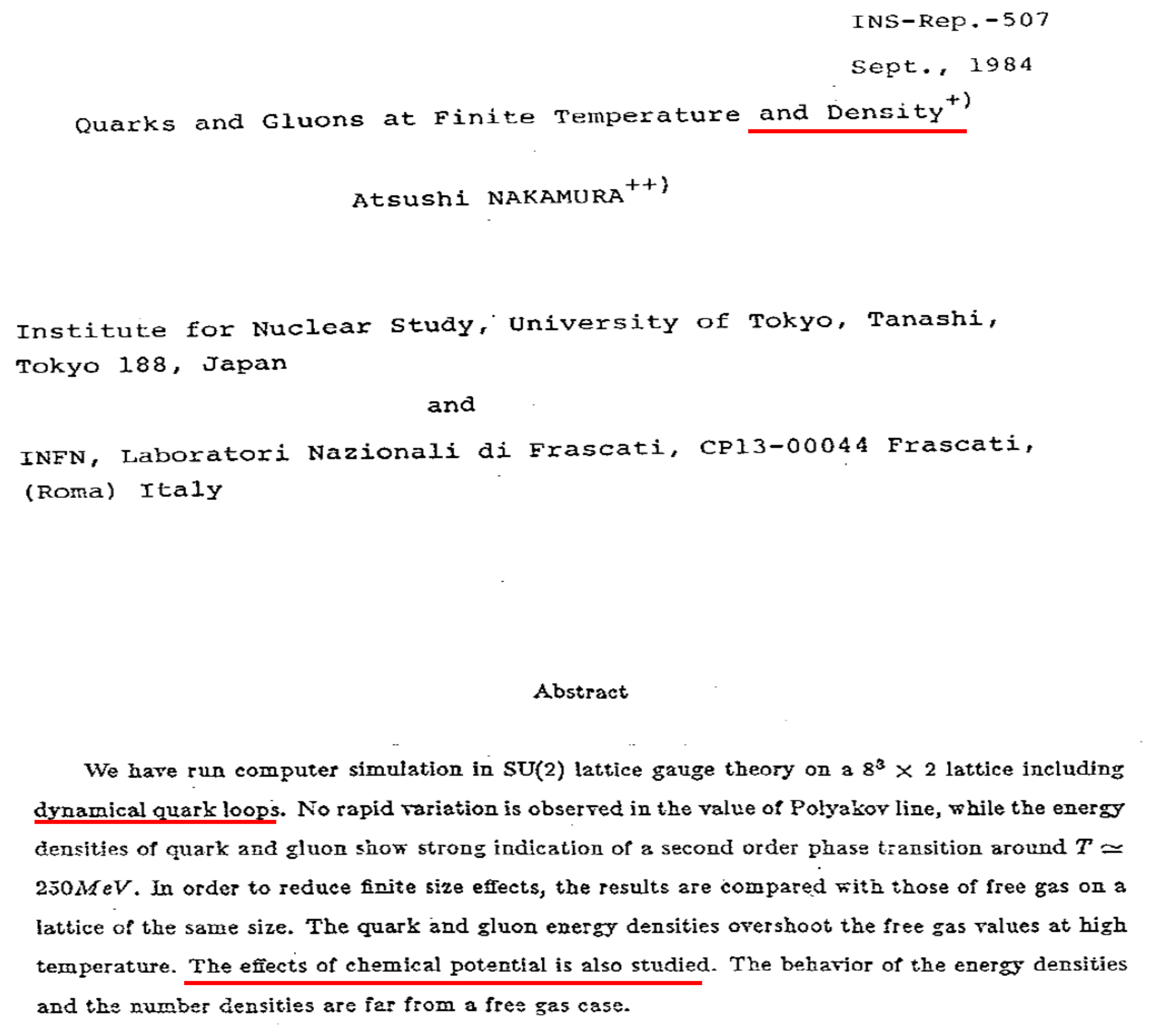


INS-Rep.-507

Sept., 1984

Quarks and Gluons at Finite Temperature and Density+)

Atsushi NAKAMURA++)

Institute for Nuclear Study, University of Tokyo, Tanashi,
Tokyo 188, Japan

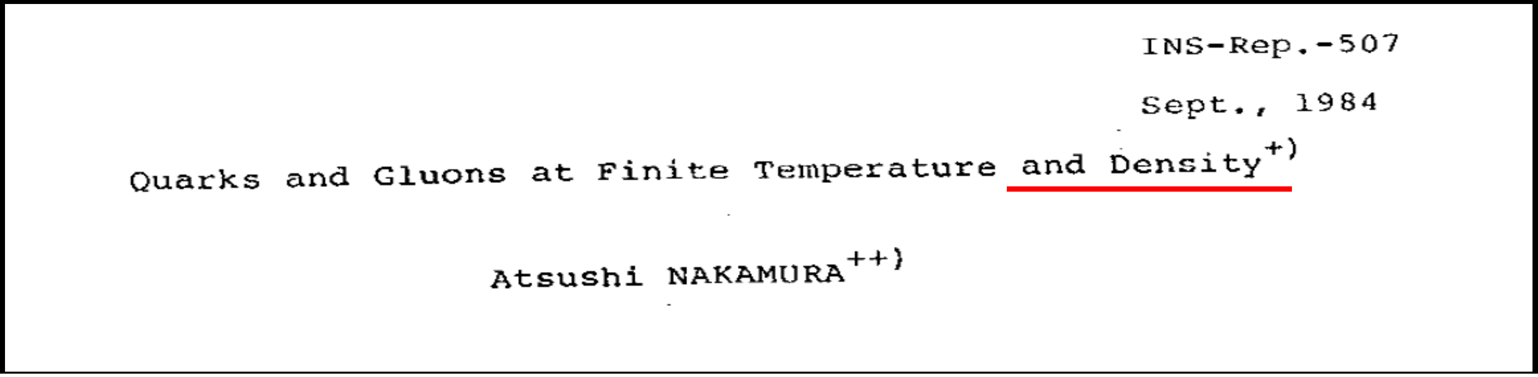
and

INFN, Laboratori Nazionali di Frascati, CP13-00044 Frascati,
(Roma) Italy

Abstract

We have run computer simulation in SU(2) lattice gauge theory on a $8^3 \times 2$ lattice including dynamical quark loops. No rapid variation is observed in the value of Polyakov line, while the energy densities of quark and gluon show strong indication of a second order phase transition around $T \simeq 250 MeV$. In order to reduce finite size effects, the results are compared with those of free gas on a lattice of the same size. The quark and gluon energy densities overshoot the free gas values at high temperature. The effects of chemical potential is also studied. The behavior of the energy densities and the number densities are far from a free gas case.

A. Nakamura, Phys.Lett.B149, 391 (1984).



INS-Rep.-507

Sept., 1984

Quarks and Gluons at Finite Temperature and Density+)

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A. Nakamura, Phys.Lett.B149, 391 (1984).

Fig.0a

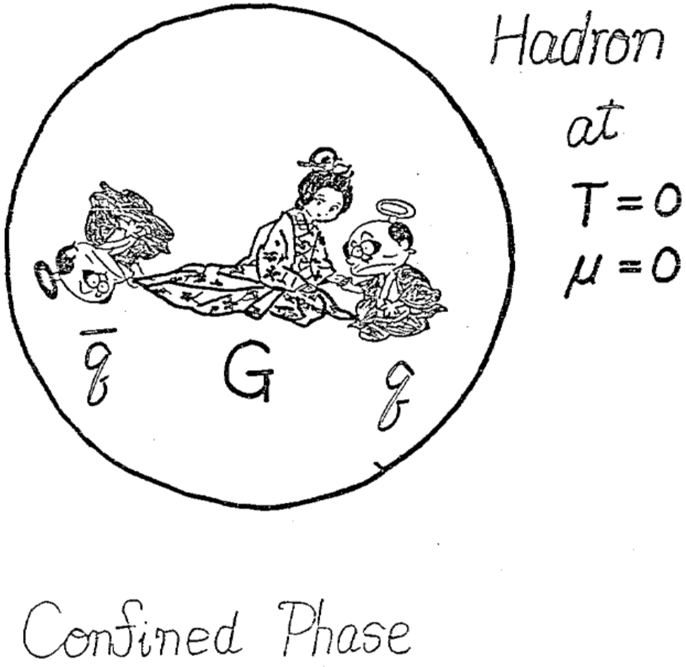
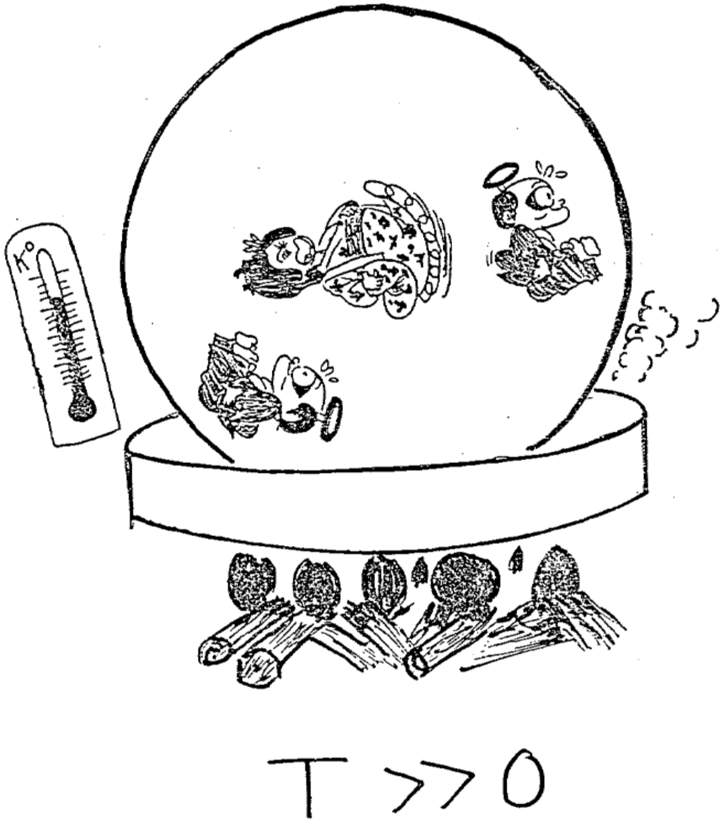
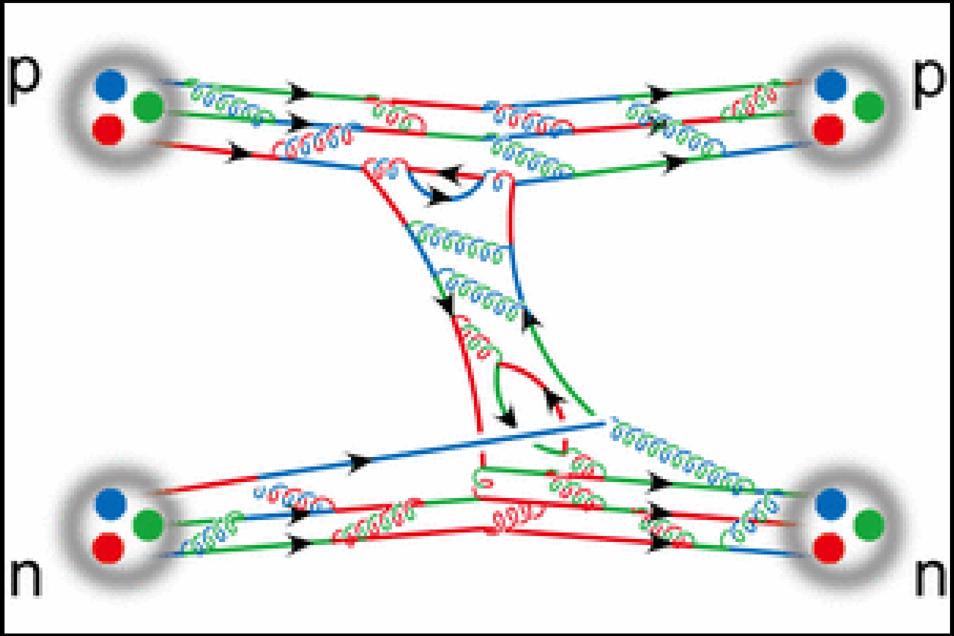


Fig.0b



Nuclear Force from Lattice QCD



[1] Why nuclear force ?

[2] NN force from lattice QCD

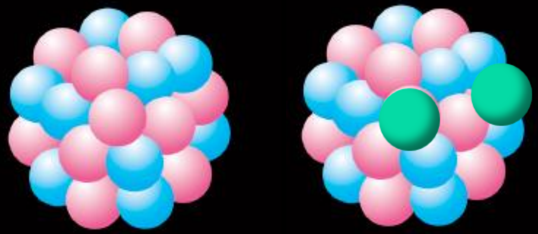
[3] BB and BM forces from lattice QCD

[4] Summary and Future

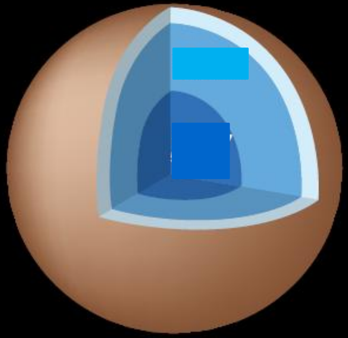
Tetsuo Hatsuda (Univ. Tokyo)
Nakamura-fest, March 13, 2010

The nuclear force is a basis for understanding …

- Structure of ordinary and hyper nuclei

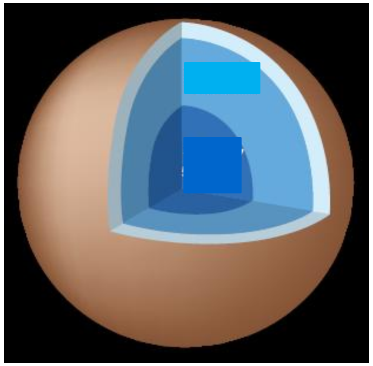


- Structure of neutron stars

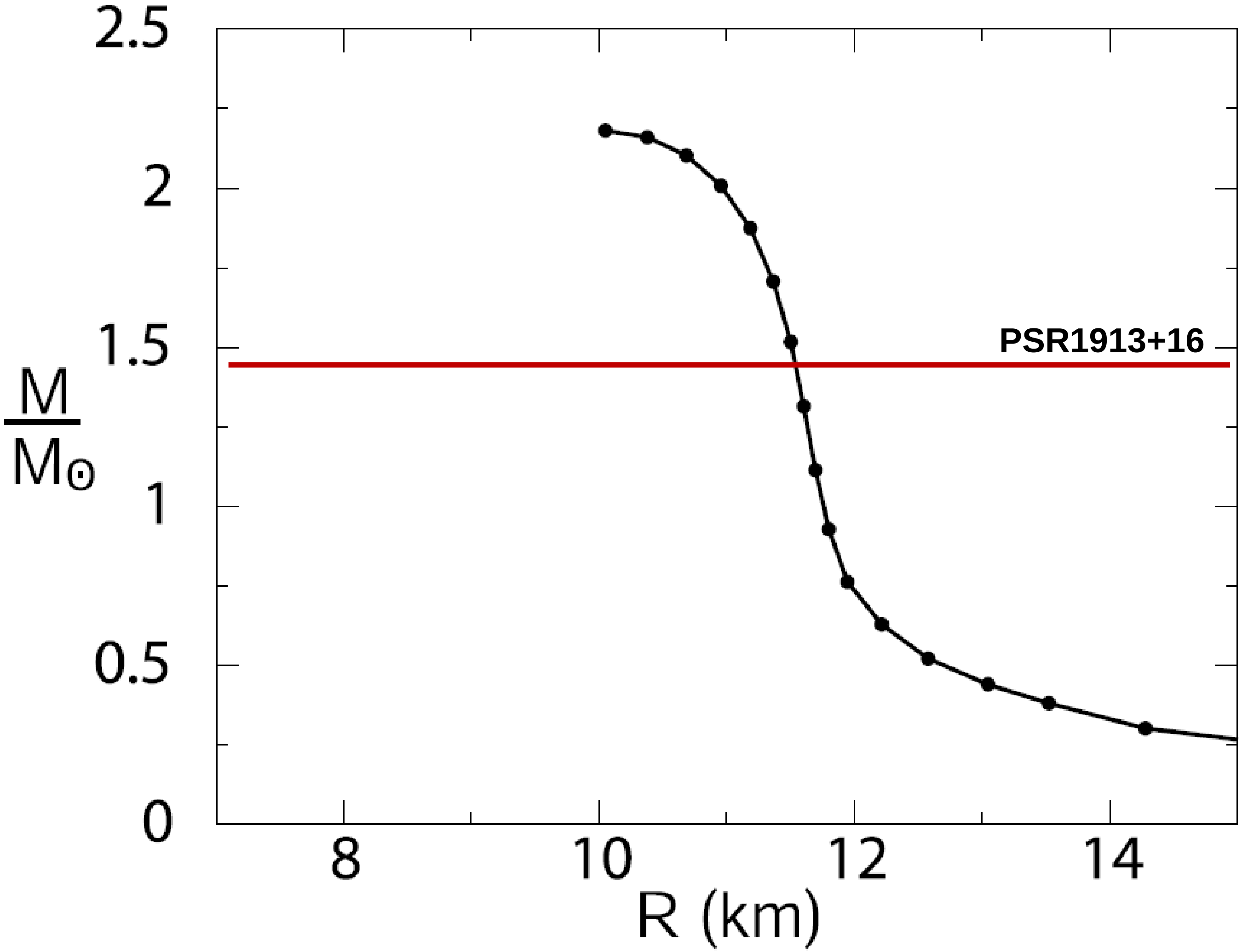


- Ignition of Type II supernovae

Observed neutrons star cannot exist without short range repulsion

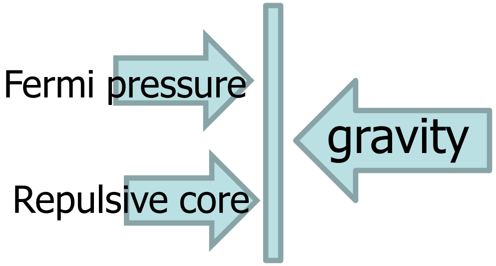


$(\rho_{max} \sim 6\rho_0)$



Neutron star binary

Pressure balance



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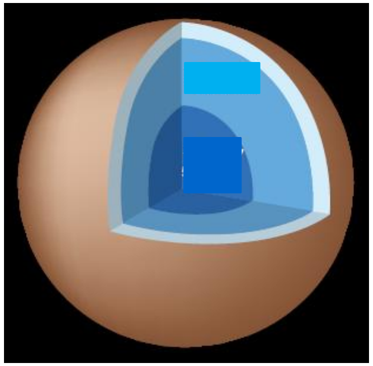

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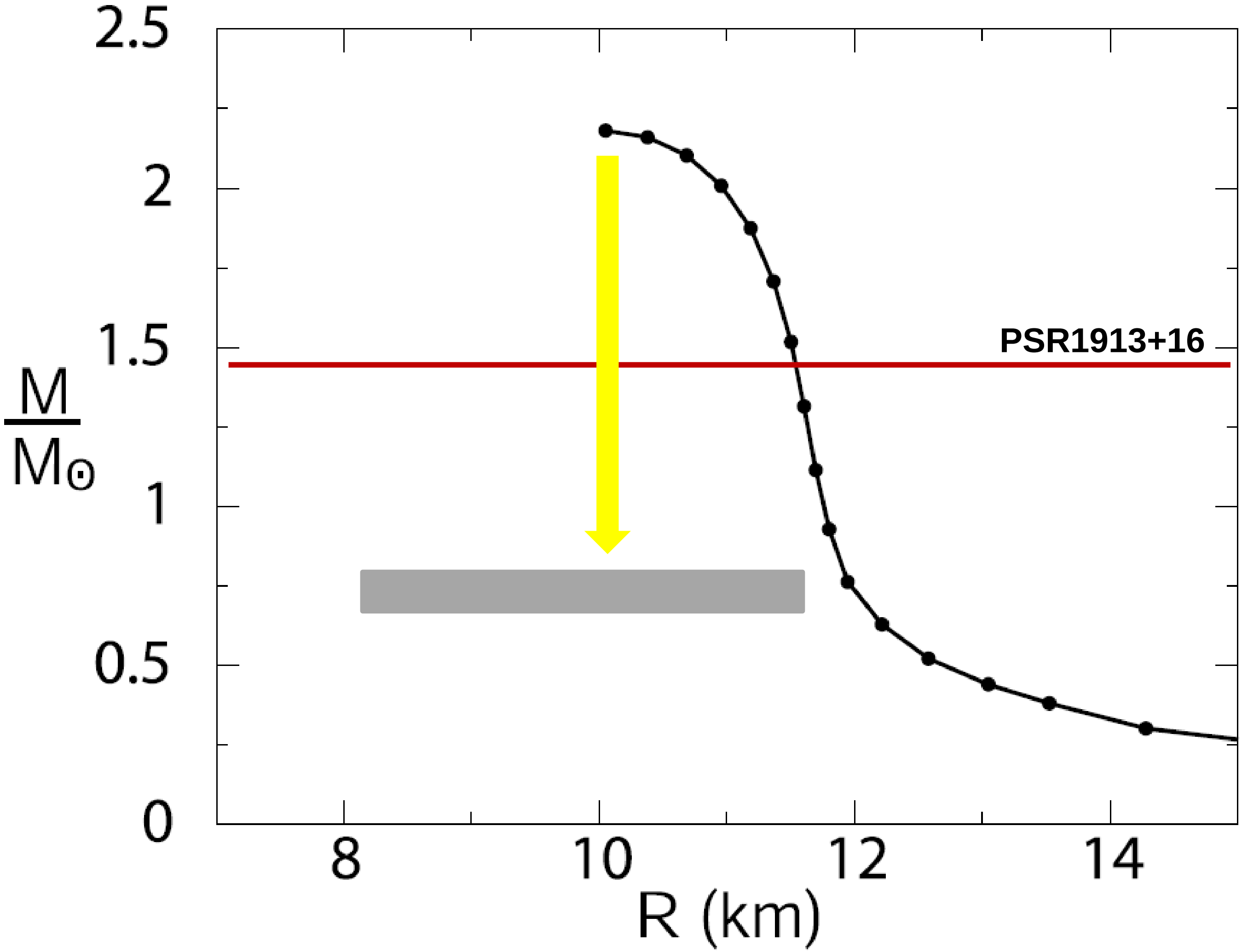
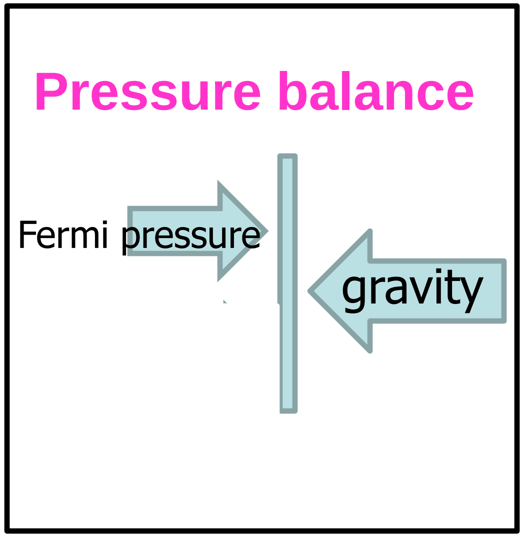
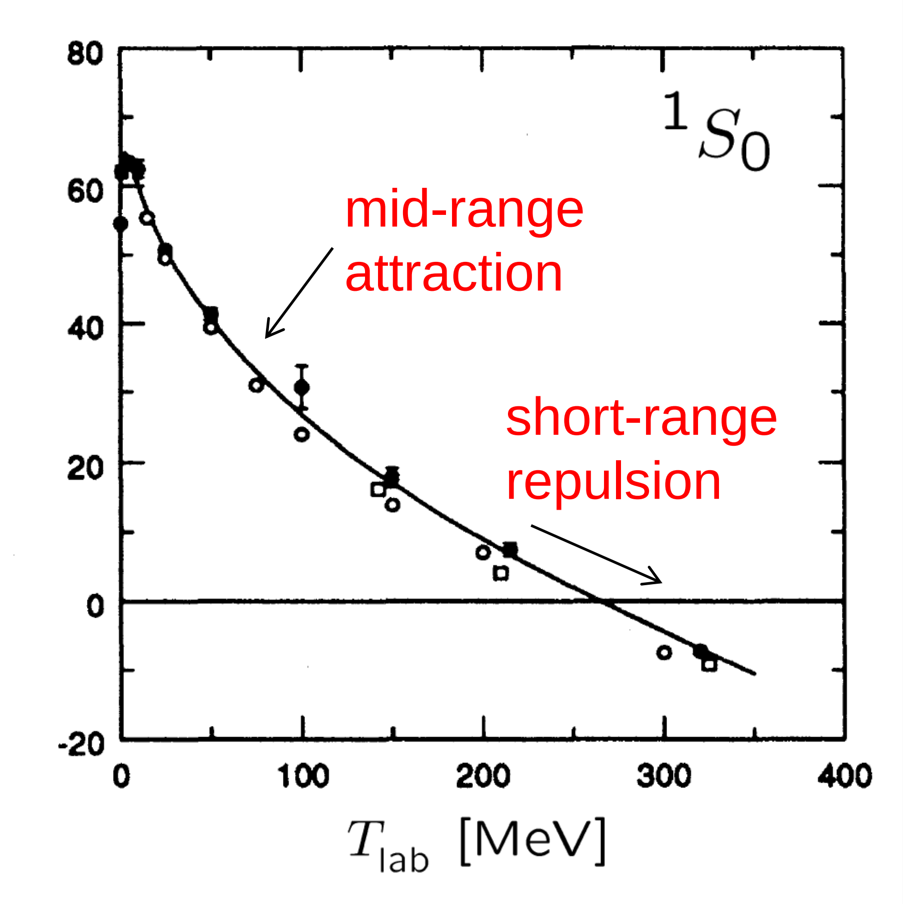


Neutron star binary



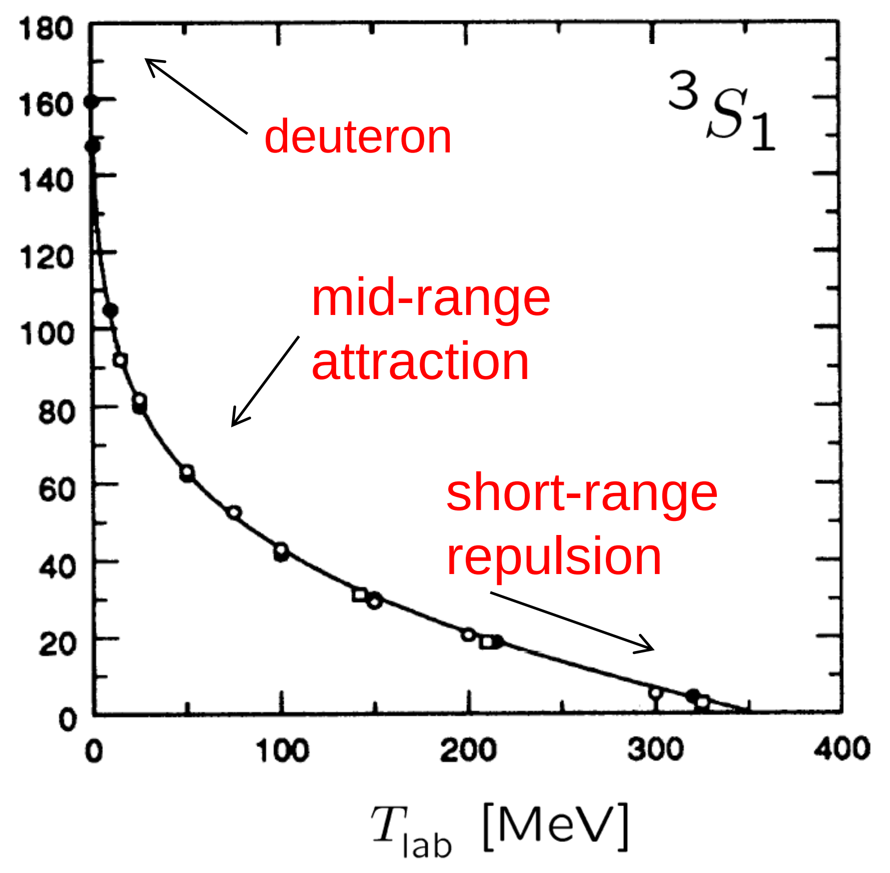
Pressure balance

NN phase shifts



mid-range attraction

short-range repulsion

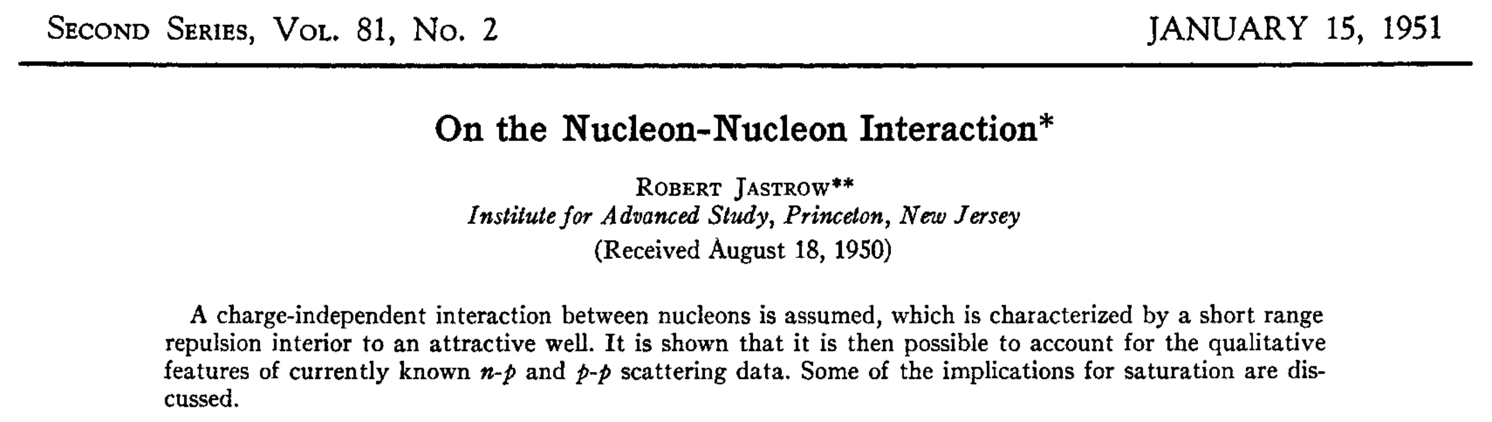


deuteron

mid-range attraction

short-range repulsion

Nijmegen partial-wave analysis,
Stoks et al., Phys.Rev. C48 (1993) 792



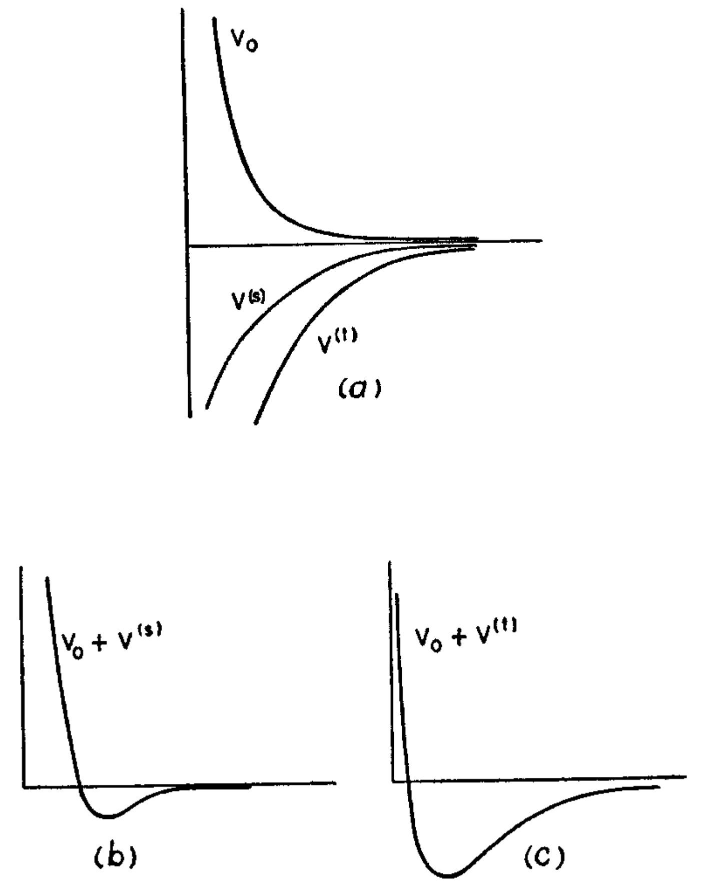
SECOND SERIES, VOL. 81, NO. 2 JANUARY 15, 1951

On the Nucleon-Nucleon Interaction*

ROBERT JASTROW**
Institute for Advanced Study, Princeton, New Jersey

(Received August 18, 1950)

A charge-independent interaction between nucleons is assumed, which is characterized by a short range repulsion interior to an attractive well. It is shown that it is then possible to account for the qualitative features of currently known *n-p* and *p-p* scattering data. Some of the implications for saturation are discussed.



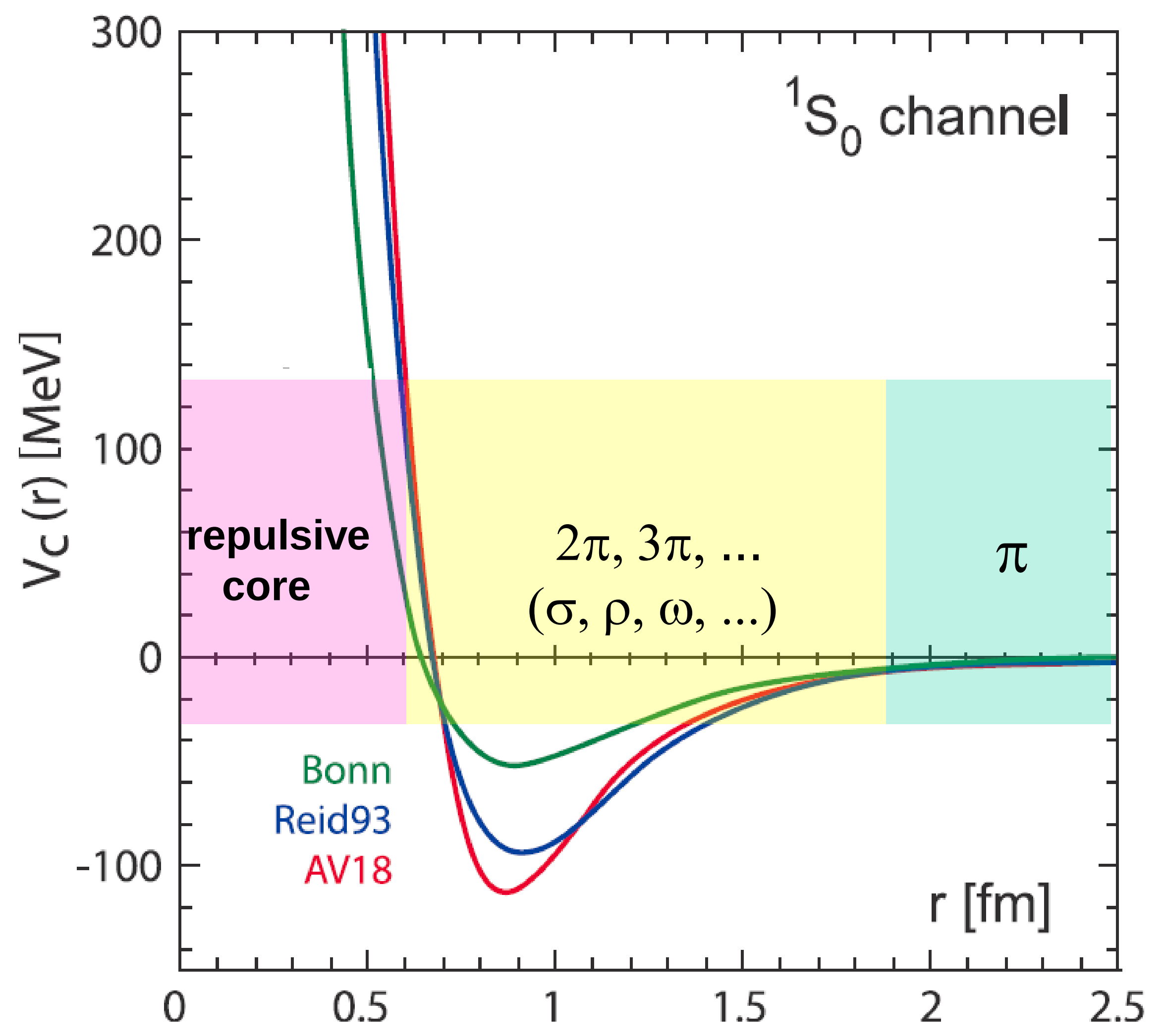
So I got up in the question period and I said, "Maybe the reason is that inside the nuclear force of attraction, which holds nuclei together, there's a very strong short-range force of repulsion, like a little hard sphere inside this attractive Jell-O."

I'll never forget, Oppenheimer got up, he liked to needle the young fellows and he said, very dryly, "Thank you so much for, we are grateful for every tiny scrap of help we can get." But I ignored his needle and pursued my idea, and actually calculated the scattering of neutrons by protons. I showed that it fit the data very well. Oppenheimer read my paper for the Physical Review and took back his criticisms. This work became a permanent element of the literature of physics.

http://www.marshall.org/article.php?id=30

Phenomenological NN potentials

(~40 parameters to fit 5000 phase shift data)



One-pion exchange

by Yukawa (1935)

Multi-pions

by Taketani et al. (1951)

Repulsive core

by Jastrow (1951)

High precision phenomenological NN potentials

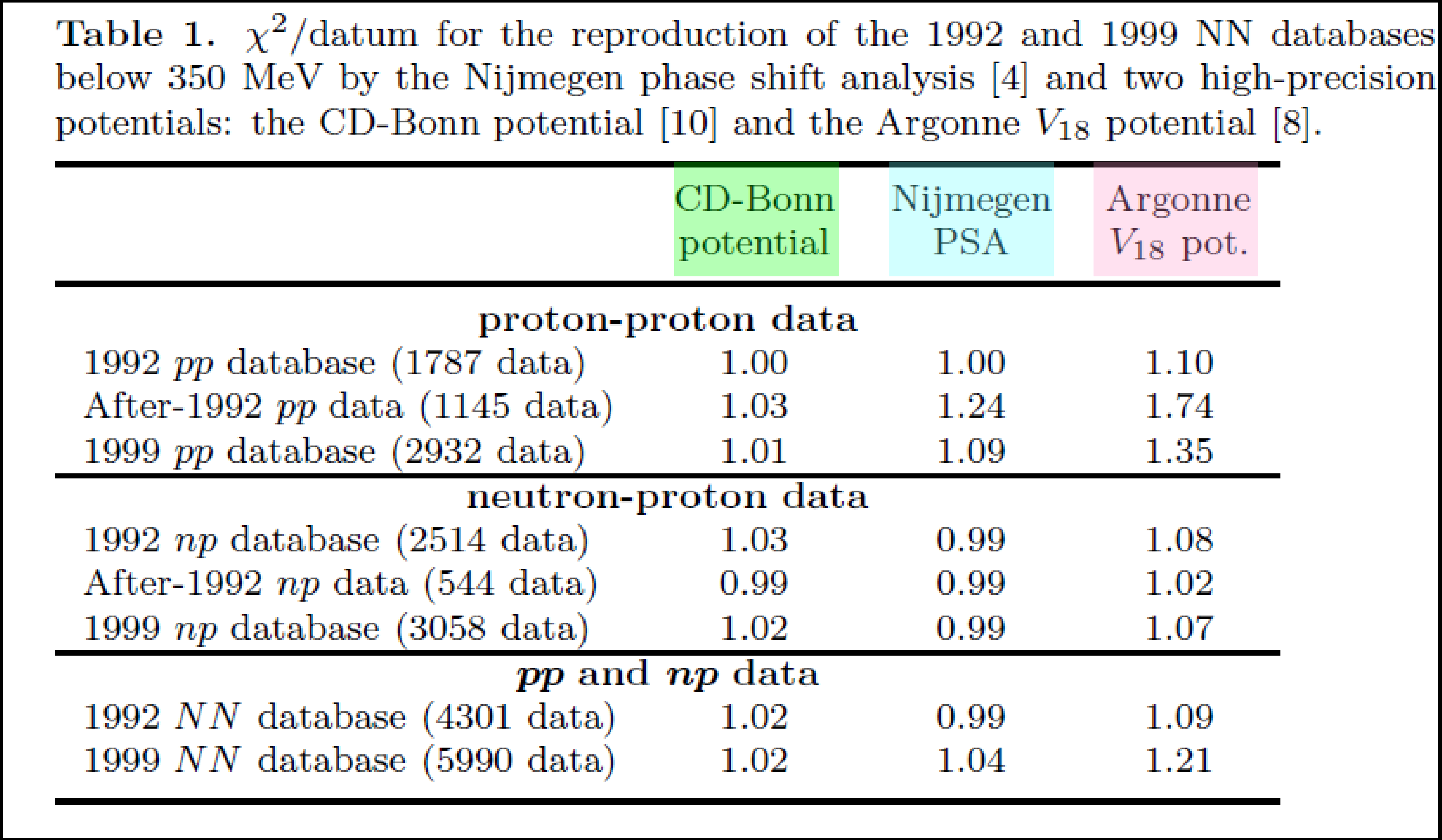
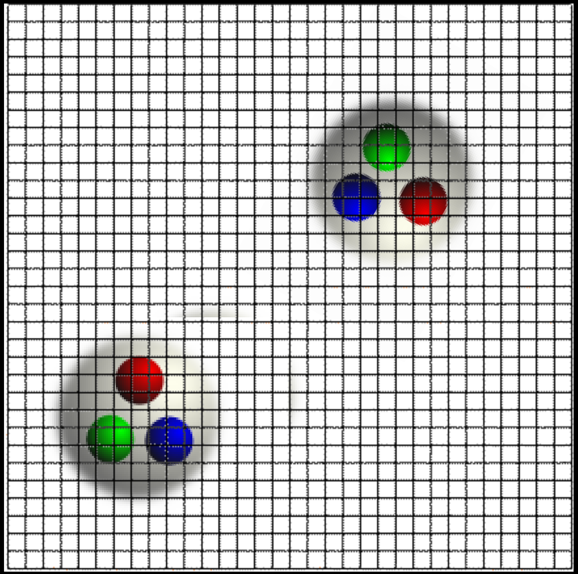


Table 1. χ^2/datum for the reproduction of the 1992 and 1999 NN databases below 350 MeV by the Nijmegen phase shift analysis [4] and two high-precision potentials: the CD-Bonn potential [10] and the Argonne V_{18} potential [8].

	CD-Bonn potential	Nijmegen PSA	Argonne V_{18} pot.
proton-proton data			
1992 *pp* database (1787 data)	1.00	1.00	1.10
After-1992 *pp* data (1145 data)	1.03	1.24	1.74
1999 *pp* database (2932 data)	1.01	1.09	1.35
neutron-proton data			
1992 *np* database (2514 data)	1.03	0.99	1.08
After-1992 *np* data (544 data)	0.99	0.99	1.02
1999 *np* database (3058 data)	1.02	0.99	1.07
***pp* and *np* data**			
1992 NN database (4301 data)	1.02	0.99	1.09
1999 NN database (5990 data)	1.02	1.04	1.21

Machleidt and Entem, nucl-th/0503025

Nambu-Bethe-Salpeter Amplitude and Nuclear Forces in Lattice QCD

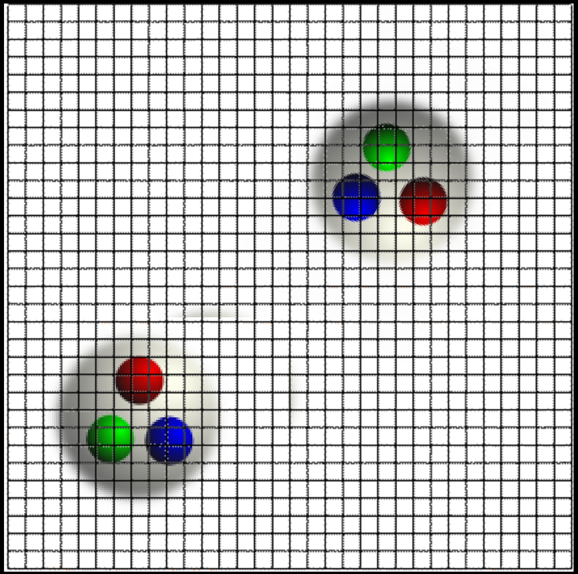


How to extract
the NN force at low energies
in lattice QCD ?

Y. Nambu,
“Force Potentials in Quantum Field Theory”,
Prog. Theor. Phys. 5 (1950) 614.

K. Nishijima,
“Formulation of Field Theories for Composite Particles”, Phys. Rev. 111 (1958) 995.

Nambu-Bethe-Salpeter Amplitude and Nuclear Forces in Lattice QCD



How to extract
the NN force at low energies
in lattice QCD ?

↕

T. Hatsuda, Y. Ikeda, N. Ishii (Tokyo)

S. Aoki, T. Doi, T. Inoue,
K. Murano, K. Sasaki (Tsukuba)

H. Nemura (Tohoku)

Hadrons to Atomic nuclei
HAL
from Lattice QCD

HAL QCD (current lattice setup)

- Exploratory studies with quenched QCD

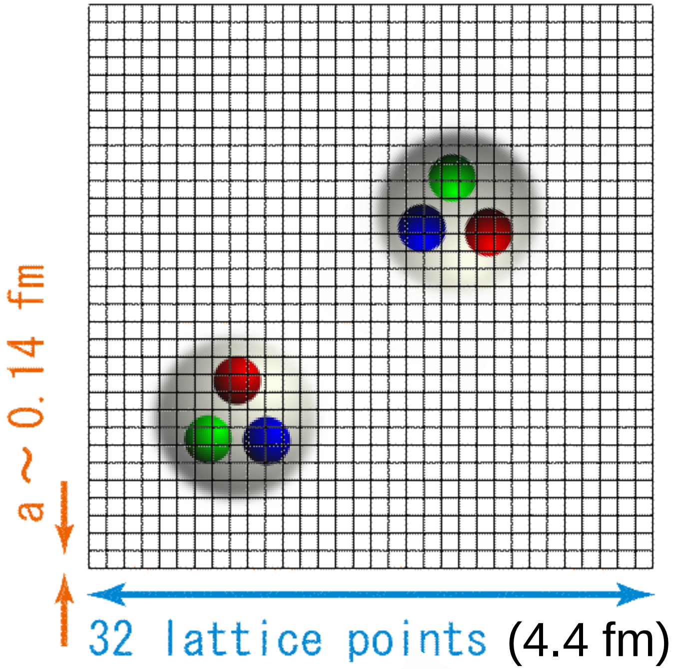
plaquette gauge action + Wilson quark

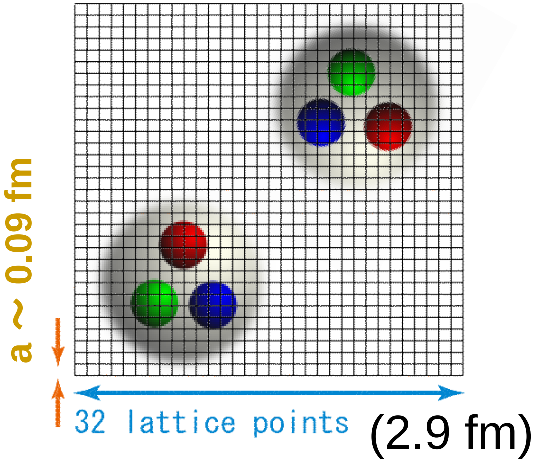
m_π = 380 − 730 MeV
a = 0.137 fm, L = 4.4 fm

- Toward real world with 2+1 flavor QCD
(using PACS-CS configurations)

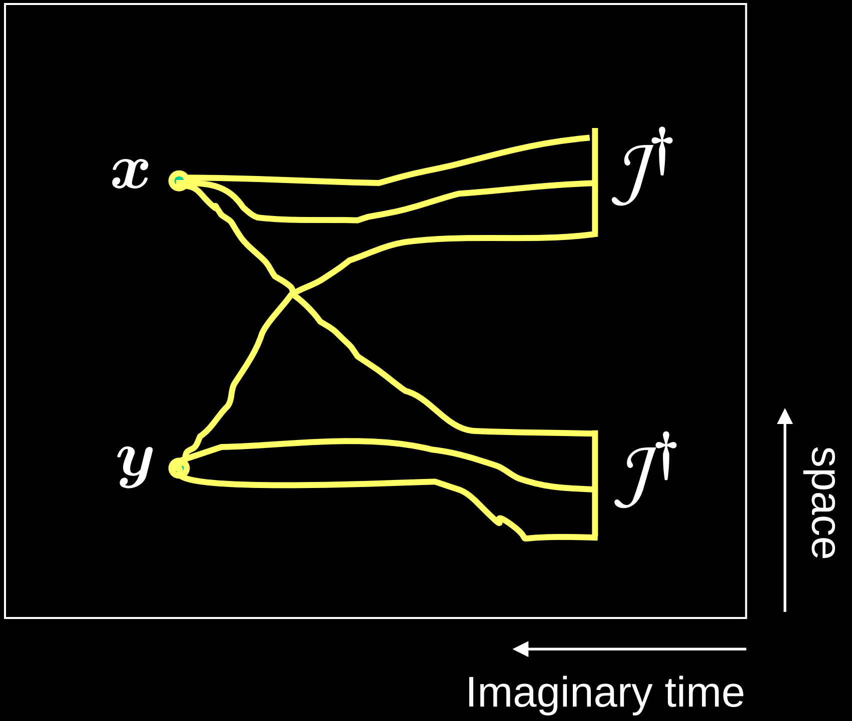
Iwasaki gauge action + clover quark

m_π = 411 – 700 MeV
a = 0.091 fm, L = 2.9 fm





Equal-time NBS amplitude **φ (r)** in lattice QCD



+ all possible combinations

$$
\begin{aligned}
C_4(\mathbf{r};t) &= \langle N_1(\mathbf{x},t) N_2(\mathbf{y},t) \mathcal{J}_1^\dagger(0) \mathcal{J}_2^\dagger(0) \rangle \\
&= \sum_n \langle 0 | N_1(\mathbf{x}) N_2(\mathbf{y}) | n \rangle A_n \, e^{-E_n t} \longrightarrow \phi(\mathbf{r}) A_0 e^{-E_0 t}
\end{aligned}
$$

φ (r > R) → phase shift : Luscher, Nucl. Phys. B354 (1991) 531

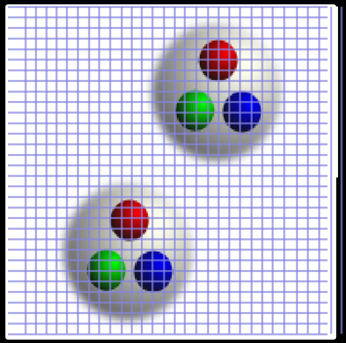
φ (r < R) → potential : Ishii, Aoki & Hatsuda, PRL 99 (2007) 022001

Central NN potentials from lattice QCD

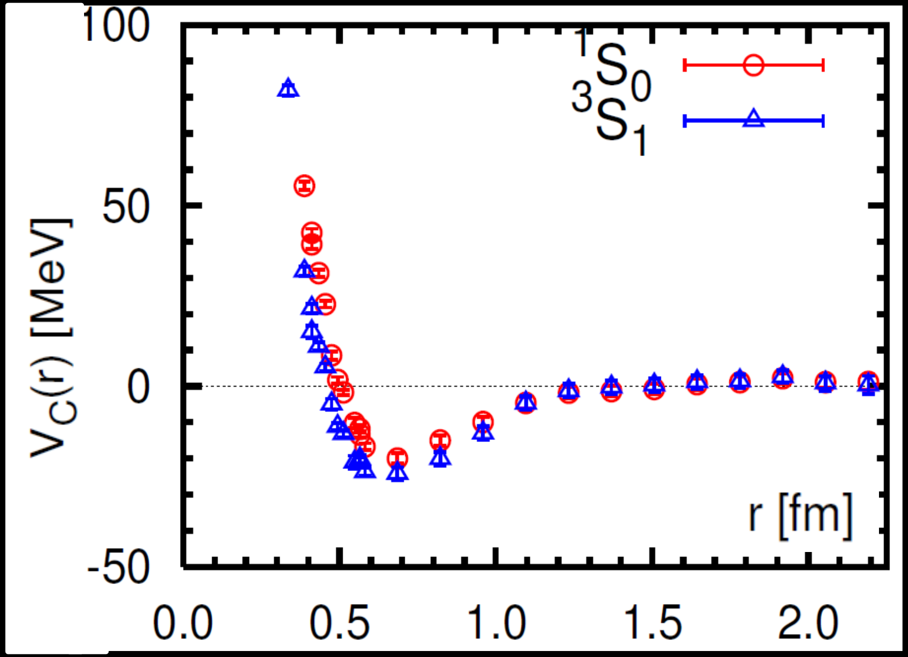
wave function

↓

NN potential

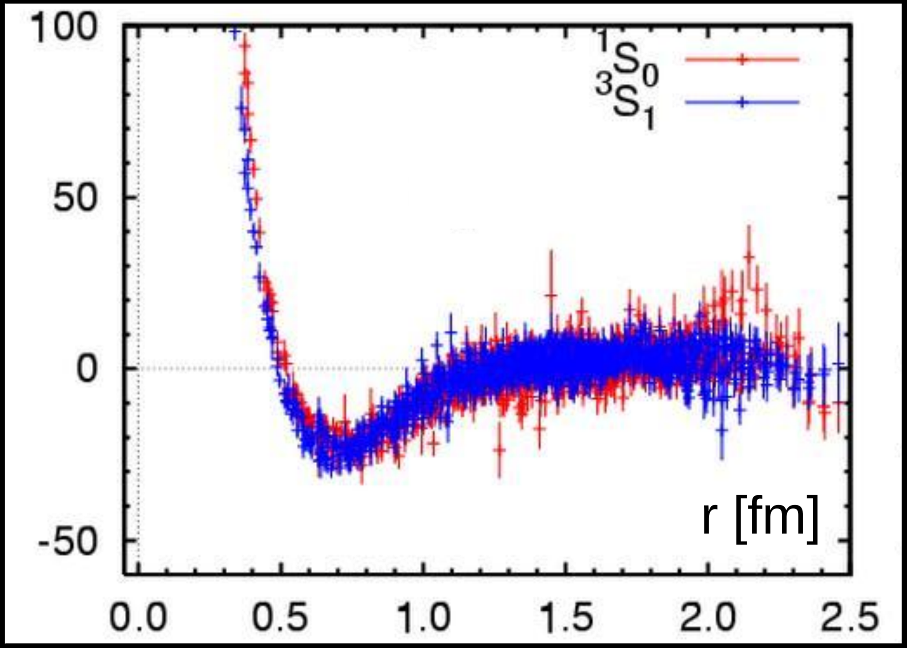


Quenched QCD
(m_π=530MeV, L=4.4 fm)



Ishii, Aoki & Hatsuda,
PRL 99 (2007) 022001

Full QCD
(m_π=570MeV, L=2.9 fm)



Ishii, Aoki & Hatsuda,
arXive 0903.5497 [hep-lat]

Systematic procedure to define the NN potential in lattice QCD

Full details: Aoki, Hatsuda & Ishii,
0909.5585 [hep-lat], PTP 123 (2010) 89-128

(i) Choose your favorite operator: e.g. $N(x) = \epsilon_{abc} q^a(x) q^b(x) q^c(x)$

- observables do not depend on the choice
- yet the local operator is useful

Nishijima, Haag, Zimmermann (1958)

(ii) Measure the NBS amplitude: $\phi(\vec{r}) = \langle 0 | N(\vec{x}+\vec{r}) N(\vec{x}) | 6q \rangle$

(iii) Define the non-local potential: $(E - H_0)\phi(\vec{r}) = \int U(r, \vec{r}')\phi(\vec{r}')d^3r'$

(iv) Velocity expansion : $U(\vec{r}, \vec{r'}) = V(\vec{r}, \nabla)\delta^3(\vec{r} - \vec{r}')$

$$\boxed{V(\vec{r}, \nabla) = V_{\rm C}(r) + S_{12} V_{\rm T}(r) + \vec{\rm L}\cdot\vec{\rm S}\ V_{\rm LS}(r) + \{V_{\rm D}(r), \nabla^2\} + \cdots}$$

LO LO NLO NNLO

Okubo-Marshak (1958), Tamagaki-Watari (1967)

(v) Calculate observables : phase shifts, binding energies etc

Key channels in NN scattering ($^{2s+1}L_J$)

$$V(\vec{r},\nabla) = V_{\rm C}(r) + S_{12}V_{\rm T}(r) + \vec{\rm L}\cdot\vec{\rm S}\, V_{\rm LS}(r) + \{V_{\rm D}(r), \nabla^2\} + \cdots$$

LO LO NLO NNLO

$^1S_0$ Central force ⟺ nuclear BCS pairing

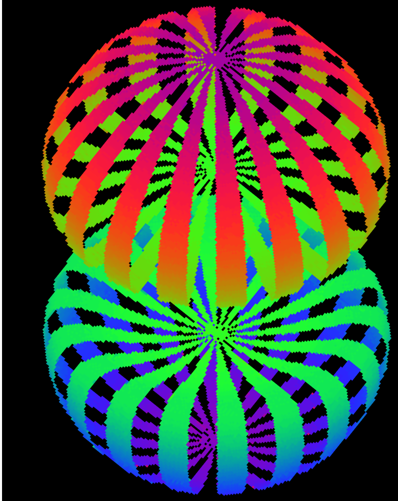
Bohr, Mottelson & Pines, Phys. Rev. 110 (1958)

$^3S_1$-$^3D_1$ Tensor force ⟺ deuteron binding

Pandharipande et al., Phys. Rev. C54 (1996)

$^3P_2$-$^3F_2$ LS force ⟺ neutron superfluidity in neutron stars

Tamagaki, Prog. Theor. Phys. 44 (1970)



Density profile of the deuteron with $S_z=\pm1$

Frequently Asked Questions

[Q1] Operator dependence of the latttice potential

[Q2] Energy dependence of the lattice potential

[A1] ($N(x)$, $U(x,x')$) is a combination to define observables

- remember,
  QM : (Φ, U) ~ (Φ', U') → observables
  QFT : (asymptotic field, vertices) → observables
  EFT : (choice of field, vertices) → observables
- local operator = a convenient choice for reduction formula

[A2] $U(x,x')$ is E-independent by construction

- non-locality can be determined order by order
  in velocity expansion (c.f. ChPT)

LO potential (central & tensor) in lattice QCD

- For $J^P = 1^+$, $|\varphi>$ comprises *S*-wave and *D*-wave,

$$|\varphi\rangle = |\varphi_S\rangle + |\varphi_D\rangle$$

where,

$$|\varphi_S\rangle = \mathcal{P}\,|\varphi\rangle = (1/24)\sum_{\mathcal{R}\in\mathcal{O}} \mathcal{R}\,|\varphi\rangle$$

$$|\varphi_D\rangle = \mathcal{Q}\,|\varphi\rangle = (1-\mathcal{P})\,|\varphi\rangle$$

- Therefore, we have 2-component Schrödinger eq.

S-wave:

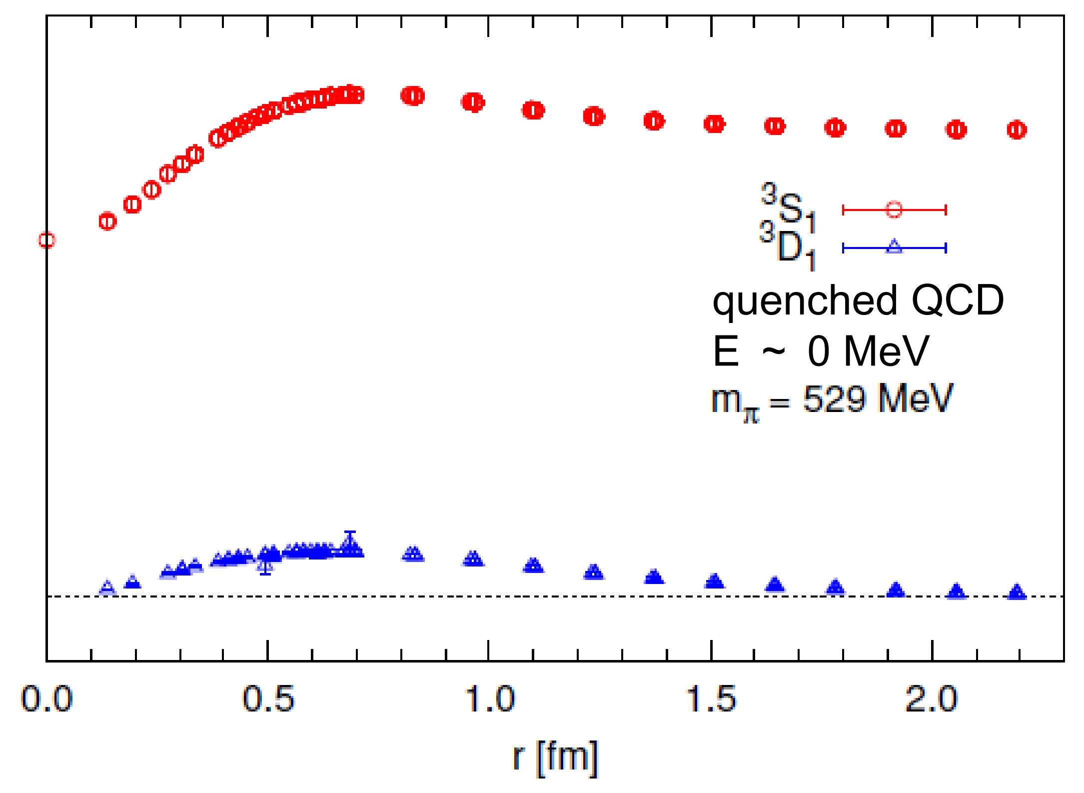
$$\mathcal{P}\,(T + \boxed{V_C} + \boxed{V_T} S_{12})\;|\varphi\rangle \;= E\,\mathcal{P}\;|\varphi\rangle$$

D-wave:

$$\mathcal{Q}\,(T + \boxed{V_C} + \boxed{V_T} S_{12})\;|\varphi\rangle \;= E\,\mathcal{Q}\;|\varphi\rangle$$

Central & tensor potentials : $V_C(r)$ & $V_T(r)$

Aoki, Hatsuda & Ishii,
0909.5585 [hep-lat]
PTP 123 (2010) 89-128



Central & tensor potentials : $V_C(r)$ & $V_T(r)$

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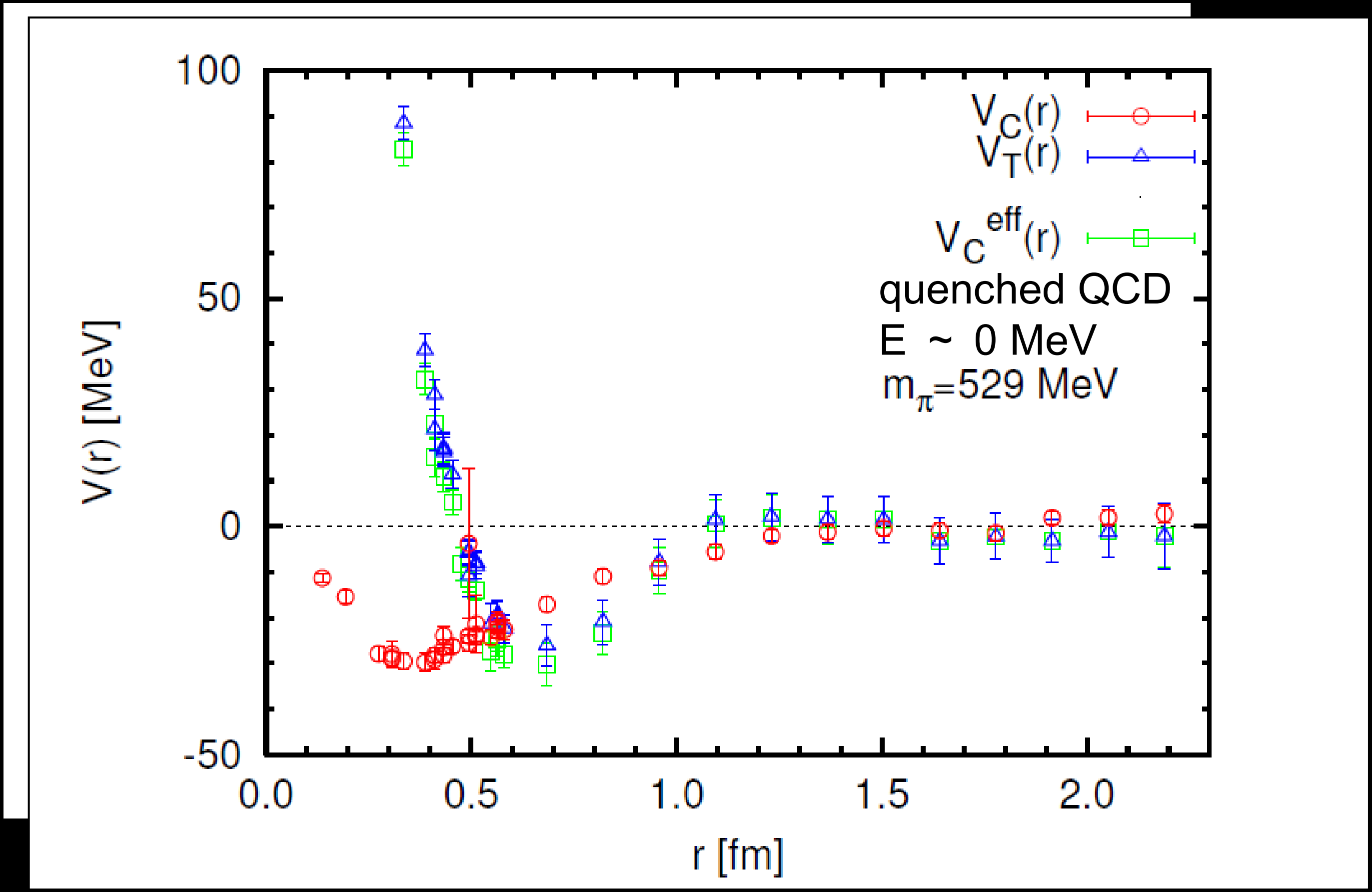

$V_c(r\rightarrow0) \sim (\log r)^\beta/r^2$, $V_T(r\rightarrow0) \rightarrow 0$
from operator product expansion
(Aoki, Balog & Weisz, arXiv:1002.0977)

Central & tensor potentials : $V_C(r)$ & $V_T(r)$

Aoki, Hatsuda & Ishii,
0909.5585 [hep-lat]
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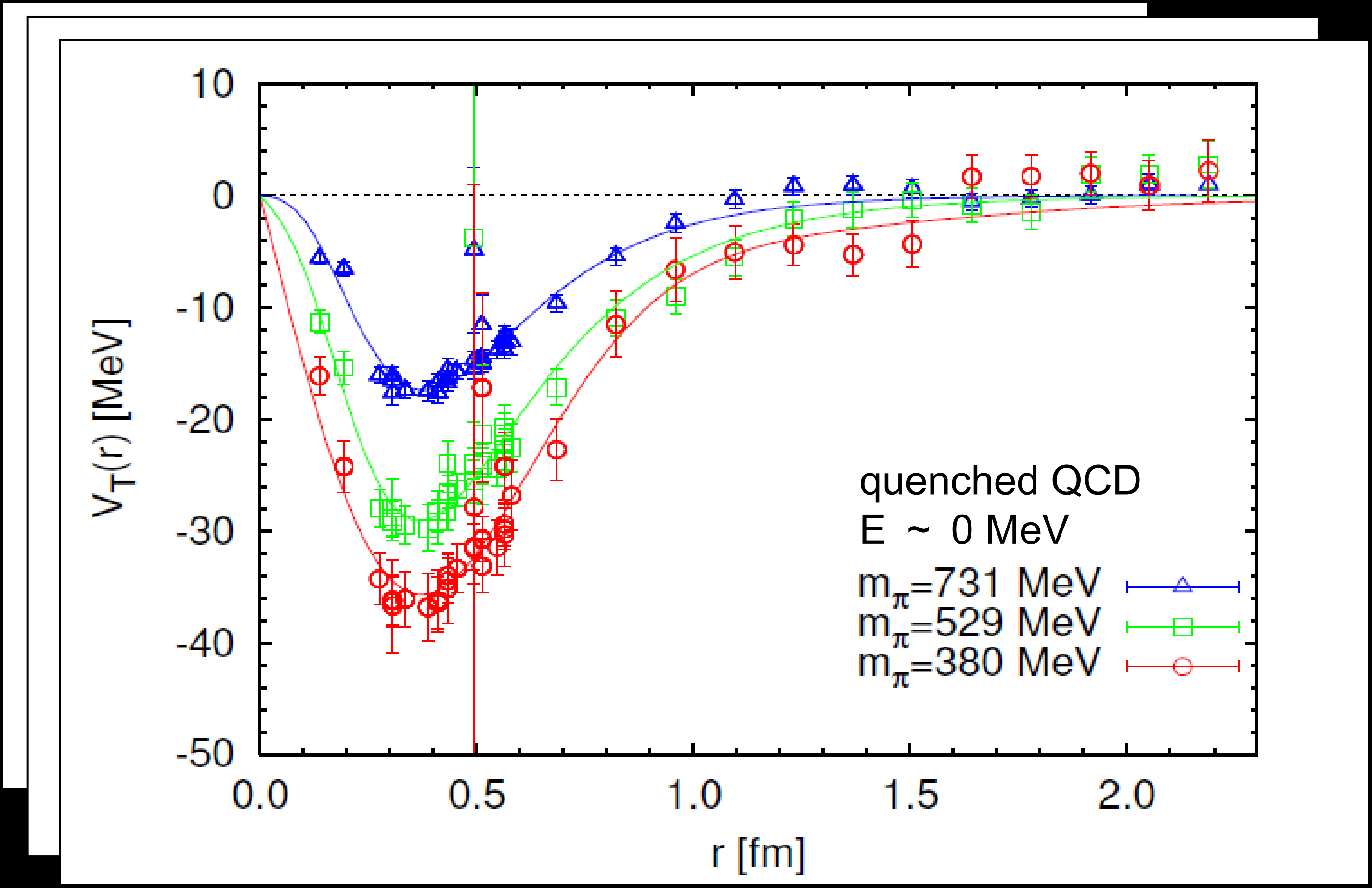
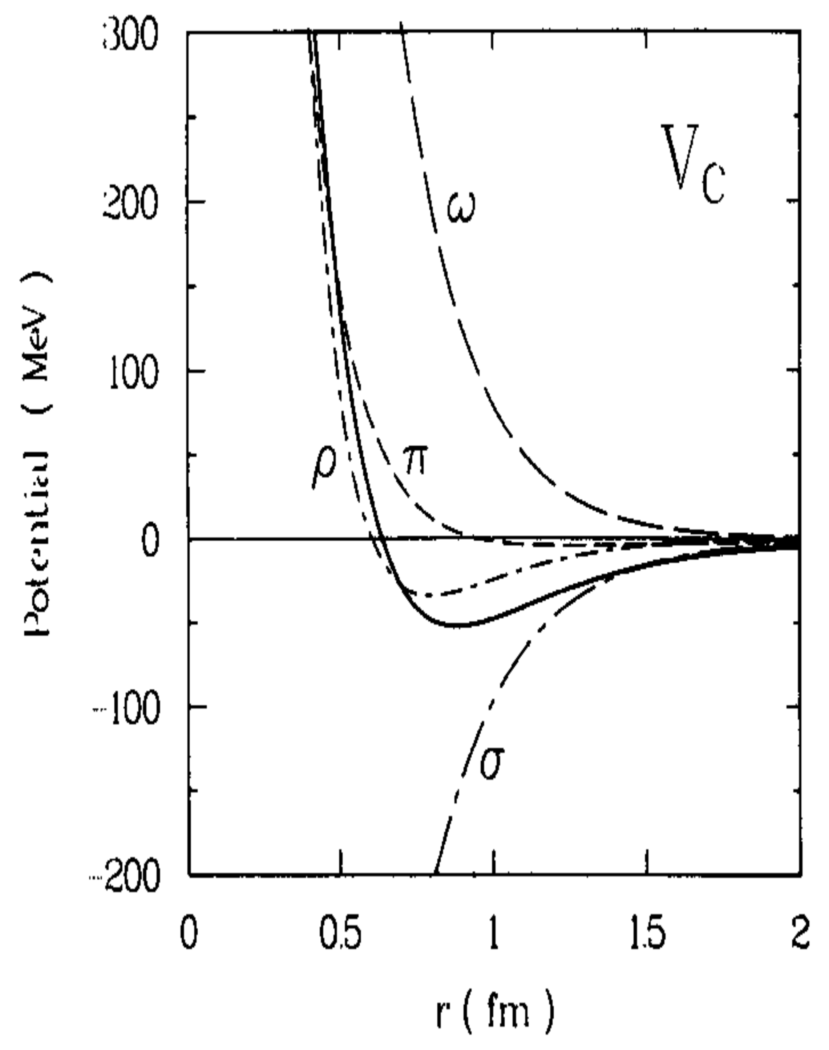


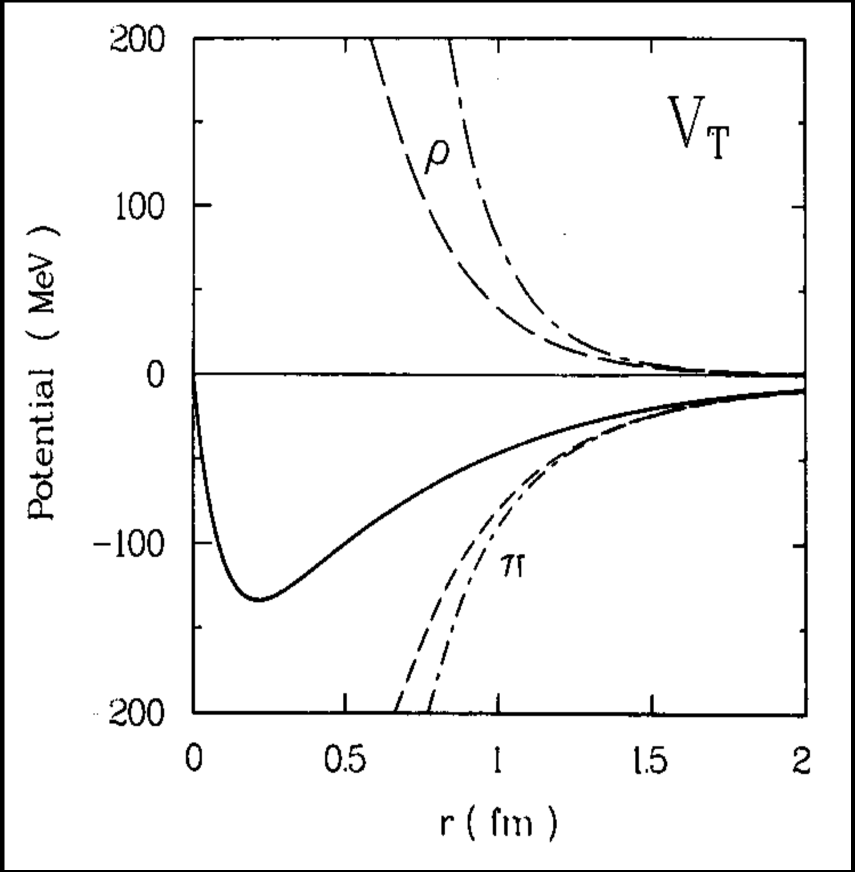
fit function

$$V_T(r) = b_1(1 - e^{-b_2 r^2})^2 \left(1 + \frac{3}{m_\rho r} + \frac{3}{(m_\rho r)^2}\right) \frac{e^{-m_\rho r}}{r}$$
$$+ b_3(1 - e^{-b_4 r^2})^2 \left(1 + \frac{3}{m_\pi r} + \frac{3}{(m_\pi r)^2}\right) \frac{e^{-m_\pi r}}{r},$$

- Rapid quark-mass dependence of $V_T(r)$
- Evidence of the one-pion-exchange

One boson exchange model : $V_C(r)$ & $V_T(r)$



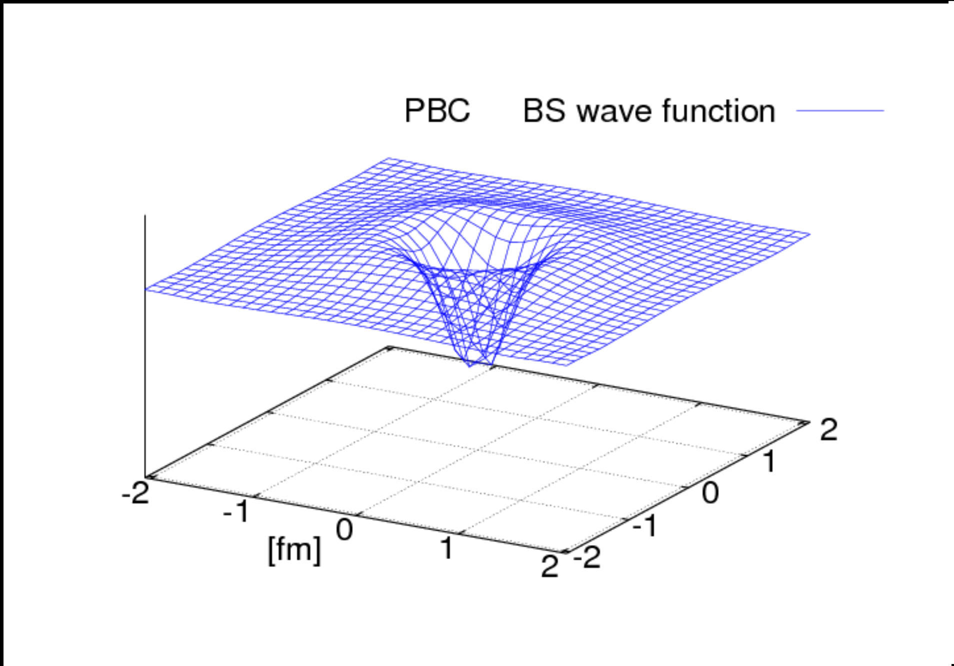


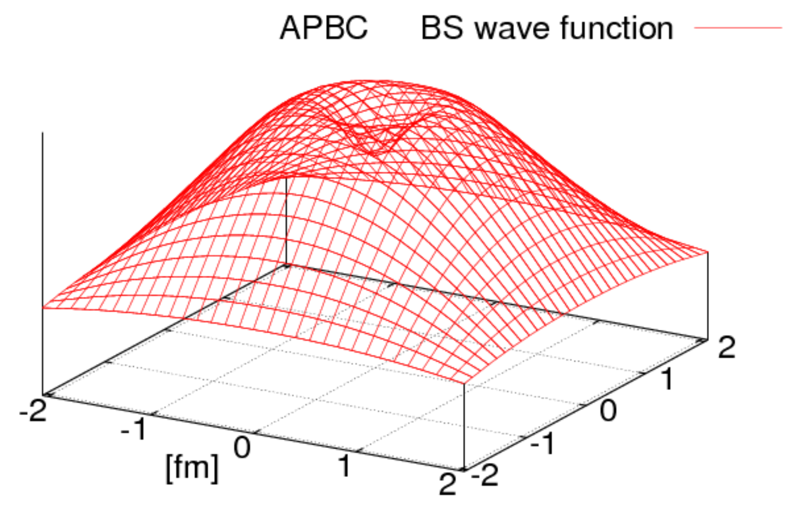
From Machleidt, Adv. Nucl.Phys.vol.19

velocity dependence of the potential (NNLO)

$$V(\vec{r},\nabla) = V_{\rm C}(r) + S_{12}V_{\rm T}(r) + \vec{\rm L}\cdot\vec{\rm S}\, V_{\rm LS}(r) + \{V_{\rm D}(r),\nabla^2\} + \cdots$$

- PBC (E∼0 MeV)
- APBC (E∼46 MeV)

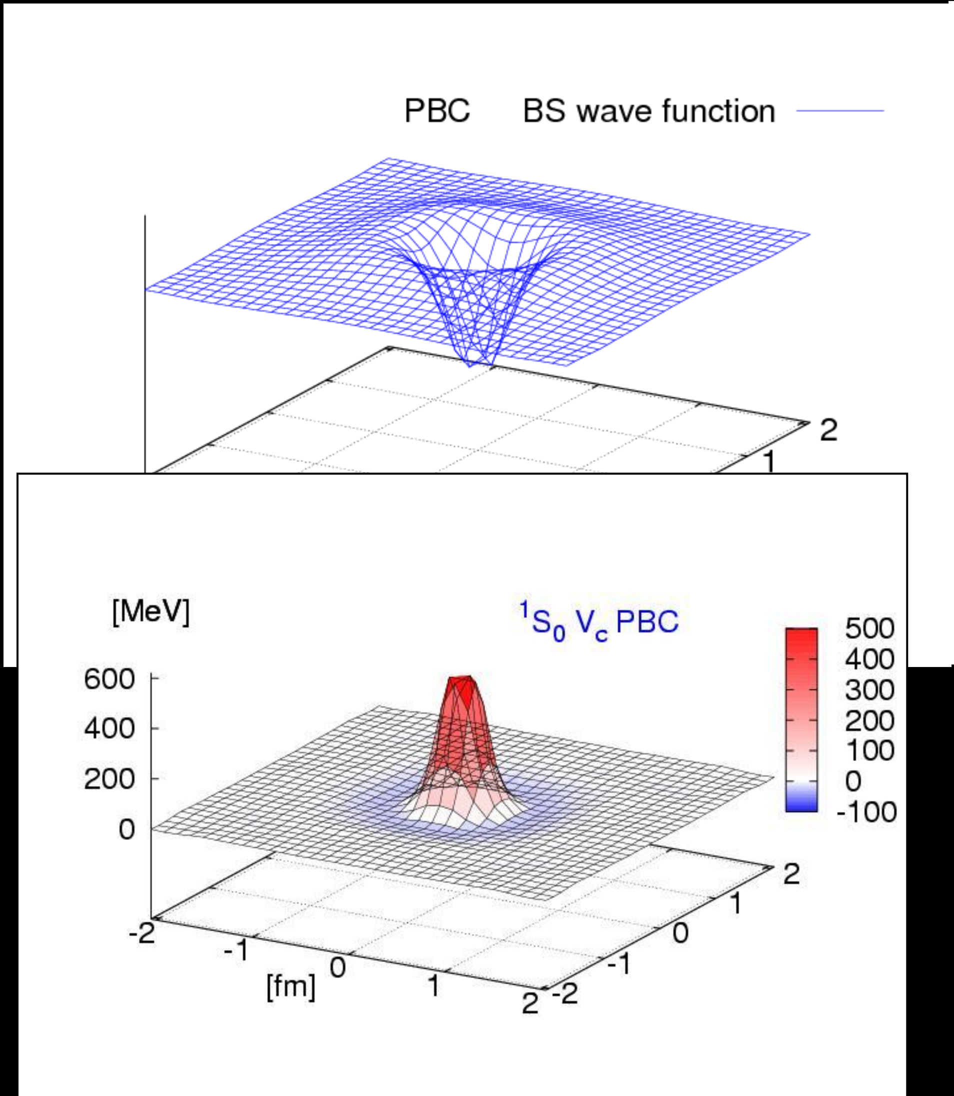


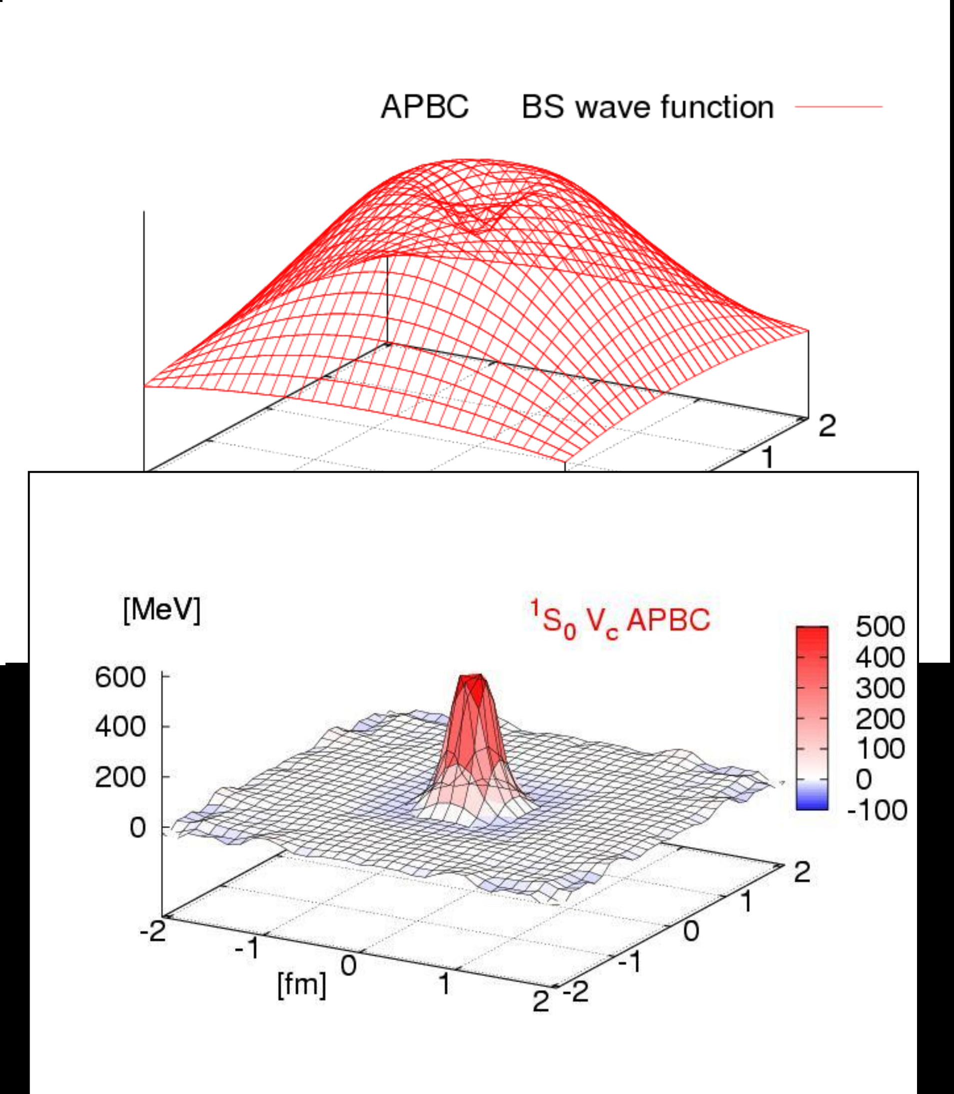


velocity dependence of the potential (NNLO)

$$V(\vec{r},\nabla) = V_{\mathrm{C}}(r) + S_{12}V_{\mathrm{T}}(r) + \vec{\mathrm{L}}\cdot\vec{\mathrm{S}}\; V_{\mathrm{LS}}(r) + \{V_{\mathrm{D}}(r),\nabla^2\} + \cdots$$

- PBC (E∼0 MeV)
- APBC (E∼46 MeV)

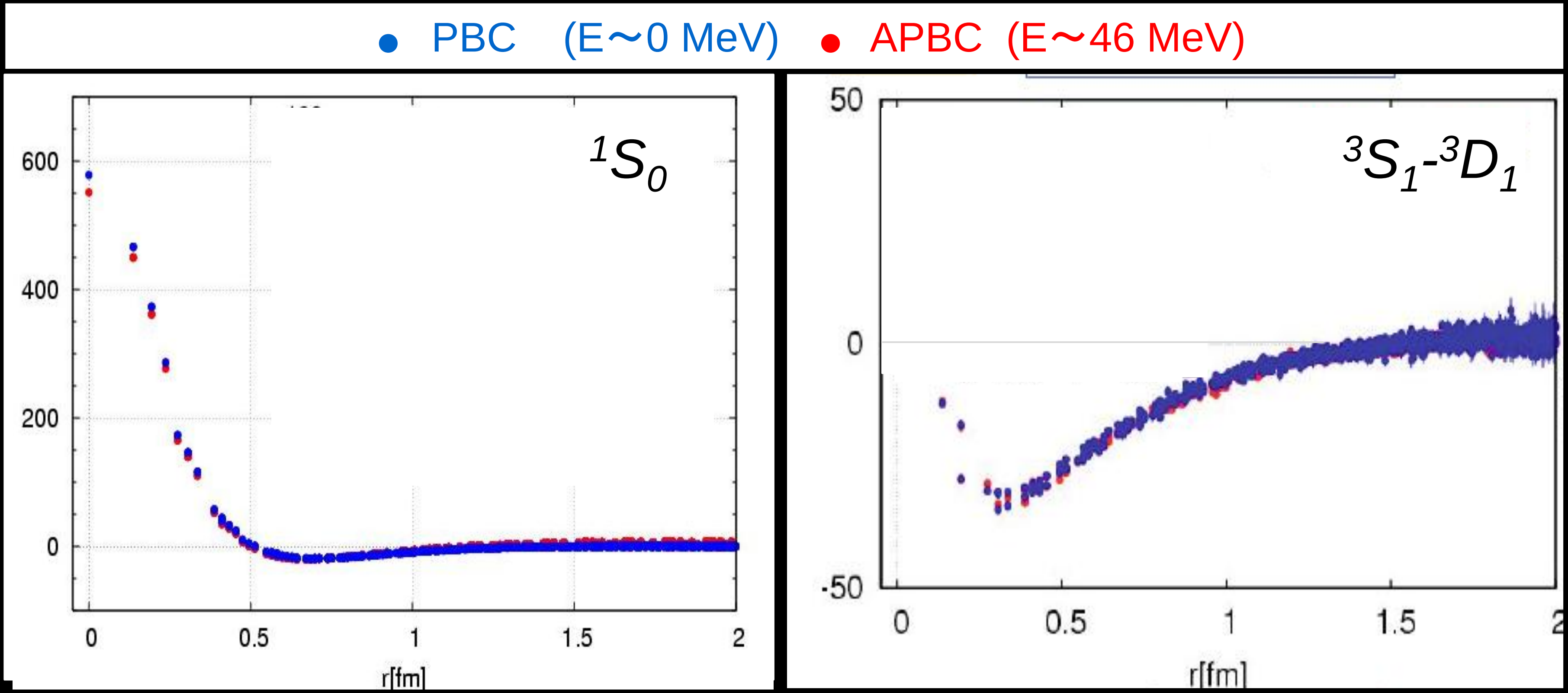




velocity dependence of the potential (NNLO)

$$V(\vec{r},\nabla) = V_{\mathrm{C}}(r) + S_{12}V_{\mathrm{T}}(r) + \vec{\mathrm{L}}\cdot\vec{\mathrm{S}}\ V_{\mathrm{LS}}(r) + \{V_{\mathrm{D}}(r),\nabla^2\} + \cdots$$

NNLO

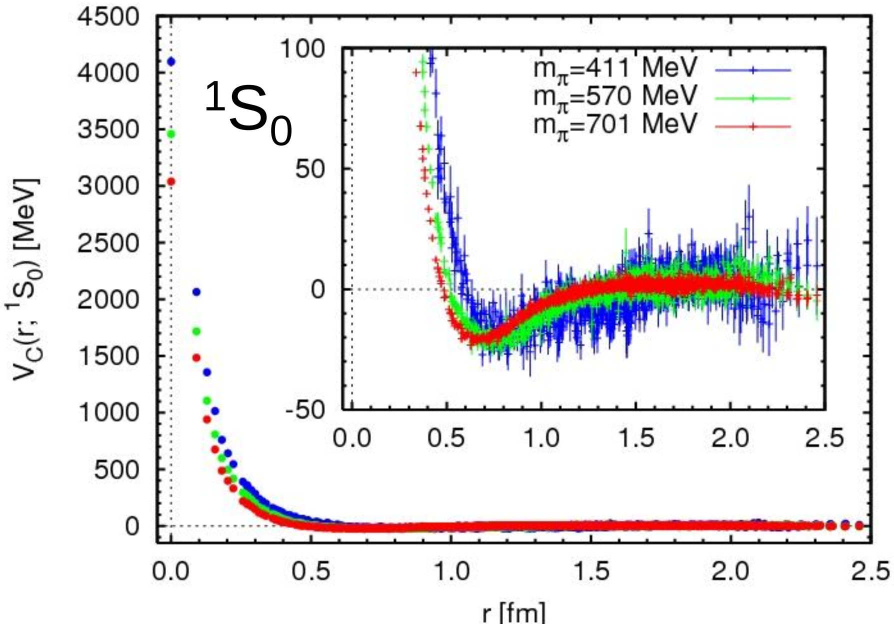


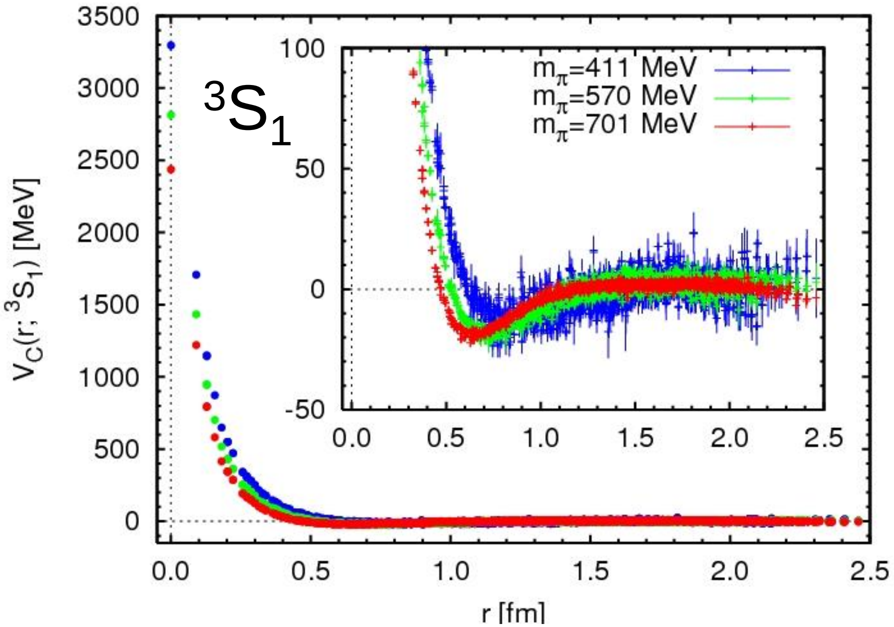
- NNLO can be determined from φ (r) for different E
- NNLO is small at least up to E_{cm} ~ 46 MeV (T_{lab} ~ 100 MeV)

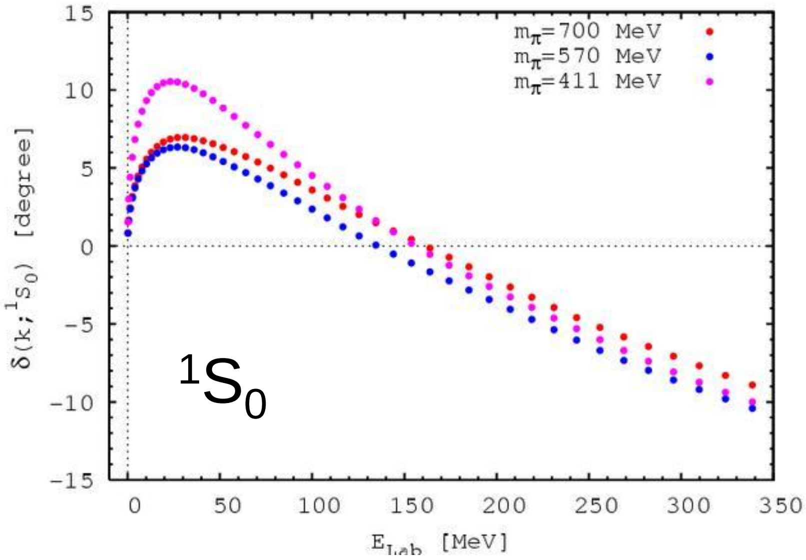
quenched QCD
m_π = 529 MeV

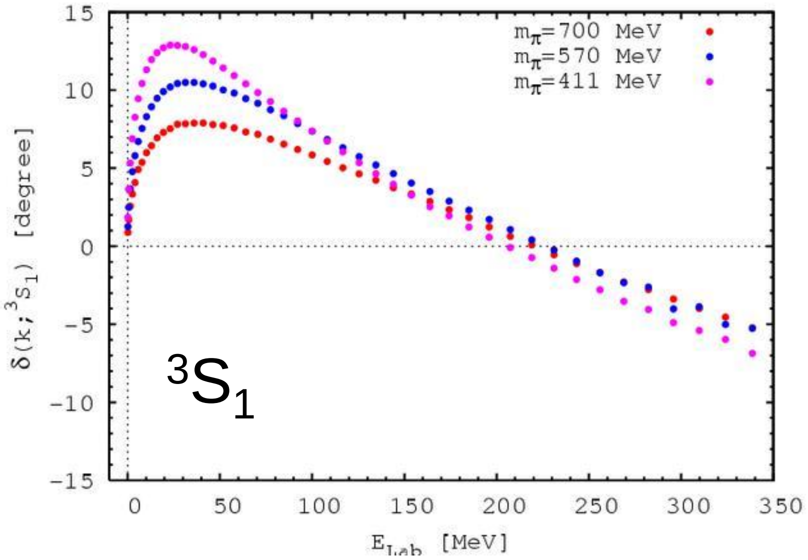
Murano et al. (HAL QCD Coll.)

Phase shift from V(r) in full QCD





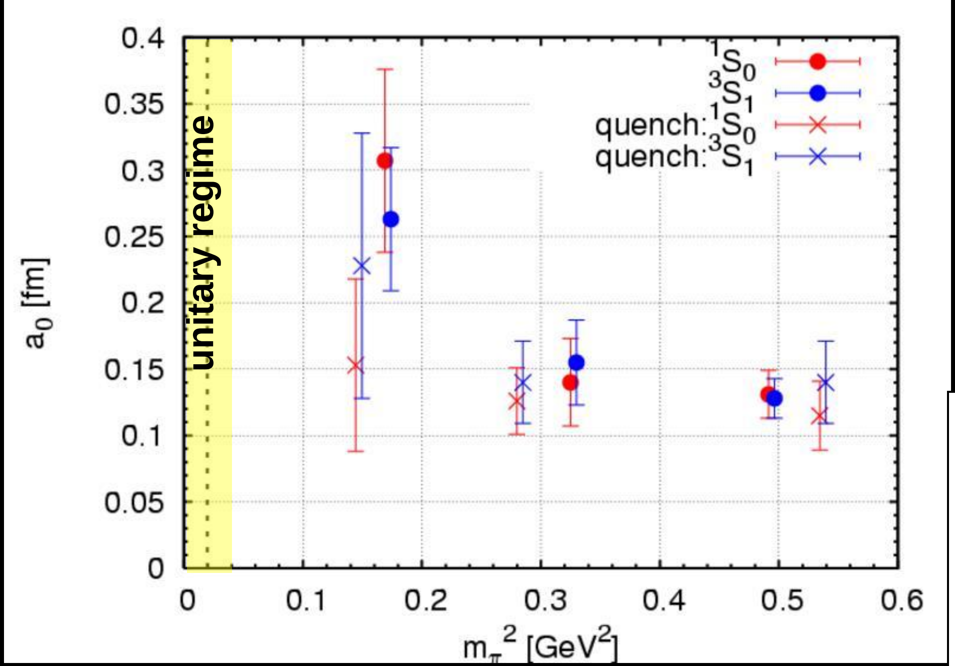




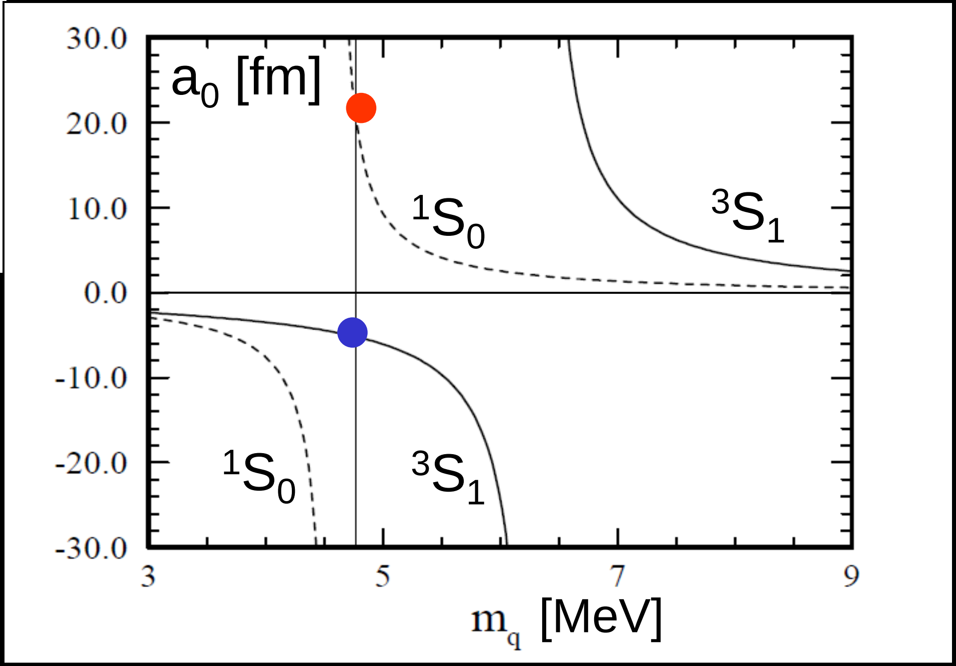
NN scattering lengths in full QCD

BS wave func. → E → Luscher’s formula

(CP-PACS method, 2005)

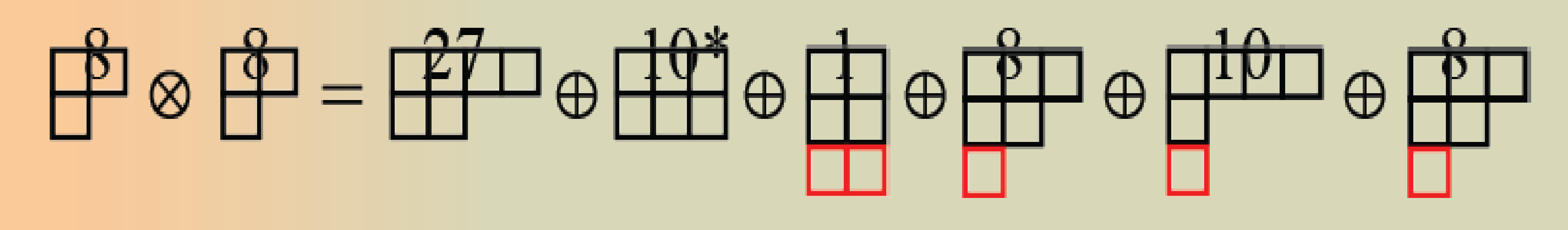


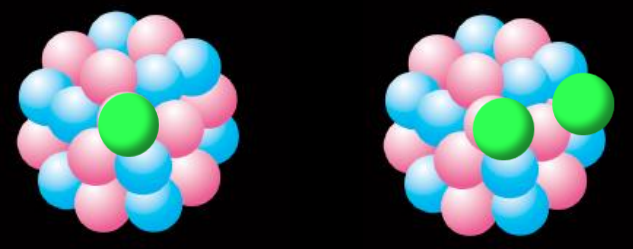
- Overall attraction
- Still far from “unitary regime”



Kuramashi Plot [hep-lat/9510025]

YN and YY interactions in lattice QCD





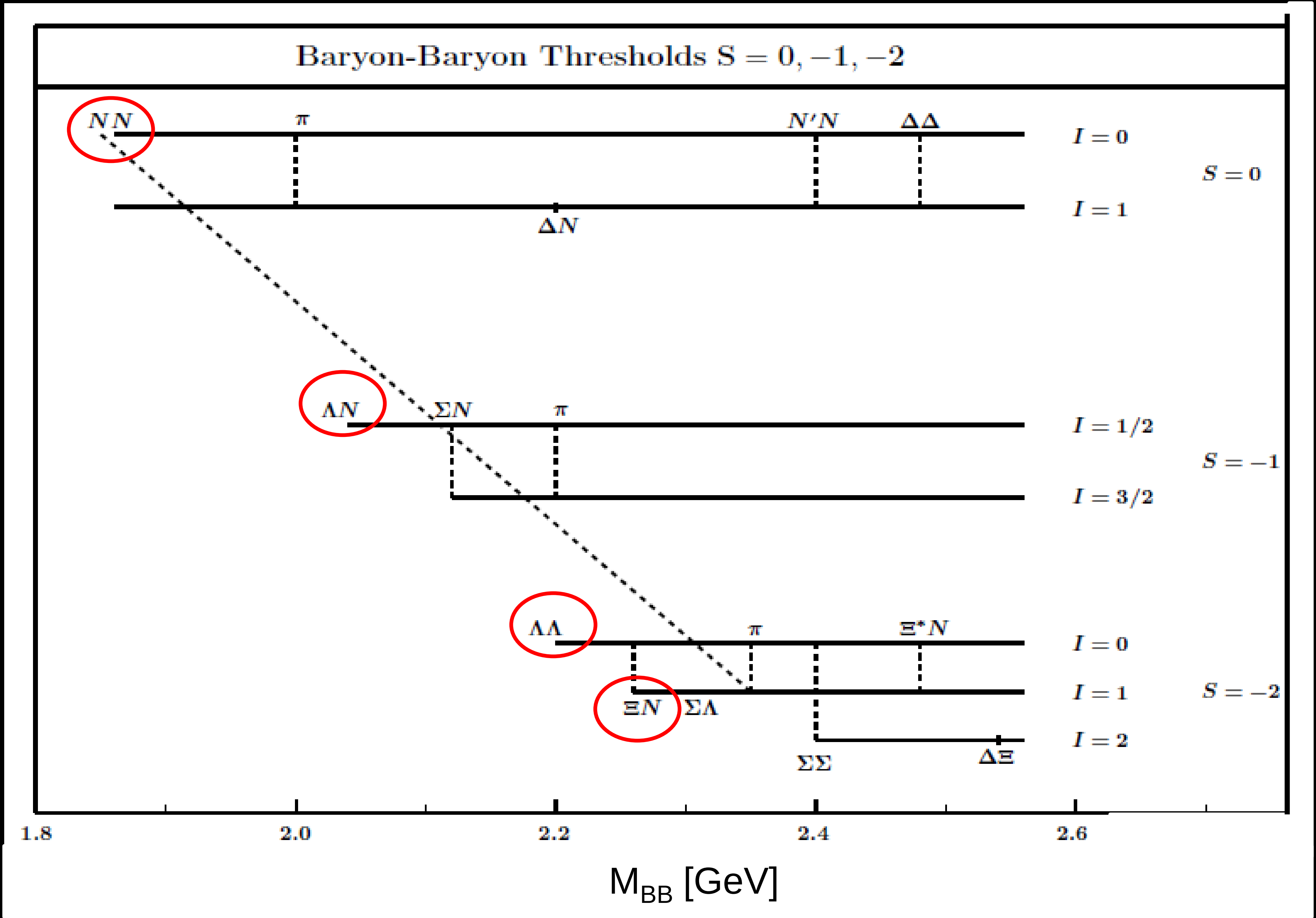

J-PARC (Tokai, Japan)

- no phase shifts available for YN and YY scatterings
- plenty of hyper-nucleus data will soon be available at J-PARC

↕

- predictions from lattice QCD ?
- difference between NN and YN ?

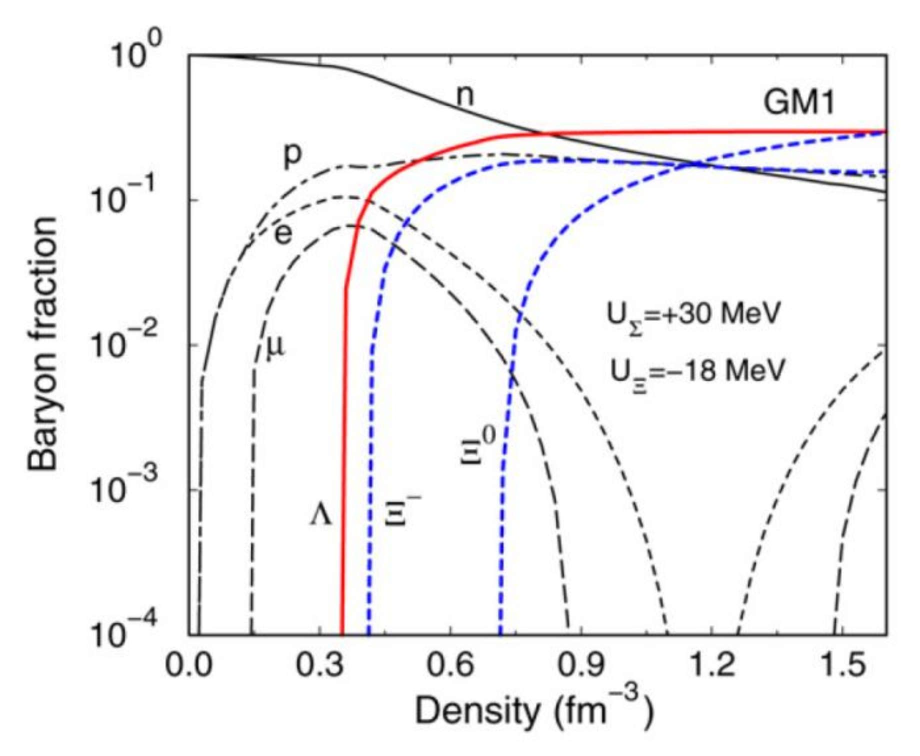
Muroya, Nakamura & Nagata, (2004)



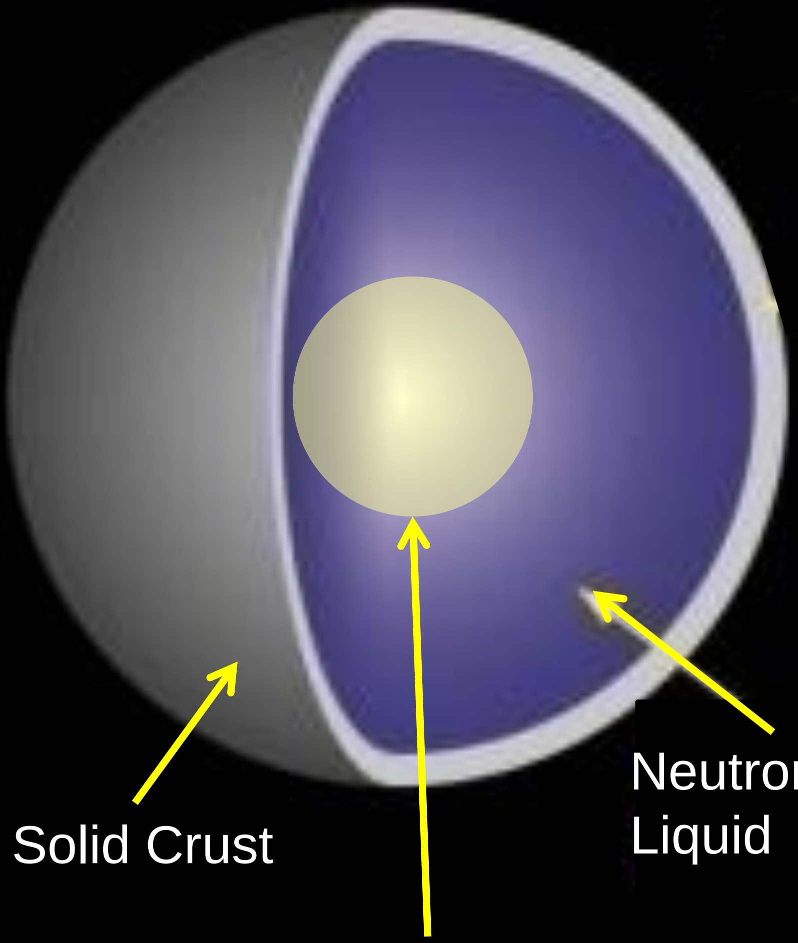
De Swart, Nagels, Rijken, Verhoeven (1971)

Hyperon Core of Neutron Stars

Radius ∼ 10 km
Mass ∼ solar mass
Central density ∼ 10^{12} kg/cm^3



Schaffner-Bielich, NP A804 (2008).

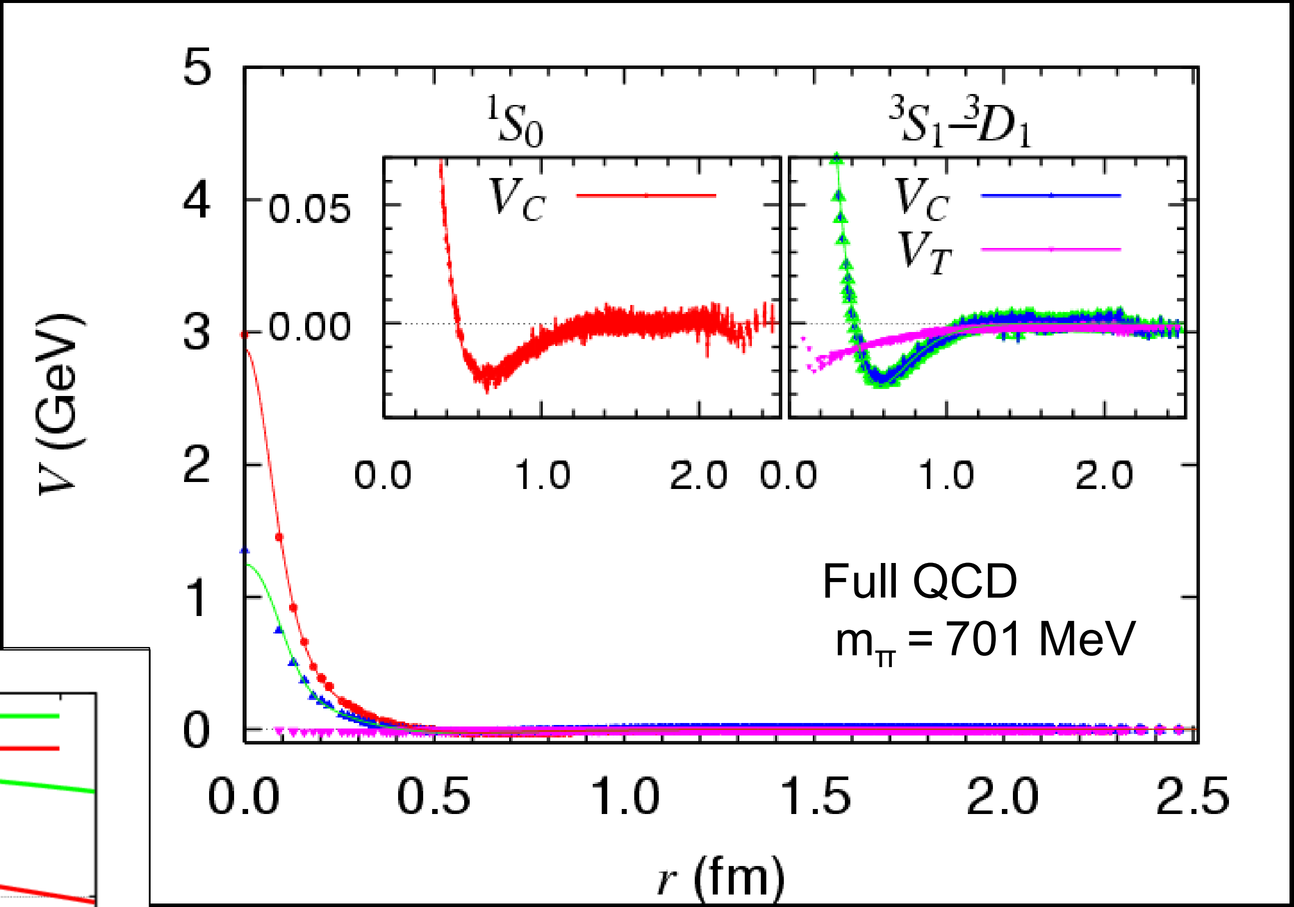


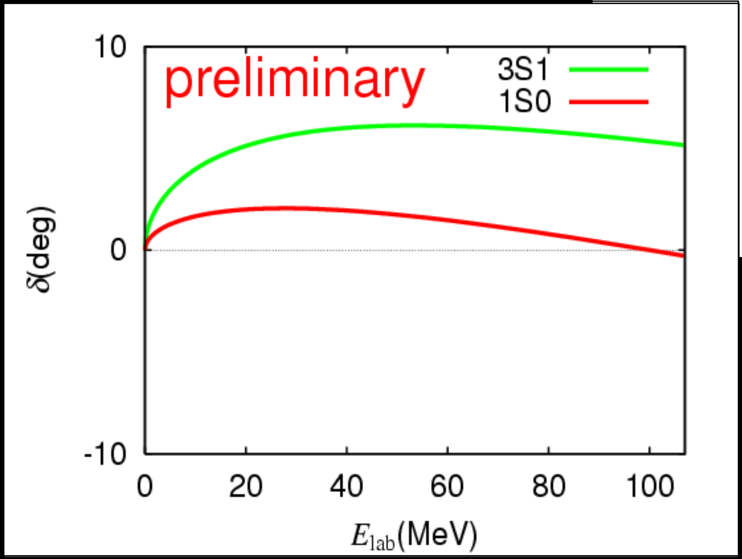
Solid Crust

Neutron Liquid

Hyperon Matter ?

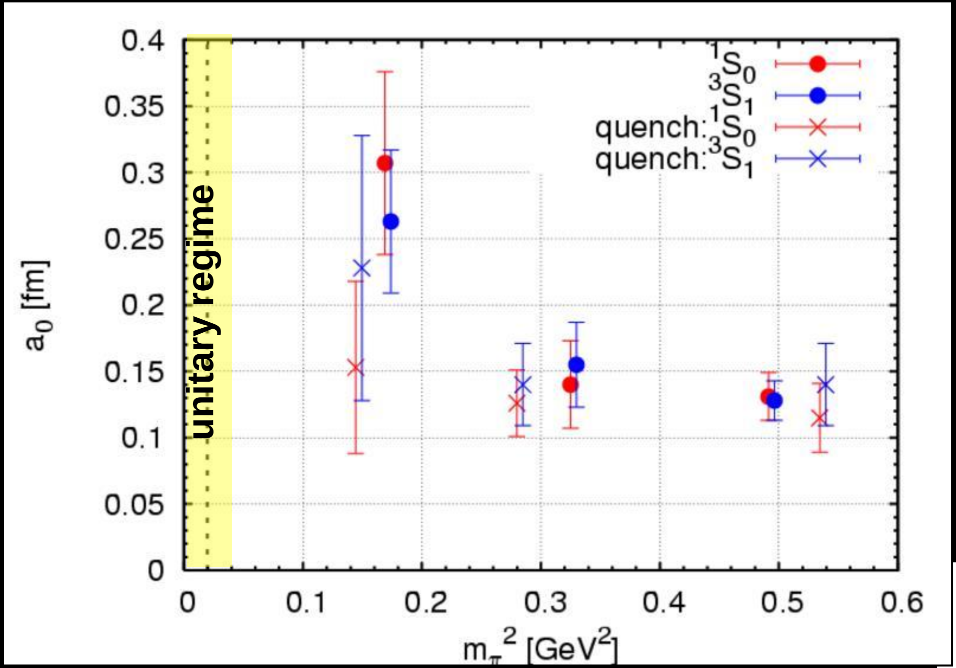
S=-1 system: ΛN interaction (I=1/2) in full QCD





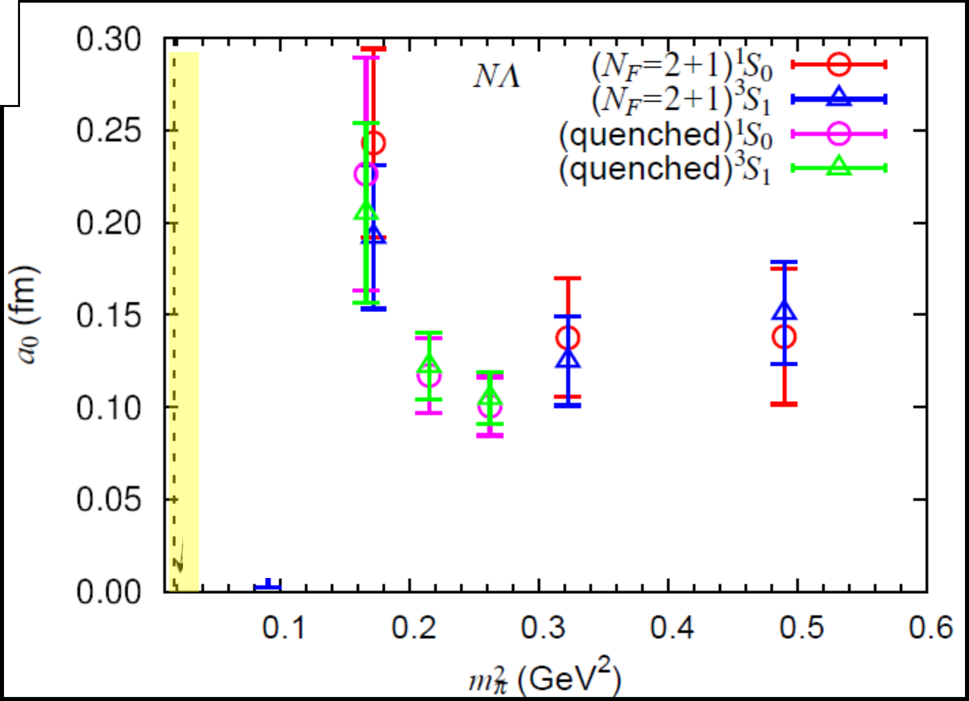
Nemura et al. (HAL QCD Coll.)

NN and ΛN Scattering lengths in full QCD



NN

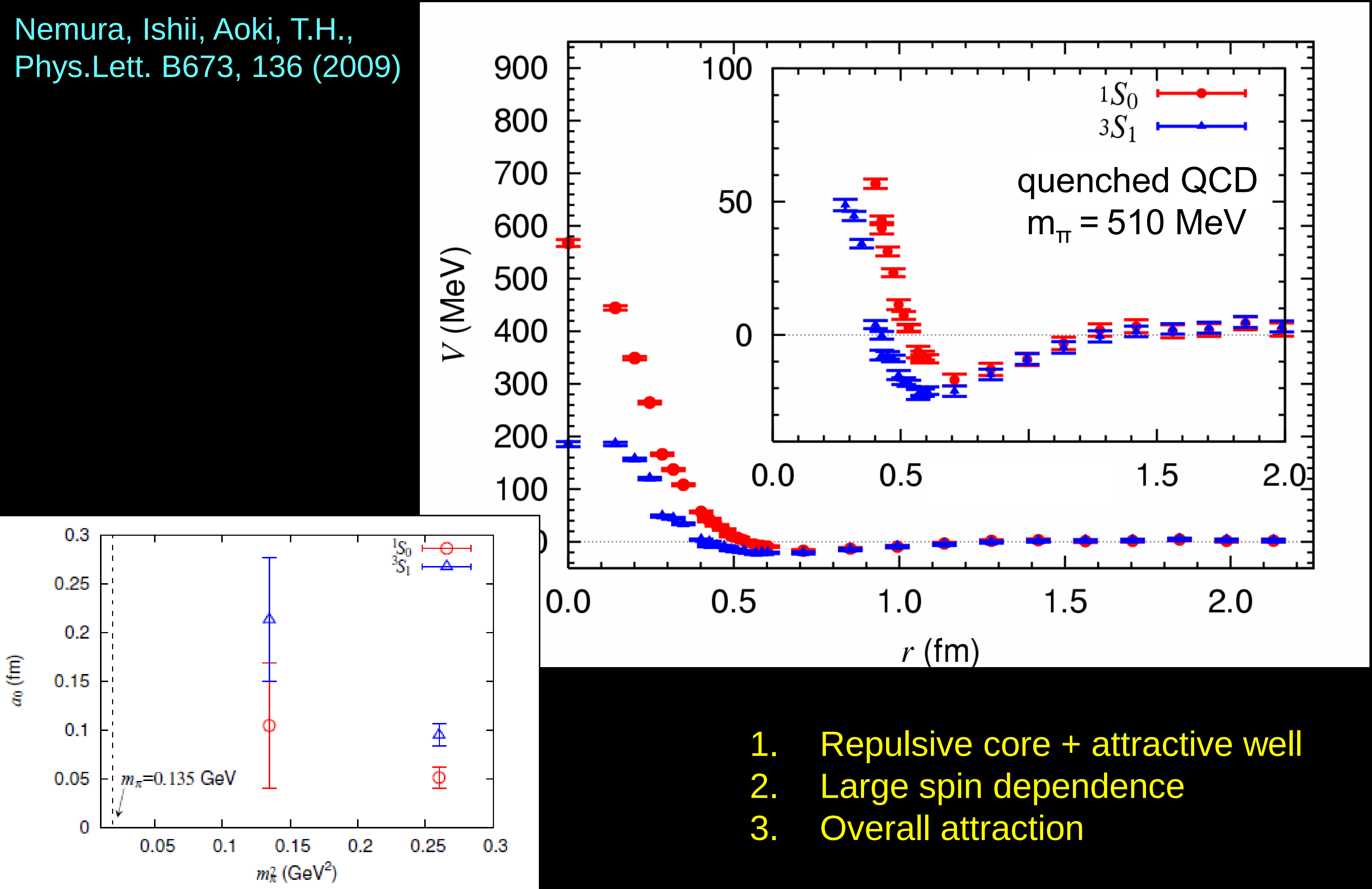
ΛN



S=-2 system: ΞN interaction (I=1)

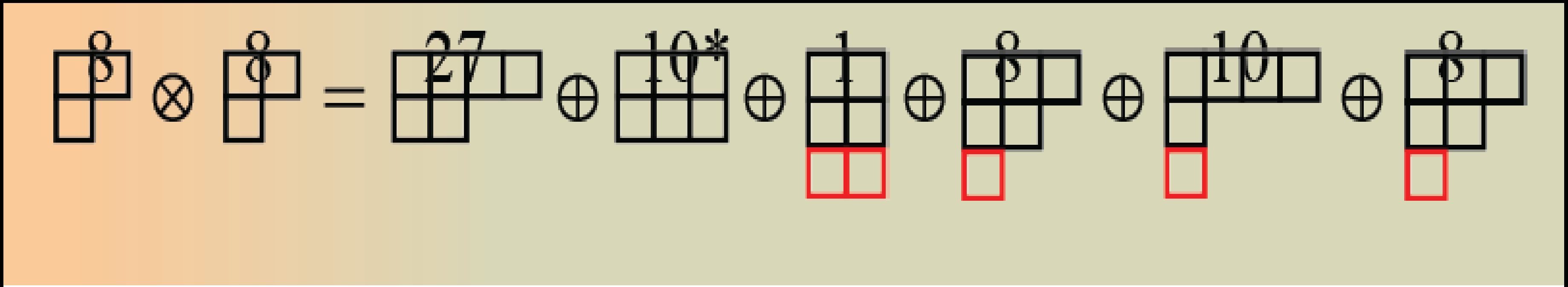
J-PARC DAY-1 exp. :
$^{12}C(K^-,K^+)^{12}Be_{\Xi}$

Nemura, Ishii, Aoki, T.H.,
Phys.Lett. B673, 136 (2009)



1. Repulsive core + attractive well
2. Large spin dependence
3. Overall attraction

BB interaction



$$8 \otimes 8 = 27 \oplus 10^* \oplus 1 \oplus 8 \oplus 10 \oplus 8$$

- We have six independent potentials for a given L.

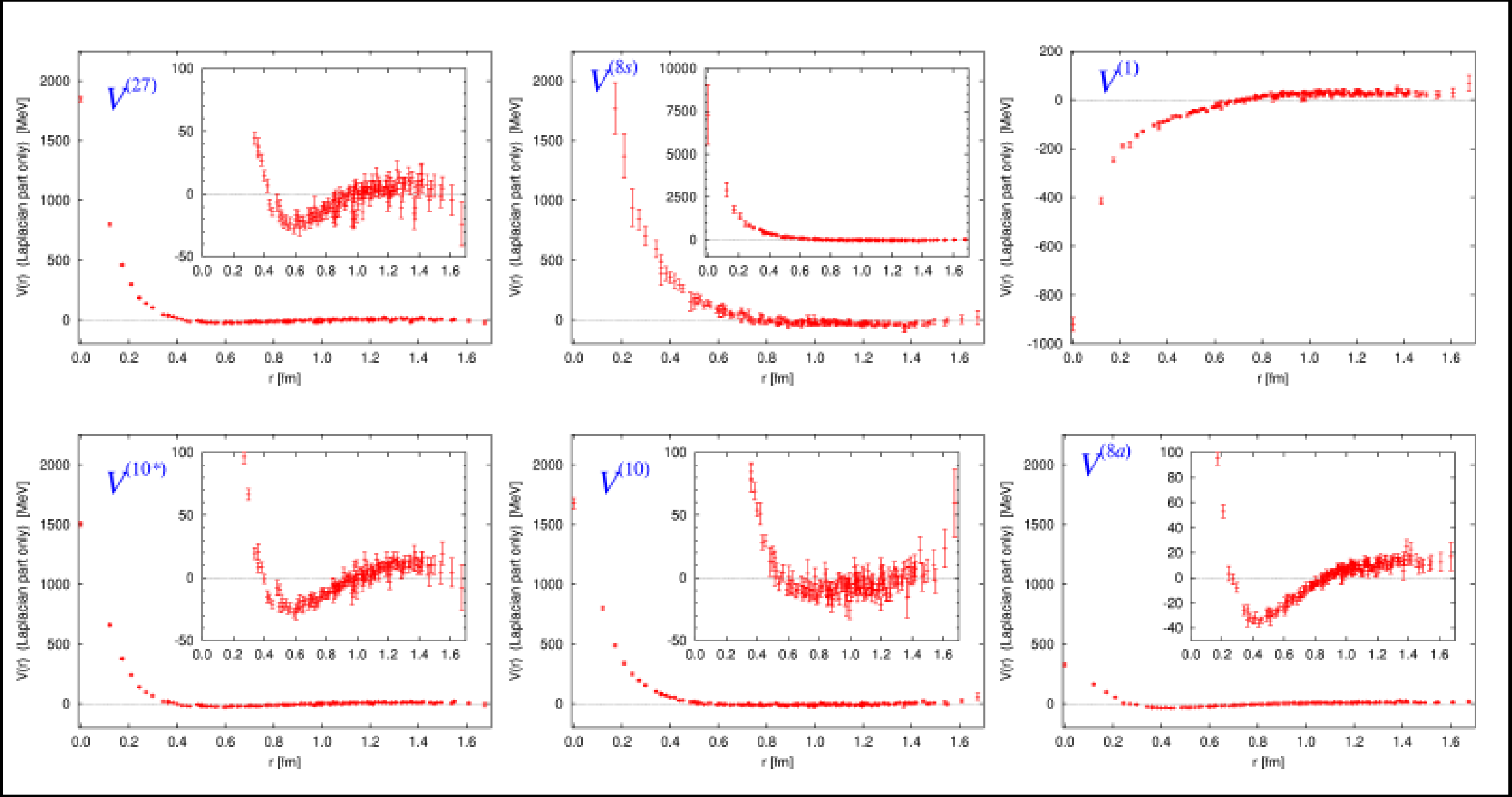
$$^1S_0 \;:\; V^{(27)}(r),\; V^{(8s)}(r),\; V^{(1)}(r)$$

$$^3S_1 \;:\; V^{(10^*)}(r),\; V^{(10)}(r),\; V^{(8a)}(r)$$

What makes the difference among these six channels ?

Pauli principle at work !?

$^1S_0$

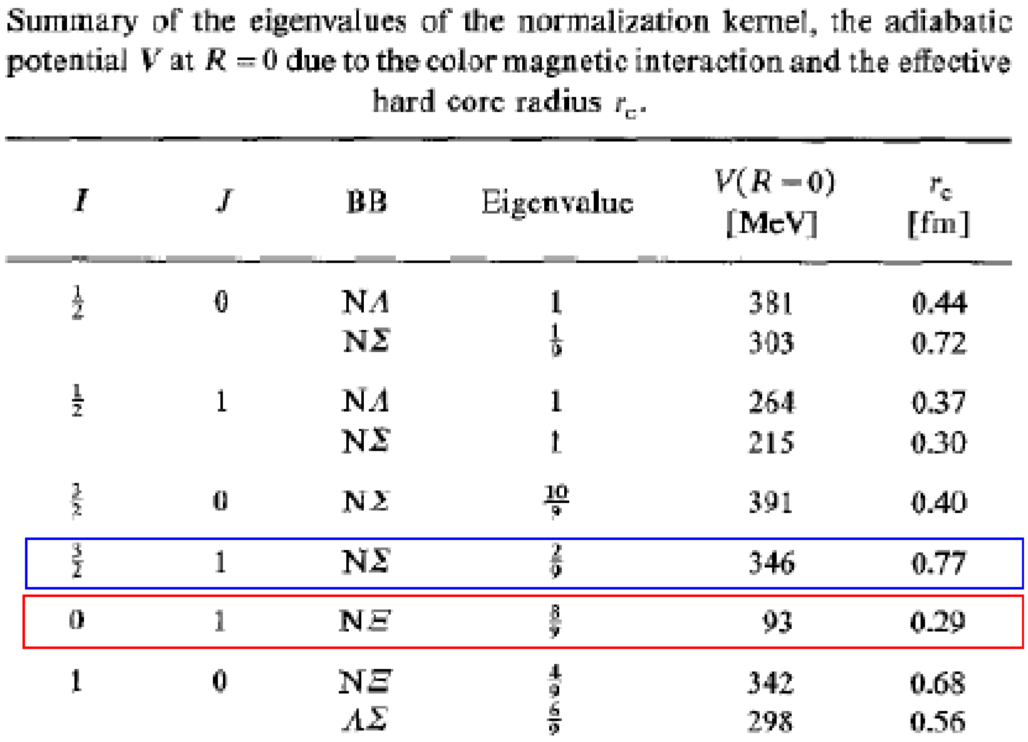


$^3S_1$

16^3x32, full (CP-PACS/JLQCD config.) a=0.12 fm, L=2fm
SU(3) limit: m_π=m_K=835 MeV, m_B=1745 MeV

Inoue et al. (HAL QCD Coll.)

Quark model prediction



Summary of the eigenvalues of the normalization kernel, the adiabatic potential V at $R=0$ due to the color magnetic interaction and the effective hard core radius r_c.

I	J	BB	Eigenvalue	$V(R=0)$ [MeV]	r_c [fm]
$\frac{1}{2}$	0	NΛ	1	381	0.44
		NΣ	$\frac{1}{9}$	303	0.72
$\frac{1}{2}$	1	NΛ	1	264	0.37
		NΣ	1	215	0.30
$\frac{3}{2}$	0	NΣ	$\frac{10}{9}$	391	0.40
$\frac{3}{2}$	1	NΣ	$\frac{2}{9}$	346	0.77
0	1	NΞ	$\frac{8}{9}$	93	0.29
1	0	NΞ	$\frac{4}{9}$	342	0.68
		$\Lambda\Sigma$	$\frac{6}{9}$	298	0.56

10-plet

8a-plet

Oka, Shimizu, Yazaki
Nucl. Phys. A464 (1987)

the eigenvalue for 8s = 0
pure-forbidden!!

semi-forbidden and
hence strong repulsive

both quark-antisym
and OGE are weak

irreducible BB source operator

$$\overline{BB^{(27)}} = +\sqrt{\frac{27}{40}}\,\overline{\Lambda}\,\overline{\Lambda} - \sqrt{\frac{1}{40}}\,\overline{\Sigma}\,\overline{\Sigma} + \sqrt{\frac{12}{40}}\,\overline{N}\,\overline{\Xi}$$

or

$$+\sqrt{\frac{1}{2}}\,\overline{p}\,\overline{n} + \sqrt{\frac{1}{2}}\,\overline{n}\,\overline{p}$$

$$\overline{BB^{(8s)}} = -\sqrt{\frac{1}{5}}\,\overline{\Lambda}\,\overline{\Lambda} - \sqrt{\frac{3}{5}}\,\overline{\Sigma}\,\overline{\Sigma} + \sqrt{\frac{1}{5}}\,\overline{N}\,\overline{\Xi}$$

$$\overline{BB^{(1)}} = -\sqrt{\frac{1}{8}}\,\overline{\Lambda}\,\overline{\Lambda} + \sqrt{\frac{3}{8}}\,\overline{\Sigma}\,\overline{\Sigma} + \sqrt{\frac{4}{8}}\,\overline{N}\,\overline{\Xi}$$

with

$$\overline{\Sigma}\,\overline{\Sigma} = +\sqrt{\frac{1}{3}}\,\overline{\Sigma^{+}}\,\overline{\Sigma^{-}} - \sqrt{\frac{1}{3}}\,\overline{\Sigma^{0}}\,\overline{\Sigma^{0}} + \sqrt{\frac{1}{3}}\,\overline{\Sigma^{-}}\,\overline{\Sigma^{+}}$$

$$\overline{N}\,\overline{\Xi} = +\sqrt{\frac{1}{4}}\,\overline{p}\,\overline{\Xi^{-}} + \sqrt{\frac{1}{4}}\,\overline{\Xi^{-}}\,\overline{p} - \sqrt{\frac{1}{4}}\,\overline{n}\,\overline{\Xi^{0}} - \sqrt{\frac{1}{4}}\,\overline{\Xi^{0}}\,\overline{n}$$

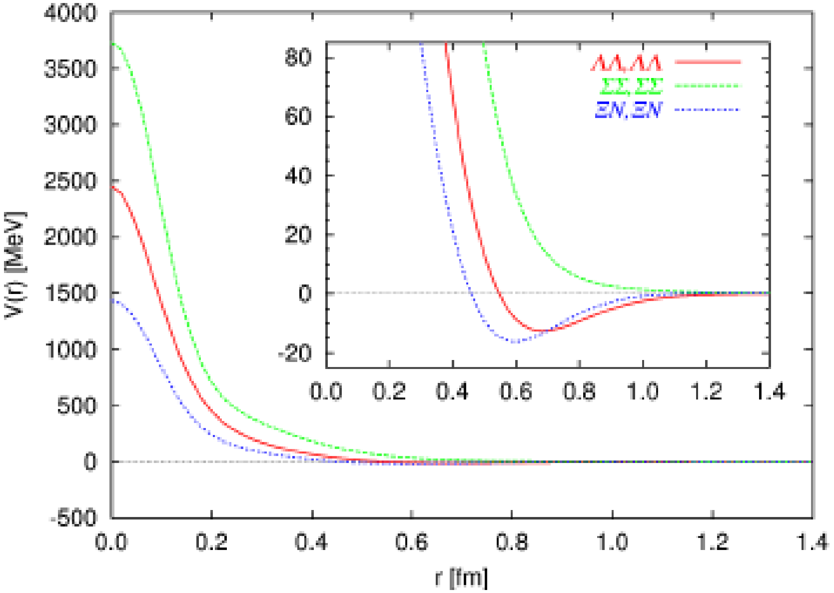
$$\overline{BB^{(10^*)}} = +\sqrt{\frac{1}{2}}\,\overline{p}\,\overline{n} - \sqrt{\frac{1}{2}}\,\overline{n}\,\overline{p}$$

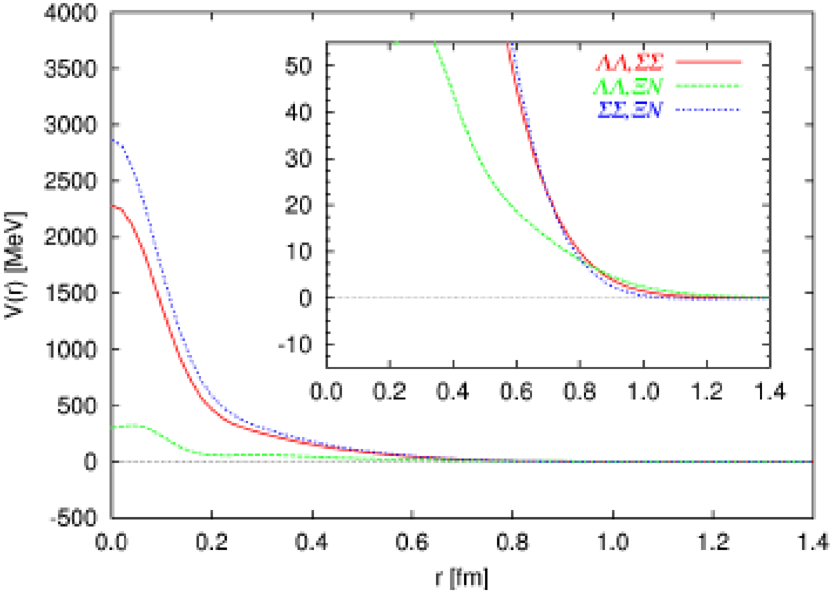
$$\overline{BB^{(10)}} = +\sqrt{\frac{1}{2}}\,\overline{p}\,\overline{\Sigma^{+}} - \sqrt{\frac{1}{2}}\,\overline{\Sigma^{+}}\,\overline{p}$$

$$\overline{BB^{(8a)}} = +\sqrt{\frac{1}{4}}\,\overline{p}\,\overline{\Xi^{-}} - \sqrt{\frac{1}{4}}\,\overline{\Xi^{-}}\,\overline{p} - \sqrt{\frac{1}{4}}\,\overline{n}\,\overline{\Xi^{0}} + \sqrt{\frac{1}{4}}\,\overline{\Xi^{0}}\,\overline{n}$$

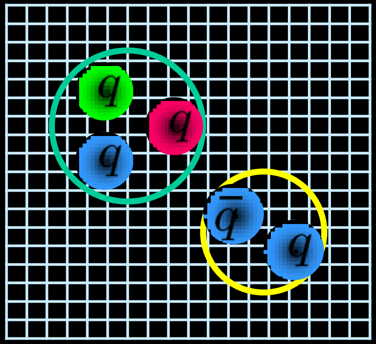
S=−2, I=0 BB $^1S_0$ potential

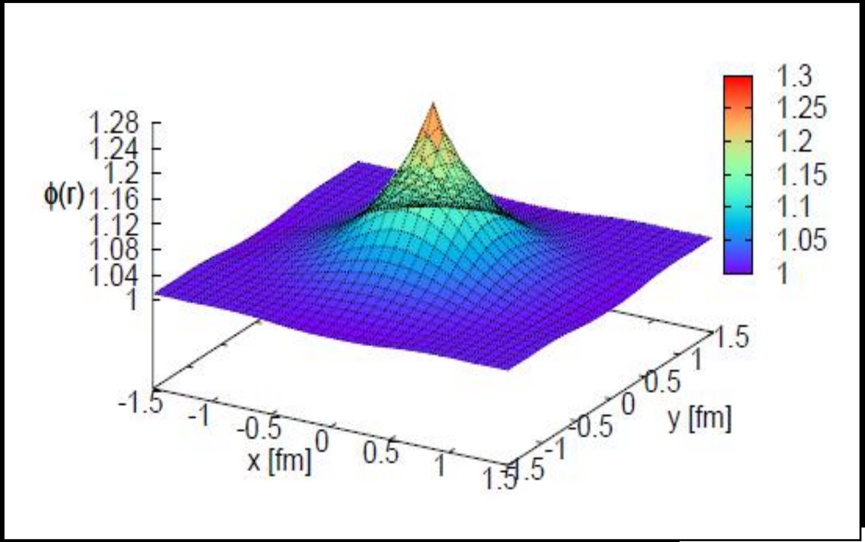
$$\begin{pmatrix} \Lambda\Lambda \\ \Sigma\Sigma \\ \Xi N \end{pmatrix} = U \begin{pmatrix} |27\rangle \\ |8\rangle \\ |1\rangle \end{pmatrix}, \quad U \begin{pmatrix} V^{(27)} & & \\ & V^{(8)} & \\ & & V^{(1)} \end{pmatrix} U^t \longrightarrow \begin{pmatrix} V^{\Lambda\Lambda} & V^{\Lambda\Lambda}_{\Sigma\Sigma} & V^{\Lambda\Lambda}_{\Xi N} \\ & V^{\Sigma\Sigma} & V^{\Sigma\Sigma}_{\Xi N} \\ & & V^{\Xi N} \end{pmatrix}$$

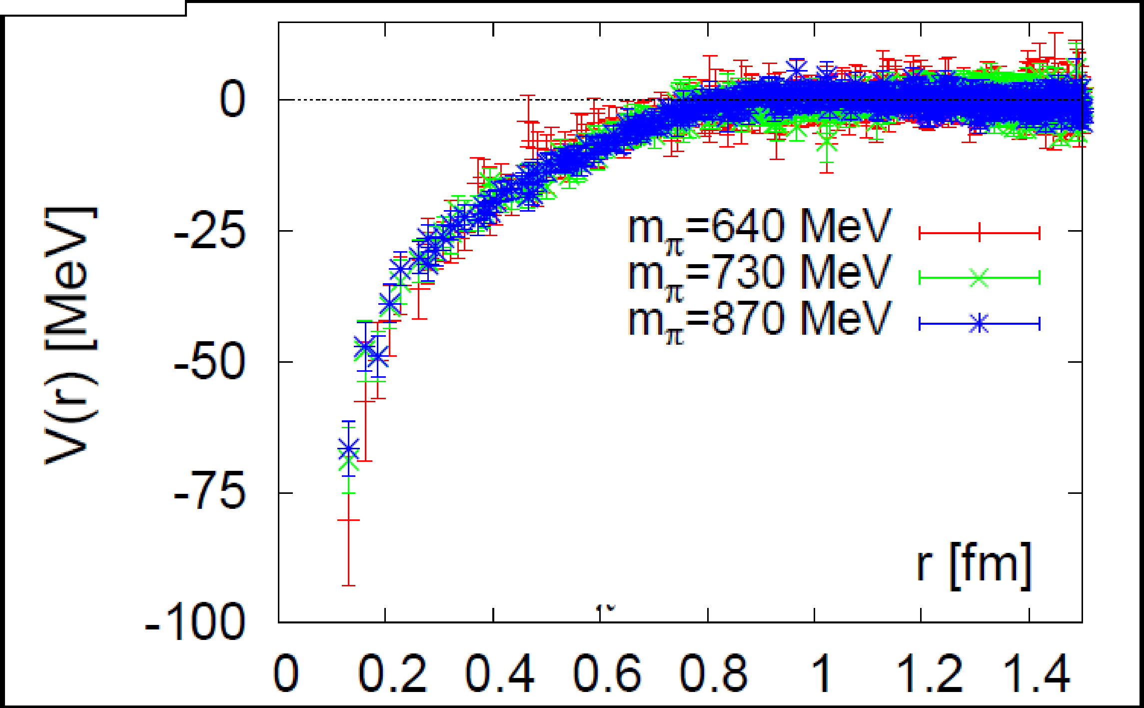




S-wave N-η_c (cc^{bar}) interaction



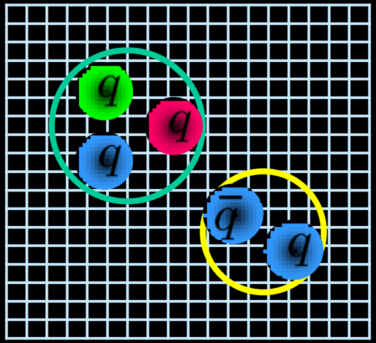


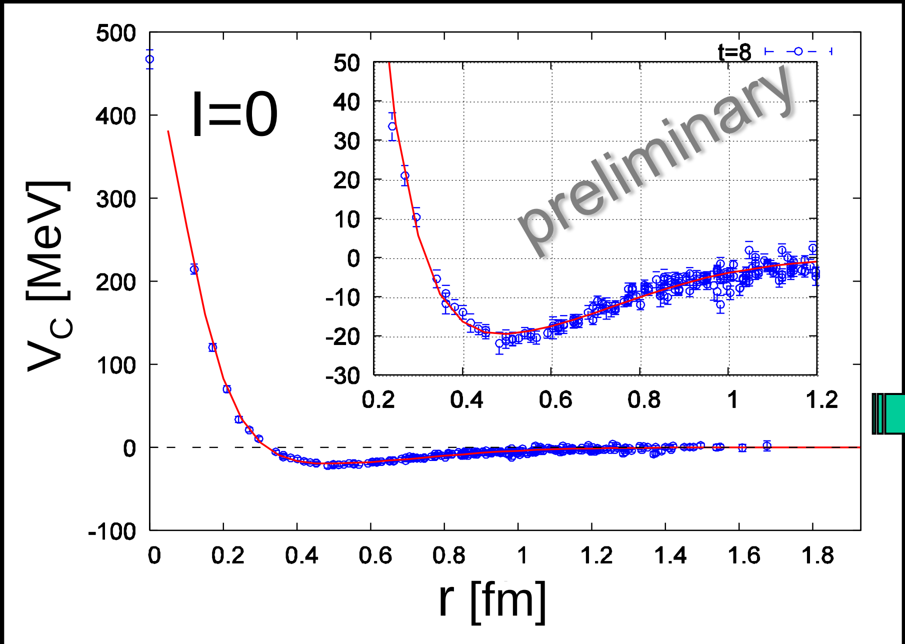


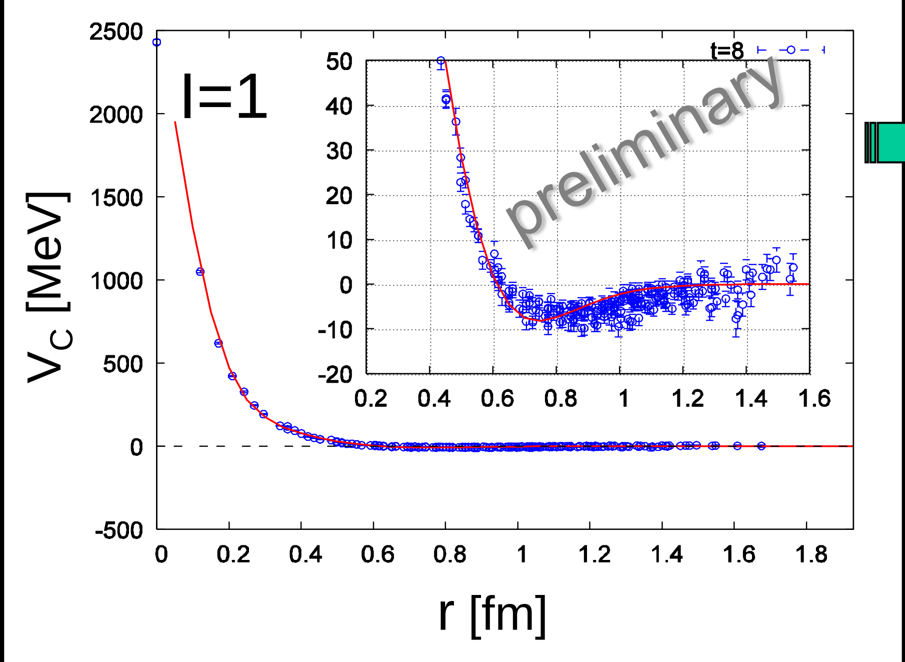
Quenched QCD
32^3x48, a = 0.093 fm

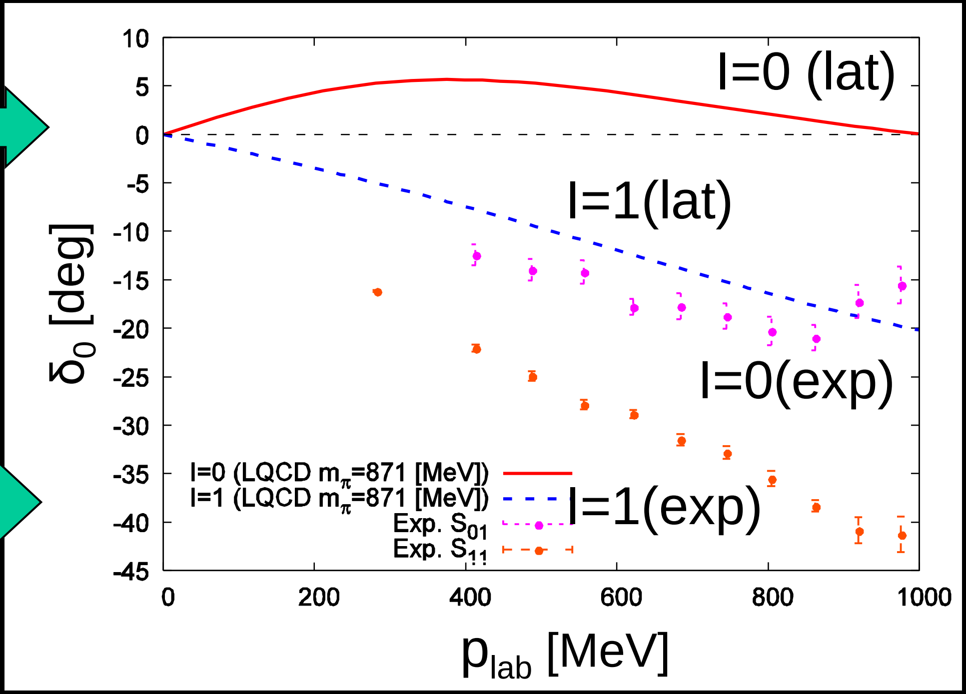
Kawanai & Sasaki (2010)

S-wave N-K^+ (us^{bar}) interaction









(2+1)-flavor full QCD
CP-PACS/JLQCD configurations
a=0.12 fm, L=1.93 fm
m_π=871 MeV

Ikeda et al. (HAL QCD Coll.)

Summary and Future

○ Nuclear forces from LQCD (HAL QCD strategy)

BS amplitude → YN, YY potentials → observables
→ exact few body calculations

○ Full QCD with m_π=140 MeV is our ultimate goal

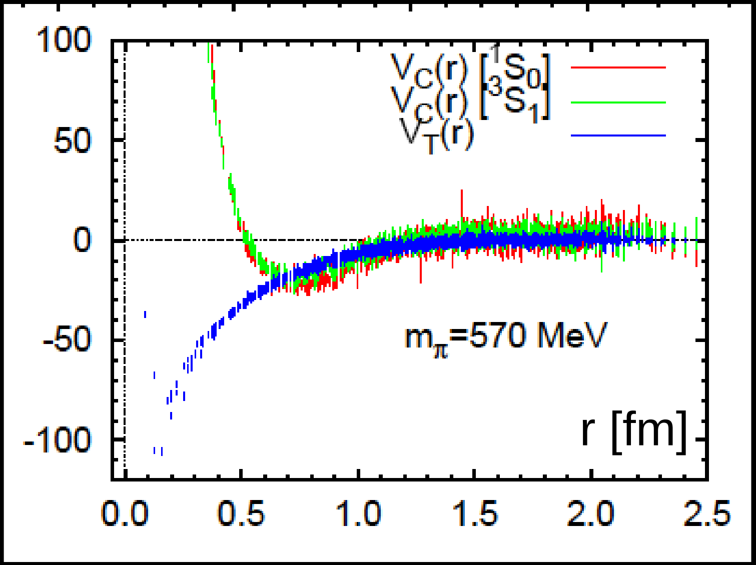
current : PACS-CS config. (N_f=2+1) with L=2.9fm & m_π = 156-701 MeV

in 1-2 years: PACS-CS config. (N_f=2+1) with L=5.8fm & m_π = 140 MeV

in 5 years: new full QCD config. on 10 Pflops machine at Kobe (2012-)

○ Current and Future targets of HAL QCD

- tensor force and π-N coupling
- LS force
- 3B forces
- light nuclei from LQCD potentials
- B-M interactions



Some Recent References

○ NN force in quenched QCD:
Ishii, Aoki & T.H., Phys. Rev. Lett. 99 (2007) 022001 [nucl-th/0611.096].

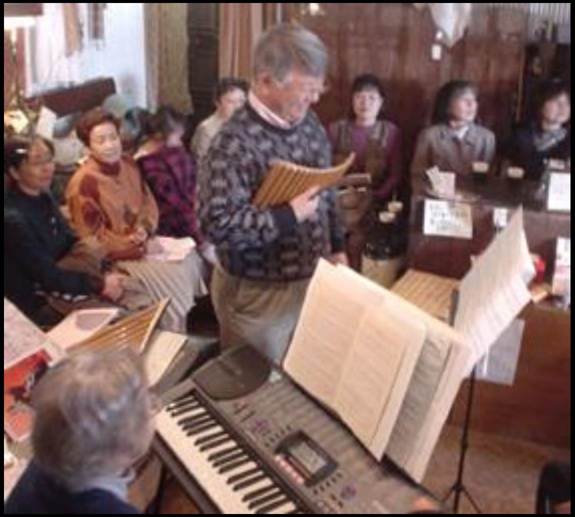
○ Theoretical foundation of the HAL formalism:
Aoki, T.H. & Ishii, Prog. Theo. Phys 123 (2010) 89-128 [arXive:0909.5585 [hep-lat]].

○ YN force in quenched QCD:
Nemura, Ishii, Aoki & T.H., Phys. Lett. B673 (2009) 136 [arXiv:0806.1094 [nucl-th]].

○ NN force in full QCD:
Ishii, Aoki & T.H. (for PACS-CS Coll.), LATTICE08 Proc. arXiv: 0903.5497 [hep-lat]

○ YN force in full QCD:
Nemura, Ishii, Aoki & T.H. (for PACS-CS Coll.), LATTICE08 Proc. arXiv: 0902.12251 [hep-lat]

Nakamura san

Congratulations
for your 60th birthday !

Pan flute

Lattice pie