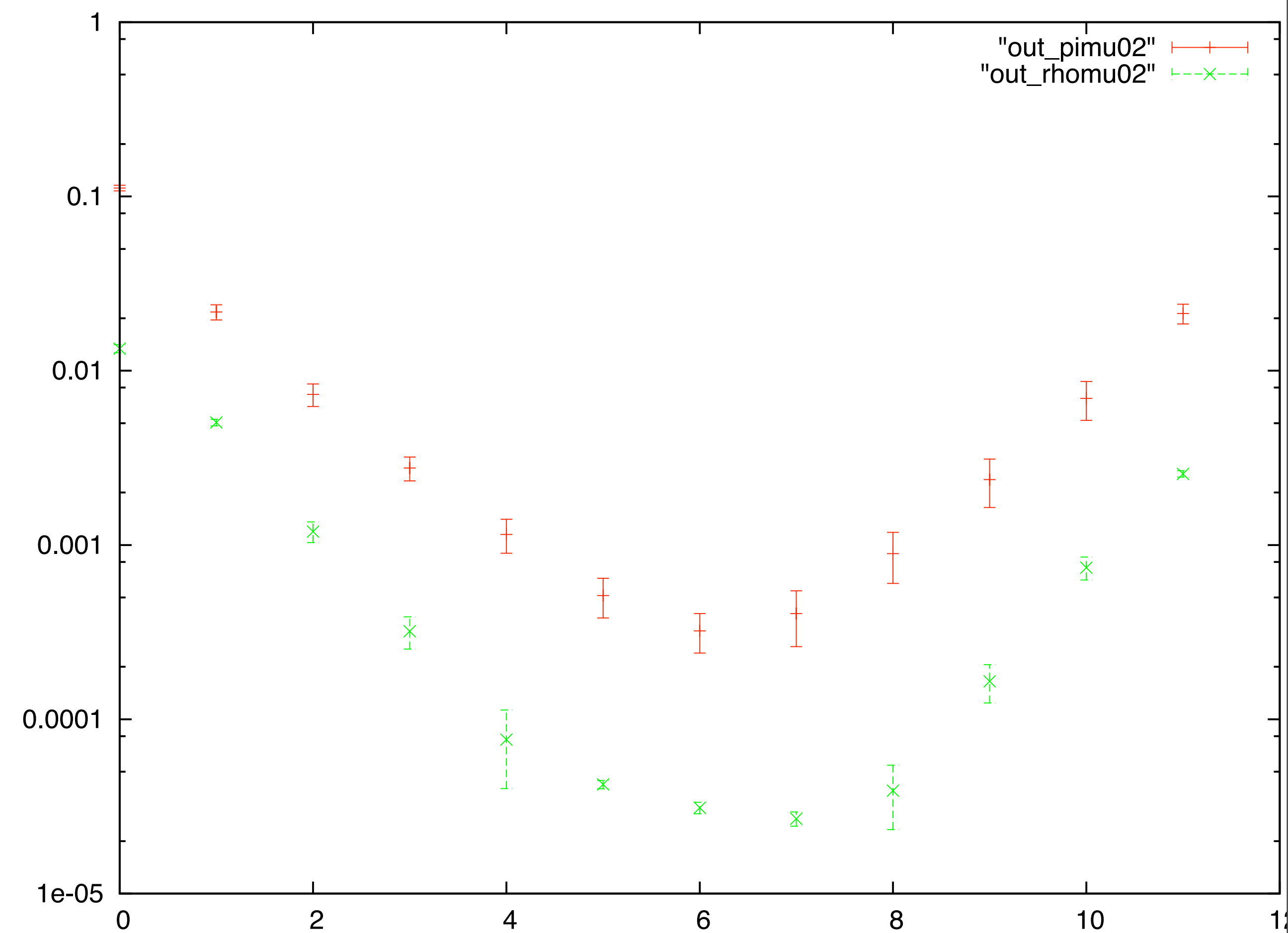
$T > T_c$

Imaginary Chemical Pot. with Improved Actions



pi and rho propagators

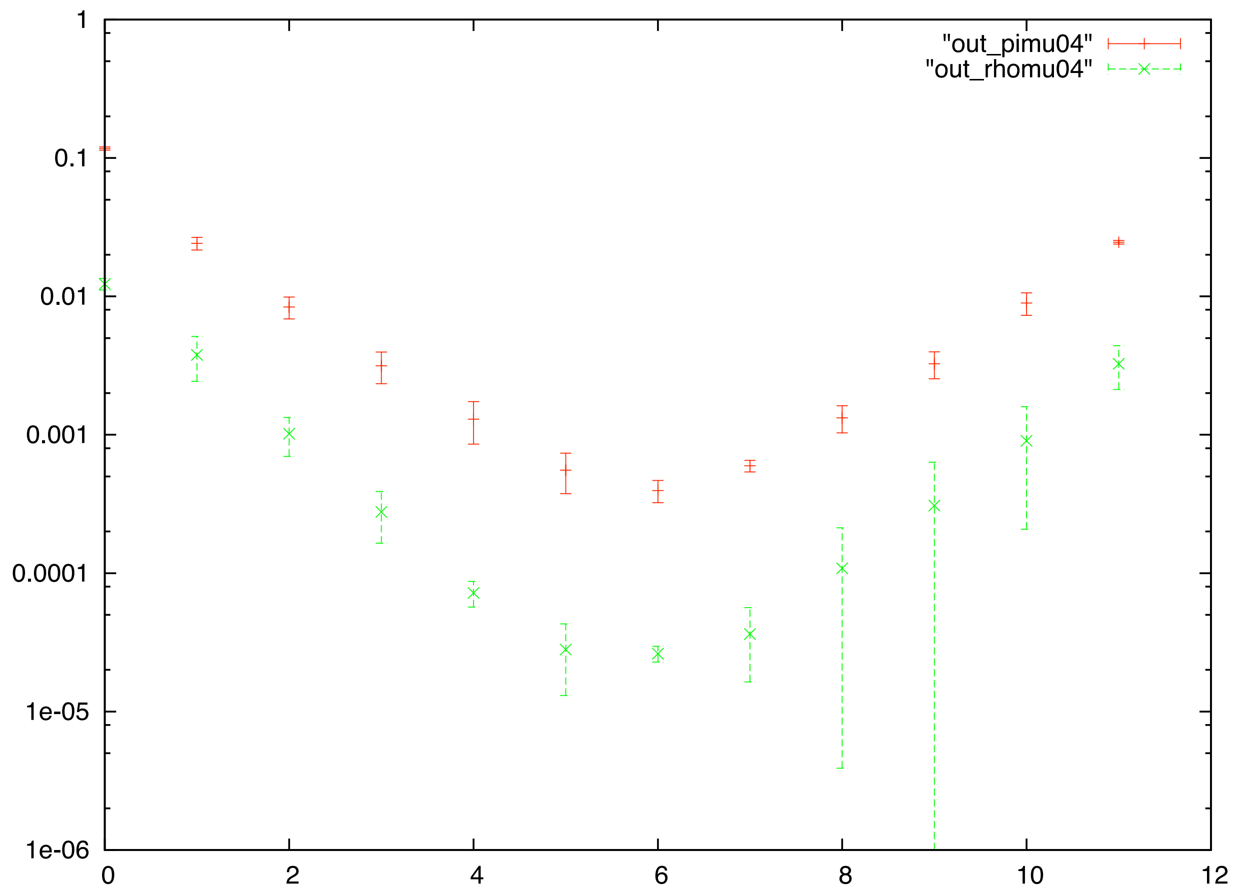


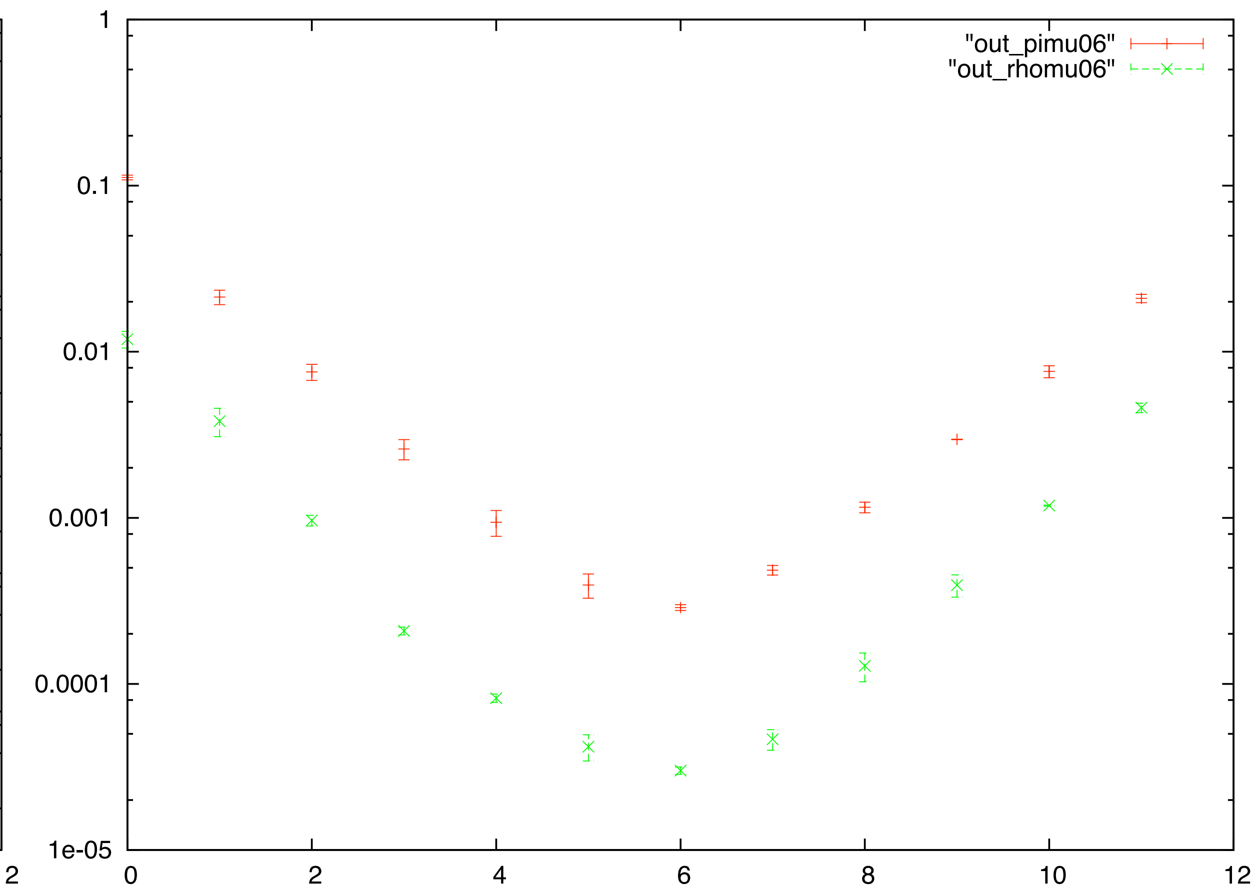


$$\frac{\mu_I}{T} = 0.0 \qquad \frac{\mu_I}{T} = 0.24$$

$$6 \times 6 \times 6 \times 12 \quad T \sim \frac{1}{3} T_c \qquad m_{PS}/m_V = 0.65 \sim 0.8$$

pi and rho propagators (2)





$$\frac{\mu_I}{T} = 0.48$$

$$\frac{\mu_I}{T} = 0.72$$

$$6 \times 6 \times 6 \times 12$$

$$\frac{\mu_I}{T} = 0.24$$

Wilson Fermions for Finite Density

- H.-S. Chen, X.-Q. Luo
  - Phys.Rev. D72 (2005) 0345041
  - hep-lat/0411023
- A.Li, X. Meng, A. Alexandru, K-F. Liu
  - PoS LAT2008:032 and 178 (arXiv:0810.2349, arXiv:0811.2112)
- C. Gattringer and L. Liptak
  - arXiv:0906.1088
- J. Danzer, C. Gattringer, L. Liptak and M. Marinkovic
  - Phys.Lett.B682:240-245,2009 (arXiv:0907.3084)

Gibbs Formula for Wilson Fermions

For Details → http://home.riise.hiroshima-u.ac.jp/~nakamura/canonical-v7.pdf

$$\begin{aligned}\Delta(x,x') &= \delta_{x,x'} \\ &- \kappa\sum_{i=1}^{3}\left\{(r-\gamma_i)U_i(x)\delta_{x',x+\hat{i}} + (r+\gamma_i)U_i^\dagger(x')\delta_{x',x-\hat{i}}\right\} \\ &- \kappa\left\{e^{+\mu}(r-\gamma_4)U_4(x)\delta_{x',x+\hat{4}} + e^{-\mu}(r+\gamma_4)U_4^\dagger(x')\delta_{x',x-\hat{4}}\right\} \\ &+ (Clover)\end{aligned}$$

$$\Delta = B - z^{-1}\kappa(r-\gamma_4)V - z\kappa(r+\gamma_4)V^\dagger$$

$$z \equiv e^{-\mu}$$

Gibbs Formula for Wilson Fermions

For Details → http://home.riise.hiroshima-u.ac.jp/~nakamura/canonical-v6.pdf

$$
\begin{aligned}
\det\Delta &= \det\frac{1}{z}\left(zB + z^2(-\kappa(r+\gamma_4)V^\dagger) + (-\kappa(r-\gamma_4)V)\right)\\
&= \det\frac{1}{z}\left(zBV + z^2(-\kappa(r+\gamma_4)) + (-\kappa(r-\gamma_4)V^2)\right)V^{-1}\\
&= z^{-N}\begin{vmatrix} -BV - z(-\kappa(r+\gamma_4)) & I \\ \kappa(r-\gamma_4)V^2 & -z \end{vmatrix} / \det V\\
&= z^{-N}\left|\begin{pmatrix} -BV & I \\ \kappa(r-\gamma_4)V^2 & 0 \end{pmatrix} - z\begin{pmatrix} -\kappa(r+\gamma_4) & 0 \\ 0 & I \end{pmatrix}\right|
\end{aligned}
$$

After long algebraic calculation

$$\boxed{\det \Delta = z^{-N} \det(T - zS)} \qquad z \equiv e^{-\mu}$$

$$T = \left(\begin{array}{c|c|cccc|c}
0 & t_1 & 0 & \cdots & & 0 \\ \hline
0 & 0 & t_2 & \cdots & & 0 \\ \hline
0 & 0 & 0 & \cdots & & \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
 & & & \cdots & t_{N_t-2} & 0 \\ \hline
0 & 0 & & \cdots & 0 & t_{N_t-1} \\ \hline
t_{N_t} & 0 & & \cdots & 0 & 0
\end{array}\right)$$

$$t_i = \begin{pmatrix} -B_i V_{i,i+1} & 1 \\ \kappa(r-\gamma_4) V_{i-1,i} V_{i,i+1} & 0 \end{pmatrix}$$

We used a formula:

$$\begin{vmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{vmatrix} = |A_{11}||A_{22} - A_{21} A_{11}^{-1} A_{12}|$$

$$\boxed{\det \Delta = z^{-N} \det(T - zS)}$$

$$S = \left(\begin{array}{c|c|cccc|c} s & 0 & 0 & \cdots & & & 0 \\ \hline 0 & s & 0 & \cdots & & & 0 \\ \hline 0 & 0 & 0 & \cdots & & & \\ \cdots & \cdots & \cdots & \cdots & \cdots & & \cdots \\ \hline 0 & 0 & & \cdots & s & & 0 \\ \hline 0 & 0 & & \cdots & 0 & & s \end{array}\right)$$

$$s = \begin{pmatrix} -\kappa(r+\gamma_4) & 0 \\ 0 & I \end{pmatrix}$$

s does not have an inverse.

$$\det \Delta = z^{-N} \det(T - zS)$$

$$T\vec{X} = zS\vec{X}$$

Generalized Eigen Value Problem

Generalized Schur Decomposition

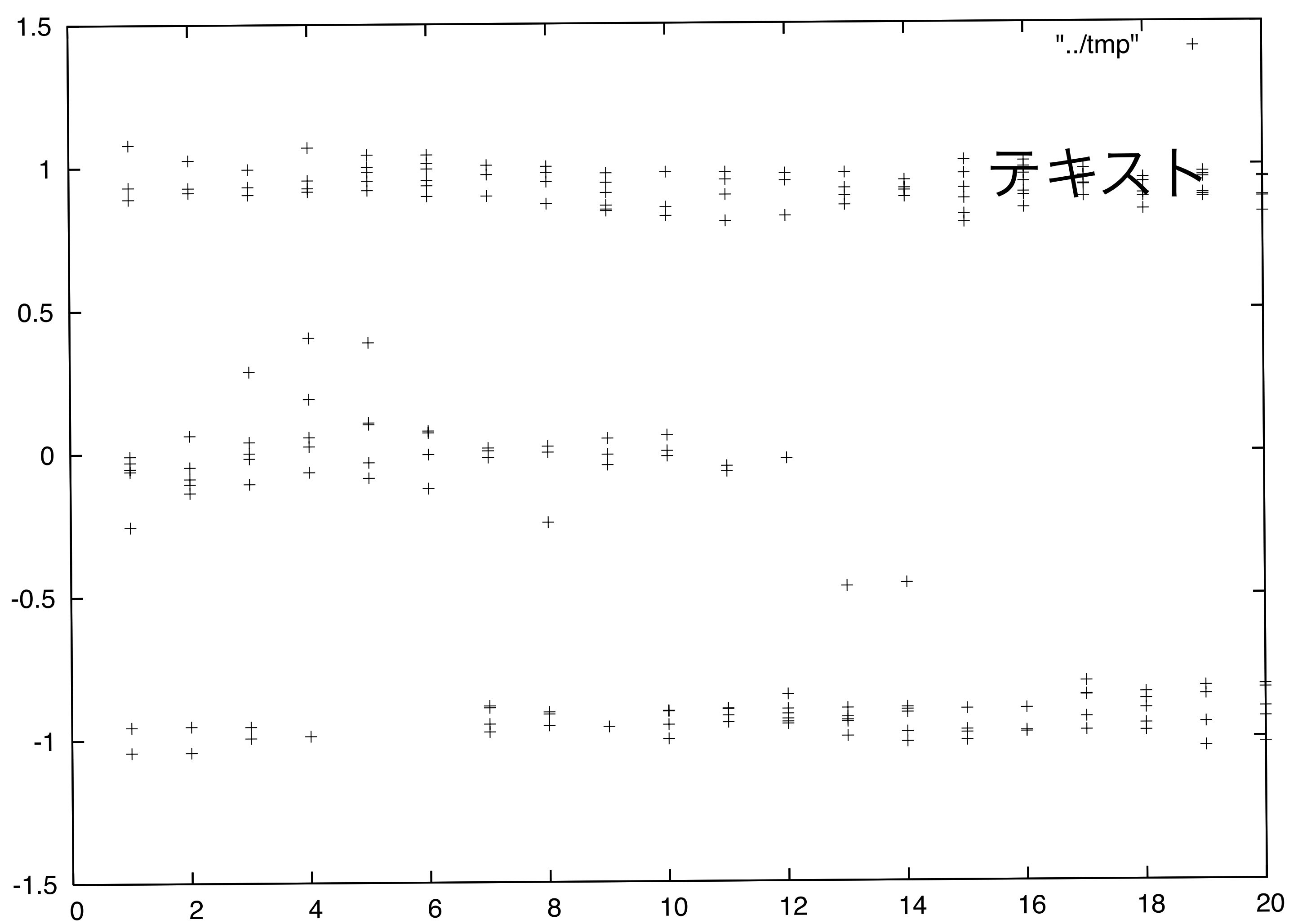
There exist unitary Q and Z such that
$Q^\dagger T Z$ and $Q^\dagger S Z$ are upper triangular.

$$
\begin{aligned}
&\det(T - zS) \\
&= \det QZ^\dagger \\
&\times \left| \begin{pmatrix} \alpha_1 & * & * & \cdots & & * \\ 0 & \alpha_2 & * & \cdots & & * \\ 0 & 0 & & \cdots & & \\ \cdots & \cdots & \cdots & \cdots & * & * \\ 0 & 0 & & \cdots & \alpha_{N-1} & * \\ 0 & 0 & & \cdots & 0 & \alpha_N \end{pmatrix} - z \begin{pmatrix} \beta_1 & * & * & \cdots & & * \\ 0 & \beta_2 & * & \cdots & & * \\ 0 & 0 & & \cdots & & \\ \cdots & \cdots & \cdots & \cdots & * & * \\ 0 & 0 & & \cdots & \beta_{N-1} & * \\ 0 & 0 & & \cdots & 0 & \beta_N \end{pmatrix} \right|
\end{aligned}
$$

$$\det \Delta(\mu) = z^{-N} \det QZ^{\dagger} \det (T - zS)$$

$$= z^{-N} \det QZ^{\dagger} \prod (\alpha_i - z\beta_i)$$

$$z \equiv e^{-\mu}$$



$$\prod(\alpha_i - z\beta_i) = C_0 + C_1 z + C_2 z^2 + \cdots$$

Direct Calculation never ends.

Didn’t you study Divide-Conquer Algorithm ?

$${}_N C_M$$

$$N = O(\text{Lattice Volume})$$

$$\mathrm{Prod}(N, M) = \alpha_1 \alpha_2 \cdots \beta_{i_1} \cdots \beta_{i_2} \cdots \beta_{i_M} \cdots \alpha_N$$

Recursive Function Prod(N,M)

$$\sum_{M1+M2=M} \mathrm{Prod}(N/2, M1)$$

$$\times \mathrm{Prod}(N/2, M2)$$

Matrix Reduction

$$\det(T - zS) =$$

$$\left|\begin{array}{c|c|cccc|c}
-zs & t_1 & 0 & \cdots & & & 0 \\ \hline
0 & -zs & t_2 & \cdots & & & 0 \\ \hline
0 & 0 & -zs & \cdots & & & \\
\cdots & \cdots & \cdots & \cdots & \cdots & & \cdots \\
 & & & \cdots & t_{N_t-2} & & 0 \\ \hline
0 & 0 & & \cdots & -zs & & t_{N_t-1} \\ \hline
t_{N_t} & 0 & & \cdots & 0 & & -zs
\end{array}\right|,$$

We multibply a matrix Q_1 from the right

$$Q_1 = \left(\begin{array}{c|c|cc|c} I & 0 & & \cdots & t_{N_t}^{-1} zs \\ \hline 0 & I & 0 & \cdots & 0 \\ \hline 0 & 0 & I & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ & & & \cdots & 0 \\ \hline 0 & 0 & \cdots & & I \end{array}\right)$$

$$\det Q_1 = 1$$

$$\det(T - zS) \quad = \quad \det(T - zS$$

$$= \left|\begin{array}{c|c|cccc|c}
t_{N_t} & 0 & & \cdots & 0 & & 0 \\
\hline
-zs & t_1 & 0 & \cdots & & & -zs\, t_{N_t}^{-1}\, zs \\
\hline
0 & -zs & t_2 & \cdots & & & 0 \\
\hline
0 & 0 & -zs & \cdots & & & \\
\cdots & \cdots & \cdots & \cdots & \cdots & & \cdots \\
 & & & \cdots & t_{N_t-2} & & 0 \\
\hline
0 & 0 & & \cdots & -zs & & t_{N_t-1}
\end{array}\right|$$

$$\det(T - zS) \quad = \quad \det(T - zS)Q_1$$

=

$$\left|\begin{array}{c|c|cccc|c} t_{N_t} & 0 & & \cdots & 0 & -zs \\ \hline -zs & t_1 & 0 & \cdots & & 0 \\ \hline 0 & -zs & t_2 & \cdots & & 0 \\ \hline 0 & 0 & -zs & \cdots & & \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ & & & \cdots & t_{N_t-2} & 0 \\ \hline 0 & 0 & & \cdots & -zs & t_{N_t-1} \end{array}\right| ,$$

=

$$
|t_{N_t}| \times \left| \begin{array}{c|ccc|c}
t_1 & 0 & \cdots & & -zs\, t_{N_t}^{-1}\, zs \\
\hline
-zs & t_2 & \cdots & & 0 \\
\hline
0 & -zs & \cdots & & \\
\cdots & \cdots & \cdots & \cdots & \cdots \\
 & & \cdots & t_{N_t-2} & 0 \\
\hline
0 & & \cdots & -zs & t_{N_t-1}
\end{array} \right|
$$

$$\det(T - zS) =$$

$$
\begin{aligned}
& |t_{N_t}| \times |\, t_1| \times \cdots |t_{N_t-2}| \\
\times \quad & |t_{N_t-1} - zs\, t_{N_t-2}^{-1}\, zs\, t_{N_t-3}^{-1}\, zs \cdots t_1\, zs\, t_{N_t}^{-1}\, zs| \\
= \quad & |t_{N_t}| \times |\, t_1| \times \cdots |t_{N_t-2}| \times |\, t_{N_t-1}| \\
\times \quad & |I - t_{N_t-1}^{-1}\, zs\, t_{N_t-2}^{-1}\, zs\, t_{N_t-3}^{-1}\, zs \cdots t_1\, zs\, t_{N_t}^{-1}\, zs \\
= \quad & |P| \times |\, I - z^{N_t} t_{N_t-1}^{-1}\, s\, t_{N_t-2}^{-1}\, s\, t_{N_t-3}^{-1}\, s \cdots t_1\, s\, t
\end{aligned}
$$

$$P \equiv t_1 t_2 \cdots t_{N_t}$$

$$\frac{\det \Delta(\mu)}{\det \Delta(\mu=0)} = z^{-N} \frac{\det\left(I - z^{N_t} Q\right)}{\det\left(I - Q\right)}$$

$$Q \equiv t_{N_t-1}^{-1}\, s\, t_{N_t-2}^{-1}\, s\, t_{N_t-3}^{-1}\, s \cdots t_1\, s\, t_{N_t}^{-1}$$

If we diagonalize Q

$$Q = \begin{pmatrix} q_1 & 0 & \cdots & & 0 \\ 0 & q_2 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & q_{L-1} & 0 \\ 0 & 0 & \cdots & 0 & q_L \end{pmatrix},$$

$$\frac{\det \Delta(\mu)}{\det \Delta(\mu = 0)} = z^{-N} \frac{\prod_{l=1}^{L}(1 - z^{N_t} q_l)}{\prod_{l=1}^{L}(1 - q_l)}$$

Alternative (version 2)

Take the representation such that γ_4 is diagonal:

$$\gamma_4 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

Then we can rewrite S as

$$S = \begin{pmatrix} S_{11} & 0 \\ 0 & 0 \end{pmatrix}$$

Using this base, we write

$$T = \begin{pmatrix} T_{11} & T_{12} \\ T_{22} & T_{22} \end{pmatrix} \qquad \vec{X} = \begin{pmatrix} \vec{X}_1 \\ \vec{X}_2 \end{pmatrix}$$

Then

$$\begin{pmatrix} T_{11} & T_{12} \\ T_{22} & T_{22} \end{pmatrix} \begin{pmatrix} \vec{X}_1 \\ \vec{X}_2 \end{pmatrix} = z \begin{pmatrix} S_{11} & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \vec{X}_1 \\ \vec{X}_2 \end{pmatrix}$$

Or

$$\begin{aligned} T_{11}\vec{X}_1 + T_{12}\vec{X}_2 &= zS_{11}\vec{X}_1 \\ T_{21}\vec{X}_1 + T_{22}\vec{X}_2 &= 0 \end{aligned}$$

Or

$$W\vec{X}_1 = z\vec{X}_1$$

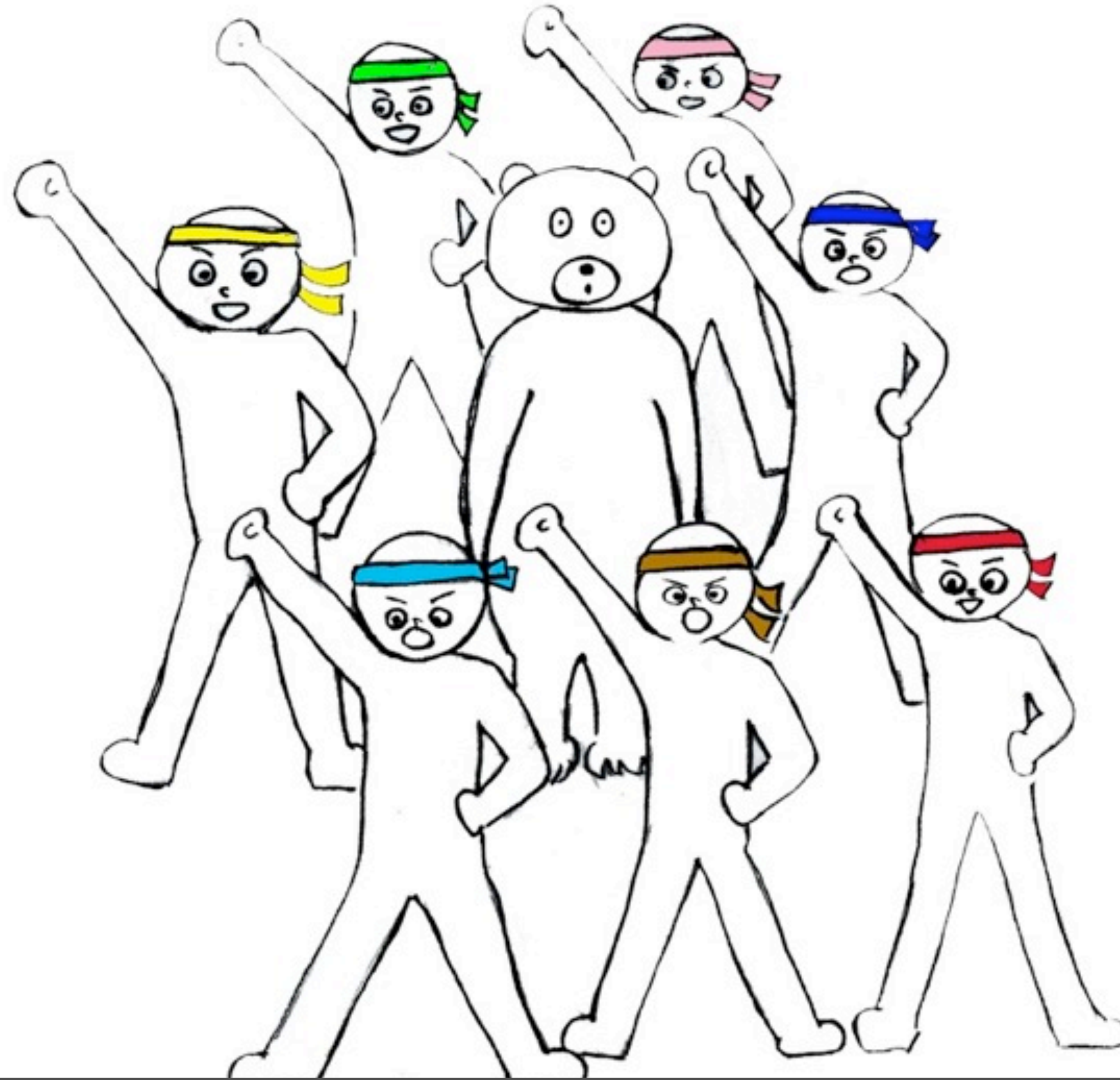
where

$$W \equiv S_{11}^{-1}(T_{11} - T_{12}T_{22}^{-1}T_{21})$$

Relation to other atempts ?

- A. Borici
  - Prog.Theor.Phys.Suppl.153:335-339,2004
  - Proceedings of Nara Workshop
- C. Gattringer, L. Liptak
  - arXiv:0906.1088

It is great to have Good Friends in the Lattice QCD at finite T and finite Density.



Let me continue to run !